An Examination of the Relationship between the Problem-Solving Behaviors and Achievements of Students in Cooperative-Learning Groups.

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An Examination of the Relationship Between the Problem-Solving Behaviors and Achievements of Students in Cooperative-Learning Groups

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AN EXAMINATION OF THE RELATIONSHIP BETWEEN THE PROBLEM-SOLVING BEHAVIORS AND ACHIEVEMENTS OF STUDENTS IN COOPERATIVE-LEARNING GROUPS

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This study examined the problem-solving behaviors, strategies, and achievement of college students enrolled in a one-semester College Algebra and Statistics course, with respect to the content areas of quantitative literacy, connections between algebraic and graphical representations, and mathematical modeling. Four instructional units of this course were chosen - two in which the students were assigned to cooperative-learning groups and two in which the students worked independently. The findings suggest that students who work in cooperative learning groups clearly exhibit important problem-solving behaviors such as persistence and a willingness to explore alternative solutions; however, they still experience difficulty explicating the connections between mathematical actions and/or processes and the mathematical concepts.

Conceptual Framework

Cooperative-learning strategies have been credited with the promotion of critical thinking, higher-level thinking, and improved problem-solving ability of students. Current research that examines behaviors that occur during group problem-solving sessions seem to indicate that groups engage in behaviors that are similar to those exhibited by expert mathematicians when they solve problems (Artz & Newman, 1990; Schoenfeld, 1987); that is, they engage in monitoring their own thoughts, the thoughts of their peers, and the status of the problem-solving process. Researchers who have studied cooperative learning at the college level generally have found that students learn just as well as in more traditional classes and often develop improved attitudes toward each other and toward mathematics (Dees, 1991; Slavin, 1995; Breghting & Hirsh, 1977; Chang, 1977; Davidson, 1971; Olsen, 1973; Shaughnessy, 1977; Treadway, 1983). Although it is not clear which components of cooperative learning are responsible for improvements in higher-level thinking, attempts have been made to identify the components. One conjecture is that dealing with controversy may be such an element. Smith, Johnson, and Johnson (1981) report on a study in which they suggest that higher results on achievement and retention of the students in the “controversy group” may be attributed to the “cognitive rehearsal of their position and the attempts to understand the opponents position” (Smith, Johnson, & Johnson, 1981, p. 652). This work provides important information concerning the efficacy of cooperative-group learning and the key components which contribute to higher-level thinking. However, research which considers both the problem-solving behaviors, strategies, and achievement of college students enrolled in traditional courses also needs to be conducted. In this study, we not only examined the problem-solving behaviors, strategies, and achievement of college students assigned to cooperative learning groups, but designed problem-solving experiences consonant with the course curriculum that
focused on the connections between mathematical actions and/or processes and
the underlying mathematical concepts.

Methodology

The subjects chosen for this study consist of 108 students enrolled in four
instructional units of College Algebra and Statistics at a major university located
in the Mid-Atlantic states. Two experimental groups and two control groups were
randomly selected. An attitude scale and a pre-test of algebraic ability were ad-
ministered to both the experimental and control groups on the first day of class. In
the experimental sections, students were assigned to cooperative learning groups
based on their performance on the pre-test (each group contained four students - 1
high score, two middle scores, and 1 low score). In the control sections, students
were told that they could work with fellow students on the various activities/labs,
but were not specifically assigned to groups. Throughout the semester, problem-
solving behaviors, strategies, and achievement were assessed through four tasks
which focused on the connections between the mathematical actions and processes
and the mathematical concepts. The first and fourth tasks took place in a regular
classroom setting and consisted of problem sets devoted to the topics of quantita-
tive literacy and modeling exponential functions. The second and third tasks took
place in the computer laboratory setting and consisted of computer labs devoted to
exploring the connections between algebraic and graphical representations of lin-
ear functions (set in the context of depreciation) and determining the best math-
ematical model for a particular set of data. Each of the four tasks were videotaped
(some problem-solving activity was recorded for each group) and audiotaped (the
problem-solving discourse was recorded for each cooperative-learning group within
both experimental sections). In addition, the initial and final problem-solving ses-
sions devoted to quantitative literacy and exponential modeling were also video-
taped for the control sections. The written work that accompanied each of these
tasks was also analyzed with respect to their ability to explicate the connections
between actions and/or processes and mathematical concepts. At the end of the
semester, the attitude scale with some additional open ended questions concerning
cooperative learning groups was again administered to all of the students.

Results and Conclusions

The results of the students MSAT, the pre-test of algebraic ability, and the
questions concerning the number of years of high-school mathematics and their
previous university mathematics history were analyzed to determine between-group
similarities and differences. Seventy-eight students comprised the experimental
group and thirty students made up the control group. The results are summarized
in the table at the top of the next page.

These results indicate that the base-line assessments were consistent within
each group. The MSAT, pre-test of algebraic ability, and the number of years of
high school mathematics all indicate that the experimental sections were more
capable and more experienced mathematically than the control sections. In addition,
more of the students in the control sections had taken and passed the non-credit remedial algebra course prior to enrolling in the current course.

The attitude scale, a 10-item Likert scale administered both prior to instruction and at the end of the semester, assessed the students' views about learning in general, the role of the teacher, and whether they learn mathematics better while working alone or with other students. Prior to instruction, both groups favored working with other students as the better way to learn mathematics. At the end of the semester, students' responses to the question, "I found working in cooperative-learning groups to be (please elaborate)...," ranged along a continuum from "extremely enthusiastic" to "helpful, but..." to "not at all useful." The results of this question are indicated in the following table:

Table 2. Responses Concerning Cooperative-Group Learning

<table>
<thead>
<tr>
<th>Categories of Responses</th>
<th>Extremely helpful</th>
<th>Helpful, but...</th>
<th>No comment</th>
<th>Not useful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>53%</td>
<td>28%</td>
<td>5%</td>
<td>14%</td>
</tr>
<tr>
<td>Experimental Group</td>
<td>60%</td>
<td>13%</td>
<td>7%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Following instruction, both groups still favored working in cooperative-learning groups. However, significant differences emerged with respect to the role of the teacher. The number of experimental group members who strongly agreed that "the role of the teacher is to facilitate learning" increased dramatically. Members of the control group remained ambivalent concerning the role of the teacher. The comments of many of the students with respect to working in cooperative-learning groups are represented by this excerpt from the Attitude and Cooperative Learning Assessment.

At first I didn't like working with other people because I usually study and work alone in order to memorize and teach myself information (which is hard to do with others). But by the end of the semester I enjoyed working with my group and I studied with 3 others for the final exam.
With respect to whether working in cooperative-learning groups has affected problem-solving strategies, the student writes:

I have listened to and heard other strategies and learned new solving and thinking patterns.

The course grades indicate how well each group did with respect to the standardized achievement criteria and are recorded in the following table:

Table 3. Final Course Grades

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>A or A-</th>
<th>B+,B,B-</th>
<th>C+,C,C-</th>
<th>D+,D,D-</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>30%</td>
<td>33 1/3%</td>
<td>26 2/3%</td>
<td>3 1/3%</td>
<td>6 2/3%</td>
</tr>
<tr>
<td>Experimental</td>
<td>21%</td>
<td>29%</td>
<td>35%</td>
<td>14%</td>
<td>1%</td>
</tr>
</tbody>
</table>

It is interesting to note that the control group did much better in the course than the experimental group although all of the baseline assessments indicated otherwise. There are several possible explanations. One possible explanation is the differences between the two groups in terms of the number of students who took and passed the remedial algebra course prior to taking the current course. Another possibility centers around the differences between the problem-solving activities and/or labs and the standard exam questions.

Throughout the course, the results of the videotapes, audiotapes, and written work were analyzed to determine the nature of the problem-solving behaviors, strategies, and achievement of both groups. Of particular interest were the videotapes of the cooperative learning groups working on the lab devoted to examining the connections between graphical and algebraic representations of linear depreciation functions. In this laboratory activity, students were asked to estimate graphically, identify the graphical feature they utilized to answer the question, algebraically answer the question, and describe how the graphical and algebraic representations were related. Results indicate that those students in the cooperative-learning groups, like those of the "controversy group" identified by Smith, Johnson, and Johnson engaged in the type of mathematical discourse that would enable them to form connections between graphical and algebraic representations. Results of the written responses on this lab were significantly higher for the students in the cooperative-learning groups than those in the control group. However, students in the cooperative-learning groups still exhibited some difficulty explicating the connections between mathematical actions and/or processes and the mathematical concepts. Furthermore, the standardized assessments (multiple-choice questions with some free-response parts) did not emphasize forming these connections. All of these factors could contribute to the differences in the standardized achievement of the two groups.

These findings provide convergent evidence concerning both the problem-solving behaviors and achievements of cooperative learning groups and suggest
the kinds of group activities which may facilitate higher-level thinking and improved problem-solving ability.

References


