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ABSTRACT

This study investigated the nature of secondary school students' and teachers' conceptions of what mathematics is, the purposes of school mathematics, and the outcomes of school mathematics. Interviews were conducted with a sample of grades 10, 11, and 12 students (n=40), teachers (n=19), counselors (n=2), and administrators (n=2) from a large government secondary school and an independent girls' school in the metropolitan area of a large Australian city. Analysis of the interview data revealed many students and teachers held broad views of the content or discipline of mathematics, while their interpretations of mathematics within a wider sociocultural context reflected additional, influential, personally and socially derived factors. Four main interwoven and overlapping factors were identified: social status of mathematics, utility of mathematics, career aspirations, and interest or disinterest in mathematics. The existence of these factors indicates that what it means to 'understand' mathematics is related to both a context and an individual's interpretation of a context. Thus, findings indicate that what mathematics education researchers identify as relevant research problems might not be problems from the perspective of mathematics learners or practicing classroom teachers. Contains 25 references. (Author/MKR)

Secondary School Mathematics in Perspective: Conceptions of its Nature and Relevance

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This study investigated the nature of secondary school students' and teachers' conceptions of what mathematics is, the intentions of school mathematics, and the outcomes of school mathematics. Interviews were conducted with a sample of Grades 10, 11 and 12 students and teachers from a large government secondary school and an independent girls' school, in the metropolitan area of a large Australian city. Analysis of the interview data revealed many students and teachers held broad views of what is the content or discipline of mathematics, while their interpretations of mathematics within a wider sociocultural context reflected additional, influential personally and socially derived factors. Four main interwoven and overlapping factors were identified: social status of mathematics, utility of mathematics, career aspirations, and interest or disinterest in mathematics. The existence of these factors indicate that what it means to 'understand' mathematics is related to both a context and individuals' interpretations of a context. Thus, these findings indicate that what mathematics education researchers identify as relevant research problems might not be problems from the perspective of mathematics learners or practicing classroom teachers.

INTRODUCTION

Publication of the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) has provided direction for curriculum development in North America in recent years. This document was "intended to establish standards for classroom curriculum and instruction" and "was also meant to be a statement of policy for school mathematics curriculum" (Lester, 1994; p.661). More recently, in Australia, publication of *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) and preparation of *Mathematics - A Curriculum Profile for Australian Schools* (Australian Education Council, 1994) has moved the national agenda for mathematics education in the direction of outcomes based planning and assessment. Both the North American and Australian documents have elicited a number of related position and discussion papers, articles on teaching ideas, and research reports (for example, see recent volumes of *Mathematics Teacher, Arithmetic Teacher, Journal for Research in Mathematics Education*, or *Australian Mathematics Teacher*). Resultant discussion, debate and action within mathematics education communities has produced many questions and issues in relation to mathematics learning and the current state of 'knowledge' in mathematics education research (for example, see

Sierpinska, Kilpatrick, Balacheff, Howson, Sfard & Steinbring, 1993; Silver & Kilpatrick, 1994).

School-based research has indicated that learners' experiences of mathematics, both inside and outside the classroom, influence their attitudes to and outcomes of learning mathematics (Crawford, Gordon, Nicholas & Prosser, 1993; McLeod, 1992; Nunes, 1992; Resnick, 1987). In particular, there are indications that students' conceptions of mathematics affect the quality of related cognitive activities and learning outcomes. That is, how students interpret the context of their mathematical learning and hence how they relate to mathematics endeavours inside and outside school can influence mathematics performance (Crawford, 1990; Cobb, Yackel & Wood, 1992; Lave, 1988; Steffe & Cobb, 1988).

Recent studies in Western Australia have revealed a wide variety of conceptions, even amongst high ability students, about the nature and purpose of mathematics and its study (Frid & Malone, 1993; Miller, Malone & Kandl, 1992; White & Taylor, 1994). If, as is claimed by initiators and supporters of recent curriculum changes, high school mathematics is an important foundation for later learning at post-secondary institutions, and for employment, then educators need to better understand the individuality and diversity of students who are presently studying mathematics in high school. Although new high school mathematics curricula have been developed and implemented in Western Australia and elsewhere, there has been little attempt to examine student conceptions of the mathematics they are studying, their motivations for studying mathematics, or their approaches to learning mathematics. Since the cohort of students entering upper secondary school in Western Australia has expanded and changed in the last few years, and the demand for post-secondary education has increased, it is vital that neglected factors in examination of mathematics learning be considered. This study focuses on a neglected factor – the conceptions of the recipients of mathematics curricula. Thus, the main aim of this study is:

> *To investigate the nature of high school students' and teachers' conceptions of mathematics.*

Specifically, the study focuses on the following three research questions:

1. What do students and teachers think mathematics is?

2. What do students and teachers think are the intentions of mathematics study and why mathematics is included in school programs?

3. What do teachers think students think about mathematics and how do these views compare with those of students?

The importance of addressing these questions lies in the fact that until mathematics education researchers address and come to understand what mathematics is in others' eyes we are not in a position to understand the effects of mathematics courses. For example, we do not know what are students' personal views of the purpose of mathematics or if these views change over time. How does school have an effect on views? What school and outside school experiences and other factors affect these views? In what ways are curriculum developers, teachers, educational administrators, parents and the general public similar and different to students in what they understand mathematics to be? Do educators or does society ask or expect something from schools that schools are not in a position to deliver? These questions are only a sample of those that might be asked in relation to conceptions of secondary school mathematics.

Thus, by investigating students' and teachers' views, the outcomes of this study will put educators in a better position to recognise and thereby more comprehensively understand how mathematical experiences are interpreted within secondary schools. For example, until we know how students, as intended recipients of school practices, experience and interpret both the content and context of school programs, we are not in a valid position to evaluate the effects of recent trends and changes in mathematics curricula. Much research in mathematics education in relation to this issue has focussed upon designing new teaching methods or environments. The broader context of the nature of students' mathematics learning has not been examined as extensively. For example, there has been little questioning of the role of secondary schools for what they are preparing students, whether it be university or vocational studies, employment, or other endeavours. In addition, although it is documented that high school graduates do not 'know' the mathematics they are purported to know (for example, see Frid, 1992; Schoenfeld, 1992), the mathematics education literature has not adequately addressed the question of what it means to 'know' mathematics. This study will partially address these challenges.

METHOD AND DATA SOURCES

The study was interpretive in nature, and therefore, being concerned with the context of learning and related interpretations of experiences, used qualitative research methods. This approach is in line with current educational research practices as they shift away from purely quantitative, quasi-scientific experiments so that researchers can more explicitly document and analyse the experiences of teachers and learners in the broad encompassing social and academic complexities of classrooms (Moss, 1994). An inductive reasoning approach for data analysis was adopted (Glaser and Strauss, 1967; Powney and Watts, 1987).

Specifically, the study used in-depth, semi-structured interviews with students and teachers. The interview protocol asked the students and teachers to respond to a range of questions about their thoughts on: why mathematics is taught in schools, what is important about learning mathematics, what mathematics is and whether it is created or discovered, reasons why people choose particular mathematics courses, factors that facilitate mathematics learning, and specific experiences with mathematics learning or teaching. Interviews were recorded on audio tape and later transcribed.

Two high schools in the metropolitan region of a large Western Australia city were selected for the study: a large government school and an independent all female school. A total of 40 students, 19 teachers, 2 career councillors and two administrators (one principal and one deputy principal) were interviewed. Nine students were in Grade 12, sixteen were in Grade 11 and fifteen were in Grade 10. The sample of students was representative of high, middle and low achievement in mathematics, and at the government school was balanced in terms of gender. Approximately half the students were of multi-cultural background (generally Asian), in that they were immigrants or had parents who were immigrants to Australia. Some students were interviewed individually, while others were interviewed in groups of two or three. The reason for this difference was that the researchers found some students would speak more openly and comfortably with their peers than individually. Teachers were all interviewed individually. Approximately half the interviews were conducted in November and December 1993, near the end of a school year, while the others were conducted in the subsequent school year, in the middle of 1994 (the Australian school year runs from February to December). The differing times enabled reliability of the data to be established in relation to end of year examinations and their potential influences on attitudes and related conceptions of mathematics experiences.

RESULTS AND RELATED DISCUSSION

What is mathematics?

Students initially gave many 'expected' responses to the question: What do you think mathematics is? These responses were 'expected' in that they reflected views of mathematics held by students reported in other studies (for example, see Crawford et al., 1993; Chant and Galbraith, 1993), including views that mathematics is: (1) numbers, rules and formulae, and (2) a logical process or way of thinking. Many students also spoke of the applied nature of mathematics, and how numbers, rules or formulae, or logic and a way of thinking can be applied to solve problems. An additional conception of mathematics as a connected hierarchy that studies relationships or patterns was spoken of by some students, and these were generally students achieving at average or above average levels.

The question that elicited further insight into students' conceptions of mathematics was: Where did mathematics come from and what is it for? Many students responded to this question in a way that pointed to a view of mathematics as a sort of technology, that is, a human endeavour for addressing human needs and solving human problems. These needs might be due to a desire to understand one's environment, or they might arise from wanting to accomplish a specific task such as determining the amount of wallpaper to paper a room. Some examples of the sorts of responses students gave in this regard are: (I is the Interviewer)

> (Two grade 11 girls)
> Brenda: It's based on our curiosity understand and explain things.
> Lesley: . . . It just came from a need to understand things.

> (Grade 11 girl)
> Tina: I think they created it as a way of explaining relationships, like the creation story is just a way of explaining how we got here. To say how it happened. I think they created a way in which we study relationships. . . . The relationships were there and they had to create a method of studying them.

> (Two grade 10 girls)
> Anna: I think, sort of people trying to logically understand everything that's going on around us. And from that comes all these different sorts of things. Rules.
> I: Would you see mathematics as something that is discovered or something that's created?
> Anna: Bit of both. . . . It was probably created as necessity, just to, it is, it's sort of, it's created, but –
> Elaine: It can't be created from nowhere.
> Anna: We discovered things.
> Elaine: It's there to be created, sort of?
> Anna: Yes, we discover things which help us more to — I don't think you can exactly discover.
> Elaine: But sort of if you just, you don't really discover a formula or something.
> Anna You create a formula, but then again . . . somebody must have created it, before it was discovered.

This last excerpt shows how students found themselves in a state of indecision when they were probed as to whether they thought mathematics was created or discovered. There were some students who were more definitive than Anna and Elaine, deciding quite quickly that mathematics is created. However, their reasons for this decision indicated they equated creation to being 'made up'. That is, they saw mathematics as something mathematicians have made up, and the reasons they saw it that way included that they did not see much use for mathematics and did not understand it to any degree.

Some additional examples of the creation/discovery debate are given below:

(Two grade 11 girls)
Helen: Oh in the start created because . . .
Ellen: Why would they create it? I don't know. I think it was just there. . . . They stumbled across it.
Helen: It's part of life. It's part of everything. It's not just, you know, you have arts, and you have maths and science. They're all combined.
Ellen: It's hard to identify where you use it. It's just - sometimes it comes out, but I don't know. But I do think it is discovered.

(Grade 12 girl)
Tania: I don't know. Discovering it then creating it as you go. You kind of do a bit of both as you go. Yes, because things like the hypotenuse. I don't know. How would he have discovered that? Like, because that squared plus that equals that squared, or whatever. He couldn't have just created that, like: 'Oh yes!' Because it applies to everything that involves the hypotenuse and that. So probably just discovery.

What is noteworthy about these excerpts, as well as the previous ones, is they indicate students have a fairly broad view of mathematics. Their conceptions of mathematics, when they respond beyond an initial immediate response, include much more than the stereotyped view of mathematics as numbers, formulae, logic or problem solving. That is, students appeared to have formed views of mathematics that encompassed more than the topics explicitly taught and assessed. Many see mathematics more inclusively, taking into consideration aspects of the nature and purpose of mathematics. It could be argued that engendering in students an awareness and appreciation of this broader context is a goal of many mathematics curricula (for example, National Council of Teachers of Mathematics, 1989; Australian Education Council, 1991), and therefore an expected outcome of learning. However, other research into students' beliefs about mathematics has not often indicated students hold views beyond the narrow, stereotypical views of mathematics as numbers, formulae, rules, logic and problem solving (for example, Crawford, 1993; Frank, 1988; Kouba & McDonald, 1991).

It is possible that students in these other studies held broader views, but the data collection methods were not appropriate for probing beyond initial expressions of views. As will be highlighted in upcoming sections of this paper, the broader contexts within which students interpret their mathematics experiences cannot be ignored if mathematics educators are to interpret research results in ways informative simultaneously to teaching practice, curriculum development and further research.

Teachers' views of what mathematics is were similar to those of students in that they included notions of problem solving, analyzing, studying patterns and relationships, and learning that mathematical skills and reasoning can be powerful tools for solving human problems. For example, when asked what mathematics is, their responses included:

> Nancy: It is not just - let's say - it's easier to say what it's not. It's not just doing arithmetic. It's not *just* manipulation of numbers and things like that. It's understanding of how things work - patterns. Stuff like understanding patterns for things like - interest even. Bank interest, which is very boring, but there's a pattern in it. . . . For me personally it is being able to actually model real life things mathematically. I think it's all very exciting.

> Lee: 'm just trying to think. To me, I mean, I guess there's two answers. I mean, maths is numbers and, you know, your calculus and all that, but that's not really what it is. I think maths is being able to understand something and work something out. Basically I suppose going back to problem solving, using your skills, what you know, to be able to solve some kind of a problem. I think that's what maths is. . . . And that's what most maths has been developed for or from. In kind of a solution to some problem that's been presented somewhere along the line, that someone's got to work out an answer to.

Both these teachers clearly see mathematics as much more than numbers and arithmetic. The other teachers spoke similarly of mathematics, focusing many of their descriptions on mathematics as a "way of thinking" (Reg) and a subject "that gets people looking at issues and things in an analytic way, not just a numeric way" (Rachel). However, they did not believe their students held the same views. They thought that most students see mathematics as boring and irrelevant, although some noted that individual students or groups of students, particularly those who are successful with their mathematics achievement, might be more in accordance with themselves as to the nature and relevance of mathematics.

Thoughts the teachers expressed in relation to what they thought students think about mathematics included:

> Lee: Students think maths is boring, and it's a whole pile of numbers and rules and stupid things that you have to learn to keep people happy, but they don't really mean anything and they'll never use them again.

> David: Boring. Painful. Necessary. I think a lot of students think maths is necessary, but they don't like it. They think it's working with numbers. The four operations and - arithmetic. They sort of think of it perhaps as a more basic thing, you know. Arithmetic, algebra, the study of numbers, that sort of thing.

Mandy: I would say - depending on the students - a lot of them would see it as something they just have to do, that their parents perceive as being important, that they may perceive as being - 'You have to do maths even when you've got a choice not to do it.' But they don't necessarily enjoy it. I think they see it as one of the evils of school. Like, 'I've got to do this and I've got to do that, but if I had a choice then I'd pick up [unclear], or woodwork or art or something like that.'

Steve: Well, I think mathematics is probably just, to them, another - to a lot of students - just another course they're doing. Not to fill in the day exactly, but they know their day consists of a social studies, a maths, a science, and an English or whatever, and some of their other extra curriculum things that they do. So to them it's like a growing process, I suppose, and I don't think they look at maths so much as a separate thing from where it's sort of taking them. I think it's a collective thing, from year 8 to year 12. It's just one of the things that they do, and it's expected of them. And I mean it's probably never been questioned from them why they do it, but it's just everyone assumes that you will do a maths course.

What is striking in these and other excerpts is that the teachers included social factors in their comments on what they believe students think mathematics is. That is, they see students' views as related to and derived from much more that the cognitive elements of mathematics learning. Mathematics to students, as interpreted by teachers, is "just another course" (Steve) that you "just have to do" (Mandy) as a "means to an end" (Vince), even though the topics learned might not be used again. The teachers' views of students' views did not separate students from the encompassing environments in which they live. Personal factors such as enjoyment or motivation, or social factors such as school requirements or parental expectations, were not seen as isolated from more cognitively oriented factors such as learning about numbers, rules, arithmetic or algebra. In particular, Steve's comment that students do not look at mathematics as separate from reasons for studying it indicate that he is in agreement with students' own interpretations of the role of mathematics in their lives. It will be seen in upcoming discussions that what mathematics means to students, that is what they 'understand' as mathematics, is not formed in isolating the academic content of mathematics from personal and sociocultural contexts.

Why is mathematics studied?

Analysis of interview data revealed four major interwoven and overlapping contributors to conceptions of the intentions of mathematics studies: social status of mathematics, utility of mathematics, career aspirations, and mathematical interest or disinterest. These components appeared to form a web of beliefs that influence the nature of conceptions of school mathematics and what it means to 'understand' mathematics. Each of these four components will now be more clearly defined and each will be elaborated upon and supported in relation to primary data from the interview transcripts.

Social Status of Mathematics

Students' conceptions reflected a social norm that mathematics is an 'important' and essential subject to study. They saw it as a subject with much prestige in the eyes of the community, especially employers. Many had questioned the validity of this status, but had accepted that it was a social value or convention they must acknowledge in their choice of subjects for upper secondary or post-secondary studies. Some examples of what students said in this regard are:

> (Grade 11 girl)
> Elaine: Because lots of employers look at what you got for maths. It's seen as very important, I think. . . . In university, if you have a look at the pre-requisites, a lot of them are maths, to get into anything else. So it is a matter of having to do it.
> . . . Science and maths especially. More emphasis is placed on doing well in them. If you can do well in English it's not, there's not many things you can do in English. You can do journalism and things like that, but there's more areas to study if you do science and maths. Like medicine and all that kind of thing.

> (Two grade 10 girls)
> Cathy: Maths and science are always the two subjects that are made paramount.
> Lisa: I think it's - people, you know, sort of think maths and science are important because they're hard. Because smart people do them.

Cathy's and Lisa's words are their interpretations of community views. They believe society at large (ie. people in general) highly values mathematics success, and see mathematics success as something people should strive for. Employers "look at what you got for maths", put much emphasis on "doing well" (Elaine), and thereby make mathematics (and science) success paramount in decisions. These students' interpretations of the status attributed to mathematics success within society also indicated they saw such status as, at least in part, invalid. They felt that part of the reason mathematics is seen as an important and paramount subject that it is generally seen as a difficult subject, and thereby a mechanism by which to filter students. That the study of 'abstract' mathematics, as opposed to 'vegie' mathematics is a filter for segregating people was something of which the students were clearly aware. This point can be seen in Tracy's words:

(Grade 11 girl)

Tracy: You get Foundations in Maths, and then Intro Calc [Introductory Calculus], and G and T [Geometry and Trigonometry], and you get - it's the general - like people say: 'Oh, I see.' And then people, other people say: 'I'm doing G and T.' . . . and it's like a very class sort of thing. It sort of divides people up.

[Geometry and Trigonometry and Introductory Calculus are Grade 11 courses, leading to Grade 12 courses (Applicable Mathematics and Calculus) that are pre-requisites for university entrance. Foundations in Mathematics is a Grade 11 course that leads to Discrete Mathematics, a less specialized Grade 12 course that can also be used for university entrance requirements. Approximately 10% of Grade 12 students study Calculus, 25% study Applicable, 35% study Discrete, and the other 20% study a non-university entrance mathematics or do not study mathematics.]

Most teachers were as explicit and vocal as the students about what they saw as high status attributed to mathematics, status they saw as unwarranted. Their questioning of the prestige attributed to mathematics study is reflected in the following interview excerpts:

(Deputy Principal)

Reg: It's prestigious. It's considered that it leads on to occupations that will give you a high income. And I think there's a perception that it's useful for business, although why, I'm not sure. . . . I think maths - and it also applies to some of the other subjects - the TEE subjects are seen as prestigious. They are the successful people who do those. There's more prestige in getting an A in Geometry and Trig, or Calculus, than there is in getting an A in Maths in Practice or Modelling with Maths, which tends to be seen as the vegie maths. But on the other hand, there's less prestige in doing no maths, than doing vegie maths even. So, and we're told, brainwashed into thinking that everyone must have some maths. It makes us a clever country.

(Principal)

Brett: Well, I think it's - you know, I think we're - the tail wags the dog. That is, the tertiary institution wags the school, and therefore decides on the curriculum. And great fuss is made in mathematics - I don't know about elsewhere, but in Western Australia anyway. Your ability in mathematics really puts you into your social strata, your intellectual social strata. It's, therefore everybody says that unless you can do your maths, you're not going anywhere, you're a no good person. It's almost that measure. And in recent years that's happened. Previously the three R's were important. Then you moved up to the four core, where there were social studies, English, science and mathematics, but now English and mathematics have been polarised, or withdrawn or separated, as really the two elements.

I: Why do we teach mathematics at secondary school?

Louis: Because societies value it. Societies somehow have this opinion that if you can do maths you are clever. . . . apparently once you have a maths degree, one that shows you can think about a lot of extremely abstract problems, you can immediately switch over into real life stuff and - um - do it. Which I think is a very strange idea. I don't believe it is so. I mean when I look at my builder [doing house renovations] he does not use too much Pythagoras and yet he can put a building together and he keeps thinking all the time. But to try and teach the thinking he's thinking in a classroom is totally stupid, isn't it? It's just a silly idea. In fact half the qualifications he needs to be a builder is actually to completely fail at school because if he really was good at maths what would he be doing? He might be a maths teacher and he would not be living in the real world.

I: What do you see is the role of maths in high school?

Louis: You mean why do we push it? Because the system values it and it's a ticket to somewhere else. You are not going to use it. You use very little maths. . . . we teach a lot of stuff that is not going to be used again. The kids all know that.

These teachers, although they earlier gave descriptions of mathematics as a powerful, useful tool for solving human problems, here seem to be saying the outcomes of high school mathematics study have more to do with schooling requirements or society conceptions of what mathematics achievement measures, than with learning mathematics as a problem solving tool. High school mathematics, from their perspectives, is a "ticket" through a gateway "to somewhere else" (Louis). That somewhere else, for most students, is not likely to be somewhere where the actual topics studied in high school mathematics would be used.

This notion of mathematics as a valuable "ticket" to elsewhere was reflected in students' comments also, with some students seeing the status of mathematics study as related to the fact it is a necessary or 'good' subject to study. It was not always clear what all was implied by the reference 'good', but it was apparent that some students saw mathematics as a 'good' subject to study because it was valuable as a means to an end, the end being prospects for employment or further study. For example, Tracy and Grace saw mathematics as a subject essential to their Grade 12 overall assessment and subsequent acceptance into a university program:

(Two grade 11 girls)

Tracy: It's good to know but I don't use it.

Grace: The use is that you'll get your TEE [Tertiary Entrance Examination] score which will get me into uni [university].

Tracy: Like a safeguard.

Tracy's and Grace's words highlight a point made by a number of students in relation to the use or value of studying mathematics. They did not always see mathematics itself as something they might need or use. Rather, it was completion of mathematics *study* that was useful in that it is a requirement or a standard social norm. Students' notions of the use of mathematics in comparison to the use of mathematics study are discussed in more detail in the next section.

Utility of Mathematics

Students saw mathematics content as important because they saw it as something needed in their daily lives. However, what they often actually described as relevant mathematical knowledge was mathematics taught primarily in elementary school. This point is exemplified in the following interview extracts:

(Grade 10 girl)
Kath: Because you have to know it I guess. Well, you use it in like lots of things, like even when you go shopping and stuff like that. Some things like algebra I don't think you really need, or - at the moment we're doing the derivatives or something, and unless you're going to be an engineer or something you probably won't need it. But you need it in later life.

(Grade 10 girl)
I: When you are outside of school, what is mathematics to you then?
Alice: In the shops. That's about all.
I: Can you think of other places?
Alice: I suppose when you are making something you need to know measurements to make it look decent, or in cooking and that type of thing you use measurements. That's about all, I think.
I: Do you think mathematics is important to your daily life?
Alice: In a way, the things, the basics that you learn in primary school would be necessary, but I don't think anything you learn in upper school is important.
I: Then I'll ask you why you think mathematics is taught at secondary school?
Alice: I think it's just like, the teachers say that if people want to go on to subjects that include maths, they've got that thing there for it. And the others just have to do it in case they want to change their minds. Can't really think of any other reason why they do it.

Both Kath and Alice question the extent to which one might use high school mathematics in one's life. Their views were reflections of the views expressed by the other students. That is, students frequently commented that it is important to study mathematics because you need it in your daily life, and they also frequently questioned the extent to which one needs to study mathematics. They saw the basics as essential to all people, but felt that much of what was taught at high school was for university or career purposes only:

(Grade 11 girl)
Grace: But like even basic maths at school, you still have to do algebra and that, and unless you're really going to professionally use it, you're never going to use it again when you leave school.

(Grade 11 girl)
Tracy: It's mainly to prepare us for uni, I think. Everything's preparing for something else. Like in primary school we prepare for high school maths, but in high school we prepare for uni maths. . . . I don't know, but I can see the relevance of it in some careers, like engineering and stuff like that, but for other careers I don't see why some of the maths we're taught is taught. . . . Well as, as a nurse, where would you use a quadratic triangle or something? I don't see its relevance if you're a nurse, knowing how to change a quadratic formula to complete a square form or something like that. I mean it's not really going to help you to be a better nurse.

(Grade 10 boy)
I: Why do you think it [mathematics] gets taught at secondary schools?
Marc: To help people get into university and stuff like that. Because you never use it in jobs. I don't suppose you would. You're not exactly going to use the quadratic formula and all that in jobs.

I: So why do you think it's taught? Any idea?
Marc: To get into university I suppose.
I: What about university or TAFE [Technical and Further Education] courses? Why is mathematics taught there?
Marc: I don't know. To make people feel they're intelligent. Because it's not going to help them. You know, it's like a dog draw or something like that, in a job you need it. Otherwise, I haven't a clue.

Although these students' comments reflect beliefs that mathematics is useful for university or career purposes, they also reveal a sense of doubt about the actual extent to which mathematics might be used in a professional context. They question the use of specific mathematics topics, for example, as Tracy and Marc question the potential use of knowledge of quadratic equations. These students, as well as many of the others, noted they saw high school mathematics as relevant to only a few careers, and therefore an unnecessary requirement for most careers. Thus, they wondered why it was so frequently a pre-requisite course of study.

(Grade 11 girl)
I: Why do you think it's so important for jobs?
Karen: I don't know. I've always wondered about that because I don't really like it very much. But they always say you have to have it. I don't know why.

(Grade 10 girl)
Andrea: . . . last term or something we were just learning shapes and that, and I know I'll never use them because my next door neighbour had a look through my file. She was curious and she said, like she was taught most of it as well. And she said: 'I've never, ever used it.' So I thought it's pretty stupid, like wasting time, something you won't need, shapes and stuff.

(Grade 10 girl)
I: Why do you thinks mathematics is taught in high school?
Nancy: To bore the kids. . . . I don't know. I guess it's because it was taught through primary school and then high school, so you might as well keep going.

As was pointed out in the previous section on the status of mathematics, many teachers also questioned the potential use to most students of much of what is taught in high school mathematics. For example:

(Course Counsellor)
Vince: Good question. I don't know! Now, as I'm perhaps a maths teacher, with a Science degree, maybe I should be saying, 'Oh, maths is important for later on in life, and relevant, et cetera, et cetera.' But I'm perhaps an unusual type of maths teacher. I don't think it is. I mean, if you think of, even the stuff they do in Maths in Practice and Modelling and Maths, once they get into the real world, how often do you have to sit down with a calculator, or, I mean, you can go to a bank these days and they can work out your repayments to the last, you know, day almost, with the computer assistance. And I would say it's very rarely, well I do my own tax, but apart from that, I'm just trying to think when I'd use a calculator. So perhaps there isn't much relevance for maths.

Sherry: Well, that would depend where they're going to go afterwards. I mean to say, the most valuable maths a child can learn if they're going to go to the trades is what they do down in manual arts and whatever, and then we should be tailoring that to their area. I mean to say - yes, but c.nerwise the kids that are going to go on to further mathematics and study it, whether it be statistics or probability or whatever, well then, I suppose we need to expose them to those things. But I think there are a lot of kids that are exposed to things that they don't need. They will never, ever want to even think about it again. And I believe that we have to address that problem.

Vince and Sherry both question the practical utility for students of much of what is taught in high school mathematics, even what is taught in the so-called practical, non-theoretical courses. Other teachers made similar points, but some of these others then commented on the use of mathematics in a broader sense that direct applicability to or usage in one's daily life. What they saw as the 'use' of mathematics was its value beyond immediate direct applications. For example:

Nancy: . . . I don't know how much the other students, at least a lot of students, get out of most of what we do teach. They don't see a relevance to anything that they can't directly use. Mind you, I don't see that direct usage has to be the reason for teaching something. Like, you don't directly use history, or things like that. But I still think it's worth teaching. You don't directly ever use poetry, but it's still worth doing. So I think a lot of work falls into that category. But at the bottom of the sort of intellectual range, I'm not too sure why we do most of what we do, and whether we need to do it so often or so much. ... Well, I think for those students who, I think if you can understand the concepts and so on, things like calculus are firstly fascinating and secondly useful. Or useful as far as the tertiary institution is concerned at least. But for its own sake I think it's quite useful, and useful to understand that these things are there, tools to use. And to me it's kind of knowing about mathematics that's part of a good education. It's like knowing history and knowing music. It's part of our culture, part of our background needs.

Calvin: Because it [mathematics] helps put everything in a very logical sequence. Even though kids r ghtn't use algebraic methods in their everyday life, I think .elps them be logical in their thinking, in their decision making, in every aspect of their lives.

Nancy's and Calvin's notions of what constitutes the 'use' of mathematics are within a cultural context, related to use of knowing about what mathematics does, or being able to think and reason as is required in formal mathematical processes.

Overall it could be said that students' and teachers' views of the utility of mathematics fell within two distinct categories: (1) the use of basic mathematics concepts and related skills in one's daily life, and (2) the potential use of more advanced mathematics concepts and skills within professional endeavours. A third category was evident in the teacher interview data: (3) the use of mathematics as a reflection of human thinking or culture. Students and teachers often questioned the degree to which it would actually be necessary to use advanced mathematics within professional activities. However, although they questioned the reasons for mathematics to be required for entry into particular career paths or related programs of study, they also acknowledged the reality of the existence of these requirements. Thus, the relationship of mathematics study requirements to career aspirations were an integral component of their conceptions of mathematics.

Career Aspirations

Students were motivated to study mathematics to enhance their prospects for a particular profession or job, to keep their career options open and maximised, and to assist their chances of gaining acceptance at a post-secondary institution. In relation to these points they said such things as:

(Grade 11 girl)
Kath: Because I need it I guess. You need it for - I chose it because of what I wanted to do in university [physiotherapy].

(Grade 12 girl)
I: Why do you think Mathematics is taught in high school?
Nancy: Hardly any of the courses at uni which prepare you for jobs that don't want an amount of mathematics.

(Grade 11 girl)
Elaine: I have to because if I don't do that, then I won't get my full TEE marks. Then I won't be able to get into uni to study what I want to study. Because there's nothing else I want to do and they always say: 'Oh, you need maths for something.'. . Some people look at it as another subject in school that they have to go to, like in the classroom. And some people look at it as something they're going to have to do in later life. And there's just all ways of looking at it. I look at it as something that I have to do, not that I particularly want to do. But I know I have to do it anyway. And if I'm going to get somewhere in life I'm going to have to basically know what I am doing there. . . . Because like, they're telling us that: 'You're not going to get hired in a job if you don't know like how to do your basic maths,' right.

In comparison to what these students said about mathematics study as necessary for gaining university entry, students who were not aiming at university study were more explicit in their condemnation of the value of high school mathematics. However, they recognized that mathematics study was necessary if they were to keep their options open for later educational or career choices:

(Grade 11 girl)
Alice: . . . I thought it [Foundations in Mathematics] was the easiest one there and I wanted to do TEE, . . . To get a score. Because everyone says that maths is good, so I just thought do that.
I: You say everyone says maths is good. What do you think?
Alice Oh, it might help. I don't really know what I want to do later, so I just think I'll keep my options open.

(Grade 11 girl)
I: Do you think it's important that you study mathematics next year?
Andrea: Not really, because I want to be a photo journalist, so I won't really need it. So, not really.
I: Why would you choose to study it?
Andrea: Well, because as I said before, I thought everyone had to do a maths subject for next year, and I thought I'd better do one, so I thought I might as well.
I: Why might you as well?
Andrea: Oh I don't know. Just because it was there and everyone else was going to do it as well, so I thought I might as well. Because I've done, I'm not sure how many TEE subjects I've done, so I thought I had to put that one down as well.

Along with recognising the necessity of mathematics studies to gain university entry and thereby attain desired career aspirations, students were frustrated. They felt as though someone else, or the system, had imposed unnecessary or unreasonable demands upon their choices, as can be seen in Tania's words:

(Two grade 12 girls)

Tania: Deciding to do maths was one of the hardest things. I mean, you really needed it. There was really no doubt not to do it. But I think to be an actress and to be on the stage, you DON'T need to sit there and go: 'Well, if I am going to walk in this direction, then I am going to have an angle of so much in order to – ' Things like that. . . . For university entry and to get in. I mean to do a Bachelor of Arts or Performing Arts at university, you still need to have got a suitable score, and mathematics is one of the things you need in high school to get into it. A lot of courses have a pre-requisite that you have to be competent in maths.

Anita: The only reason I am doing maths is to keep my options open. . . . Yet I don't really know what I want to do. But because, you know, you need it for university.

Tania: I never really thought of not doing maths.

These students recognize that although the educational system appears to give them choice in whether or not to study mathematics, that choice is very much an illusion. It is an illusion because mathematics study is pre-requisite for entry into many career paths. They might not want to study mathematics, but are virtually forced to if they want to avoid cutting themselves off from a wide range of options in subsequent years.

Teachers also were aware of the role mathematics study plays as a pre-requisite to many career paths, and like students, they sometimes questioned the actual role of these pre-requisites:

(Deputy Principal)

Reg: . . . I think at the end of high school our brightest kids, in terms of TEE scores, will pick up medicine, and they'll pick up physiotherapy, ed[ucation], science, law - none of which are that mathematical. And they tend to steer clear of some of the mathematical ones. So I suspect our best maths students don't go on and do more maths.

Aaron: Why is it taught in secondary school? Well, I'd say employers really want it, so I think if a kid's going to get a reasonable job, he's got to have a mathematics background. Most employers rank maths as one of their number one pre-requisites, I think. So I'd say that's the main reason. Parents and employers want it. Parents want their kids to have it so that they can get a good job, and - plus it helps kids with everyday skills. You know, some of the kids actually do benefit, you know, they can go out in society and use skills like shopping and, you know, adding up grocery bills and stuff like that. It helps them with everyday skills. But I'd say the number one reason is because it's the way society is. It expects kids to have maths if they're going to get good jobs. Most, or a lot of jobs do - not all of them. It gives kids more choice with employment.

Lee: Well, if they're doing the kind of maths, and I'm thinking kind of like maybe Intro Calculus or some of the 6, you know, unit 5 and 6 stuff that the kids do, if you say to the kids: 'You have to do it because you want to go to uni and uni says you have to do it', well the kids will accept that. But I don't think they really like it very much. But they accept it because that's what uni says you have to do. And you've got to play by the rules.

(Principal)
Brett: . . . If you can do the most difficult maths, no matter what you call it or what it's used for, you're in the upper echelon and it can open up gates for you. And the sort of stuff you learn at, the sort of stuff you learn at school in your mathematics, no, it's of no value to you. I went to a talk on education in industry only last week, and they said if they stopped teaching them all that junk, and just gave kids a feel - these were people in the mining industry - and the representatives there were a couple of the world's biggest companies. . . . They were saying, 'Look, if they'd just stop all that junk and take the average kid, to give him a feel for statistics.'

Sherry: . . . But most of them only choose to do these, well, particularly higher order mathematics, because some career path dictates that they have to have it. There are a few kids that do it because of the love of it, but most of them do it because some career path dictates that they need to have it.

In summary, there was both diversity and conformity in students' and teachers' conceptions of mathematics in relation to their career aspirations. Conformity appeared in the form of recognition of the pre-requisite necessity of mathematics study for certain career paths, particularly in relation to university entry. There was also a high degree of belief that many of these pre-requisites were an unwarranted constraint. The diversity or conflicts that arose amongst students were a result of the extent to which they either needed or desired to "leave their options open". Choosing whether to study mathematics or what specific courses to study at upper secondary level then became a dilemma for some students. They did not see how mathematics would be useful to them, or did not enjoy mathematics study, yet were faced with externally imposed requirements or societal values and expectations.

Within these viewpoints was an interweaving of the three components discussed so far: status, utility and career. That is, the interdependent nature of these factors is highlighted when students' reasons for choosing mathematics study or particular mathematics courses are examined. The web of beliefs influences students as they come to understand and interact with their world, including school, societal and personal environments, and these interactions also influence the nature of the belief system. That is, students do not construct their understanding of what mathematics and mathematics study are independent of their place in a complex social and cultural context.

Mathematical Interest or Disinterest

A few students expressed an interest in mathematics related to a sense of the intellectual fascination and challenge it provided, and these students were not necessarily the most able students. In general however, few students enjoyed mathematics for its own sake. Some examples of their comments about their interest in mathematics were:

(Two grade 10 girls)
Hilary: It's always been my favourite subject since primary school.
Bev: It's challenging. . . . And I like how we have, when we have group discussions. Like you hear everyone's point of view.

(Grade 12 girl)
Anna: I sort of do it because it's challenging, and I think it's, it's just an interesting and challenging thing to do.

On the other side of the story, students' expressions of their dislike of or lack of interest in mathematics included the following:

(Two grade 11 girls)
Ellen: Like for research assignments or something, for English or something, you like research something and then you find out all these points and then later on after a while you go: 'Oh, I know about that!' You know, it's interesting. But maths isn't like that. It's just like you learn it and then that's all out the door and you forget about it the rest of your life.
Helen: . . . We sort of remember for the test and that's about it.

(Grade 10 boy)
Dean: I just shut it out of my head, just like everybody else. I think it's a boring subject.

(Grade 10 boy)
I: Why do you not like it?
Marc: Oh, it's just boring. I'm not interested in it. I can be, but some days I can't be bothered.
I: Can you think of things that could be done to help you feel it wasn't so boring?
Marc: Yes. When I'm good at it, or stuff like that, I like doing it. But when I just can't work it out, or anything like that, that's when I get frustrated and I don't want to do it.

A strong interest in mathematics was also expressed in relation to career aspirations and the social importance of mathematics, and enjoyment was achieved through being successful in relation to these other key components. In particular, there was evidence that interest in mathematics is linked with a sense of personal success in mathematics, findings that are not new within the realm of mathematics education research. For the students of this study, there was correlation between students who had been successful with mathematics and those who were intending to or who were already enrolled in university entry mathematics courses.

Another aspect of interest or disinterest in mathematics emerged in relation to what peers or society see as 'acceptable' feelings about mathematics. Enjoyment or interest in mathematics are not 'acceptable' , a societal norm reflected in this teacher's account of a classroom incident:

> Sherry: . . . but I don't think there are many kids that do now get the enjoyment out of their maths just for doing it. One of my girls in my year 10 class the other day, she said, 'I love maths.' And she felt very uncomfortable about admitting that, but she was strong enough to say, 'But I love maths.' I said, 'Great, [Melissa]!' You know? In front of the class. I said, 'You don't need to feel bad about that.' I said, 'We all have things that we like to do', I said, 'and that's great to hear you say that.'

Since acceptable views are not ridiculed by others, while unacceptable ones might be, it might be that this study has not been able to examine reliably what individuals', particularly students', feelings are in relation to interest or disinterest in mathematics.

Other components of mathematics learning

Three were an number of other features of learning that students felt were influential in their mathematics learning. Each of these features are summarized below, with some supporting examples of what students said in relation to the point. The .features and examples are not discussed in detail. However, since they are essential in the full context of students' experiences of mathematics and mathematics learning, if one is to examine possible confounding influences, they are included here, and will also be commented on in the final conclusions and discussions of the implications of the study's findings. Features of mathematics learning that students felt were influential in their effect were:

- Mathematics learning is more that rote memorisation; it is necessary to understand what you are doing.

  Tracy: . . . in logarithms you're set rules and she proves that they actually do work, and the logic behind the rules. So it's not just something you learn by rote. It's something you understand.

  Grace: Like last year when we didn't - like we had a rule and you didn't know where it came from. And that makes it harder to use and harder to learn than if you understand what's behind the rule.

- Mathematics classes are "too fast", proceeding at too quick a pace for people to learn best.

  Ellen: But now I am going right down hill because I don't understand it. She just goes really fast and everyone in the class I think would agree that she just goes too fast. . . . Because she goes onto the next thing and you're just figuring out the next thing, like the first thing you did, and then you look up and there's like all different stuff and it's like you know, and it takes the whole lesson to try and figure out one thing.

- Mathematics learning is repetitive and boring and students could learn more if mathematics were taught in a way that made it more fun and more relevant.

  Alice: Well it's really different because in maths you learn something and in other subjects you learn different things. Because like Nancy said, there's more things to learn. It is not always repeating things.

  Nancy: Just like in health studies. You can only learn so much about se and everything. It's just like maths. It's boring.

  David: It's a pretty good subject, even though most people don't enjoy it. I don't really enjoy any of it. . . . I don't really enjoy doing any of it. It's just a pretty useful subject.

CONCLUSIONS

Students have 'understanding' of mathematics that is dependent upon context. That is, they define mathematics in a school sense as well as a personal sense. From a mathematics 'discipline' perspective of what mathematics is and where it comes from they generally have well-formed conceptions. However, these conceptions do not play a prominent role when students are asked to give personal meaning to mathematics in relation to their lives. Students' personal perspectives give meaning to mathematics only in relation to a number of social factors, including the social status of mathematics, the utility of mathematics, career aspirations, and interest (of lack of interest) in mathematics. That is, students incorporate into their conceptions of mathematics a wider cultural and community perspective of mathematics than that upon which mathematics educators generally focus. More specifically, students put mathematics in a whole school, career and life perspective, but mathematics education research does not presently adequately address this broad viewpoint.

Thus, it could be said that 'understanding' mathematics is neither a goal nor a necessary component of students' mathematical learning, at least not in the sense mathematics educators might define 'understanding'. Students 'understand' mathematics when they are meeting their goals as described in relation to their career aspirations and sense of the social importance of mathematics.

What also is of interest is how the four factors identified within the context of the intents of mathematics study conflict with the ideas of how mathematics educators identify problems present in mathematics education practices. For example, understanding generally is associated with a range of interconnecting, cognitive frameworks that can be utilised to explain concepts and solve problems (National Council of Teachers of Mathematics, 1989). Further self-analysis is needed by the research community into their own interpretations of what mathematics is, what it means to learn mathematics, what it means to understand mathematics, what is the nature and role of mathematics education research and how mathematics education research functions in research and other educational contexts.

Recognition is needed for how students view curriculum because this study indicates students do not separate mathematics from their personal or social contexts. They do not perceive of mathematics as one might describe mathematics as a discipline, but rather, describe mathematics in relation to a range of socially derived components. Researchers' capacities are restricted whilst they persist with disregarding the whole school context and a view of mathematics that virtually ignores students' views.

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