People find difficulty in performing syllogistic reasoning. This paper outlines some reasons for poor syllogistic reasoning. Representational systems (RS) used in presenting and evaluating syllogisms can be distinguished in terms of attributes such as specificity, expressiveness, and abstraction. Graphical RSs have pedagogical potential because of their expressivity and specificity attributes. The paper looks at four different graphical RSs for supporting syllogistic reasoning: Venn diagrams, Euler circles, Stennings Euler circles, and Tarski's World (TW). Each is described in terms of those distinguishing attributes. Claims are made about which should generate better learning outcomes as a result of different degrees of these attributes. Empirical work is underway which involves the construction of tasks which help learners to see the relationship between a diagram and the syllogism. The study will involve the pre- and post-testing of 18-year old school pupils on premises and conclusions from 10 selected prototypical syllogisms with non-linguistically biased predicates. Four figures illustrate the representational systems. (Contains 16 references.)

(Author/MAS)
Towards an analysis of visual media in learning:  
A study in improving syllogistic reasoning.

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Abstract: People find difficulty in performing syllogistic reasoning. This paper outlines some reasons for poor syllogistic reasoning. Representational systems (RSs) used in presenting and evaluating syllogisms can be distinguished in terms of attributes such as specificity, expressivity and abstraction. Graphical RSs have pedagogical potential because of their expressivity and specificity attributes. The paper looks at different graphical RSs for supporting syllogistic reasoning and describes them in terms of those distinguishing attributes. Secondly it makes claims about which should generate better learning outcomes as a result of the different degrees of these attributes. The talk will report findings of an empirical study relating outcomes from the use of these systems to the systems attributes.

1. Syllogistic Reasoning

Syllogistic reasoning, is the purest form of deduction, has been the study of cognitive scientists and philosophers for more than two thousand years since Aristotle [Aris36] and turns out, in certain cases, to be very difficult for human problem solvers to perform. A syllogism is made up of two premises that each contain a single quantifier, and a conclusion which relates the two end terms;

Some philosophers are polymaths  
All polymaths are intelligent  
Therefore, some philosophers are intelligent

Premises and conclusions of syllogisms can be in one of four distinct moods:

<table>
<thead>
<tr>
<th>Premises</th>
<th>Mood</th>
</tr>
</thead>
<tbody>
<tr>
<td>All A are B</td>
<td>affirmative universal (A)</td>
</tr>
<tr>
<td>Some A are B</td>
<td>affirmative existential (I)</td>
</tr>
<tr>
<td>No A are B</td>
<td>negative universal (E)</td>
</tr>
<tr>
<td>Some A are not B</td>
<td>negative existential (O)</td>
</tr>
</tbody>
</table>

There are four possible figures, which the premises of a syllogism can be written in. In each case, A and C are the end terms, (occurring in conclusions), and B is the middle term (occurring in both the premises). These figures are as follows;

<table>
<thead>
<tr>
<th>Premises</th>
<th>Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - B</td>
<td>B - A</td>
</tr>
<tr>
<td>B - C</td>
<td>C - B</td>
</tr>
</tbody>
</table>

The first two are asymmetrical because the middle term is located in different places in the two premises, and the second two are symmetrical because the middle term is located in the same position in both premises. Each premise can be in one of the four moods (A,E,I,O), and therefore there are 64 distinct forms in the premises. In fact according to more modern views of syllogistic reasoning there are 512 possible syllogisms, because, each pair of premises can be combined with eight possible conclusions, (four conclusions in the form A - C, and four of order C - A). This however does not alter the fact that...
only 27 of these pairs are valid. Some combinations are very simple and others, people get no better than chance results of success.

1.1. Aspects of Human Performance in Syllogistic Reasoning

People find syllogistic reasoning hard, what is still a mystery is why people find some syllogisms so hard. There are a number of hypotheses to suggest an answer to this question and they fall under the broad categories outlined below.

"A theory of syllogistic performance should at the very least account for the relative difficulty of different forms of syllogism for the figural response bias, and for the nature of the erroneous responses, including those of the type, "no valid conclusion" (interrelating the end terms). The atmosphere effect can only account for some errors and not for those of the form "no valid conclusion". It is not intended to deal with the relative difficulty or with the figural effect. The conversion theories certainly account for some errors and for some aspects of the relative difficulty of syllogisms, but they cannot explain either the figural bias or the erroneous "no valid conclusion" responses. The Euler circle theories explain some aspects of relative difficulty, but not the figural bias" [Johns92].

The following sections show an overview of relevant theoretical positions with respect to these difficulties in syllogistic reasoning, it is not intended to be an exhaustive summary of the area, but an introduction to current and recent thinking in the area.

The Atmosphere Hypothesis: Woodworth and Sells [Wood35], connoted the term "atmosphere effect" which predicts the errors in performing syllogistic reasoning to be the result of global impression produced by the premises rather than on the basis of strict logical deduction. Atmosphere is defined in terms of two dimensions, quality and quantity. The quality refers to the premise being either affirmative or negative, the quantity of the premise refers to whether the statement is universal or particular. The orthogonal pairings of values subsumed under quality and quantity yield the four moods above (A, I, E, O). The atmosphere effect can then be stated, firstly referring to quality, that: "whenever the quality of at least one premise is negative, the quality of the most frequently accepted conclusion will be negative, when neither premise is negative, the conclusion will be affirmative" and the second principle, referring to quantity, states that; "whenever the quality of at least one premise is particular, the quantity of the most frequently accepted response will be particular; when neither premise is particular, the conclusion will be universal".

Illogical Conversion: Chapman and Chapman [Chap59], have a view on the difficulty of syllogistic reasoning which they called illogical conversion, this is discussed in [News90]. The principle refers to the acceptance of the converse of either an A or an O statement when logically this is not permissible, and where the premise pair has a logically valid conclusion in another figure. This principle accounts for the main errors in AA, AE, AI, IA, AO and OA pairs.

The effects of belief on syllogistic reasoning: Possibly the most pervasive, and in some ways easily countered group of problems with reasoning go under the heading of belief related. There are three principle ways in which beliefs could affect reasoning: (1) They may distort the interpretation of the premises. (2) They may influence the deductive process, biasing which conclusions are reached. And (3) They could be used to filter out unacceptable conclusions that are produced by the deductive process.

2. Representation of Syllogistic Reasoning

There are a number of representational systems which can be used for the support of learning and solving syllogistic reasoning problems. The larger context of this work has involved analysis of many different systems, including Venn [Venn94], Johnson Ladirs mental models [John92], Euler circles [Eule72], Stennings adapted Euler cirles [Sten92(a)], Tarskis World [Barw90] and Lewis Carrols "Game of Logic" [Dodg96]. There is space here only to describe four of these systems.
2.1. Venn Diagrams

Venn diagrams can be used to represent three term quantified deduction (syllogistic reasoning). Three overlapping circles are placed within a square which counts as the universe of discourse. Each of the circles represents one of the terms used in the syllogism. The Venn diagram is used during the process of validating the syllogism by taking each premise and shading out the area where there is no possibility of there being a member within that region, and by marking with a cross those areas where it is claimed that a member exists. If a premise states that a member exists within an ambiguous area, i.e. one where there is two possible areas, then the cross is placed on the line between the two areas.

Figure 1 (a). Showing Venn diagram representation of the syllogism All B are A, Some B are C. Figure 1(b). Showing the partially specified individual Some As are Bs and either C or Not C.

In Figure 1, the syllogism 'All B are A, Some B are C' is shown using the Venn RS. The Venn RS is capable of substantially more abstraction than Euler Circles. Venn's system is usually used with a notation for expressing disjunctions (either a chain of 'x's or placing symbols on borders). This means it can express partially specified individuals (things which are A and B and either C or not C), and so clearly has less 'specificity' [Sten93]. Figure 2 demonstrates exactly this partially specified individual using the second of the conventions for expressing disjunctions.

2.2. Euler Circles

Euler circles can be used to represent syllogisms. However there are at least two variants on the Euler circle system. The more traditional system, (see e.g. [Cera71]), is described here. In the next section, a more novel interpretation of Euler due to [Sten92] is given. In this traditional system the initial premises are given as the relation between two circles either separated, overlapping one contained with the other, the second contained within the first, or one identified with the other. These relations are known as the Gergonne relations. One of the interesting results of Euler circle RS is that they can give some indication as to the complexity of the syllogism [Cera71], that is, the possible A - C, C - A combinations. It is possible that people when presented with written or spoken syllogisms, believe there is only one diagrammatic representation and therefore one model which relates the premises. If then, when they have constructed a diagram they are unable to immediately read off the conclusion, subjects will have no reason to construct further diagrams. On this theory, subjects will fail when there is more than one Euler representation of the premises and where a conclusion derivable from the first diagram was not derivable from further diagrams which where not constructed.
In Figure 2 we see the possible Euler representations for the syllogism - All B are A, Some B are C. The reader will note that in (2), the universal quantification is restricted to the B - A relation. (2) is also the diagram with the "maximal number of regions". With the added convention of some shading in the region where there is certainly some membership, this diagram approaches the adapted system of the next section.

2.3. Stennings Euler circles

Although Stenning [Sten93c] insists his system is a more reasonable interpretation of the Euler system than e.g. [Cera71], and indeed the author is inclined to agree, it is useful to separate the two. Conventionally, the four possible premises (AIE0), could be represented by 5 possible circle configurations, (A by 2, I by 4, E by 3 and O by 1). Stennings system with its added conventions for interpretation has an exactly one to one mapping for the premise configurations to diagrams. There are therefore four diagrams and four premises where one premise is represented by one diagram only. Figure 5. shows the Stenning representation of the syllogism All B are A, Some B are C, Some C are A.

The important features of Stennings system are in the representation of each of the premises and the systematic combination of the premise pairs. So in terms of the premise representation, looking at the second premise in figure 3 (Some B are C), where in the Gergonne relations there would have been four possible diagrams, there is now only one. The added convention of the shading, indicating the existence of a member of that region, now allows for a much more simple representation. The difference can also be demonstrated by a linguistic comparison. Whereas the traditional Euler circles may have allowed for Some As are Some Bs, Some and indeed All As are All Bs, Some As are All Bs and lastly Some but only Some As are Some and only some Bs, the Stenning model permits only the last of these. In terms therefore of specificity, Stennings system is far more specific, less abstract and equally expressive than its traditional counterpart. The traditional Euler system required that the solver created all possible combinations of all possible premise representations, and then to solve the syllogism the solver would have to propose a solution and check that it was consistent with all the resulting diagrams. The enormity of this task can be understood, when, in the case of the syllogism "Some A are B, Some B are C" 16 conclusion diagrams are generated. It is suspected that, although the Stenning systems is a much more simple system to understand and read conclusions from, the process of registration of the two premise diagrams is not so well adopted by the learner. This process is developed more fully in

Figure 3. Adapted Euler representation of the syllogism All B are A, Some B are C.
Rather simplistically, the process begins with development of characteristic diagrams of each premise, these are given. Registration is a more complex process, the goal being to combine the middle terms circle with as many regions as possible relating the end terms. When this is complete it remains to observe the shaded areas from the component premises, if any of these remain non-intersected, and no more than one premise was negative then there is a valid conclusion. The formulation of the conclusion depends on the quantification of this non intersected area.

2.4. Tarski’s World (TW)

Written by Jon Barwise and Jon Etchemendy of Indiana University and Stanford Universities respectively, TW is an interactive program designed to be used with the book “The Language of First Order Logic” [Barw90]. The program is described by [Gold92], and basically supports an interpreted first order logic for a first order language with equality. The program has three components, a world module, a sentence module and a keyboard module, each in separate windows. The world module displays certain objects, (cubes, tetrahedra and dodecahedra) and certain relationships e.g. Large(x), Larger(a,b) and Leftof(c,d). Although the system is not designed either to teach or support syllogistic reasoning, it is possible to use TW to display states of a world and look at syllogistic inference.

![Figure 4. Showing Tarski’s World Representation of blocks world.](image)

Materials have been developed which are to be used in association with TW, that help learners to see the relationship between the diagram and a syllogism. The important ideas which can be learned from TW are those associated with the contingency of truth upon the world. In TW where the world is very restricted this kind of message is quite easily presented. When a pair of premises are presented to a student, he may attempt to construct a conclusion. The relationship between the premise and the world is such that there are many worlds in which the premise may be true. If we say, for example, that \( \exists x (\text{Large}(x) \land \text{Tet}(x)) \), there may be one, two, three or however many large tetrahedra, there may be many cubes, no cubes, dodecahedron etc., in order for the premise to be true. The validity of a conclusion rests on it being impossible to generate a diagram in which the premises are true and the conclusion false. TW fills a further role, which is to show in a step-wise fashion, by the playing of the Hintikka game, the validity of any claim a student makes about the truth of a sentence.

3. Empirical Work

Empirical work is under way which will involve the construction of similar tasks which can all be performed using the systems outlined. The method will be to pre and posttest subjects with premises, and conclusions from ten selected prototypical syllogisms with non-linguistically biased predicates and
to ask subjects to validate conclusions. The test population will be sixth form 18 year old school pupils. The results will refer the learned outcomes to the attribute profiles of each of the representational systems as shown in table 1. (below). Item analysis will reveal the importance of each RS, and attributes of each system in terms the of the various groups of biases from the literature.

<table>
<thead>
<tr>
<th></th>
<th>TW</th>
<th>Euler</th>
<th>Sten</th>
<th>Venn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expressivity</td>
<td>High</td>
<td>Low</td>
<td>Lowest</td>
<td>Low</td>
</tr>
<tr>
<td>Specificity</td>
<td>High/Low</td>
<td>Med</td>
<td>Med</td>
<td>Med</td>
</tr>
<tr>
<td>Naturalness</td>
<td>Med/Low</td>
<td>Med</td>
<td>Med</td>
<td>Med</td>
</tr>
<tr>
<td>Abstraction</td>
<td>High/Low</td>
<td>Med</td>
<td>Med</td>
<td>Med</td>
</tr>
</tbody>
</table>

Table 1. Showing qualities of representational systems for solving Syllogistic problems

The results will provide a profile of the most critical factors of the RSs for learning. It is likely that this attribute profile will be transferable to more ill-formed domains, and this will be the subject of later follow-up studies.

5. Acknowledgments

I should like to thank Tim O'Shea (British Open University) for his invaluable help in the development of the ideas and methods included in this paper, Keith Stenning (HCRC, University of Edinburgh) for comments on drafts and Paul Lefrere (British Open University) for his encouragement. This work was enabled with the assistance of an Open University Research Award.

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