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ABSTRACT

Open-ended tasks were used to examine gender differences in complex mathematical problem solving. The results of this study suggest that, overall, males perform better than females, but the gender differences vary from task to task. A qualitative analysis of student responses to those tasks with gender differences showed that male and female students exhibited many similarities in their solution processes, such as making similar types of mathematical errors and using similar strategies and representations. This study suggests not only the complexity of the issue of gender differences, but also the feasibility and usefulness of using open-ended tasks to explore the issue. Contains 13 references. (Author/MKR)

Exploring Gender Differences in Solving Open-Ended Mathematical Problems

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EXPLORING GENDER DIFFERENCES IN SOLVING OPEN-ENDED MATHEMATICAL PROBLEMS

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Open-ended tasks were used to examine gender differences in complex mathematical problem solving. The results of this study suggest that overall, males perform better than females, but the gender differences vary from task to task. A qualitative analysis of student responses to those tasks with gender differences showed that male and female students had many similarities in their solution processes of solving these problems, such as making similar types of mathematical errors and using similar strategies and representations. This study suggests not only the complexity of the issue of gender differences, but also the feasibility and usefulness of using open-ended tasks to explore the issue.

Gender differences in mathematics performance have been a popular but unresolved issue in educational research (Fennema & Leder, 1990). Since Sells (1973) expressed the concern that early decisions not to study mathematics in schools might be excluding students, and especially female students, from higher paying occupations such as those in science, engineering, and medicine, there has been an increased interest in research about gender and mathematics (Fennema & Leder, 1990). In particular, researchers have focused on investigations of gender-related differences in mathematical performance and have provided different theoretical models to explain the gender differences in mathematics performance.

In the early 1970s, Maccoby and Jacklin (1974) conducted a comprehensive review of research on gender differences and concluded that "boys excel in mathematical ability" (p. 352). Others indicated that there were no gender differences in the earlier years of elementary school, but in upper elementary, junior high, and senior high school, males outperformed females in mathematics. A review of research on gender differences in mathematics by Hyde, Fennema, and Lamon (1990) suggests that gender differences are declining. However, females continue to express less confidence in their mathematical ability and a lower perception of the usefulness of mathematics to them in the future (Lindquist, 1989). Even among the mathematically gifted students, females have lower educational aspirations in mathematics and sciences than do males (Benbow, 1992). As Leder (1990) indicates, "the issue of gender differences in mathematics learning is complex and there are many perspectives from which it can be explored" (p. 21).

Most of the previous studies used multiple-choice tasks to examine the gender-related differences in solving routine mathematical problems (Marshall, 1983). How male and female students differ in solving more complex mathematical problems remains to be investigated. The purpose of this study is to use open-ended tasks to explore the gender-related performance differences in solving complex mathematical problems. The open-ended problems allow students to display their solution processes, so male and female students' thinking and reasoning can be examined beyond the correctness of the numerical answers. Thus, this study is intended to provide more in-depth information about male and female students' thinking and reasoning in solving complex mathematical problems.

Method

Data Source

The data used in this study were from an earlier research project (Cai, in press). In particular, 227 sixth-grade students (96 females and 131 males) from the Pittsburgh area participated in this study. Subjects were asked to complete seven open-ended problems within a regular classroom setting (about 40 minutes). These open-ended problems involve a variety of important mathematical content areas, such as number sense, pattern, number theory, pre-algebra, ratio and proportion, estimation, and statistics. The appendix shows two of the seven tasks. These problems were from the QUASAR project (Silver, 1993).

Data Coding and Analysis

The data were coded and analyzed according to two analysis schemes: quantitative analysis (Lane, 1993) and qualitative analysis (Cai, Magone, Wang, & Lane, in press; Magone, Cai, Silver, & Wang, 1994). In the quantitative analysis, each student response to an open-ended problem was scored using a five-point scale (0 - 4) with 0 = no understanding, 1 = beginning understanding, 2 = some understanding, 3 = nearly complete and correct understanding, and 4 = complete and correct understanding. In the qualitative analysis, each student's response is examined in detail in terms of cognitive aspects of solving the open-ended problems, such as solution strategies, mathematical errors, mathematical justifications, and representations. These cognitive aspects are the focus of the qualitative analysis since they have been identified as important and significant dimensions in cognitive psychology in general and in mathematical problem solving in particular. An elaborate description of the framework for the qualitative analysis can be found in Cai (in press).

Inter-rater Agreement

In order to ensure a high reliability of coding student responses to open-ended problems, two raters independently coded about 50 student responses to three of the open-ended problems. The inter-rater agreements for the quantitative analysis range from 84–89%. Inter-rater agreements for the qualitative analysis range from 86–98%.

Results

Quantitative Results

Overall, male students have significantly higher aggregated mean scores than female students (M_{males} =18.79, $M_{females}$ =16.36; $t = 2.43$, $p < .01$). The gender differences were also examined for each open-ended problem. Table 1 shows the mean scores of male and female students on each open-ended problem. Males have significantly higher mean scores than females on four of the problems. This implies that the overall difference between male and female students is mainly due to the differences on the four problems. It is interesting to note that the four problems on which there are statistically significant gender differences require computation, but the others on which there are no significant gender differences require less computation.

Table 1.
Mean Scores of Male and Female Students on Each of the Open-ended Problems

	Division* Problem	Estimation Problem	Average Problem	Number Theory* Problem	Pattern Problem	Ratio &* Proportion Problem	Pre-algebra* Problem
Male(n=131)	3.15	2.15	2.18	2.27	3.05	2.90	3.09
Female(n=96)	2.79	2.19	2.07	1.66	2.96	2.08	2.61

* For this problem, the difference in mean scores between male and female students is statistically significant ($p < .05$).

Qualitative Results

Because of the space limitation, the qualitative results based on the first three of the problems for which there exist gender differences are reported in this paper. Of these problems, two of them (Division Problem and Number Theory Problem) appear in the appendix.

Division Problem. The first open-ended problem which shows gender differences is a division-with-remainder story problem (see appendix). In solving the division problem, one not only needs to apply and execute division computations correctly (computation phase), but also one needs to interpret the computational results with respect to a given story situation (sense-making phase). The qualitative analysis of the Division Problem was conducted from four aspects: (1) solution process, (2) execution of procedures, (3) numerical answer, and (4) interpretation.

Over 90% of the male and female students selected the appropriate procedure (e.g., long division) to solve the problem. However, a significantly larger percentage of male (86%) than female students (75%) executed the procedure flawlessly ($z = 2.01$, $p < .05$). A larger percentage of male (70%) than female students (61%) provided the correct answer of 13, but the difference is not statistically significant. For those who had incorrect answers, both male and female students frequently gave 12 or 12 with a whole number remainder (i.e., 12 R 8) as their answers. Only a few students expressed their numerical answer as 12 with a decimal remainder or 12 with a fractional remainder. Similarly, although a larger percentage of male (50%) than female students (44%) provided appropriate interpretation of their answers, the difference is not statistically significant. Therefore, the qualitative results of the Division Problem suggest that there is a significant gender difference in the computation phase of solving the problem (favoring males), but gender difference does not exist in the sense-making phase.

Number Theory Problem. This problem (see appendix) assesses student number sense and the ability to use basic concepts of number theory to solve a problem. It allows for multiple correct answers. In particular, the number 1 and any multiple of 12 plus 1 are correct answers (i.e., 1 + 12n, for n = 0, 1, 2, ...). Each student response was coded with respect to: (1) numerical answer, (2) solution strategy, (3) mathematical error, and (4) representation.

A significant larger percentage of male (61%) than female students (43%) had correct numerical answers ($\underline{z} = 2.69$, $\underline{p} < .01$). For those male and female students who had correct answers, 83% of female and 86% of male students had the correct answer of 13. The remaining 17% of female and 14% of male students had a correct answer other than 13, including 25, 49 etc. This implies that for those who had correct answers, female and male students tend to provide similar types of correct answers.

In solving the Number Theory Problem, eight different solution strategies were identified. Some students used common-multiple strategies to solve the problem. For example, a student might find 12 as a common multiple of 2, 3, and 4 by direct computation (2 X 6 = 12, 3 X 4 = 12, 4 X 3 = 12), and then found the answer by adding one to the common multiple. Another example is that a student listed the multiples of 2, of 3, and of 4; then identified the common multiple; and added one to find the answer, as shown below:

2, 4, 6, 8, 10, <u>12</u>, 14, 16, ...
3, 6, 9, <u>12</u>, 15, 18, ...
4, 8, <u>12</u>, 16, 20, 24, ...
12 + 1 = 13, so the answer was 13.

Other students used guess-and-check strategies to solve the problem. Only a slightly larger percentage of male students (53%) than female students (48%) had a clear indication of using one of the solution strategies, with the difference being not statistically significant. For those who had a clear indication of using one of the strategies, female students tended to use common multiple strategies more frequently than males; while male students tended to use guess-and-check strategies more frequently than females (χ^2 (1, $\underline{N}$ = 116) = 4.93, $\underline{p} < .05$).

Male and female students made similar mathematical errors and used similar representations. The most frequent error (about 30%) made by both male and female students is that students manipulated numbers unreasonably. For example, they simply added the given numbers together to get the answer without any mathematical justification. The same percentage of male and female students (42%) used mathematical expressions to show their solution processes; 17% of female and 14% of male students used pictorial representation to show their solution processes; and 42% of female and 44% of male students used written words to show their solution processes.

Ratio and Proportion Problem. This problem assesses student problem-solving skills in a map-reading context that requires knowledge of ratio and proportion. In particular, students were given that 3 centimeters in a map represents 54 miles in actual distance, and they were asked to use proportional reasoning to determine the actual distance that 12 centimeters represents on the map. Each student's response was coded with respect to: (1) numerical answer, (2) solution strategy, (3) mathematical error, and (4) representation.

A larger percentage of male (69%) than female students (48%) had correct numerical answers ($z = 3.19$, $p < .01$). Similarly, a larger percentage of male (82%) than female students (60%) had a clear indication of using one of the identified solution strategies ($z = 3.68$, $p < .01$). However, both male and female students most frequently used a "unitary strategy" to solve the problem. One of the examples of using unitary strategy is like:

> 12 ÷ 3=4. Since 3 centimeters represents 54 miles, 4 X 54 = 216, so it is 216 miles.

Another example of using unitary strategy is like:

> Since 3 centimeters represents 54 miles, 54 ÷ 3 = 18, 18 X 12 = 216. So it is 216 miles.

Two male students used formal proportional reasoning strategy to solve the problem; no female students did so. Moreover, male and female students used similar representations in their solutions. In fact, over 90% of the male and female students used symbolic representations to show how they found their answer. The most frequent error made by both male and female students in solving this problem was that they manipulated numbers unreasonably; this finding is similar to what was found for the Number Theory Problem.

Brief discussion

This study used open-ended tasks to examine gender differences in solving complex mathematical problems. The results of this study suggest that overall, male students perform better than female students, but the gender differences vary from task to task. Gender differences appear to be significant on tasks requiring computation, but the difference dramatically decreases on tasks not necessarily requiring computation. In particular, in solving the division-with-remainder problem, males outperform females in the computation phase, but not in the sense-making phase

For those tasks showing significant gender differences, a more elaborate qualitative analysis of student responses was conducted. Although, a larger percentage of male than female students provided the correct answer, male and female students showed many similarities in the solution processes used to solve these open-ended problems, such as making similar types of mathematical errors and using similar strategies and representations. The results of this study suggest not only the complexity of the issue of gender differences in mathematics, but also the feasibility and usefulness of using open-ended problems to explore this issue.

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Appendix

Division Problem

Students and teachers at Miller Elementary School will go Spring sightseeing by bus. There is a total of 296 students and teachers. Each bus holds 24 people. How many buses are needed?

Show your work.
Explain your answer.

Answer:

Number Theory Problem

Yolanda was telling her brother Damian about what she did in math class.

> **Yolanda said, "Damian, I used blocks in my math class today. When I grouped the blocks in groups of 2, I had 1 block left over. When I grouped the blocks in groups of 3, I had 1 block left over. And when I grouped the blocks in groups of 4, I still had 1 block left over."**

Damian asked, "How many blocks did you have?"
What was Yolanda's answer to her brother's question?
Show how you found your answer.

Answer:
