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ABSTRACT

This paper is a reaction to a plenary address, "A Research Base Supporting Long Term Algebra Reform?" by James Kaput (SE 057 182). The reactions fall into three categories: comments on Kaput's dimensions of algebra reform, a brief discussion of algebra and algebra reform from the viewpoint of a curriculum developer of the Connected Mathematics Project (CMP), and some concerns about Kaput's three stages of reform. (MKR)

A Response to A Research Base Supporting Long-Term Algebra Reform

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A RESPONSE TO A RESEARCH BASE SUPPORTING LONG-TERM ALGEBRA REFORM

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My reactions to Kaput's paper fall into three categories: comments on the Dimensions of algebra reform, a brief discussion of the algebra reform from the view point of a curriculum developer, and finally, some concerns about the three stages of reform.

Some Musings on the Dimensions of Algebra Reform

Kaput offers three dimensions in which to measure change in algebra reform: Breadth, Integration, and Pedagogy. In the discussion of Breadth he describes five aspects of algebra. He claims that the first two aspects, *Algebra as Generalizing and Formalizing Patterns & Constraints, especially, but not exclusively Algebra as Generalized Arithmetic Reasoning and Algebra as Generalized Quantitative Reasoning*, (1), and *Algebra as Syntactically-Guided Manipulation of Formalisms*, (2); give rise to all the others—*Algebra as the Study of Structures Abstracted from Computations and Relations*, (3); *Algebra as the Study of Functions, Relations, and Joint Variation*, (4); *Algebra as a Cluster of Modeling Languages and Phenomena-Controlling Languages*, (5).

There has been a great deal of effort and time devoted to categorizing, describing, or defining school algebra. Kaput's five aspects of algebra are yet another, but not dissimilar, cut on school algebra. Most recently, the NCTM Algebra Working Group (NCTM, in prparation) wrestled with these same questions of school algebra and settled on four themes, *Functions and Relations*, *Modeling*, *Structure*, and *Language and Representations*, around which to organize discussions of "algebra for all" in the K–12 curriculum.

Recent discussions of reform in school algebra have tended to broaden the view of school algebra, which has caused some lively reactions. Some people have argued that function, which is common to both Kaput's and the NCTM Algebra Working Group's descriptions of school algebra, is not algebra, but analysis. Many view school algebra as being closely related to abstract algebra at the college level. For example, at the Algebra Initiative Conference (Lacampagne, Blair & Kaput, 1995) over 60 mathematics educators and mathematicians met for three days to discuss algebra in the K–16 curriculum. Most of the research mathematicians present were algebraists. If school algebra is to be categorized as a study of functions, then shouldn't research analysts be involved with discussions of school algebra? The study of functions was usually allocated to a course called precalculus or analysis—a course for mathematics and science majors. Functions is a recent addition to the school algebra curriculum, in part due to the accessibility and implementation of graphing utilities into the curriculum.

Some people have also argued that modeling cuts across all areas of mathematics as does structure. The NCTM Working Group also proposed that the organizing themes could be developed by studying important ideas in change and growth

(analysis), size and shape (geometry), uncertainty (probability), number, data, etc. This led one reviewer to question, "How does algebra differ from the other content areas in mathematics?"

Is there a danger that school algebra is becoming too broad? For whom are these categories helpful? It is important for teachers, curriculum developers and mathematics educators to have a working definition of algebra—even if it is very broad and encompassing. Teachers need to have a sense of the "big picture" of algebra to help them make decisions about the curriculum and student's understandings. Curriculum developers need a vision of algebra to develop a coherent and balanced curriculum. Researchers need a framework around which to organize their research. What view does the general public have of algebra? According to Wheeler (1991, as reported in Romberg & Spence, 1995), "proponents of the current reform movement argue for a particular perspective that is different from that held by diverse individuals, including the perspective of many (if not most) working mathematicians." Romberg & Spence (1995) claim that the current perspectives about algebra from an absolutist perspective are about mathematics in general. Does it make a difference if we all select different themes, strands, or definitions to guide our thinking? Do all roads lead to Rome?

Is there a simpler answer that could help guide these discussions and reform efforts? Romberg & Spence (1995) claims, "For students, algebra should be a way to express real-world phenomena in mathematical language. Their experience of algebra should include many and varied problems from the real world so they will gain understanding of the power and usefulness of algebraic notations and conventions." Romberg & Spence's (1995) claims together with Kaput's strands 1, 2, and 5 suggest that language and representation for expressing generalization and formalization of mathematical ideas could be a main organizing theme or strand of algebra. Kaput devotes a large part of his paper discussing "language and representations" and "generalizations and representations."

If language and representations are the organizing theme of school algebra, then the focus of algebra reform shifts to "why one needs a mathematical language," "which language," and "how one learns the language." This theme would allow a rich and dynamic language (symbols, graphs, tables, pictures, computer languages, simulations, etc.) to develop as students study engaging problems. In turn, the problems would lead to the development of powerful reasoning strategies and understanding of important mathematical ideas in arithmetic, analysis, geometry, statistics, probability, or even abstract algebra. The language becomes the means to represent the ideas and reasoning—or the means to represent the "generalizations and formalizations." With this categorization of algebra, functions and structure are still important—perhaps, more so. The important ideas of functions and structure can emerge on their own. Mathematics education researchers whose interests are the development of functions or structure would continue their research under the umbrella of analysis or abstract algebra (or just functions and structure).

It is the development of a mathematical language that is both brief and general to encode mathematical ideas and reasoning that has been a cornerstone in the

development of mathematics. However, this suggestion of school algebra as a language for generalizing and formalizing mathematical ideas is not new and it too will cause controversy—partly because this has been the perceived dominant theme of traditional school algebra. It may be too close to a "drill and kill" curriculum. If "language and representations" and "generalizations and formalizations" are the dominant themes of school algebra, the emphasis should not be on a single course devoted to practicing isolated unrelated skills. Instead, language should be developed along with the mathematical ideas of function, geometry, data, and so on.

While all of Kaput's strands, as well as those offered by the NCTM Algebra Working Group and others, are all important ideas in mathematics, are they school algebra? Are they too broad? Is there a broad view for mathematics educators interested in algebra and another for the general public? The suggestion of "language and representation" and "generalizations and formalizations" as organizing strands for school algebra is offered as a middle ground for the various interpretations of algebra.

Perspectives of Algebra and Algebra Reform from a Curriculum Developer

This section contains a brief description of a curriculum project and the implications of this project for research in school algebra.

Description of CMP

The Connected Mathematics Project (CMP) is a middle school curriculum project funded by the National Science Foundation (Lappan, et al., 1995) that is being developed at Michigan State University (W. Fitzgerald, G. Lappan, and E. Phillips) together in conjunction with the University of Maryland (J. Fey) and the University of North Carolina (S. Friel). The developers of the CMP curriculum believe that observations of patterns and relationships lie at the heart of acquiring deep understanding in mathematics. Therefore, the CMP curriculum is organized around interesting problem settings—real situations, whimsical situations, or interesting mathematical situations. Students solve problems and in so doing they observe patterns, and relationships; they conjecture, test, discuss, verbalize, and generalize these patterns and relationships. The mathematical strands of number, measurement, geometry, probability and statistics, and algebra are developed across the middle grades.

Algebra in the CMP Curriculum

If mathematical concepts are developed from a problem situation or context, then the variables in the situation and how they are related become ideas that permeate all the units. Thus "generalizing and representing" these relationships is part of all the CMP units, including those units designated as algebra. For example, in an early two-dimensional measurement unit a sequence of activities leads to a generalization of a strategy for finding the area of a circle. One of problems in the sequence has students investigating which measures are most closely related to

the price change in pizzas—circumference or area or radius or diameter. This problem seeks a relationship between the measures of a circle and the cost of a pizza. Eventually the sequence of activities ends in a generalization about the area of a circle given its radius. In a geometry unit on two dimensional shapes, students investigate the relationship between the angle measure (or the number of sides) on the shape of a plane figure. In the data units students decide which variables and which relationships to investigate, and how to represent these relations. When mathematics flows from the study of problems or contexts, then variables and the manner in which they are related, naturally arise. Furthermore, in such situations there may be more than two variables, and students must decide which variables to study and then discuss possible effects of the other variables.

While variables and patterns are part of each unit, they come to the foreground in a unit called *Variables and Patterns.* The focus is on looking at a variety of situations and more formal ways to represent these situations. Pictures, words, tables and graphs together with some algebraic symbolic representations are studied. Moving freely among the representations takes time to develop and hence is also an important part of all the units. Three other units, *Moving Straight Ahead* (linear functions), *Growing, Growing...* (exponential functions), and *Launching Rockets and Leaping Frogs* (quadratic functions), investigate patterns of regularity among the rate of change between the variables. It is the concept of "rate of change" that helps students identify, represent and reason about linear, exponential, or quadratic functions. Students do some work with symbols, which are temporally free of context, for the purpose of investigating the general characteristics of a specific function. While symbols are used to represent these situations, along with other representations, symbol manipulation is not the focus of the units—modeling and functions are the foci. Another unit, *Say it With Symbols*, looks more closely at ways to represent problem situations, symbolically—particularly those that give rise to different, but equivalent expressions— while another unit, *Thinking with Mathematical Models,* looks more closely at modeling.

Research in the Connected Mathematics Project

Of utmost concern to the CMP curriculum developers are what students will be able to do and know at the end of three years. The research component of CMP consists of several stages: videotapes, student work, interviews, teacher and student surveys, and observations have been conducted throughout the project, primarily to guide the development of the teacher and student materials. To assess students' understanding and reasoning, pretesting and posttesting of both CMP and non-CMP 6th, 7th, and 8th grade classes are currently going on. The tests consist of a pretest and posttest using both the Iowa Test of Basic Skills and a test designed by an outside evaluator that reflects the recommendations of the *NCTM Standards* and three authentic assessment tasks from the Balanced Assessment Project (Schoenfeld 1995) administered at various times during the year. The Iowa Test of Basic Skills was strongly promoted by the CMP Board. They felt that the public, regardless of any other evidence, needed to be convinced that students

participating in the program would not do worse on tests of basic skills. Otherwise, these new curriculums may vanish on the vine (see further comments under the following section on the Three Stages of Reform). Since the set of units designated for 7th grade in CMP has a strong proportional reasoning theme running throughout the units, the principal investigators are also conducting research on students proportional reasoning abilities at the end of 7th grade.

The *NCTM Standards* based test does not give a complete picture of students' knowledge or reasoning. It does begin to paint a picture of students' reasoning and problem solving abilities as well as their ability to make connections and communicate. The research described above falls into Kaput's short or intermediate stages of reform. During the long-term phase of reform some of the questions that need to be addressed are:

- Does the CMP curriculum give students more power to solve morecomplex problems?
- Is it possible to build a complete mathematics program based on explorations of interesting problems? What are the strengths and weaknesses? What misconceptions might arise from these curriculum reform efforts?
- How long must students be engaged with an important mathematical idea so that the student carries understanding of the idea into the next grade or level? How many years must a student be involved with a "reformed" classroom to reap the benefits? What are the implications for students who go from a reform based curriculum at one level to a standard curriculum at the next level?
- What transitions, and over what period of time, do students need to make connections? What kind of transfer activities do students need to move from a problem based setting to a symbolic based setting or to otherrepresentational schemes?
- What algebraic reasoning do students develop? What knowledge of algebra (or any other area of mathematics) do they carry over into high school? Does the CMP curriculum provide deeper insights into algebraic forms?
- Will the possible loss of personal manipulative skills be a longer term stumbling block to mathematics development? (This question is perhaps more important as students move to the high school.) How much symbolic skill (arithmetic and algebraic) is necessary for students to model a situation or to manipulate an expression to reveal new information about a situation? (A similar question could be asked about other representational schemes.)
- How much help do teachers need to implement a new curriculum that requires a different view of mathematics and a different peda-

gogy? What kind of support and at what levels do teachers need this support? What mathematics do teachers learn by teaching these new curricula?

- What kind of linkages among teaching, learning, and assessment do these curriculum projects provide?

The CMP curriculum, as well as other curriculum projects, are based on the best available research from mathematics education and the cognitive sciences. However, none of the research to date has been conducted in settings where students have been engaged in significant mathematics in classrooms and have developed their understandings and reasonings as a community of scholars over several years. These new rich curriculum projects provide a unique opportunity to carry on significant research over a long period of time that has not been available since the new math—research on teaching, learning, and assessment. Kaput suggests that the long-term research efforts should begin in the elementary grades. There is enormous potential for developing students' mathematical power with these new curriculum projects in Grades 6–12. Algebra reform should begin on several fronts.

Comments on Kaput's Three Stages of Reform

Kaput's discussion of the three stages of reform, on the surface, appear reasonable. It makes sense to tinker with short-term reform—these efforts could also inform the long-term efforts at reform. Much of the reform in algebra that has been going on since the release of the *NCTM Standards* has been short-term reform. Most of these reform efforts have been "add ons," such as the use of graphing calculators, computer software, or manipulatives, to the existing curriculum.

There is a danger to these short-term efforts. First, most of these efforts ignore the weaknesses and deficiencies of the present curriculum. There is at the same time a tendency to be a bit caviler about the benefits of graphing software packages. To use graphing software utility effectively requires a deeper understanding of functions and relations than is currently in the curriculum. Many students do not understand functions and consequently mimic the procedures needed to use a graphing calculator without understanding what they are doing. There is a danger that we could be replacing abstract symbol manipulation with equally abstract algorithmic techniques on how to use the graphing calculator (or computer). Furthermore, some people interpret statements like, "with graphing calculators and symbolic algebras there is little need for work with symbol manipulations" as meaning no need for skill development. Putting graphing calculators into the hands of students requires very careful reorganization and re-conceptualization of the algebra curriculum and research to support these long-term efforts. How effective these technologies are will have to wait for Kaput's long-term stage of reform.

However, short-term efforts may torpedo future reform efforts. For example, the implementation of graphing calculators into the college algebra courses at one Big Ten university was perceived as weakening the algebraic skills of students going into the calculus class and has since been forbidden at this university. At the

heart of this conflict about algebra reform is the role of symbol manipulation. The general public and many mathematicians perceive the mastery of symbolic manipulations as an important part to learning algebra. Students need to model situations using a variety of representations, including algebraic symbols. They need to show that different expressions for the same situation are equivalent. Students need to transform equations or expressions into equivalent forms that can be entered on a computer or graphing calculator. Further reasoning with equivalent symbolic forms can very often reveal information that is not apparent in graphs or tables. Mindless drill and practice has not worked. But what is the role of symbol manipulation? How much understanding and skill with symbolic transformations or manipulations are needed to reason effectively with symbols? These questions need careful research to convince the general public about the needs and benefits of reform in algebra.

K–12 teachers are key players in the algebra reform movement and what they do or do not do is closely tied to public and university approval. Any efforts at reform must help teachers understand the proposed changes. If teachers are convinced that such efforts will lead to greater understanding and reasoning for their students, they will support the reform activities. However, even these teachers ask for help to convince administrators and parents that these changes will not be harmful—and that these changes will help.

Without some fundamental changes that Kaput describes in *Breadth, Integration, and Pedagogy*, these short-term reform efforts will have little effect and in fact may offer a real roadblock to the needed long-term reform. The general public also has a short attention span—there is a tendency in this country for quick solutions. Will they have the patience to withstand the efforts needed to implement long-term reform? The backlash has already begun. The Michigan State Board of Education is recommending that the *Standards* as described in their State Frameworks not be mandatory. In addition, the State Board inserted stronger standards on skills in many of the subject areas, deleted some that appeared to imply value judgment, and only narrowly voted down a proposal to make the teaching of the creation science mandatory for all students. Similar backlashes are occurring in other states. The strengths of short-term reform efforts must be advertised with the promise to look at both the gains and losses of such efforts.

So my questions are: (1) Do the relative benefits of short-term reform out weigh the more dangerous backlash that occurs when the public perceives these efforts as detrimental to student learning (even when there has been no real decline of skills)? (2) If the short-term reform is seen as a failure, what does this say about the intermediate and long-term reform? (3) Will we have time to carry out the reform? (4) What can we as a community of mathematics educators do to provide the time and opportunity for reform to progress?

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