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ABSTRACT

This paper explores the process by which mathematical knowledge is socially constructed. Interactional analysis of a lesson on fact families shows how one third-grade mathematics class negotiated the meanings of writing a number sentence for a picture and a fact family. In the course of classroom interactions, teacher and students shifted the lesson's focus from number sentences that represented physical images to permutations of 3 numerals around 2 operation symbols. (Author)

Fact Families as Socially Constructed Knowledge

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FACT FAMILIES AS SOCIALLY CONSTRUCTED KNOWLEDGE

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This paper explores the process by which mathematical knowledge is socially constructed. Interactional analysis of a lesson on fact families shows how one third grade mathematics class negotiated the meanings of writing a number sentence for a picture and of a fact family. In the course of classroom interactions, teacher and students shift the lesson's focus from number sentences that represented physical images to permutations of 3 numerals around 2 operation symbols.

There is a large body of theoretical work on the social construction of knowledge as it applies to mathematics teaching and learning. Some studies focus on individuals' construction of mathematical knowledge while participating in classroom interactions, others describe the development of communal definitions of what it means to do mathematics, and still others focus on the influence of cultural symbols on knowledge development.

Bauersfeld, Krummheuer, & Voigt (1988) apply the theory of symbolic interactionism to the analysis of interactions in mathematics classrooms. They argue that the meanings of objects, words, and actions lie in the meanings that individuals attribute to them in the course of social interaction. Voigt (1992) argues that, "In classroom life the meanings of mathematical concepts and the validity of mathematical statements are socially accomplished. . . . (E)specially in introductory situations, we cannot presume that the learner would ascribe specific meanings to the topic by themselves — meanings which are compatible to the mathematical meanings the teacher wants the student to ascribe" (p. 5). As teacher and students work toward mutual understanding of a mathematical idea, they may reach what Krummheuer calls a "working interim" where both parties come to believe that they understand each other while, from the observer's perspective, they have created consistent, but not completely compatible, understandings of the topic at hand. In studying classroom interactions, the observer could therefore infer a particular individual's knowledge of, say, fact families, from observations of his/her interactions with the objects or with other individuals, and similarly, one could infer the collective knowledge of fact families that is constructed by the group through their attempts to communicate. The collective understanding that emerges may differ from that of individual participants.

Building on this work, this paper describes the dynamic process by which collective mathematical knowledge in a 3rd grade classroom community is constructed. Through analysis of one mathematics lesson, this paper further attempts to provide an example of how the students as well as the teacher influence the nature of the knowledge developed.

The objective of the lesson examined here, according to the required textbook, was "to use fact families to recall addition and subtraction facts" (Eicholz et al., 1985, p. 10). However, as teacher and students interact, the collective meaning of "fact family" and the purpose of the lesson change. As the class moves through the 4 phases of the lesson, introduction, practice activities, written seatwork, and a final challenge problem, the lesson intended to focus on relationships among facts becomes a lesson in symbol manipulation.

Analytic Technique

This lesson was selected from data collected for a larger project that provided qualitative descriptions of the interaction patterns that emerged in a 3rd grade text-book-based mathematics class (McNeal, 1991). This particular lesson seemed to be a striking illustration of the theories currently under discussion among researchers in mathematics education and educational psychology. No claims are made that this textbook lesson is typical.

Data from the larger study included field notes, video recordings and transcripts of 28 mathematics lessons over the first 8 weeks of instruction. Based on the work of Bauersfeld, Krummheuer, and Voigt (1988), individual transcripts were analyzed line by line, in chronological order, for patterns that would illuminate the mathematical meanings and communicative practices of this community. Assertions developed from each lesson were then compared with those from each of the previous lessons. Exceptions to emerging patterns were also tested against the entire body of data following analytic procedures of Erickson (1986). Interpretation of the following transcript is thus based on analysis of the entire corpus of data, rather than on the one episode alone.

The Lesson

The following mathematics lesson occurred on September 1 during the 6th class session for the year. After about 20 minutes of problem solving, the class began the textbook portion of the lesson. The actions described took 42 minutes, and were followed by afternoon recess.

Following the suggestion in the textbook, Mrs. Rose (all names are pseudonyms) used pictures of dominos to elicit from the class the definition of a fact family.

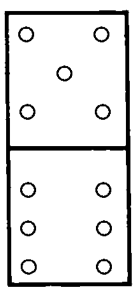

Figure 1

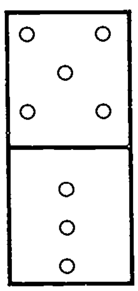

Figure 2

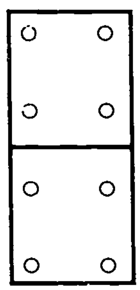

Figure 3

1 Mrs. R: Notice the domino boys and girls. [pointing to Figure 1 on the overhead projector] How many spots do you see on the top of the domino here?

2 Students: 5.

3	Mrs. R:	[pointing] How many do you see on the bottom?
4	Students:	6.
5	Mrs. R:	OK. I would like for someone just to give me, ah, an addition number sentence for these, for this domino right here. An addition number sentence. Who can give me one. [calls on one student whose hand is raised]
6	Student:	5 plus 6 equals 11.
7	Mrs. R:	All right. [writes 5 + 6 = 11, then makes a side comment] 5 + 6 equals 11. Who can give me another addition number sentence for this? Chris? [no response] Up there. We have one number sentence, 5 + 6 equals 11, what else could we do? What else could we use? Use those numbers up there.
8	Chris:	6 plus 5?
9	Mrs. R:	Wonderful. 6 plus 5 equals 11. [writes 6 + 5 = 11] Who can give me a subtraction number sentence using these dominos? Betty.
10	Betty:	6 take away 5.
11	Mrs. R:	How many do we have altogether, Betty?
12	Betty:	[after a short pause] 11.
13	Mrs. R:	11.
14	Betty:	Take away 5.
15	Mrs. R:	11 take away 5 equals what, Betty?
16	Betty:	6.
17	Mrs. R:	6. Very good. [writes 11 - 5 = 6] Who can give me another one? Another subtraction number sentence? Karl.
18	Karl:	11 take away 6 equals 5.
19	Mrs. R:	Now, look here [pointing to number sentences], boys and girls. How many, How many facts do we have there?
20	Students:	4. Oh! 4.
21	Mrs. R:	4 facts. How many numbers, Chris, did we use? How many numbers?
22	Chris:	3...2.
23	Mrs. R:	How many numbers did we use?
24	Chris:	3.
25	Mrs. R:	We used 3. We just made what we call a fact family.
26	Student:	A [fact or fat?] family. [laughs]

27	Mrs. R:	A fact family is 4 facts made out of 3 numbers. [shows Figure 2] Let's look at this domino right here. Let's see if we can think of 2 addition number sentences for it. [calls on Jennie whose hand is raised]
28	Jennie:	[starts to go to the board] Um, I know one for the top one [Figure 1].
29	Mrs. R:	Just, you just tell me. Just tell me. For this right here.
30	Jennie:	16 minus 5. Equals 11.
31	Mrs. R:	OK. Jennie, did we have 16 . . . dots?
32	Jennie:	[makes a face] No.
33	Mrs. R:	[laughs] All right. Who can give me a number sentence; who can give me two addition number sentences for [Figure 2]?

Mrs. Rose focused the class on the important features of the domino (lines 1-5), and they quickly produced the first addition fact (line 6). When she called for a second, Chris seemed unsure what she meant, but made the expected interpretation, and no discussion was warranted. Mrs. Rose therefore did not realize that the domino representation might produce multiple interpretations until Betty (line 10) indicated her understanding that the task required using the 2 numbers shown in any number sentence. Although Betty's interpretation was consistent with her classmates' responses, it was not compatible with the intended task. This prompted Mrs. Rose to give the class more information, implying that students should use the *total* number of dots (line 11). As she started to move on (line 27), Jennie volunteered another fact for Figure 1, having misunderstood both the definition of the mathematical task and the social cue that the group had finished collecting facts for this domino. Her sentence included more than the number of dots shown, and suggested that she understood the task to mean: Create a sentence using numbers made from the two given. (This was confirmed later when she explained how she had come up with her numbers.)

In the remainder of the introduction, Mrs. Rose led the class through a similar sequence for Figure 2, and then used Figure 3 to illustrate the special case of a family with only two facts. She then gave individual students some practice activities. These exercises required students to make fact families for three numbers given without a picture. When students produced inappropriate number sentences, Mrs. Rose prompted them to check that they had used only the given numbers. For example, she wrote 1, 5, and 6 in a circle and called two students to the board, "Make a fact family out of these numbers. Quick as you can. (to the class) You boys and girls see if they're correct." When Nan wrote 11 + 6 = 17, Mrs. Rose stopped her as she wrote 17, "What's the number you just wrote?" She then asked, "Do you see 17 on here?" and reminded her, "Using these three numbers." Finally, only John and Annie were still working: They had found three facts for 2, 6, and 8, but were struggling to find the fourth. Mrs. Rose wanted to move on so the class would have sufficient time to complete their written assignment, so she came to assist them.

Mrs. R: You've got 8 - 2 is 6 then you have 6 + 2 = 8, 2 + 6 = 8, what do we still need? We've got 2 pluses, we've got one minus, what do we still need? Do we need another plus or do we need a minus? We've already taken away 2, now what are we gonna take away? Good. Very good.

Mrs. Rose then quickly reviewed the instructions for each problem on the assigned textbook pages. For example, the first problem was: 8 + 5 = __, 5 + 8 = __, 13 - 5 = __, and 13 - 8 = __. When Mrs. Rose called on Mike for 5 + 8 = __, he replied, "5 + 8 equals . . . 15?" Pointing to something in his book, Mrs. Rose reminded him, "We're only gonna use these three numbers, 13, 8, and 5." His response was inaudible to the observer, but Mrs. Rose went on, "These are the 3 numbers we're using for the fact family. We just said 8 + 5 is 13, now what's 5 + 8?"

Mrs. Rose asked the students to copy the number of the problem and then write answers only. When they began to work, most of the students spoke with Mrs. Rose about the format or instructions for their written work. When she and the students talked about fact families, she assisted them by saying, "Only use these three numbers," or "Remember a family has four facts." In one case, the following exchange occurred.

Mrs. R: See these 3 numbers you're gonna use? OK, you got 8 + 5, right? There's 8, there's a 5, what number do we need now? [response is inaudible] 13. Put 13 right there. Now look at these numbers, there's 5, there's 8, what other number do we need? [no response] There's the 5, there's the 8. 13. Put 13 right under there. OK, now there's the13, there's the 5, now what number do we need here? [response is inaudible] Very good.

Mrs. Rose then put a "challenge problem" on the blackboard that she had taken from the book for the students to try when they had completed their work: "Use the 'Addition on Venus' symbols shown to write four fact-family number sentences: $\Delta + \Sigma = \Omega$" (Eicholz et al., 1985, p. 10). Mrs. Rose copied only the three symbols, Δ, Σ, Ω, and asked the students to create a fact family for these figures. The task therefore looked like the triples of numbers presented during the practice activities. Only one student, John, challenged the teacher's task saying, "Those aren't numbers. You can't make a fact family." No other students joined his protest, showing that his interpretation of the task differed from theirs.

Discussion

This lesson illustrates the social construction of knowledge in two ways. First, the group negotiated what it meant to write a number sentence for a picture. Seeing the domino according to the conventions of school mathematics involved learning to see only what was in the picture (16 - 11 = 5 was inappropriate for Figure 1), and learning that the entire quantity shown must be maintained (6 - 5 was also inappropriate). Assuming this interpretation to be self-evident put students in the position of guessing what the teacher had in mind. This helped reproduce the elicitation pattern (Voigt, 1985) seen here.

Second, as the lesson about related facts referred to as fact families proceeded, the purpose and meaning of these changed. In the introduction, Mrs. Rose intended building on the students' contributions, using the domino as a concrete representation of the relationships that she believed a fact family described. The students, however, translated the number of spots on each half of the domino into the numbers to be used in composing arithmetic facts. For them, this first part of the lesson was not about related facts, as they did not know the term "fact family" until they were done. It was about figuring out what the task was. Their unexpected responses and her desire to avoid stating the definition in turn obligated Mrs. Rose to point to features of the picture that implied the meaning she had in mind. Although several students offered number sentences that were deemed inappropriate (6 - 5, 16 - 5, 3 - 8 for Figure 2, and 4 -4 = 0 for Figure 3) as they tentatively tried to make sense of her expectations, at no time did they challenge her constraints or ask for explicit clarification of the task. At this stage of the lesson, there were at least three different under-standings of the representation and hence of the task, but the collective understanding was that the lesson was about creating number sentences from given numbers.

During the practice activities, students continued to test their understanding of the task and of fact families against the teacher's. Having seen several examples completed by this time, they became less tentative as they received immediate feedback. The exercises that presented three numerals without a picture caused Mrs. Rose's response to inappropriate number sentences to shift from references to a picture ("How many do we have altogether?") to references to a list of numbers ("Is that one of the 3 numbers?"). The collective understanding of the lesson also shifted: Fact families were permutations of three numbers around the + and - symbols. When the students' confusion persisted in a lesson that seemed simple to her, and none of her previous forms of assistance were sufficient, Mrs. Rose suggested that John and Annie check the number of addition and subtraction facts they had.

During seatwork, instructions to provide only answers further separated the definition of a fact family from relationships among the facts. This was compounded by the exchange, audible to the whole class, in which Mrs. Rose effectively shifted Mike's attention from computing 8 + 5 = __, to filling in the blank by process of elimination.

Despite John's protest that creating fact families from symbols with no conventions for relating them was unreasonable, no one, including the teacher, recognized the validity of his claim. This was the final phase in the evolution of the meaning of fact families from a set of useful relationships to a set of permutations of three numerals around two operation signs.

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