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ABSTRACT

This study investigated the beliefs about mathematics held by two secondary preservice teachers as they participated in a teacher education program that promoted the National Council of Teachers of Mathematics (NCTM) Standards and the use of technology. Of particular interest was what the teachers believed and how those beliefs were structured. Theoretical perspectives developed by Green (1971), Perry (1970), and Belenky, Clinchy, Goldberger, and Tarule (1986) were particularly helpful in this analysis. Analyses of data taken over a 15-month period of time indicated that both the teachers' beliefs and the structures of their beliefs differed. Recognition of these various structures is of considerable importance when developing teacher education programs that promote reflection and adaptive teaching. (Author)

On the Notion of Secondary Preservice Teachers' Ways of Knowing Mathematics

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ON THE NOTION OF SECONDARY PRESERVICE TEACHERS' WAYS OF KNOWING MATHEMATICS

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This study investigated the beliefs about mathematics held by two secondary preservice teachers as they participated in a teacher education program that promoted the NCTM Standards and the use of technology. Of particular interest was what the teachers believed and how those beliefs were structured. Theoretical perspectives developed by Green (1971), Perry (1970), and Belenky, Clinchy, Goldberger, and Tarule (1986) were particularly helpful in this analysis. Analyses of data taken over a 15 month period of time indicated that both the teachers' beliefs and the structures of their beliefs differed. Recognition of these various structures is of considerable importance when developing teacher education programs that promote reflection and adaptive teaching.

This study focuses on prospective secondary teachers' abilities and confidence to do mathematics and the beliefs they express about mathematics as they progress through a four quarter sequence in mathematics education. The sequence consists of two courses in mathematics education, followed by a quarter of student teaching, and concluding with a post student teaching seminar. This study was conducted in the context of the NSF supported project Research and Development Initiatives Applied to Teacher Education (RADIATE).[1] We will explicate three different aspects of knowing 1) what the teachers seem to be able to do mathematically, 2) what beliefs they seem to hold about mathematics and how those beliefs are structured, and 3) the implications of their knowledge and beliefs about mathematics for the teaching of mathematics. To illustrate these different aspects of knowing, we will concentrate on two informants, Harriet and Kyle, who were two of the students who participated in the teacher education program. Data for the study consisted of an initial survey that included mathematical tasks, questions about the teaching of mathematics, and the selection of similes that reflected their views about mathematics and its teaching; eight interviews including a card sort of participant-identified statements from previous interviews; four tests administered during the first two courses; numerous journal entrees in which the informants responded to specific questions related to course activities; and observations of their field experiences including student teaching. The teacher education program placed considerable emphasis on alternate teaching methods, including an extensive use of technology, and daily opportunities for the teachers to engage in various reflective activities.

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Theoretical Perspectives

It is our intent to study what the preservice teachers knew and thought about mathematics and also to consider various theoretical perspectives for describing the ways in which the teachers held their knowledge and beliefs. To guide our analysis, a variety of theoretical perspectives were used. Ernest (1991) identified five different belief systems that included theories of mathematics, learning mathematics, teaching mathematics, assessment in mathematics, and aims of mathematics education. We were primarily interested in our informants' theories of mathematics and the teaching of mathematics as we found the line between these theories to be quite blurred in the reality of studying their beliefs. We considered schemes created by Perry (1970) and Belenky, Clinchy, Goldberger, and Tarule's (1986) for considering the way that the teachers' knowledge was held. In particular we were interested in the question of whether the teachers see mathematics from a dualistic perspective where there is some sort of absolute system and the learner is either right or wrong, or whether they see mathematics from a more relativistic perspective where mathematics is dynamic and the learner is influenced by personal as well as community constructions (Cobb, 1994). To provide further dimensionality to the understanding of teachers' beliefs, we considered Green's (1971) metaphorical analysis of the structure of beliefs. In particular, we were interested in the intensity or centrality of the teachers' beliefs (Green's notion of psychologically central beliefs versus peripheral beliefs), possible logical connections among beliefs (Green's notion of primary beliefs versus derivative beliefs), and finally how various beliefs are clustered, noting in particular which beliefs seem isolated from others. By considering the structure of beliefs as well as the substance of beliefs, we can better understand possible entree points for influencing the teachers' beliefs and how those beliefs potentially influence classroom practice. Such an understanding is fundamental to developing teacher education programs that enable teachers to realize constructivist orientations which serve as foundations for in the NCTM *Standards*.

The Substance of Harriet and Kyle's Beliefs and Knowledge of Mathematics

Harriet and Kyle entered their mathematics education sequence with considerable similarity in their mathematical backgrounds as determined by courses taken and grades earned. They both maintained B+ averages in collegiate work. Although they differed in gender and ethnicity, they were both from middle class families and each had a mother who was a school teacher. Harriet frequently spoke with authority in class offering her perspective, occasionally challenging other view points, and sharing her experiences as an African American female. Kyle rarely initiated responses in class but spoke freely when asked for a response or when he worked in his family group. In one interview he commented on his privileged position as a White student.

On the surface, both students seemed to have comparable ideas about mathematics. They each spoke of learning mathematical concepts and the importance of establishing mathematical relationships. However further analysis of interviews, surveys, and class products suggested that Harriet and Kyle held different views about the nature of mathematics and understood mathematics differently. Kyle valued mathematics that could be applied. "I think real world is very important because it's hard to learn something that you can't apply. ... it's got to seem useful in order to learn it." He could easily give examples of applications of mathematics. He enjoyed problem solving and working challenging problems, often drawing from a variety of applications including woodworking, sports, and physics. Harriet also spoke of the importance of applications but when asked for examples she repeatedly referred to the same examples of using knowledge about percents to calculate the price of an item on sale or a gratuity. Harriet explained that she enjoyed mathematics because it was easy and did not involve reading. She spoke of mathematics as being right or wrong and argued that it was important for the teacher to tell students if they are correct or incorrect. She chose an assembly line metaphor for mathematics (written survey, 3/29/94) explaining that, "You start out with a problem, certain parts of your brain perform certain functions and you produce a product of 'answer'." Kyle saw learning mathematics as building a house, where "everything must be thought in advance or else you may have to build and tear down over and over. Same is true for math, one needs to learn it well the first time." Harriet seemed to restrict her view of mathematics to mathematics that she understood and considered appropriate content for high school students. Harriet explained that she wanted "to make sure that I help those [students] and teach as much as I can correctly to young adolescents." Kyle seemed to have two distinct kinds of mathematics. Like Harriet, Kyle believed school mathematics should be primarily rules, formulas, abstractions, and well-defined concepts. However, Kyle also enjoyed a rich mathematics outside of school that helped him solve problems from a variety of perspectives. He struggled with these two different types of mathematics in the classroom. "I think that applications and real world will be real good in high school, but in this sense, they were real good and helped me to learn a lot, but they weren't preparatory for those higher abstract levels of math."

Harriet and Kyle seemed to have different competencies in mathematics. Harriet responded to mathematical questions with vague language, sometimes misinterpreting the question or providing an unusual or incorrect response. Kyle used standard mathematical vocabulary and often provided specific examples to support his answers. The initial survey (3-29-94) posed a question about how to respond to a student who claimed that the area of a rectangle increases as the perimeter increases and provided examples to support this contention. Harriet accepted the student's generalization as correct, but objected to the student's use of a square as an example of a rectangle . Kyle offered a counterexample that disproved the student's generalization. In a later survey (11-4-94) we again see marked differences in how Harriet and Kyle discuss mathematics. In response to the question, "When someone says 'Geometry' what comes to your mind?" Harriet replied by listing names of geometric figures whereas Kyle talked about "finding surface area, volume, and perimeter of shapes", noting that "geometry is extremely useful in the real world." When asked, on another survey (10-26-94), what "transformation" means and what, if any, experiences contributed to your understanding of "transformation", Harriet's answer was brief: "The word transformation means change to me. Vocabulary in my high school English classes contributed to my understanding of transformations." Kyle's responses, however, were more detailed as he talked explicitly about mathematical reflections and rotations, and briefly connected these notions to an experience in his calculus class involving vectors. This pattern of Harriet being vague and general when given the opportunity to talk about mathematics, and Kyle being quite specific and contextual when provided such opportunities prevailed throughout the interviews.

Harriet and Kyle both agreed that a good mathematics teacher should be attentive to the needs of all students. Harriet emphasized "adapting pace of skills to student ability", and Kyle emphasized "teaching a class to *understand* [emphasis his] math, not just memorize ideas" (initial survey, 3-29-94). Harriet seemed to be much more confident in her ability to teach mathematics than was Kyle. Despite the fact that Harriet did not exhibit a strong knowledge of mathematics on tests, projects, interviews, or in her journal discussions, she expressed confidence in her ability to teach it. She was confident that she could relate to students and understand their needs. She was confident that she had command of the mathematics she anticipated teaching. Kyle was concerned about his mathematical content knowledge and how it might affect his teaching of mathematics. He frequently referred to the importance of developing a mathematical foundation which seemed to consist of important rules and facts. Since he could not recall all the rules and facts, he anticipated problems in his teaching. Kyle was confident in his problem-solving ability, but he seemed unsure about how this would help him teach.

Considering the Structure of Harriet and Kyle's Beliefs and Knowledge

We can see similarities in what beliefs Harriet and Kyle share about mathematics and its teaching (e.g., placing an emphasis on relating mathematics to the real world) and also how their beliefs differ (e.g., the way that mathematics should be related to the real world). We can gain further insight into their beliefs about mathematics and mathematics teaching by examining the structure of their beliefs. Of the two, Harriet held a more dualistic orientation. Initially she was critical of opportunities to engage in reflective activities. While she later expressed the view that she enjoyed sharing ideas with others, her orientation was more multiplistic than relativistic. In the main, she relished certainty. Her mother was perhaps the most significant factor in influencing her beliefs about teaching. Indeed, across more than six hours of interviews, Harriet only identified four statements (during the card sort interview) that she thought represented what she believed to be particularly important; two of these involved testimonies about her mother. In some sense her beliefs were held non-evidentially in that she tended to accept as evidence those things that conformed to her perceptions about how mathematics should be taught. This circularity tends to preclude reflection. A possible counterpoint to this orientation was Harriet's perceptions about technology. Initially, she posited the view that it was foolish to spend time using the computers given that computers generally were not available for classroom use. But toward the end of her program, she took quite a different perspective, claiming that technology was going to fundamentally change the teaching of mathematics. Indeed, in her card sort interview, she rejected the notion that students couldn't learn mathematics if they hadn't learned the basic skills because of the availability of technology. This shift may suggest the beginnings of a more relativistic orientation, but one has to wonder about the psychological strength of this belief given that it appears isolated from her other belief structures.

Kyle seemed to appreciate contextuality as suggested by the following statements he identified during the card sort interview: "I would use cooperative learning, but not all the time. Don't make all your examples from the book. You can tell a lot about way they know about a problem by the mathematical terminology they're using. The stuff that I thought was important I would stress and the stuff that I thought was unimportant, would kinda go through quickly." In a summary statement during the card sort interview, he emphasized the importance of learning in groups, receiving ideas from class, and criticism. Kyle's experiences were "connected" in that they fit, generally speaking, into a core belief that mathematics should be made interesting for students by enabling them to see connections between mathematics and the real world. While Kyle felt some tension between the importance he placed on basic skills and his orientation toward real world connections, the fact remains that these two views were not totally isolated. This sense of connectedness suggests a relativistic orientation that may account for why Kyle seemed to prosper from the reflective class activities in a way that Harriet did not.

Since it appears that Kyle's beliefs were held evidentially, there is reason to believe that his beliefs will likely be modified over time. The fact that he was concerned about "holes" in his mathematical background was actually predicated on a more pluralistic or relativistic perspective. That is, he saw mathematics from a broader perspective than did Harriet and consequently was more keenly aware of what he didn't know. The fact that he was better able to integrate various voices (e.g., his two different views of mathematics) about mathematics provides a context in which he both enjoyed and profited from the reflective journal entrees he was asked to write. While Harriet may not have been a received knower, neither was she an integrated knower. Her "filtering system" for what she accepted as evidence for believing what she did was much less permeable than Kyle's. Her apparent confidence in teaching mathematics was likely predicated on her understanding of students rather than her formal mathematics as she tended to shy away from unfamiliar mathematics. Thus her "glitches in mathematical knowledge" did not seem to concern her for she did not see them as impeding her ability to teach the mathematics she anticipated teaching. This isolation helps explain her resistance to engage in reflective activities involving her understanding of mathematics.

We can see that Harriet and Kyle not only hold different beliefs about mathematics and vary in their ability to do mathematics, but the structure of their beliefs varies as well. This difference in structure has considerable implications for their ability to realize the NCTM *Standards*. Harriet's isolation of beliefs, her reliance on authority (e.g., her mother's voice), and her non-relativistic conception of mathematics tend to isolate her from the reflection needed for an adaptive means of teaching. Our evidence indicates that she may be moving toward a more relativistic perspective, as suggested by her views on technology. For Kyle, who believes as does Harriet in the importance of emphasizing basic skills, we see a teacher who appreciates contextuality, thereby suggesting a potential exists for changing and reforming his teaching over time.

A Concluding Remark

By considering both the substance and structure of teachers' beliefs, we provide a certain dimensionality that captures the intensity and interconnections among beliefs. While it is well established that teachers' beliefs influence practice, it may be even more important to consider the means by which those beliefs are structured. Recognition of the way beliefs are structured provides us with the potential for seeing how isolated beliefs can be related to beliefs more strongly held—thus ensuring their endurance when buffeted by the usual obstacles teachers face. By considering both the substance and structure of beliefs, we have the potential for eliminating the random effectiveness often associated with our attempt to reform the teaching of mathematics.

References

Belenky, M.F., Clinchy, B.N., Goldberger, N.R., & Tarule, J.N. (1986). *Women's ways of knowing: The development of self, voice, and mind.* New York: Basic Books.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13-20.

Ernest, P. (1991). *The philosophy of mathematics education.* London: Falmer Press.

Green, T. (1971). *The activity of teaching*. New York: McGraw-Hill.

Perry, W. (1970). *Forms of intellectual and ethical development in the college years: A scheme*. New York: Holt, Rinehart, & Wilson.