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ABSTRACT

The main focus of the study was to describe how students of various ages established the truth of their ideas in school geometry. Thirty-two students in grades 2, 5, 7, and in a high school geometry class were interviewed. Formal proof was used in less than 1% of the student arguments. Second and 5th graders were most likely to convince themselves or others by using a basic image process or by drawing pictures. High school students and 7th graders were more likely to convince themselves and others using an Intuitive Affirmation. In addition, their arguments were more elaborate and propositional in nature than arguments given by 2nd and 5th graders. (Author/MKR)

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How Students Establish the Truth of Their Ideas in School Geometry

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HOW STUDENTS ESTABLISH THE TRUTH OF THEIR IDEAS IN SCHOOL GEOMETRY

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The main focus of the study was to describe how students of various ages established the truth of their ideas in school geometry. Thirty-two students in grades 2, 5, 7, and in a high school geometry class were interviewed. The study found that formal proof was used in less than 1% of the student arguments. Second and 5th graders were most likely to convince themselves or others by using a basic image process or by drawing pictures. High school students and 7th graders were more likely to convince themselves and others using an Intuitive Affirmation. In addition, their arguments were more elaborate and propositional in nature than those arguments given by 2nd and 5th graders.

The NCTM *Curriculum and Evaluation Standards* subscribe to a constructivist view of mathematics learning and teaching in which students learn mathematics meaningfully as they personally construct mental structures and operations that enable them to deal with problematic situations, organize their ideas about the world, and make sense of their interactions with others (National Council of Teachers of Mathematics (NCTM), 1989). The constructive process occurs as students reflect and make sense of their interactions with the world and their peers. In this new view of mathematics learning and teaching, primary responsibility for establishing the truth of mathematical ideas lies with students. Teachers and textbooks are no longer viewed as the providers of mathematical truth. In essence, each student is seen as a mathematician, somebody who is responsible for solving mathematical problems by making conjectures and establishing the validity of those conjectures within the classroom culture. As we place such a heavy responsibility on students, it behooves us to know how they cope with it. We know how mathematicians formally establish the truth of conjectures—they use proofs. But how do students do it? How does the notion of justification evolve in students? Furthermore, if we want to help students learn increasingly sophisticated ways of justifying their mathematical conjectures, we must understand their current ways of justifying ideas.

Procedures

Age and relevant knowledge have been found to be important variables in much of the research on reasoning. Eight second graders, ten fifth graders, eight seventh graders, and six high school students, were randomly selected from a pool of volunteers in two similar school systems. Approximately half of those selected at each grade level were females. To assure that students selected had an adequate knowledge of mathematics, only students with standardized mathematics test scores above the fiftieth percentile were included.

A set of nine problems involving concepts in geometry was selected from a variety of sources so that the problems could be easily understood by students at all age levels in the study (i.e., the problem did not require much formal geometry knowledge or terminology to understand). For example, problem number 7 gave the students a figure of a triangle inscribed in a circle and asked "Is it true that for every triangle that there is a circle that passes through each of the vertices (the three points) of the triangle?"

After solving each problem, students were asked two follow-up questions. The first question was "What makes you think that your answer is correct?" The second was "When I gave this problem to some other students, some of them gave answers different from the one you gave me. If each one of you that gave me a different answer has a chance to convince a group of students (in the same grade) that your own idea is correct, how would you get the others students to believe you?" A tenth interview item explicitly asked students how they establish truth of their ideas in mathematics.

All of the interviews were conducted by the author. The interviews were audio taped, then later transcribed. During the interviews the interviewer wrote on the interview form as the answers were given. In addition to student verbal responses, the interviewer kept track of student drawings used in conjunction with verbal responses.

Coding of Student Justifications

After the students were interviewed and tapes were transcribed, student responses were analyzed. An initial set of the primary components was created from both relevant research and student responses in this study. The first result of the study was an elaboration of the set of primary components in order to accurately describe important lines of reasoning used by the students. Coding began with two people using the set of primary components, each coding a sample of student arguments. These codings were compared and disagreements were discussed. Seventeen primary components were identified and refined (see Table 1 for abbreviated list of primary components) in this study. When this process was completed, a third person, not involved in the development of the primary components, coded a random sample of student responses. The codings were compared with the refined codings, and a rate of 90% agreement was computed. After this process was complete, the primary components were used to complete a final coding of all student responses.

Findings from primary component analysis

Approximately 70% of component usage was accounted for by four components: Draws or Proposes to Draw a diagram (DM), Intuitive Affirmation (IA), Basic Image Process (IS), and Statement of Fact (SF). Furthermore, formal proof (UP) accounted for less than 1% of the 896 arguments given (see Figure 1).

Primary component preference differed somewhat by grade level. The use of Intuitive Affirmation (IA) was greatest among high school students, and the use of both Basic Image Process (IS) and Draws or Proposes to Draw (DM) were most popular with second and fifth graders. The findings also indicated an increased use of Statement of Fact (SF) as grade level increased. Table 2 on the next page

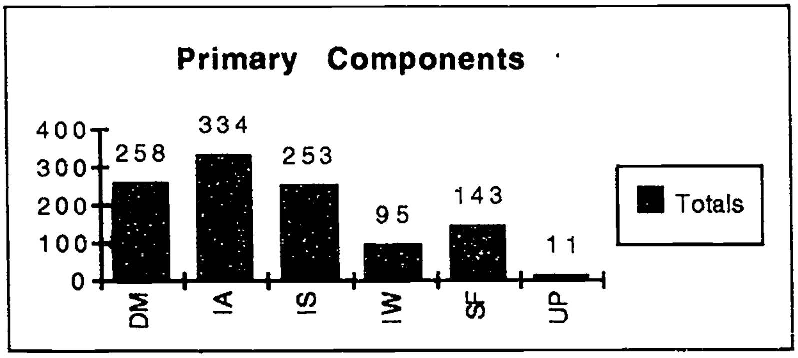

Figure 1. Most frequent primary components used in student responses

displays the percent of use of each component over all student responses by grade level. That is 23% for second grade indicates that of all components used by second graders, 23% of them were Basic Image processes (IS).

Almost 80% of students' argument chains (the string of primary components used in an entire student response to a question) were one or two components in length, with the most popular being the singleton chains Intuitive Affirmation (IA) and Basic Image Process (IS). While brevity is a respected quality of mathemati-

Table 1. *Primary Components; an abbreviated list*

Code	Name	Description of Primary component
DM	Draws, proposes to draw	The student draws or proposes to draw objects or does some kind of manipulation (folding, cutting, or the like) to support his or her argument.
IA	Intuituve affirmation	The student makes a statement that he or she accepts as certain and self evident. No validation of the statement is attempted.
IS	Basic image process	(with direct support of diagrams) - The student draws inference from generating, transforming, or inspecting pictures he or she creates or finds.
IW	Advanced image process	The student draws an inference by generating, transforming, or inspecting images without drawings or existing diagrams.
SF	Statement of (or appeal to) fact	The student refers to facts, definitions, or formulas that he or she assumes is common knowledge.
UP	Uses (or proposes to use) proof	The student presents a set of statements each being justified by theorem, axiom, or definition. In proposing a proof, the student presents an outline or explicit direction of how a proof would be constructed.

Table 2. *Most frequently used Primary Components used by grade level*

Grade	Primary Components				
	DM	**IA**	**IS**	**IW**	**SF**
Second	18%	21%	23%	9%	7%
Fifth	22%	18%	21%	7%	7%
Seventh	17%	23%	17%	5%	11%
High School	12%	32%	7%	4%	15%

Table 3. *Number of incorrect components and total components used in incorrect chains*

	DM	IA	IS	IW	SF	UP
Total incorrect components used	1	45	91	35	11	0
Total Components used	122	81	127	41	44	0

cal proof, it is so only when such arguments are sufficiently rigorous. The student chains did not in most cases constitute acceptable rigor. Thus, in this study, the brevity of the arguments seemed to indicate a weakness rather than a strength.

Errors in student arguments

In addition to coding student arguments, each primary component and the argument as a whole were judged to be correct or incorrect. For example a student may have used a Statement of Fact as part of their argument, but stated the fact incorrectly. If this incorrect fact caused the argument as a whole to be incomplete or incorrect, the whole argument was coded incorrect. This data was used to determine what types of errors caused arguments to be faulty.

Fifty percent of all the arguments given by students were incorrect or incomplete. The cause of many of the errors in student arguments was the incorrect or insufficient use of imagery. The most common error that students who were using visual thinking made is that they failed to utilize appropriate propositional knowledge to constrain their thinking or recognize its possible inadequacies. Conclusions from this study indicate that imagery is productively used in problem solving when it is guided and constrained by appropriate propositional knowledge. There is evidence in this study to suggest that as students move up through the van Hiele levels, they don't necessarily stop using visual reasoning. Instead, their visual reasoning becomes more sophisticated, incorporating into it increasingly more sophisticated propositionally stored knowledge. Their visual thinking is different at different van Hiele levels because, at each level, it is constrained by totally different knowledge structures.

Conclusions

I believe this study provides essential information for guiding instructional strategies aimed at promoting and refining students' geometric reasoning because if we wish to help students refine how they reason, we must first understand their current methods of reasoning.

The preference for more verbal rather than visual arguments for the high school geometry students and seventh graders seems consistent with the notion that they were thinking about geometric ideas at van Hiele's second or property-based level. The preference of younger students for visual arguments suggests that the students in the second and fifth grade were thinking about their ideas at van Hiele's first or visual level. Students at the visual level believe in geometric ideas because they "just see it" (van Hiele, 1986) or because of visual transformations (Battista, 1994). Students at higher levels reason based on more elaborate, property-based knowledge. Thus, in addition to visually stored knowledge, students at the higher levels have propositionally stored, property-based knowledge that can be included in their arguments.

The frequent uses of Intuitive Affirmations and Basic Image Processes and the errors in arguments caused by these primary components in this study suggest the need to present students with situations which help them to build and coordinate the use of propositional knowledge with visual knowledge. I believe that exploring geometry using manipulatives or computers, creating conjectures, and then arguing about those conjectures with classmates is essential in helping students developing these mental processes. This study also suggests that having students try to convince others of their mathematical ideas not only forces them to reflect on their ideas, but to elaborate these ideas, making them more mathematically explicit. This finding thus supports constructivist notions of the value of class discussions in which students must argue and support their mathematical ideas. That is, students must be given opportunities to create mathematical ideas, and most importantly, to decide for themselves if these ideas are mathematically sound.

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