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ABSTRACT

This paper proposes that post-reform mathematics teaching may be characterized as "improvisational." Observations of an extended mathematical investigation from a summer institute for elementary teachers are used to examine four aspects of improvisational practice: (1) the structuring of the activity; (2) planning and preparing that is both reflective and anticipatory; (3) an attentiveness and responsiveness in the moment; and (4) an improvisational understanding of the content itself. It is concluded that this conceptual framework may help both teacher educators and researchers understand better how to help teachers learn this way of teaching because it preserves rather than simplifies its complexity. (Author/MKR)

Beyond Recipes and Behind the Magic: Mathematics Teaching as Improvisation

Annette M. Sassi and Lynn T. Goldsmith

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BEYOND RECIPES AND BEHIND THE MAGIC: MATHEMATICS TEACHING AS IMPROVISATION

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This paper proposes that post-reform mathematics teaching may be characterized as "improvisational." It uses observations of an extended mathematical investigation from a summer institute for elementary teachers to examine four aspects of improvisational practice: 1) the structuring of the activity; 2) planning and preparing that is both reflective and anticipatory; 3) an attentiveness and responsiveness in the moment; and 4) an improvisational understanding of the content itself. The paper concludes that this conceptual framework may help both teacher educators and researchers understand better how to help teachers learn this way of teaching because it preserves rather than simplifies its complexity.

In response to the mathematics education reform movement, many teachers are grappling with how to reconstruct their mathematics teaching. The reforms being advocated take away much of the certainty of teaching mathematics that teachers have known in the past and replace it with an indefiniteness that often leaves teachers feeling that they must invent their math classes on the fly. Yet, while these reforms challenge traditionally structured classrooms, they are not meant to suggest that teachers abandon all sense of organization and order.

The challenge for teacher educators is to communicate and model how post-reform teaching might look when one of its major characteristics is its lack of prescription. While there are no recipes for creating these new forms of teaching, neither is it a matter of teaching solely by intuition or "feel." There are pedagogical and epistemological issues to which teachers must learn to attend closely: for instance, how to recognize an opportunity for a rich discussion that wasn't planned; how to determine if a child's mathematical argument is rich enough to explore more deeply; how to anticipate the kinds of questions that will get students engaged in a substantive mathematical inquiry. It is crucial to help teachers develop a deep sense of what this teaching is about so that they do not feel as if they've abandoned certainty in favor of a free fall into a pedagogical abyss.

To succeed at this task we need conceptual frameworks that preserve rather than collapse the complexity of attending to the particularities of individual classrooms—one of the hallmarks of "constructivist" teaching. While the new pedagogy encourages teachers to confront this complexity in their classrooms, it offers few theoretical constructs of what might be entailed in responding to it. In this light, work in philosophy on practical reason and judgment is especially relevant. Nussbaum (1990) in particular, provides a rich analysis of the situated complexity of deliberating and choosing well. She argues for the priority of the particular and

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holds that good deliberation must take into account the contextual features of the situation. She argues further that the person engaged in practical deliberation must inevitably improvise, balancing her own general knowledge with the particularities of the given situation. Several writers have offered practical insights into the workings of improvisational activity in different domains. In extending this notion to teaching, we have drawn from analyses of musical improvisation (Sawyer, 1992; Sudnow, 1978) and discussions of improvisational qualities of other largely interactive endeavors such as qualitative research (Oldfather & West, 1994) and play (Sawyer, in press).

This paper uses observations of an extended mathematical investigation from a *Mathematics for Tomorrow* (MFT) summer institute for elementary teachers to explore the claim that teaching is improvisational. Our analysis focuses on teachers of teachers, rather than on the practice of schoolteachers. While the elementary teachers participating in the MFT project are just beginning to consider new ways to teach, the practice of the MFT staff exemplifies many of the key tenets of this new pedagogy. Since our interest lies in understanding some important dimensions motivating and organizing constructivist teaching we felt we needed to turn to mature (though still developing) examples of such teaching to explore its character.

Method

The mathematical investigation serving as the focus of analysis is called *Starfish.* This is an adaptation of *Xmania*, an exploration of number systems used at SummerMath for Teachers (Schifter & Fosnot, 1993). The exercise was part of the July 1995 Summer Institute for MFT teachers, and was the first extended mathematical investigation that teachers undertook as participants in this two year, professional development program.

Data for the analysis presented here comes from the materials used during the Starfish activity, field notes of observations of teachers' work during the Starfish exploration and of teacher educators' interactions with them, and notes of a post-institute debriefing meeting with the instructors regarding their teaching during the Starfish exploration.

The investigation itself begins with the instructors posing a problem within the context of a fantasy society. Teachers are told that a famous mathematician of Starfish society unexpectedly died just as she was to make public her newly invented number system. The existing system represented the quantities between Ø and 26 with the symbols Ø, A, B, C, D , E, F,...Z, and then referred to any quantity larger than Z as "lots." While the details of the new system died with the professor, she left some sketchy information about it: the system can represent any quantity exactly, it can be used to perform any arithmetic operation, and it uses only the symbols A, B, C. D, and Ø. Teachers are then asked to develop a system consistent with these claims and with the aid of some artifacts which the Professor left behind. There is no explicit discussion of how teachers might use these "artifacts" (which are base 5 blocks), although they are instructed to use these materials as an integral part of constructing the system. With this introduction, teachers begin their explorations in groups of three to five.

The goals of this activity are several. It aims for teachers to investigate fundamental issues about number systems and place value and to experience some of the issues children encounter as they learn our base 10, place value system. These goals form a constant mathematical structure around which the teacher educators design specific exercises. The activity also aims to give teachers experience in collaborative mathematical inquiry, and the chance to see the range of ideas that develop as different people work on the same problem.

Analysis

Our reading of the literature on improvisation and our prior observations of the MFT teacher educators lead us to propose four key factors characterizing improvisational practice: 1) the structuring of the activity; 2) planning and preparing that is both reflective and anticipatory; 3) attentiveness and responsiveness in the moment; and 4) understanding of the content itself. This paper considers these four factors in the context of the Starfish investigation.

Structuring for possibilities. Consider first what characteristics of this activity allow it to have "improvisational potential?" An insight came during a discussion when two teachers asked why the teacher educators structured it so tightly. Why weren't they simply given some beans and ask to construct number systems? A number of the teachers felt that such constraints bound, rather than facilitate, creative explorations. Yet, improvisational arts have very defined structures. In improvisational music, for instance, certain musical structures such as a fixed chord progression bound where the improvisations can—or can't—go at particular times in the piece.

Similarly, structuring the place value investigation by providing materials which suggest grouping by fives, and using alphabet letters in place of Hindu-Arabic numerals effectively pulls people away from the terrain that they know (organizing quantities into units of 10) and moves them into more unfamiliar territory (thinking about how to organize and represent quantity). As another teacher later pointed out, had they been given just beans they would have been tempted to construct a base 10 system because they already knew how to group by powers of 10. Had they been able to rely on this understanding, they would not have been challenged as deeply to build from scratch their understanding of units, groupings, and naming of units. Because teachers were charged with describing their systems in terms of both the base 5 blocks and the symbols, they were constrained from using the symbols in a base 10 form—for instance, using upper and lower case letters to get nine symbols. By structuring out an easy reliance on the familiar, the teacher educators opened up possibilities for the deep exploration of the unfamiliar.

The organization of an improvisational activity also has to allow for creativity in the moment. The structuring of Starfish has such characteristics. When the teacher educators planned the activity they did not know what, exactly, would emerge from teachers' work. The particular materials and their presentation purposely leave open the possibility of inventing many number systems. By providing four symbols (in addition to zero) and four block sizes, the teacher educators construct a mathematical ambiguity that can be resolved in a number of ways. Should the symbols represent numbers of blocks (A blocks, B blocks... A rods, B rods, etc.), or should they represent the various sized blocks themselves (A is a unit, B is a rod, C is a "flat")? One choice may lead to the construction of a base system, another to a Roman numeral-type system.

Planning and Preparing. While the teacher educators had a good sense of the kinds of discoveries that teachers were likely to make, they still needed to be ready for surprises. This readiness to respond to the ongoing work involves a kind of planning and preparing that incorporates considerations of possible scenarios and responses to them. By analogy, we can turn again to improvisational music. Preparing for an improvisational performance does not involve running through an exact arrangement. (In fact, this is impossible by definition, since there is no exact arrangement of the music to practice, only sketches.) Instead, the musician anticipates what might happen with other musicians, tries out possible families of responses to them, and investigates new musical spaces in anticipation of confronting them in the performance.

In many ways, preparing for a math activity like Starfish requires similar planning and preparation. The teacher educators used past experiences to anticipate different kinds of outcomes and directions, and conjectured about possible responses to these circumstances. For example, teachers study the different kinds of systems that different groups created and talk as a whole group about the different systems. The teaching staff want the teachers to consider some key issues about number systems, for example: what is gained or lost mathematically with different systems; how important is efficiency; what resemblances to each other do different systems bear? But the teaching staff are never sure what systems will be created. Consequently, in anticipating the discussion the teacher educators imagine a range of possible scenarios. In the event that only base five systems would have emerged, for instance, the staff imagined that they would stimulate consideration of the questions above by asking teachers to talk about their false starts—the systems that people started to create but abandoned because they felt that they weren't working.

Furthermore, planning for Starfish is ongoing. For example, while the teacher educators planned to have the teachers do worksheets involving arithmetic operations in the different invented systems, they did not finalize the particular problems for the worksheets until they could assess the range of group understanding about the different systems. This year the instructors designed the worksheet to incorporate some specific mathematical misconceptions about place value they observed in this particular group of teachers; the worksheets had never before included these kinds of problems, because they had not surfaced as requiring attention until now.

Mindfulness and Responsiveness. To work well, this kind of planning and preparing requires teachers to be especially attentive and responsive to their students in the moment. One cannot plan an activity with contingent elements and then fail to listen and watch for contingent outcomes. In the case of constructing the worksheets on operations, the teacher educators were able to incorporate the teachers' misconceptions precisely because they were alert and attentive to such things. In designing these exercises the teacher educators were especially responsive to the unusually wide range of mathematical abilities represented in this particular group of participants. While some teachers were struggling with counting in the different systems, others were trying to figure out how to develop a division algorithm for base 5. The teacher educators' response to this range was to develop different worksheets to challenge people at different levels of understanding: one dealing with issues about counting; one focusing on understanding addition and subtraction; and one challenging teachers to think about division, fractions, and "pentimals." While the latter was appropriate for some of the teachers in this group, it would not be appropriate for all groups of teachers who explore Starfish. In fact, this was the first time in dozens of experiences teaching Starfish that several participants made extended investigations of division and fractions.

Content Knowledge and Context. Teaching which predicates basing instructional experiences on the current state of students' knowledge cannot be done well unless the teacher, herself, understands the content in deep and flexible ways. To draw another analogy to music, the improvisational musician's knowledge of music is fluid and situational; she understands the structure and meaning of the music and knows how to mobilize this understanding to compose in the moment. In a similar way, these teacher educators know mathematics in a fluid, non-fixed way. They are able to see the various elements of the domain as interrelated elements of a continually constructed and re-constructed discipline rather than as discrete facts, operations, or procedures. (cf. Schifter 1994).

This kind of knowledge is necessary to be improvisationally responsive to what arises in the moment. The teacher educators need to know the intellectual terrain well enough to follow what others are doing and to grasp, in the moment, if students' thinking is consistent with the terrain or not. They also need to understand enough about how understanding develops so that they can image a student's likely path of understanding (Fennema, Carpenter, & Loef, 1993). Finally, they need to understand the kinds of mathematical and pedagogical interventions that will help students to develop these ideas (Shulman, 1987).

In the case of the Starfish activity, the teacher educators were able to help participants work through ideas by following their reasoning, asking questions and posing situations which were designed to push on their thinking, and redirecting investigations when they felt it necessary. This was possible to do because the teacher educators were prepared mathematically for the multiple directions the activity could go. Had they been less "fluent" in base 5, less familiar with the kinds of understandings that teachers typically construct when engaged in this work, or less able to make connections between teachers' thinking and the important mathematical ideas inherent in the exploration, they would not have been able to help keep participants' thinking as active and focused on moving toward greater understanding than they were.

Conclusion

Capturing what teaching within the new pedagogy entails is especially difficult because this kind of teaching has no easy recipes or prescriptions. Furthermore, to codify it would effectively undermine the principles upon which it is based. This paper has taken to task the challenge of articulating some of the fundamental characteristics of teaching from a "constructivist" perspective by characterizing it as "improvisational" and looking at four aspects of improvisational practice in the context of an extended mathematical investigation. If the characterization of post-reform teaching as improvisational proves through further investigations and refinements to be robust, then this framework may indeed contribute to a greater specificity of what this kind of teaching entails and what is required to learn it.

References

Fennema, E., Carpenter, T., & Loef, M. (1993). Learning to use Children's Mathematical Thinking: A Case Study. In C. Maher & R. Davis (Eds) *Relating Schools to Reality.* Needham Heights, MA: Allyn & Bacon.

Nussbaum, M. C. (1990). The Discernment of Perception: An Aristotelian Conception of Private and Public Rationality. *Love's Knowledge: Essays on Philosophy & Literature.* Oxford: Oxford University Press.

Oldfather, P. & West, J. (1994). Qualitative Research as Jazz. *Educational Researcher.* 23(8): 22-26.

Sawyer, K. (in press). *The Performance of Pretend Play: Improvisational Creativity and Conversation in the Nursery School.*

Sawyer, K. (1992). Improvisational Creativity: An Analysis of Jazz Performance. *Creativity Research Journal.* 5(3): 253-263.

Schifter, D. (1994). Teachers' Changing Conceptions of the Nature of Mathematics: A Developmental Model. *Proceedings of the Sixteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, (296-302), Baton Rouge, LA: PME-NA.

Schifter, D. & Fosnot, C.T. (1993). Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform. New York: Teachers College Press.

Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review.* 57(1): 1-22.

Sudnow, D. (1978). *Ways of the Hand: The Organization of Improvised Conduct.* New York: Harper & Row.