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ABSTRACT

This report describes the changes in a freshman-level calculus course that occurred as a consequence of adopting the Harvard Consortium Calculus text. The perspective is that of the lecturer. The course is intended as an introduction to calculus for liberal arts students, that is, students who will not be expected to use calculus as a mathematical tool in their area of major study. The perceptions of the instructor about global changes that occurred in the course include that learning was different, the material was appropriate to students' needs and level of sophistication, students came away with different attitudes about mathematics, the use of the graphing calculator opened up new ways of understanding and representing mathematics, the use of cooperative groups is an important technique in promoting student involvement in learning, and the increased use of writing was critical to students' learning conceptually rather than mechanically. (MKR)

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Impact of Calculus Reform in a Liberal Arts Calculus Course

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IMPACT OF CALCULUS REFORM IN A LIBERAL ARTS CALCULUS COURSE

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This report describes the changes in a freshman-level calculus course, Survey of Calculus, that occurred as a consequence of adopting a reformed calculus text, *Calculus* by Deborah Hughes-Hallett, Andrew Gleason et al. (better known as the Harvard Consortium Calculus or HCC text). The perspective is that of the lecturer.

The course is intended as an introduction to calculus for liberal arts students, that is, students who will not be expected to use calculus as a mathematical tool in their area of major study. The course exists because of a faculty belief that calculus is one of the great intellectual achievements of humankind, has been a major factor in the development of western civilization and should be part of every liberal education. Prior to the adoption of the Hughes-Hallett/Gleason text however, these reasons for calculus as a part of a liberal education were nowhere apparent in the course. Rather, the course was a shadow of the mainline scientific calculus, emphasizing development of skills with computational elements of calculus. Given an audience whose interests are non-scientific and whose skills with symbolic manipulation are not strong, the course left students with a feeling that mathematics is a collection of formulas and procedures to be memorized and then forgotten. (One of the principal motivations for the calculus reform movement was concern about students learning to manipulate symbols rather than understanding concepts that form a basis for general analysis in problem solving (Douglas, 1986).

Research Questions

- What factors motivated one mathematics professor to make changes in his Survey of Calculus course?
- What changes were made in curriculum, pedagogy, and assessment as a result of these motivations?

Method

Study Participants. Students enrolled in MATH 117 A Survey of Calculus for both the Winter (n = 104) and Autumn (n=40) Quarters, faculty (n=1), graduate teaching associates (GTA) (n=2), and a random sample of students (n=15) selected for interviews across two 10-week quarters.

Data Collection. A cyclical process of questioning, observing, and hypothesis generating occurred throughout the study. Major data sources included weekly interviews, daily observations, field notes, and collected artifacts.

Results and Discussion

For successful reform, the text materials must change, the instructor must change, and the assessment must change. We will look at each of these aspects of the Survey of Calculus course and describe the changes in each.

The Hughes-Hallett/ Gleason text was selected because it is quite different from traditional texts.[1] The essence of the change is captured in the following quote from the preface of the text:

- At every stage, this book emphasizes the meaning (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect You will often be asked to explain your ideas in words or to explain an answer using graphs...
- There are few examples in the text that are exactly like the homework problems, so homework problems can't be done by searching for similar-looking "worked-out" examples. Success with the homework will come by grappling with the ideas of calculus.
- Many problems in the book are open-ended. This means that there is more than one correct approach and more than one correct solution...
- This book assumes that you have access to a calculator or computer that can graph functions, find (approximate) roots of equations, and compute integrals numerically. There are many situations where you may not be able to find an exact solution to a problem, but can use a calculator or computer to get a reasonable approximation. An answer obtained this way is usually just as useful as an exact one. However, the problem does not always state that a calculator is required, so use your own judgment...
- This book attempts to give equal weight to three methods for describing functions: graphical (a picture), numerical (a table of values) and algebraic (a formula). Sometimes it's easier to translate a problem given in one form into another.... It is important to be flexible about your approach: if one way of looking at a problem doesn't work, try another. (Hughes-Hallett, Gleason, et al., 1994, p. xiii)

One of the features most appealing for the liberal arts audience is given in the last point: the text makes strong use of graphical and numerical representations and (in the portion of the book used in the course) downplays the importance of formulas. For an audience whose algebraic skills are modest, this emphasis was highly beneficial, allowing them to examine concepts without being required to carry out extensive algebraic computations.

Another important feature of the text are the examples and problems that require thoughtful application of the concepts. Below is a problem from the text to illustrate the type of thinking students were asked to do (Hughes-Hallett, Gleason, et al., 1994, p. 34).

> Values of three functions are contained in Table 1.16 (The numbers have been rounded to two decimal places.) Two are power functions and one is an exponential. One of the power functions is a quadratic and one is a cubic. Which one is exponential? Which one is quadratic? Which one is cubic?

x	f(x)	x	g(x)	x	k(x)
8.4	5.93	5.0	3.12	0.6	3.24
9.0	7.29	5.5	3.74	1.0	9.01
9.6	8.85	6.0	4.49	1.4	17.66
10.2	10.61	6.5	5.39	1.8	29.19
10.8	12.60	7.0	6.47	2.2	43.61
11.4	14.82	7.5	7.76	2.6	60.91

Another example, taken from the chapter on differentiation, illustrates how a traditional topic can be treated in a new and intriguing manner.(Hughes-Hallett, Gleason, et al., 1994, p. 128)

> Table 2.13 shows the number of abortions per year, A, performed in the US in year t (as reported to the Center for Disease Control and Prevention). Suppose these data points lie on a smooth curve A=f(t).

Table 2.13
Abortions reported in the US (1972-1985)

Year, t	1972	1976	1980	1985
Number of abortions reported, A	586,760	988,267	1,297,606	1,328,570

(a) Estimate dA/dt for the time intervals shown between 1972 and 1985.

(b) What can you say about the sign of d^2A/dt^2 during the period 1972-1985?

Typically the second derivative is introduced as an exercise in differentiating the first derivative of a function f(x) and is used to identify the concavity of the function, presumably useful in sketching the graph of f(x). Those who have done this know that points where the concavity changes are very difficult to identify on the graph. Graphing calculators make this use of the second derivative obsolete. In class discussion of this example, students were asked tox construct arguments both for and against legislation limiting access to abortion. The side arguing for limitations used the fact that the number of abortions was increasing (i.e., $dA/dt > 0$). The side arguing against limitations used the fact that the number of abortions was increasing at a decreasing rate (i.e., $d^2A/dt^2 < 0$). In another example illustrating the use of the second derivative, the authors quote a member of Congress who during the 1985 Defense Department budget hearings complained, "It's confusing to the American people to imply that Congress threatens national security with reductions when you're really talking about a reduction in the increase" (Hughes-Hallett, Gleason, et al., 1994, p.127). Examples of this sort lend credence to the argument that calculus should be a part of a liberal education.

Change on the part of the lecturer was motivated by several factors. The Hughes-Hallett / Gleason text provides abundant opportunities to examine and discuss problems that probe conceptual understanding and this encouraged the lecturer to increase time spent interacting with students (a non-trivial task in large lecture sections) in examining ideas. Another motivation for change came from the NCTM reforms that encourage the use of cooperative groups. Parts of the lecture hour were regularly used to put students in groups of three or four to work together on specially prepared problems that were turned in at the end of class. It was a humbling experience for the lecturer to observe how his beautifully prepared, carefully organized lecture presentation seemed to have made no impression on the students as they struggled to construct their own understanding while working on the problems. The ease with which a graphing calculator overhead unit could be used in class and the integration of technology into the text was a third motivation for change; increased use of technology during the lecture. A TI-82 graphing calculator was used in almost every lecture. An illustration of a place where the calculator was particularly effective is in the authors' introduction to the derivative in the form of a thought experiment in which a grapefruit is tossed up into the air and then falls back to the ground with its height at time t recorded in a table. Using the calculator, it was possible to reproduce the data for height and time in one second increments as a table in the calculator. From the initial table, the class was able to calculate average velocities over one second intervals about time t_0. The table increment was then changed to present the height and time in half second increments and average velocities about t_0 were again computed. The process was repeated one more time with a table having time increments of one tenth of a second. From the three sets of calculations of average velocities, the limiting value was clear to the students. This sort of demonstration is not feasible with blackboard and chalk; the time required to generate the table so dominates the process that the calculation becomes the important issue and the use of the data is obscured.

Evaluation strategy changed in several ways. First, written homework assignments from the text replaced one of the mid-term exams, making written problem solutions approximately 22% of the total marks for the class. Second, the format of exam problems changed; they were more open-ended and asked for explanation of the reasoning used to arrive at a solution. An example of such a problem is the following:

> You're home for a long weekend and your little sister, who is taking Advanced Placement Calculus in high school, tells you that she is failing because she doesn't have a clue as to what a

derivative is, or how to find a derivative or what the derivative does. Write a paragraph describing what you would do to help her learn calculus.

On a problem such as this, the expectation was that the student would mention something about the derivative as a way to measure the rate at which a function changes, would give an example in which different representations (numeric, graphical or symbolic) were used as a means of measuring rate of change, would give a description of the mathematical definition of derivative (along the lines of "the limiting values of the average rates of change over increasingly smaller and smaller intervals about a specified value of the variable"), and, perhaps, give an example of a problem in which the derivative would be used. As mentioned earlier, a third change was use of part of the lecture hour for problem solving by students working cooperatively in small groups. About once a week, during the final twenty-five minutes of the lecture hour, students were asked to work in groups of three or four. The problem was generally a topic from that day's lecture or something from the text that had been studied recently. The group was to come to consensus on a solution to the problem and to write a solution together. This work constituted less than 5% of the total marks for the class and was used as a bonus, added to mid-term scores.

The use of written homework assignments, the change in exam questions and the group problems in lecture all contain elements of reform efforts to develop students' facilities with written communication of mathematical ideas. The Harvard materials provide a rich variety of problems that require careful examination of concepts and the application of that understanding to new situations. It was important that students regarded this as a central part of their activity in the course, which meant including it as part of the grade.

What are the perceptions of the instructor about the global changes that occurred? The learning was different; the material was appropriate to students' needs and level of sophistication. Students came away from the course with different attitudes about mathematics. The use of the graphing calculator opens up new ways of understanding mathematics, new ways of representing mathematical objects and aids students in their studies of mathematics. The use of cooperative groups is an important technique in promoting student involvement in learning. Increased use of writing in mathematics is critical to students learning conceptually rather than mechanically.

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