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ABSTRACT

As teachers begin to implement mathematics curricula that capitalize fully on computing technology and that are focused on concepts and applications instead of on execution of by-hand symbolic manipulation routines, their well-established routines of thinking about mathematics and its teaching no longer apply in seamless fashion. This case study, a part of which is reported here, examines the ways that an experienced teacher who participated in Computer-Intensive Mathematics Education (CIME), a 4-week program on the teaching and learning of mathematics in technology-intensive environments, confronted some of the mathematical issues inherent in technology-intensive mathematics. This report gives some insight into one teacher's understanding of functions, independent variables, and parameters, and the ways that this understanding interacts with her use of the new computing tools. (Author)

The Interplay of Mathematical Understandings, Facility with a Computer Algebra Program, and the Learning of Mathematics in a Technologically Rich Mathematics Classroom

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Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

THE INTERPLAY OF MATHEMATICAL UNDERSTANDINGS, FACILITY WITH A COMPUTER ALGEBRA PROGRAM, AND THE LEARNING OF MATHEMATICS IN A TECHNOLOGICALLY RICH MATHEMATICS CLASSROOM

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As teachers begin to implement mathematics curricula that capitalize fully on computing technology and that are focused on concepts and applications instead of on execution of by-hand symbolic manipulation routines, their well-established routines of thinking about mathematics and its teaching no longer apply in seamless fashion. This case study, a part of which is reported here, examines the ways that an experienced teacher who participated in CIME, a four-week program on the teaching and learning of mathematics in technology-intensive environments, confronted some of the mathematical issues inherent in technology-intensive mathematics. This report gives some insight into one teacher's understanding of functions, independent variables, and parameters, and the ways that this understanding interacts with her use of the new computing tools.

Researchers (Fennema and Franke, 1992) have suggested important components of teachers' knowledge that impact on their teaching and their students' learning: knowledge of mathematics (Ball, 1988; Lampert, 1989) and mathematical representations (Hiebert and Wearne, 1986), pedagogical knowledge (Clark and Peterson, 1986; Shulman, 1986), and knowledge of how students come to understand mathematics (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). In computer-intensive environments, additional components of teachers' knowledge that impact on their teaching and their students' learning may include knowledge of the use of technology for the exploration of mathematics and knowledge of the technology itself.

The Empowering Mathematics Teachers in Computer-Intensive Environments project (National Science Foundation award number TPE 9155313) is a multiple-year teacher enhancement/research project which focused on developing secondary mathematics teachers' abilities to implement computer-intensive mathematics curricula. Teachers involved in the project (Computer-Intensive Mathematics Education or CIME) completed several courses connected with their teaching of *Computer-Intensive Algebra*[1] (CIA) (Fey, Heid, et al., 1991), a radically reformu-

[1] The *Computer-Intensive Algebra* Project was funded by grants from the National Science Foundation under award numbers DPE 84-71173 and MDR 87-51500 to The University of Maryland (Principal Investigator: James T. Fey) and award number MDR 87-51499 to The Pennsylvania State University (Principal Investigator: M. Kathleen Heid). The CIME project was funded by the National Science Foundation to The Pennsylvania State University through award number 91-55313 (Principal Investigators: M. Kathleen Heid and Glendon W. Blume; Faculty research associate: Rose Mary Zbiek). Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation. First distributed through the Office of Technology Liaison at The University of Maryland, *Computer-Intensive Algebra* is currently being distributed as Fey, J. T. & Heid, M. K. with Good, R., Sheets, C., Blume, G., Zbick, R. M. (1995). *Concepts in Algebra: A Technological Approach*. Dedham, MA: Janson Publications.

lated beginning algebra curriculum that is built around the concept of function, employs calculators and computers as tools for student exploration, and develops fundamental concepts of algebra (e.g., variable, function, equivalence, system) through mathematical models of realistic situations. The CIME course experiences (one four-week course the summer prior to their teaching CIA and one one-week course the following summer) had three integrated components mathematics; assessing students' understandings in technologically rich mathematics classrooms; and issues of teaching and learning in computer-intensive environments.

As teachers begin to implement mathematics curricula that capitalize fully on computing technology and that are focused on concepts and applications instead of on execution of by-hand symbolic manipulation routines, they find that their well-established routines of thinking about mathematics and its teaching no longer apply in the same seamless fashion. The case study reported here examines the ways that an experienced teacher who participated in the CIME program thinks about the new mathematics, the ways she interacts with computing tools, the ways she attempts to understand what her students are understanding, and the ways she converts her new experiences into a teaching/learning situation for her students.

Subject and data

The focus of the case study was Sara, a teacher who had taught mathematics, almost always first-year algebra, for over 20 years in the same rural high school. The primary data used as a basis for this case study consists of verbatim transcripts from a variety of sources over a thirteen-month period: task-based, scenario, and documentation interviews with Sara, eight observation cycles focused on the CIA class Sara taught, small group sharing sessions in which Sara participated during the summer courses, and sessions during both summers during which Sara helped plan and execute task-based interviews with a ninth grade student who had completed a *CIA* course.

We conducted three types of interviews[2] with Sara during the summers preceding and following her first year of teaching CIA. Task-based interviews (TBI at the beginning of Summer 1, TBII at the end of Summer 1, and TBIII during Summer 2) were designed to get a picture of Sara's understanding of mathematical concepts underlying CIA and her use of technological tools to explore those concepts. Scenario interviews (SCI at the beginning of Summer 1, SCII at the end of summer 1, and SCIII during summer 2) were designed to tap Sara's abilities to understand students' mathematical understanding as seen through interview transcripts provided for her. A documentation interview (DOC) during the second summer provided data on Sara's perception of teaching CIA.

We conducted four rounds of observations of the CIA class that Sara taught. Each round consisted of several days of observations. Pre-observation conversation and post-observation conferences along with the observations, were focused

[2] Interviews and observations were designed and conducted by M. Kathleen Heid, Glen Blume, and Rose Mary Zbiek. Analysis was aided by Mathematics Education doctoral students Barbara Edwards, Wilhelmina Mazza, and Barbara Edwards.

on Sara's instructional decision-making. Finally, we analyzed portions of what happened during the summer courses. We analyzed what Sara said about teaching CIA during small group sessions, and we studied the ways in which she attempted to assess student understanding through task-based interviews she and several others conducted both summers.

Results

Analysis of the data is currently ongoing, but preliminary results suggest possible tensions related to teaching mathematics in technologically rich environments. Several results address the effects of a teacher's developing understanding of mathematical concepts, of the use of computing tools, and of new ways to think about teaching and learning. An example of these effects is discussed below.

As Sara thought about, talked about, and taught a functions-oriented algebra course, her personal understanding of function came to the fore. Sara saw little use for function notation, often using explicit function rules rather than more generic function notation. Early in December, for example, Sara was beginning a total class discussion of a CIA problem which involved attendance at a talent show as a function of the price of a ticket. The function rule with which the class was working was $a(t) = 1.05(800 - 50t)$ and the class was finding the ticket price that yielded various attendance values. The following interchange ensued:

Sara: What was the input variable in this situation?

S1: Ah,...the input variable was the price of a ticket.

Sara: Okay. Okay, S2, what was the output variable?

S2: Attendance.

Sara: Okay, the output variable was attendance. And S3, do you remember what another form for the rule was that we were looking for yesterday?

S3: **a** equals one point oh five times the quantity eight hundred minus fifty t...

Sara: Okay, **a** equals ... so we know that instead of writing **a** of t, we can also just write that as **a** equals when we're wanting to find the attendance. When would you write it as simply **a** equals? Which command would you be using when you would do that?

S: The solve command.

Sara is suggesting to her class that they should find the attendance for a given price (say $8) by "solving" the equation $a = 1.05 (800 - 50*8)$. Even though the program with which Sara's class was working would have allowed the user to ask the program to "evaluate" a(8), Sara prefers not to use function notation and finds a way to get the numerical answer without such notation. Interestingly, Calc T/L II[3] is a program especially designed to force the user's attention on the objects with

[3] Calc T/L II by J. Douglas Child is distributed by Brooks Cole; Pacific Grove, CA.

which they are working. Before asking for some particular symbolic manipulation, the user must choose the object with which he or she is working. To evaluate the function a(t) for $t = 8$, the user would (1) select "function," (2) define the function, a, from the function window, then (3) select "expression," (4) write a(8) from the expression window, and (5) evaluate it. Sara was proposing what was, for her purposes, a shorter method: (1) select "function," (2) define the function, a, from the function window, (3) redefine the function a as $a = 1.05 (800 - 50*8)$, and then (4) write resulting redefined function, which would be displayed in evaluated form. The fact that the Computer Algebra System her CIA class used was predicated on a function as object approach was no help to Sara since she was reticent to explore the computer program and used it to get answers even if the methods producing those answers made little conceptual sense. Her use of the function concept suggested a "process" rather than an "object " concept. This tendency to view function as process along with her aversion to function notation played itself out as Sara encountered families of functions.

Prior to teaching CIA, during the first CIME summer Sara was just beginning to deal with families of functions, at first allowing only families with familiar names (e.g., linear, quadratic). She took a "function as process" approach to exploring families of functions with which she had no previous familiarity. For example, in investigating the effects of a on $f(x) = f(a,x) + bx + c$, Sara started by assuming a b-value of 5 and a c-value of -5. She continued, saying "Well, let's just let x be 2, okay?" She then calculated the value of the resulting expression, $f(a,2) + 5(2) - 5$, for $a = -2$ and $a = -4$, and concluded that the function decreases as a decreases since $f(-4,2) + 5(2) - 5 < f(-2,2) + 5(2) - 5$. The fact that she took a numerical instead of a graphical approach to her exploration may have been thought to be related to her relative inexperience at that time with graphics programs and their use in teaching algebra. The following summer, however, after having taught CIA to a low-ability group of ninth graders for a year, her approach to exploring functions was not very different from the first summer's approach. In exploring the function $f(x) = a^x + f(b,x) + c$, she decided to let b be equal to 1, let c be equal to 0, and let x be equal to 2. She then calculated and examined values of $a^2 + f(1,2)$ as the value of a increased in a manner similar to her exploration the previous summer. She continued the exploration, this time seeming to reverse the role of the parameter and the independent variable completely, graphing $f(x) = x^2 + f(1,2)$, and treating the original a as if it were the independent variable instead of the parameter. Because Sara has fixed the value of the original independent variable at $x = 2$, she is examining a different function than was originally intended and concludes that changing b and c have the same effect on the function. She noted that, in so doing, "whether I change the b or the c, it has the same effect." Because the function notation itself has little meaning for Sara, the fact that she is examining $f(x) = a^x + f(b,x) + c$ is no different from her examining $f(a) = a^x + f(b,x) + c$.

For Sara, teaching mathematics in a technology-intensive environment meant encountering new mathematics or encountering old mathematics that takes on new importance. In many traditional mathematics textbooks, there was no confusion about the meaning of the function notation. In $f(x) = 4^x + f(5,x) + 6$, it was clear that the independent variable was x. In those environments, Sara and her students would have had to confront the meaning of the function notation. In the technology-intensive environment surrounding the teaching of CIA, both Sara and her students were confronted with situations in which clearer understandings of functions and function notation were needed. Perhaps because Sara was not one to explore the tool on her own and because she could find ways, however conceptually inappropriate they might have been, to generate numerical answers without using appropriate notation, her understanding and use of function notation seemed not to improve substantially over the year.

Sara's emerging understanding of functions and avoidance of function notation, her reluctance to explore the capacity of the computer, and her lack of experience with families of functions combined to produce a confusing perception of one of the central CIA mathematical concepts. Other data suggests that this set of circumstances had significant effects on her students' mathematical understandings.

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