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ABSTRACT

This case study investigates an experienced secondary school mathematics teacher's understanding of mathematics ("her" math) and decisions she makes about her students' classroom experiences ("their" math). This report focuses on the competing roles of the teacher's growing understanding of novel technology-rich mathematics and her decisions about activities and expectations in an algebra course in light of her beliefs about learning and teaching. Data document developments in her mathematical understanding and classroom practice during her first 13 months of teaching Computer-Intensive Algebra as a participant in the Empowering Secondary Mathematics Teachers in Computer-Intensive Environments project (CIME). (Author)

# Her Math, Their Math: An In-Service Teacher's Growing Understanding of Mathematics and Technology and Her Secondary Students' Algebra Experience

**Rose Mary Zbiek**

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# HER MATH, THEIR MATH: AN IN-SERVICE TEACHER'S GROWING UNDERSTANDING OF MATHEMATICS AND TECHNOLOGY AND HER SECONDARY STUDENTS' ALGEBRA EXPERIENCE

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This case study investigates an experienced secondary school mathematics teacher's understanding of mathematics ("her" math) and decisions she makes about her students' classroom experiences ("their" math). This report focuses on the competing roles of her growing understanding of novel technology-rich mathematics and her decisions about activities and expectations in this algebra course in light of her beliefs about learning and teaching. Data document developments in her mathematical understanding and classroom practice during her first 13 months of teaching Computer-Intensive Algebra[1] as a participant in the Empowering Secondary Mathematics Teachers in Computer-Intensive Environments project (CIME).[2]

## Framework

A practicing teacher's understanding of school mathematics includes a blend of her knowledge and beliefs about formal mathematics, about pedagogy, and about how people learn mathematics. Numerous prior studies (cf. Ball, 1991; Carpenter, Fennema, Peterson, & Carey, 1988) document the impact of practicing teachers' understandings and beliefs on their classroom decision-making. Other works investigate the relationship between teachers' understanding of mathematics and their students' achievement. This literature suggests that teachers' understandings of mathematics affect their classrooms and their students' learning environment in complex ways.

As Fennema and Franke (1992) note, when positing a framework for research in teacher knowledge, there is a need for research that explores the relative roles of knowledge of mathematics, pedagogy, and learning with respect to beliefs and current context of the teacher. If technology is an integrated part of school mathematics and curricula change to reflect its presence, teachers are teaching mathematics that is new to them with a focus on process rather than product in a technology-intensive mathematics classroom filled with open-ended activities and mathematical explorations (NCTM, 1989). Technology then suggests a need to study a teacher's understanding and doing of mathematics with technology and the classroom learning environment she fosters.

[1] A revised version of Computer-Intensive Algebra is now distributed by Janson Publications as Concepts in Algebra: A Technological Approach.

[2] CIME is funded by the National Science Foundation under award number TPE-9155313, M. K. Heid and G. Blume, Principal Investigators. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

## Methodology

**Context.** The current study follows one teacher through her entire experience as one of 60 participants in Empowering Secondary Mathematics Teachers in Computer-Intensive Environments (CIME). CIME is a teacher enhancement project that implements and tests a model for the continuing education of secondary school mathematics teachers. The CIME experience begins with a four-week summer institute that concentrates on developing teachers' mathematical understandings and technology knowledge and on engaging them in using alternative forms of assessment to better understand students' conceptions of mathematics. The first summer workshop also introduces them to *Computer-Intensive Algebra* (CIA) as one example of an innovative, technology-intensive first-year algebra curriculum. This curriculum – built around the function concept, mathematical modelling, use of multiple representations, open-ended exploration, and constant access to computer algebra systems – differs greatly from teachers' prior experiences with first-year algebra. CIME teachers return to their schools to implement CIA and then attend a one-week institute during the next summer.

**Instruments.** Data reported here come from four sources as collected over 13 months. Ten interviews address understanding and doing of mathematics in the presence of computing tools, understanding and beliefs about how learning occurs and about what it means to understand mathematics, and documentation of classroom activities. Six series of observations (four in CIA classes and two in geometry classes) with pre- and post-observation interviews address the nature of mathematics in the classroom and the teacher's changing practice. The remaining data are a set of one journal entry per week written by the teacher and copies of course materials created and used by the teacher.

**Analysis.** Analysis of data began with the coding of all interview transcript passages that included any discussion of mathematics. Patterns arising from these yielded tentative hypotheses about the subject's understanding and beliefs about mathematics and her perceptions of students' understandings and learning of mathematics. The findings exemplified here are hypotheses that survived comparison with results of a similar analysis of the classroom observation and journal data.

## Subject

"LeAnne" is certified to teach secondary mathematics and taught for 22 years before her CIME experience and this study began. She has undergraduate degrees in both mathematics and elementary education, mathematics certification at both the elementary and secondary levels, and a master's degree in secondary mathematics education. LeAnne teaches in a suburban/rural high school of approximately 850 students in grades 9 through 12. Teaching only at the high school level for the last decade, LeAnne had a fairly stable teaching assignment consisting almost exclusively of geometry courses. She never taught an algebra course using CIA materials prior to the CIME experience but verbally espoused CIA goals of technology use and exploration.

## Findings and Discussion

In the paragraphs that follow, a summary of LeAnne's expressed views about learning, doing, and teaching mathematics followed by a description of her classroom environment lead to a seeming contradiction between what she espouses and what actually occurs in her classroom. Subsequent consideration of the LeAnne's views and actions however address the extent to which she seems to alleviate the contradiction as her understanding and beliefs grow and change throughout the academic year.

**Expressed views**. Throughout the year, LeAnne expressed consistent views about mathematics learning, teaching and curriculum. She frequently spoke of learning as discovery and teaching as facilitating, as exemplified in her assessment of a sample teaching scenario during one interview:

> [The teacher in this scenario is] questioning them...she's not saying...what the answers are or what they need to do to find them...she's having them, ah compare their answer with what information they have...to maybe help them think through what they should get. And this is what I do a lot;...very few times do I actually give the kids answers...But I ask them questions for them to think through what the answer should be.

The importance of reasoning through mathematical problems as opposed to simply knowing outcomes also came through clearly in her stated goals for the geometry course. These expressed values appear consistent with the goals of CIA and CIME and set expectations about how LeAnne herself would approach computer-intensive mathematics and orchestrate her classroom. What then seems to be the nature of LeAnne's mathematics and what characterizes her students' classroom experience?

**"Her" mathematics**. LeAnne herself never used a computer algebra system (CAS) to solve real-world problems, complete modelling tasks, or investigate function families prior to CIME and the current study. However, she spent many hours preparing for the class by using Calculus T/L II[3] (the CAS available in her classroom) and working through the computer labs in the CIA curriculum materials. She became adept at using the technology and used it during interviews in ways and for mathematics tasks that transcended as well as matched the CIA curriculum. At the end of the year, LeAnne could quickly use Calculus T/L II to produce and use symbolic rules, tables and graphs of functions, to edit these things and to re-organize these images on the screen. She also developed a deeper sense of exploring both situations modeled by a function and families of functions presented in the abstract. Evidence of the level of expertise she achieved is in an interview task at the end of the year: Describe the effects of changing the values of

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[3] Calculus T/L II is distributed by Brooks/Cole Publishing Company, Pacific Grove, CA.

$a$, $b$, and $c$ on the graphs of functions of the form $f(x) = a^x + f(b,x) + c$. She begins with $a=b=c=1$ and enters $f(x)=1$^$x+1/x+1$; the display appears as $f(x) = 2 + f(1,x)$. She noted:

> 1 to any power will give you 1, so it's adding the 1 plus 1... *[She creates s table as in Figure 1.]* Okay, at 0 I get undefined, and at half, 4. Then it keeps on going down. *[She produces graph in Figure 2.]* So if I graph it; it's a hyperbola (sic). Okay. Ah, what I'm going to do is change *a* to maybe 2 and see what happens. *[Teacher entered $f(x) = 2$^$x+1/x+1$ and produced a table and then the graph in Figure 3, while losing the definition of the first function, $f(x) = 2 + f(1,x)$.]*...I should have called [this new function] a different function name because I lost the original.

LeAnne sketched and labeled each of these two graphs. She then tested $a = -2$ by producing the table and graph in Figure 4. She noted that in this table "there are more 'undefineds' here." After she created the graph, she discussed the conflict between the graph and the table, sketching what she claimed is a better graph (Figure 5). She then zoomed in on the portion of the graph for $0.5 \leq x \leq 2$ but got an error message and noted:

> But yet on my table it's saying that ah, let's see. At 1 *y* should be a 0; so there should be a point at 1. Hmm. Okay, let's. Change this [value of *a* to] -1; see what happens.

LeAnne explicitly compared the graph and table, noting a discrepancy and predicting the pattern for negative values. She then tested one more value of $a$ ($a = -1$) and concluded:

> When it's positive you get ah, two hyperbolas (sic), and it keeps increasing which means making a smaller one [on the right side]. And then when it's negative it keeps going off into infinity every other number. I mean ah, whole number it gives you a point and then the next *x* which is, I've got it set at 2.5 at half then it goes undefined.

LeAnne seemed to have a feasible attack to the problem. She used the tool fluently to achieve her goals, occasionally editing her previous work or ideas. When the tool produced unexpected outcomes, she stopped and interpreted them or pursued them further. She explained in detail what she did with the CAS and why her actions and results made sense. Her agile use of the CAS was apparent. However, she consistently stated conclusions based on two examples (e.g., try $a = 1$ and $a = 2$ and conclude about all $a \geq 0$). If asked to further justify conclusions, LeAnne relied on one additional example (e.g., $a = -1$) and only occasionally reasoned abstractly. This was "her math."

**"Their" mathematics**. In September, students met in the lab and worked through explorations. LeAnne described the lab as a noisy place where students

| x | y |
|---|---|
| 0 | undefined |
| 0.5 | 4 |
| 1 | 3 |
| 1.5 | 2.66667 |
| 2 | 2.5 |
| 2.5 | 2.4 |
| 3 | 2.33333 |
| 3.5 | 2.28571 |
| 4 | 2.25 |
| 4.5 | 2.22222 |
| 5 | 2.2 |

*Figure 1.*

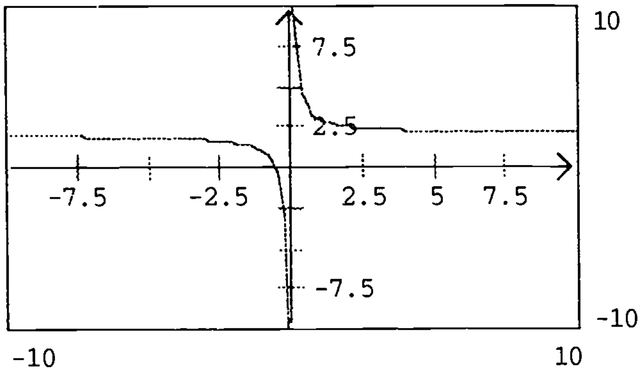


*Figure 2.*

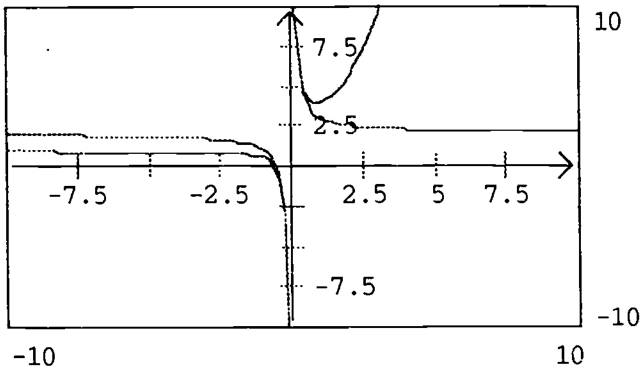


*Figure 3.*

| x | y |
|---|---|
| 0 | undefined |
| 0.5 | undefined |
| 1 | 0 |
| 1.5 | undefined |
| 2 | 5.5 |
| 2.5 | undefined |
| 3 | -6.66667 |
| 3.5 | undefined |
| 4 | 17.25 |
| 4.5 | undefined |
| 5 | -30.8 |

*Figure 4a.*

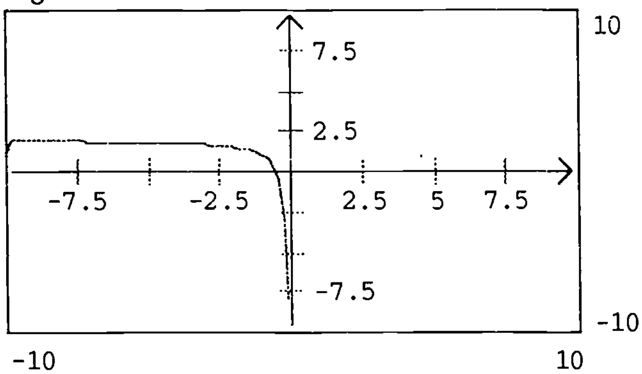


*Figure 4b.*

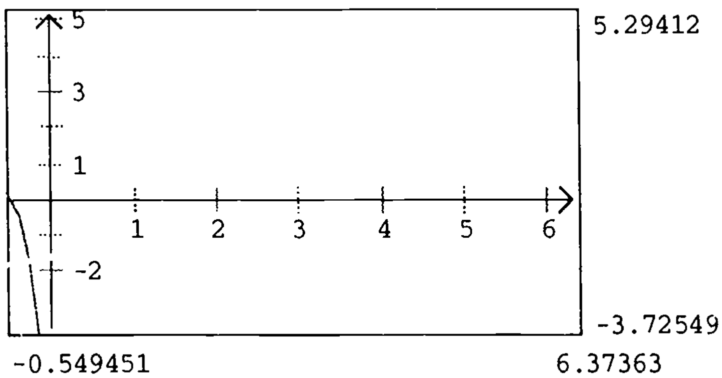


*Figure 4c.*

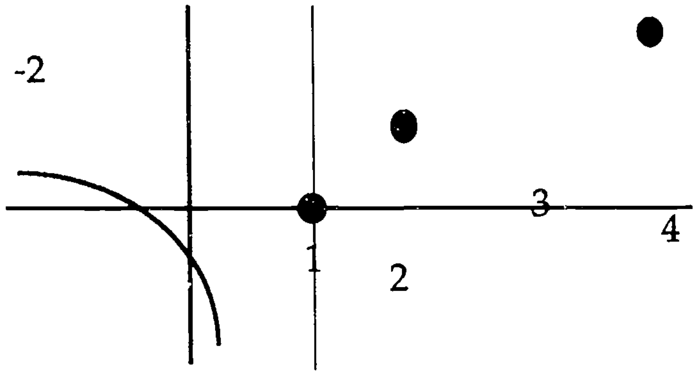


*Figure 5.*

asked many questions, puzzled over mathematical tasks and occasionally struggled with tool syntax – challenges LeAnne experienced while working through activities to prepare for class.

By the middle of the school year, LeAnne made some changes. Class began in a non-lab classroom where, with occasional computer demonstrations, LeAnne presented carefully prepared notes about how to use the CAS in the lesson. The notes prescribed keystrokes and commands that students would need to answer almost every problem they would encounter. Students then went to the lab and smoothly executed the lesson. For example, one CIA lab explores temperature in degrees Celsius as a function of temperature in degree Fahrenheit, with F(C) = f(9,5)C + 32. Students began this "exploration" by assembling in the non-lab classroom. LeAnne gave notes about using direct solve commands to determine the value of C given the value of F(C) and using computation commands to compute F(C) given the value of C. LeAnne and her students referred to these as "Finding Celsius" and "Finding Fahrenheit," respectively. Exchanges between teacher and student lab pairs during the lab experience then fell mainly into one of two predictable teacher-led patterns: determining whether a problem required "Finding Celsius" or "Finding Fahrenheit," and dwelling on the keystrokes needed. One example is LeAnne's exchange with two students as they sought F(56):

| | |
|---|---|
| L: | *They're looking for Fahrenheit. Don't you have Fahrenheit? [She scrolls up the screen to F(C)=f(9,5)C + 32.] Right. So let's reuse this, Fahrenheit. [She clicks on it.]* |
| S1: | *I think so.* |
| L: | *Then go into EXPRESSION. Okay?* |
| S2: | *Okay.* |
| L: | *EXPRESSION. [Student clicks on EXPRESSION option.]* |
| S1: | *Okay, for this we put, ah, 56 and then this. [LeAnne points to syntax notes for computing F(C). Student enters F(56) and the CAS responds with f(664,5).] 664 over 5.* |

In addition to the notes, LeAnne created supplemental worksheets to provide practice with the tool. One worksheet showed printouts of LeAnne's CAS work to answer CIA questions. The students' task was to replicate her work, checking that they got the same results. The classroom mathematics experience moved from experimentation and diversity of approaches to precision and rapidity of task completion.

**Initial comparison**. Although LeAnne talked about valuing exploration, conceptual development and reasoning why, her students spent class time taking notes on LeAnne's explorations and then following these notes algorithmically. She knew the open-ended, exploratory mathematics environment reflecting her CIA goals, but she needed a less chaotic classroom. Her need for orderliness influenced on-going changes in her classroom. The result was a blend of *her* exploration and *their* organized activity.

## Conclusion

In LeAnne's classroom, "their" math was "her" math at the beginning of the course. By the end of the year, their math became a well-orchestrated march through her tool-based tasks to lead to her mathematical conclusions. A myopic view of this observation however neglects to recognize the growth over one year in LeAnne's understanding of mathematics, agility with technology, and awareness of key aspects of innovative and radically different school mathematics curricula.

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