DOCUMENT RESUME

ED 387 342 SE 056 784

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TITLE	Role of Self-Efficacy and General Mental Ability in Mathematical Problem-Solving: A Path Analysis.
PUB DATE	Apr 95
NOTE	37p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 18-22, 1995).
PUB TYPE	Speeches/Conference Papers (150) -- Reports - Research/Technical (143)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Affective Behavior; High Schools; *High School Students; *Mathematics Achievement; Mathematics Anxiety; Mathematics Education; Path Analysis; *Self Efficacy; *Student Attitudes; *Student Motivation

ABSTRACT

According to social cognitive theorists, people's judgments of their own capabilities to accomplish specific tasks strongly influence human motivation and behavior. Path analysis was used to test the influence of math self-efficacy and general mental ability on the math problem-solving performance of 329 high school students. A model that also included math anxiety, gender, and math background accounted for 61% of the variance in performance. Ability and self-efficacy had strong direct effects on performance. Ability also had a strong direct effect on self-efficacy, which largely mediated the indirect effect of ability and background on performance. Self-efficacy had a strong direct effect on anxiety, which, in turn, had a weak direct effect on performance. Although girls and boys did not differ in ability, self-efficacy, or performance, girls reported higher anxiety. Most students were overconfident about their mathematics capability. Results support the hypothesized role of self-efficacy in Bandura's (1986) social cognitive theory. Contains 51 references. (Author/MKR)

Running Head: MATH SELF-EFFICACY

Role of Self-efficacy and General Mental Ability In Mathematical Problem-solving: A Path Analysis

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Paper presented at the meeting of the
American Educational Research Association, San Francisco, 1995
Session 37.24

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Abstract

Path analysis was used to test the influence of math self-efficacy and general mental ability on the math problem-solving performance of 329 high school students. A model that also included math anxiety, gender, and math background accounted for 61% of the variance in performance. Ability and self-efficacy had strong direct effects on performance. Ability also had a strong direct effect on self-efficacy, which largely mediated the indirect effect of ability and background on performance. Self-efficacy had a strong direct effect on anxiety, which, in turn, had a weak direct effect on performance. Although girls and boys did not differ in ability, self-efficacy, or performance, girls reported higher anxiety. Most students were overconfident about their mathematics capability. Results support the hypothesized role of self-efficacy in Bandura's (1986) social cognitive theory.

Role of Self-efficacy and General Mental Ability In Mathematical Problem-solving:

A Path Analysis

According to social cognitive theorists, people's judgments of their own capabilities to accomplish specific tasks strongly influence human motivation and behavior (Bandura, 1986). In part, this is because these self-efficacy judgments are said to mediate the influence of other predictors of behavior on a particular performance. In academic settings, for example, the confidence that students have in their own ability helps determine what they do with the knowledge and skills they possess. Consequently, the influence of actual ability on some academic performance is due, at least in part, to what students actually believe they can accomplish. Prior determinants such as ability and previous performance attainments help create self-efficacy perceptions and are also strong predictors of subsequent performance. However, because "people's perceptions of their efficacy touch, at least to some extent, most everything they do" (Bandura, 1984, p. 251), self-efficacy judgments should retain predictive value when these determinants are controlled, and the determinants should influence subsequent performance indirectly through their effect on these judgments.

Self-efficacy beliefs are hypothesized to influence the choices students make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience. A high sense of efficacy may serve students well when solving math problems, not because it "causes" them to be better problem solvers, but because it engenders greater interest in and attention to working the problems, increased effort, and greater perseverance in the face of adversity. Such students are also likely to feel less apprehensive about their mathematical capabilities. For these reasons, Bandura (1986) described self-efficacy as a mediating mechanism of personal agency--mediating between the sources of its creation and subsequent outcomes. Factors such as the anxiety associated with specific academic areas are considered common mechanisms of personal agency, for they, like self-efficacy, also influence academic outcomes. Bandura suggested, however, that the influence of these mechanisms on academic performances is largely due to the confidence with which students approach related academic tasks.

The predictive and mediational role of self-efficacy has received support from a growing body of findings from diverse fields (see Maddux, Norton, & Stoltenberg, 1986; Multon, Brown, & Lent, 1991). In addition, researchers who have investigated the relationship between math self-efficacy and various mathematics outcomes report significant correlations and strong direct effects (e.g., Collins, 1982; Hackett, 1985; Hackett & Betz, 1989; Pajares & Miller, 1994, in press; Siegel, Galassi, & Ware, 1985). For example, Hackett and Betz (1989) found that math self-efficacy beliefs were highly predictive of undergraduates' choice of major, even when variables such as math aptitude and anxiety were controlled. Pajares and Miller (1994) used path analysis to investigate mathematics problem-solving from a social cognitive perspective and found that self-efficacy to solve math problems was more predictive of that performance than were prior determinants such as gender or math background or than common mechanisms such as anxiety, self-concept, and perceived usefulness of mathematics. Self-efficacy also mediated the effects of gender and math background both on the common mechanisms and on the performance task. Men and women differed in performance, but these differences were mediated by the students' self-efficacy perceptions. That is, the poorer performance of women were largely due to lower judgments of their capability. These findings were consistent with those of Hackett (1985).

Bandura (1986) cautioned that, because efficacy judgments are task-specific, a self-efficacy measure must assess the same skills called for in the performance task with which it is to be compared, and it must be administered as closely as possible in time to that performance. These guidelines are seldom followed, and so the mismeasurement of self-efficacy often produces poorly defined constructs, confounded relationships, and ambiguous findings. For example, Cooper and Robinson (1991) found that a regression model with math anxiety, aptitude scores, and math background showed that self-efficacy did not account for a significant portion of the variance in performance. However, this was likely due to self-efficacy and performance being assessed according to different criteria--judgments of confidence to succeed in math courses were compared with scores on a performance measure that consisted of solving mathematics problems. Pajares and Miller (in press) have shown that, because judgments of self-efficacy are task-specific, to increase prediction, measures of self-efficacy should be tailored to the criterial task being assessed and the domain of functioning being analyzed.

Other variables that have figured prominently in the study of mathematics outcomes are math self-concept, math anxiety, math background, and gender (Reyes, 1984). As regards anxiety, Bandura (1986) contended that only when people cannot predict or exercise control over events do they have reason to fear them. Thus, anxiety is largely determined by the confidence individuals bring to a task. Efficacy beliefs predict "how well people cope with threats and how much fear arousal they experience" (p. 321). Self-efficacy should predict performance even when the effects of anxiety are controlled, but the influence of anxiety should dissipate when efficacy percepts are controlled. Numerous studies report a negative correlation between math anxiety and math performances (see Schwarzer, Seipp, & Schwarzer, 1989, for meta-analysis). In most cases, the relationship weakens when variables such as self-efficacy are controlled (Hackett, 1985; Llabre & Suarez, 1985; Meece, Wigfield, & Eccles, 1990).

One drawback to including a variable such as self-concept in investigations of self-efficacy is that the conceptual difference between the two constructs is not always clear. Self-concept beliefs are generally defined in terms of judgments of self-worth associated with one's perceived competence. Beliefs regarding judgments of capability are considered part of an individual's self-concept, although Bandura (1986) argued that the two represent different phenomena and must not be mistaken for each other. Previous investigators have often referred to judgments of capability as self-concept of ability (e.g., Nicholls, 1990; Reyes, 1984)

Pajares and Miller (1994) found that math self-concept correlated both with self-efficacy (.61) and with problem-solving performance (.54), but the direct effect of self-concept on performance was reduced to a modest .163 when it was part of a path model that included self-efficacy, which had a direct effect of .545. Results also suggested that the relationship between self-concept judgments and problem-solving performance was largely a result of noncausal covariation due to the influence of math self-efficacy. Nonetheless, the conceptual interrelatedness of the constructs is such that separating them empirically may well be a futile enterprise, and we chose not to include math self-concept in the present investigation.

As noted earlier, students form efficacy perceptions as they attempt and complete tasks. Bandura (1986) argued that, because individuals interpret their experiences, such interpretations are likely to be more influential in predicting behavior than are the attainments themselves. Prior experiences influence subsequent behavior largely through their effect on self-efficacy beliefs, which, in turn, can influence performance "independent of past behavior" (p. 424). Researchers have reported significant correlations between math background and performance (Cooper & Robinson, 1991) as well as significant direct effects of background on math outcomes such as choice of math-related majors (Hackett, 1985) and problem-solving (Pajares & Miller, 1994). Researchers who have investigated the relationship between background and self-efficacy report both significant correlations and direct effects (Cooper & Robinson, 1991; Hackett, 1985). Pajares and Miller (1994) found that the high school math background of college undergraduates had significant direct effects on their math self-efficacy (.419) and on problem-solving performance (.099), as well as an indirect effect on performance through self-efficacy (.276).

Early findings regarding the relationship between gender and math performance showed that boys and girls did not differ during elementary school but that differences appeared in middle school and increased throughout high school (see Fennema & Sherman, 1978). When affective variables such as confidence were controlled, however, performance differences disappeared, leading researchers to suspect that these affective variables were the source of differences. When differences in math background were also controlled, even smaller differences on math achievement are found (Lapan, Boggs, & Morrill, 1989). Researchers have reported declines over the last two decades in gender differences in mathematics, and only at the highest levels do men continue to outperform women (Feingold, 1988). The relationship between gender and math self-efficacy has not been explored as thoroughly as that between gender and math performance. Initial studies suggested that boys were more confident in their math skills (Fennema & Sherman, 1978). More recent findings show that boys and girls have equal confidence during elementary school, but, by middle school, boys are more confident (Pintrich & De Groot, 1990).

Purpose of the Study

Pajares and Miller (1994) used path analysis to investigate mathematics problem-solving and constructed a path model with relationships hypothesized from Bandura's (1986) social cognitive perspective. In addition to self-efficacy and problem-solving performance, the model included prior determinants (gender and previous experience) and common mechanisms (self-concept and perceived usefulness of mathematics). We sought to replicate and extend Pajares and Miller's study in several important ways. The most substantive effort to extend their findings involved the inclusion in the path model of a measure of general mental ability, or psychometric g. We chose an assessment of psychometric g because it accounts for the single largest component underlying individual differences in mental ability (see Carroll, 1993, for a review of factor analytic literature on cognitive abilities; and Thorndike, 1984) and because of the general acknowledgement that psychometric g is a strong predictor of academic performance (Hunter, 1986; Jensen, 1984; Thorndike, 1986).

Previous investigations of the influence of math self-efficacy on math outcomes have not included a measure of general mental ability (e.g., Dew, Galassi, & Galassi, 1984; Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1991). Instead, researchers have examined scores on the quantitative section of standardized aptitude tests, such as the Scholastic Aptitude Test (SAT) or the American College Test

(ACT), with samples of college students. This is problematic for two reasons. First, scores on math aptitude tests are confounded by disparities in math background and by other extraneous attitudinal and anxiety factors related to mathematics (Dew et al., 1984; Hackett & Betz, 1989). Second, college students' scores on the SAT and ACT are usually restricted in range. Because students with lower scores are screened out by the college admission's process, correlations between math performance and aptitude measures in college samples are often attenuated. Pajares and Miller (1994) did not include ability in their path model but they acknowledged, as did reviewers of their manuscript, that its exclusion may have influenced the effects found. Consequently, they recommended that a future model include such a measure with an eye to testing their findings. A nonverbal, untimed measure of general mental abilities such as Raven's Advanced Progressive Matrices is less influenced by educational background than are aptitude tests or other ability measures. Thus, it minimizes the confounding inherent in such measures and provides a better control for ability in a path model testing the mediational role of self-efficacy.

As have most self-efficacy researchers, Pajares and Miller (1994) conducted their research with college undergraduates; our investigation focused on the confidence of high school students in public school. The use of this sample overcomes the problem of range restriction in ability assessment and broadens the generalizibility of our findings to public school settings and high-school-aged students. A third refinement of Pajares and Miller's design was our use of a modified Mathematics Confidence Scale as the assessment of math self-efficacy. Changes in the scale were made in light of factor analytic results from Langenfeld and Pajares (1993). For the reasons earlier explained, we also chose to include math anxiety rather than math self-concept as a codeterminant of self-efficacy and a control variable in the model. In addition, Pajares and Miller (1994) suggested that accuracy of self-perception is an important variable in the study of self-efficacy. Consequently, our investigation was designed to explore the nature of students' calibration, that is, the extent to which they accurately perceived their capability to solve mathematics problems as well as the relationship between calibration and the other variables under investigation. Last, our sample included African American students. Graham (1994) acknowledged that variables such as self-efficacy are important factors in the study of motivation but noted that these factors have been too sparsely examined in either race homogeneous or race heterogeneous studies. Consequently, we investigated race differences in mathematics judgments and performance attainments.

The purpose of this study, then, was to use path analysis to test Bandura's (1986) hypotheses regarding the predictive and mediational role of self-efficacy in the area of mathematics problem-solving and in a high school setting. We sought to determine whether the confidence with which high school students approach the solving of math problems made an independent contribution to the prediction of problem-solving performance when other variables that have received extensive study in the area of mathematics--math anxiety, math background, gender--and, of primary interest in this investigation, general mental ability are part of the path model. We also tested whether self-efficacy mediates the effect of gender, prior experience, and ability on anxiety and problem-solving performance. Our primary goal was to determine whether, and to what extent, mathematics self-efficacy would maintain an effect despite the expected strong correspondence between general mental ability and performance. Guided by previous research (e.g., Hackett, 1985; Pajares & Miller, 1994) and based on social cognitive theory, a model of the relationships among the variables was hypothesized and is illustrated in Figure 1.

Method

Participants and Procedures

Participants consisted of 329 high school students from two public schools in the South (100 in grade 9, 106 in grade 10, 81 in grade 11, and 43 in grade 12; 150 girls, 180 boys; 67 African American, 258 non-Hispanic White). One of the schools was a university laboratory school that enrolls students in numbers representative of the community as regards race and SES. Instruments were group-administered in individual math classes during two class periods. During the first period, students were asked to complete the general mental ability measure. During the second class period, students first completed the self-efficacy and anxiety measures. Directions on the self-efficacy instrument informed students that they would be asked to solve the problems on which their confidence was being assessed. After these instruments were completed and collected, the math performance measure was administered.

Variables

General Mental Ability. Raven's Advanced Progressive Matrices (Raven, Court, & Raven, 1983) is a nonverbal test of general reasoning ability that has repeatedly been shown to be a good marker test of psychometric g (Jensen, 1987). The APM was group-administered with the standard instructions. Students were given 50 minutes to attempt all items, which was ample time for all to complete the instrument. Raven et al. (1983) reported a test-retest reliability coefficient of .91 over a 6-8 week interval. Internal consistency estimates for the APM are above .90.

Math Self-Efficacy. The Mathematics Confidence Scale (MCS) (Dowling, 1978) consists of 18 problems that represent three components of mathematics (arithmetic, algebra, and geometry), three levels of cognitive demand (computation, comprehension, and application), and two problem contexts (real and abstract). (Sample item: "There are three numbers. The second is twice the first and the first is one-third of the other number. Their sum is 48. Find the largest number.") Dowling reported a correlation of .57 between the MCS and the confidence scale of Fennema and Sherman's (1976) Mathematics Attitude Scales. Langenfeld and Pajares (1993) obtained a Cronbach's alpha coefficient of .91 in a factor analytic study of 520 undergraduates. The factor analysis also revealed that item #18 did not load on the scale. Consequently, that item was replaced with another geometry item from Dowling's alternate forms test of the MCS. Pajares and Miller (in press) used the MCS with a sample of 391 undergraduates and obtained a Cronbach's alpha coefficient of .90. We expanded the 5-point Likert scale to 6 points to remove the "uncertain" choice and to better determine overconfidence, underconfidence, and calibration between efficacy judgments and performance. We obtained a coefficient of .92 on the adapted instrument with our high school sample. Item-total score correlations ranged from .30 to .74.

Math anxiety. The Mathematics Anxiety Scale (MAS) (Betz, 1978) consists of 10 items--five positively worded and five negatively (Sample item: "I get really uptight during math tests") Betz reported a split-half reliability coefficient of .92. Correlations of about .70 have been reported between the MAS and the 98-item Mathematics Anxiety Rating Scale (Cooper & Robinson, 1991). Hackett and Betz (1989) obtained Cronbach's alpha values ranging from .86 to .90, and Dew, Galassi, and Galassi (1983) reported test-retest reliability of .87 over a two-week interval. Frary and Ling (1983) found that all items loaded highly on the factor they defined as math anxiety. We obtained a Cronbach's alpha coefficient of .90. The MAS was scored such that a high score is indicative of high anxiety.

Math background. Some researchers have defined high school math background as the number of years during which students take high school mathematics (e.g., Hackett & Betz, 1989). However, some students can take four years of mathematics and never progress beyond a basic course, whereas others can take two courses in one year. It seems reasonable to assume that a student who has progressed through Calculus will have developed greater knowledge of, and success with, mathematics than will a student who progressed through Algebra I in the same number of years. Consequently, we operationalized math background as the maximum math level that students had attained by the time of their participation in our study. Levels ranged from 1 (Applied Math) to 7 (Calculus). Note that our emphasis was on exposure to course content rather than on the success or failure associated with this exposure, a choice consistent with that of other researchers using this variable (Frary & Ling, 1983; Hackett & Betz, 1989; Pajares & Miller, 1994).

Overconfidence, Underconfidence, and Calibration. Similar to Pajares and Miller (1994) we defined overconfidence as marking an item 4, 5, or 6 on the self-efficacy Likert scale and then incorrectly answering the item; underconfidence was defined as marking an item 1, 2, or 3 and getting the item correct. Students also received a calibration score that reflected the number of items on which their confidence judgment and performance attainment concurred; that is, the number of items on which they expressed confidence (by marking 4, 5, or 6) and answered correctly plus the number of items on which they expressed lack of confidence (by marking 1, 2, or 3) and answered incorrectly. The calibration score was, in essence, a measure of accuracy of self-perception.

Math Problem-solving Performance. Consistent with Bandura's (1986) guidelines, the 18 problems on which performance was assessed were the same as those on which confidence was measured. Dowling (1978) reported a KR 20 reliability coefficient of .79 and a mean item difficulty of .29 on her original instrument. For an item to be included in the final instrument, several criteria had to be met: (a) percent correct between .30 and .70, (b) point biserial correlation coefficient greater than .50, (c) discrimination index greater than .40, and (d) significant corrected phi coefficient. Pajares and Miller (1994) used the problems with undergraduates but concluded that the high mean (14.1 out of 18) suggested that the instrument might be more suitable for high school students. Langenfeld and Pajares (1993) obtained a KR 20 of .87 with 520 undergraduates. We obtained a KR 20 reliability coefficient of .83 on the instrument revised in accordance with factor analytic results and with our high school sample.

Data Analysis

Path analysis techniques examine the direct and indirect effects between variables. Cook and Campbell (1979) wrote that such techniques are especially appropriate when "theoretical, empirical, and commonsense knowledge of a problem" (p. 307) provides a defensible mapping of the latent variables and their probable causal links. Path analysis is appropriate in nonexperimental designs only when hypothesized relationships have strong theoretical and empirical support. We emphasize, however, that, although path analytic procedures have the potential to explore complex directional relationships and rule out models that demonstrate a poor fit with the obtained data, they do not prove a model correct. Subsequent studies using competing models, as well as LISREL techniques to test goodness of fit, are required to further clarify and resolve theoretical tenets.

The path model tested was as follows: Gender was hypothesized to influence all variables; general mental ability mediated the influence of gender and influenced the remaining variables; high school math level mediated the influence of gender and ability and influenced the remaining variables; math self-efficacy mediated these hypothesized prior determinants on both the performance task and math anxiety; math anxiety was hypothesized to influence performance directly (see Figure 1). Like math self-efficacy, math anxiety is considered a common mechanism of personal agency in social cognitive theory and a joint predictor of math-related outcomes. However, because "math anxiety . . . is viewed as a consequence of efficacy expectations" (Hackett, 1985), we placed it accordingly in the path model. The model was fully specified; that is, a variable was hypothesized to influence all variables preceding it in the model. Moreover, no competing model was hypothesized or tested. Consequently, no tests for goodness of fit were necessary. Additional analyses were conducted to examine relationships and mean differences on variables not included in the path model but which, nonetheless, provided interesting and relevant insights.

Results

Table 1 presents the means, standard deviations, and Pearson-Product moment correlations for all variables in the study. At the time of our study, the average math level of the students in our sample was that of geometry. Math self-efficacy scores ranged from 18 to 108, the possible minimum and maximum, and averaged 81.5 on the total instrument (4.5 per item on the 6-point Likert scale). Students were largely confident about their ability to solve the problems. Anxiety scores ranged from the minimum of 10 to the maximum of 50 and averaged 25.8 (2.6 per item on the 5-point Likert scale), almost exactly the mid-point. On the general mental ability measure, the mean of 19 falls within the average range in comparison to peers of approximately the same age (Raven et al., 1983). Compared to results of a study equating the Advanced Progressive Matrices test with the nationally standardized Otis-Lennon test, the mean of the sample is equivalent to an "IQ" score of 104 (Jensen, Saccuzzo, & Larson, 1988). Also, the standard deviation of the raw scores for the sample (5.9) is comparable with that of groups of similar age (5.5) (Raven et al., 1983). Scores on the 18-item performance measure averaged 9.4 and ranged from 1 to 18.

Note that gender was significantly correlated only with anxiety. Consistent with results of recent investigations, boys and girls did not differ in ability, level, or math problem-solving performance. The low correlation with self-efficacy, however, differs from those reported by researchers investigating these variables at the college level. The magnitudes of the correlations between all other variables were also consistent with those of previous investigations.

Table 2 provides a decomposition of effects from the path analysis. The independent variables accounted for 61% of the variability in problem-solving performance, $F(5,323) = 99.1$, $p < .0001$. With the expected exception of the ability model, each of the regression models was significant. Figure 1 illustrates the fully specified model with nonsignificant paths removed. <u>Note that this is the full model and not a reduced model recomputed with nonsignificant relationships removed</u>. Figure 1 also shows the residual path coefficients (R) that represent factors affecting a specific variable but that are not measured or accounted for in the model--the square root of the unexplained variation in the dependent variable.

Path coefficients from self-efficacy, ability, anxiety, and math level to performance were significant. The key finding from the path analysis was that the direct effect of self-efficacy on performance was as strong ($\beta = .348$) as was the expected powerful effect of ability ($\beta = .321$), a finding that resolves the substantive question of the investigation. As expected, the influence of gender on performance was not significant. The strong influence of self-efficacy on anxiety ($\beta = .411$) supports the theoretical view that math anxiety is, at least in part, a by-product of efficacy perceptions.

Although boys and girls did not differ in performance, ability, math level, or even self-efficacy perceptions, girls reported higher anxiety. These results are consistent with Hackett's (1985) findings regarding gender, self-efficacy, and anxiety. Consistent with social cognitive theory, the strong direct effects of ability (β = .398) and math level (β = .198) on self-efficacy suggest that they are important sources of efficacy information. Table 3 provides an overview of direct and indirect effects on math performance and math self-efficacy. As expected, ability and math level influenced performance directly, but their indirect effects suggest that part of their influence was mediated by students' self-efficacy perceptions. The interactive effects of gender were tested and found nonsignificant.

Confidence and Performance Calibration

Past findings suggest that most students are overconfident about their capability to solve math problems (see Hackett & Betz, 1989; Pajares & Miller, 1994). Note on Table 4 that 86% of the high school students overestimated their performance, whereas only 9% underestimated. Of the 329 students in our sample, only 4 correctly predicted their responses to all 18 math problems. The magnitude of overconfidence was also much greater than that of underconfidence. Students overestimated their performance by an average of 5.5 problems and underestimated it by 1.3 problems. Using a similar test, Pajares and Miller (1994) reported that undergraduates overestimated performance by an average of only 1.91 problems and underestimated by 1.06.

Students in the underconfidence group were better calibrated; that is, their confidence judgments and performance attainments were more closely in line. For example, students in the overconfidence group erred more often than did those in the underconfidence group: the 86% that overestimated did so on 6.2 problems, whereas the 9% that underestimated did so on only 3.5 problems. Moreover, students in the underconfidence group had higher performance scores (10.6 to 9.0) and higher calibration scores (12.9 to 10.8).

The calibration scores themselves provided interesting information. For example, calibration scores significantly correlated with math level (.38) with general mental ability (.42) and especially with math performance (.67) (see Table 1). Also, students with better calibration reported higher self-efficacy and lower anxiety. There were no gender differences in calibration, but African American students had lower average calibration scores than did White students (9.6 to 11.6). Girls and boys were equally over- or underconfident, and we found no significant relationship between gender and level of confidence, χ^2 (3, N = 329) = 2.72, α = .05; observed χ^2(3, N = 329) = .09.

Race Differences

Due to the absence of clear theoretical guidelines and prior research results, the construct of race was not included in the path analysis. However, results of independent samples t-tests with alpha level adjusted using the Dunn method of planned pairwise comparisons provided some insights regarding the relationship between this construct and those in the path model (see Table 5). There were differences between African American and non-Hispanic White students across several variables. African American students had lower performance scores. However, although their self-efficacy was also lower than that of Whites, African American students were more overconfident about their math capability. The combination of lower performance and higher self-efficacy meant that the calibration score of African American students was substantially lower than that of White students. High confidence judgments despite lower performance also meant that African American students were overconfident on a greater number of items. African American students were less confident in their correct answers than were White students, but in approximately similar proportions to their respective performance scores. There were no differences in anxiety and no interactive effects of gender and race.

On the strength of these results, a path model was tested with race as an exogenous variable hypothesized to influence all endogenous variables in the previous path model. The regression model with performance as the outcome measure showed only a 1% increase in R^2 (.62), suggesting that the effects of race were mediated largely by ability scores. The R^2 for the path models with self-efficacy, anxiety, or level as the dependent variables did not increase; R^2 for the ability model increased from .00 to .10.

Discussion

The primary aim of this study was to discover whether the self-efficacy beliefs of high school students about their mathematical problem-solving played the predictive and mediational roles ascribed to them by social cognitive theory when general mental ability was controlled. To this end, we used path analysis techniques and constructed a path model that included math self-efficacy, general mental ability, math anxiety, previous background in mathematics, and gender, with relationships hypothesized from prior research findings and social cognitive theory. Our results showed that students' self-efficacy beliefs about their math capability had strong direct effects on math anxiety and on mathematics problem-solving performance even when general mental ability was controlled. These are striking findings in light of the particularly stringent test of the influence of self-efficacy that inclusion of a general mental ability measure in the path model provides in an investigation of mathematics problem-solving (see Zimmermann, Bandura, & Martinez-Pons, 1992). In addition, self-efficacy partially mediated the effect of ability and math background both on anxiety and performance scores.

Also, although students' math anxiety correlated significantly with problem-solving performance, results indicated that the influence of anxiety was primarily a result of noncausal covariation largely due to the effect of self-efficacy. Similar findings have been reported by researchers exploring mathematics and other academic areas (see Alexander & Martray, 1989; Hackett, 1985; Hackett & Betz, 1989; Pajares & Johnson, 1994; Pajares & Miller, 1994).

We found also that the high school students in our sample were even more overconfident about their mathematical capabilities and more poorly calibrated than were college undergraduates in previous investigations (see Hackett, 1985; Pajares & Miller, 1994). It may be, as Bandura (1986) suggested, that individuals progressively become more accurate in appraising their own abilities. Another explanation may be that metacognitive capabilities such as calibration are related to ability and to competence in specific academic areas. Indeed, we found that students in higher math levels were better calibrated and that there were strong correlations between calibration and both ability and performance. In any case, it is reasonable that college students, who represent the highest academic strata of the student population, either possess or develop more accurate self-perceptions than do high school students.

As regards gender differences, our results are consistent with recent findings reporting small or no gender differences in mathematics performances or confidence judgments at the high school level (Hyde, Fennema, Ryan, Frost, & Hopp, 1990). Despite equal general mental ability and mathematics competence, however, girls in our sample reported higher math anxiety, suggesting that factors are still at work in decreasing certain mathematics self-beliefs of young women. There were no differences in level of under- or overconfidence between girls and boys. Not only were girls as overconfident as boys, but they were overconfident to the same degree. There was also no gender difference in calibration. Our results differ from those of Lundeberg, Fox, and Punĉochaŕ (1994), who reported that undergraduate men tended to be more confident in their incorrect answers than were women. This may be because the researchers conducted their study at the college level and in psychology classes. It may be that differences are particular to a college population or that differences develop as students get older.

Additional results showed that the math self-efficacy of African American students was substantially lower than that of their peers. Differences in level and performance demonstrated that a sizeable proportion of these students also had weaker mathematics skills than did White students. We emphasize, however, that, despite differences across these constructs, African American students reported above average confidence judgments. In fact, after controlling for lower performance attainments, African American students were overconfident on a greater number of items than were White students, and they were also more confident on items they did not solve correctly. These findings are consistent with Graham's (1994) summary of the literature on self-perceptions of ability of African American students--that they "maintain undaunted optimism and positive self-regard even in the face of achievement failure" (p. 103). Our finding that African American students were more poorly calibrated suggests that self-efficacy research focusing on this metacognitive process would be beneficial.

Some self-efficacy researchers have suggested that teachers would be well served by paying as much attention to students' perceptions of competence as to actual competence, for it is the perceptions that may more accurately predict students' motivation and future academic choices (see Hackett & Betz, 1989). Clearly, self-efficacy is a strong predictor of academic performances, and assessing students' self-efficacy can provide teachers with important insights. For example, researchers have demonstrated that self-efficacy beliefs strongly influence the choice of majors and career decisions of college students (see Hackett, 1985). In many cases, inaccurate perceptions of mathematics capability, not lack of capability or skill, are responsible for avoidance of math-related courses and careers. If this is so, efforts to identify and alter inaccurate judgments should prove beneficial. And, if math self-efficacy beliefs are a primary cause of math anxiety, then interventions designed to improve skills by decreasing anxiety may be useful to the extent that they increase students' confidence in their capability.

Bandura (1986) argued that some overestimation of capability is useful because it increases effort and persistence. Although most students in our study overestimated their competence, some underestimated it. Students who lack confidence in skills they possess are not likely to engage in tasks in which those skills are required, and they will more quickly give up in the face of difficulty. We find it regrettable, for example, that the girls in our sample were as capable as the boys but, nonetheless, reported higher anxiety. We wonder if this is the case across academic areas at the high school level, and, if it is, why this difference should exist in the face of equal ability and performance. We recommend that our study be replicated at lower academic levels, especially those in which these sorts of self-beliefs begin to be created.

But how much confidence is too much confidence? The vast number of students we tested demonstrated strong confidence in their ability to solve mathematics problems, but this confidence was not matched by reciprocal competence. Bandura (1986) argued that successful functioning is best served by reasonably accurate efficacy appraisals, although the most functional efficacy judgments are those that slightly exceed what one can actually accomplish. The efficacy perceptions our students reported were neither reasonably accurate nor slightly excessive. Nonetheless, we are uncertain as to when overconfidence may be characterized as excessive and maladaptive. Bandura (1986) also argued that "the stronger the perceived self-efficacy, the more likely are persons to select challenging tasks, the longer they persist at them, and the more likely they are to perform them successfully" (p. 397). We wonder if the strong self-efficacy that most students demonstrated ultimately results in these benefits. We strongly discourage efforts to lower students' efficacy percepts, but we see value in developing intervention strategies and instructional techniques aimed at helping students develop more accurate self-appraisals rather than raising already overconfident beliefs. Improving students' calibration will require helping them to better understand what they know and do not know so that they may more effectively deploy appropriate cognitive strategies during the problem-solving process, but the challenge is to accomplish this without lowering their confidence and optimism. Additional research should be aimed at exploring the nature of the relationship between efficacy judgments and calibration. Insights from other quarters of cognitive research may be useful in this regard (e.g., Bransford, 1979, 1992; Duffy & Roehler, 1989; Glenberg, Sanocki, Epstein, & Morris, 1987; Pressley, Borkowski, & Schneider, 1989).

We recommend also that future path models include other common mechanisms of personal agency--for example, prerequisite prior knowledge and knowledge of strategies to solve problems. Additional studies might explore the relationship between self-efficacy judgments and attributions of success and failure, as well as the relationship between self-efficacy and variables such as goal setting and attainment (see Zimmerman et al., 1992). Our model with self-efficacy as the dependent variable accounted for only 27% of the variance. Consequently, research is needed regarding prior determinants and sources of efficacy information other than ability or level placement as well as how perceptions of efficacy mediate the influence of these sources on subsequent performance. We again remind the reader that causal modeling analyses on which inferences of causality are made are not without controversy (see Freedman, 1987), and, although they provide the most powerful statistical tools we presently have with which to explore the nature of causal relationships in nonexperimental studies (see Bentler, 1987; Duncan, 1975; Grusec & Lytton, 1988) they are at the mercy of the relationships hypothesized to exist before the model is constructed. As such, they reflect the theoretical orientation that undergirds a study and the researcher's interpretation of the theoretical directional interplay among the variables. These interpretations must be made carefully and modestly. Quantitative efforts should be complemented by qualitative studies aimed at exploring how efficacy beliefs are developed and how students perceive that these beliefs influence their academic attainments and the academic choices that they make.

Findings from this study strengthen Bandura's (1986) claim that self-efficacy beliefs play an influential role in human agency. They also support the work of prior investigators reporting a significant relationship between self-efficacy and related academic performances. The clear implication that arises is that researchers and school practitioners should be looking to students' beliefs about their academic capabilities as important predictors of other affective variables and of academic performances and that efforts should be made to identify these beliefs and, where necessary and feasible, alter them, for they are important components of motivation and behavior (Schunk, 1989, 1991).

References

Bandura, A. (1984). Recycling misconceptions of perceived self-efficacy. Cognitive Therapy and Research, 8, 231-255.

Bandura, A. (1986). Social foundations of thought and action: A social cognitive theory. Englewood Cliffs, NJ: Prentice Hall.

Bentler, P. M. (1987). Structural modeling and the scientific method: Comments of Freedman's critique. Journal of Educational Statistics, 12, 151-157.

Betz, N. E. (1978). Prevalence, distribution, and correlates of math anxiety in college students. Journal of Counseling Psychology, 25, 441-448.

Betz, N. E., & Hackett, G. (1983). The relationship of mathematics self-efficacy expectations to the selection of science-based college majors. Journal of Vocational Behavior, 23, 329-345.

Bransford, J. D. (1979). Human cognition: Learning, understanding, and remembering. Belmont, CA: Wadsworth.

Bransford, J. D. (1992). The IDEAL problem solver. Belmont, CA: Wadsworth.

Carroll, J. B. (1993). Human cognitive abilities: A survey of factor-analytic studies. New York, NY: Cambridge University Press.

Collins, J. L. (1982, March). Self-efficacy and ability in achievement behavior. Paper presented at the meeting of the American Educational Research Association, New York.

Cook, T. D., & Campbell, D. T. (1979). Quasi-experimentation: Design & analysis issues for field settings. Boston: Houghton Mifflin.

Cooper, S. E., & Robinson, D. A. G. (1991). The relationship of mathematics self-efficacy beliefs to mathematics anxiety and performance. Measurement and Evaluation in Counseling and Development, 24(April), 4-11.

Dew, K. M. H., Galassi, J. P., & Galassi, M. D. (1983). Mathematics anxiety: Some basic issues. Journal of Counseling Psychology, 30, 443-446.

Dew, K. M. H., Galassi, J. P., & Galassi, M. D. (1984). Math anxiety: Relations with situational test anxiety, performance, physiological arousal, and math avoidance behavior. Journal of Counseling Psychology, 31, 580-583.

Dowling, D. M. (1978). The development of a mathematics confidence scale and its application in the study of confidence in women college students. Unpublished doctoral dissertation, Ohio State University, Columbus.

Duffy, G. G., & Roehler, L. R. (1989). Why strategy instruction is difficult and what we need to do about it. In C. B. McCormick, G. E. Miller & M. Pressley (Eds.), Cognitive strategy research: From basic research to educational applications (pp. 133-154). New York: Springer-Verlag.

Duncan, O. D. (1975). Introduction to structural equation models. New York: Academic Press.

Feingold, A. (1988). Cognitive gender differences are disappearing. American Psychologist, 43, 95-103.

Fennema, E., & Sherman, J. A. (1978). Sex-related differences in mathematics achievement and related factors: A further study. Journal for Research in Mathematics Education, 9, 189-203.

Frary, R. B., & Ling, J. L. (1983). A factor-analytic study of mathematics anxiety. Educational and Psychological Measurement, 43, 985-993.

Freedman, D. A. (1987). As others see us: A case study of path analysis. Journal of Educational Statistics, 12, 101-128.

Glenberg, A., Sanocki, T., Epstein, W., & Morris, C. (1987). Enhancing calibration of comprehension. Journal of Experimental Psychology: General, 116, 119-136.

Graham, S. (1994). Motivation in African Americans. Review of Educational Research, 64, 55-118.

Grusec, J. E., & Lytton, H. (1988). Social development: History, theory, and research. New York: Springer-Verlag.

Hackett, G. (1985). The role of mathematics self-efficacy in the choice of math-related majors of college women and men: A path analysis. Journal of Counseling Psychology, 32, 47-56.

Hackett, G., & Betz, N. E. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. Journal for Research in Mathematics Education, 20, 261-273.

Hyde, J. S., Fennema, E., Ryan, M., Frost, L. A., & Hopp, C. (1990). Gender comparisons of mathematics attitudes and affect: A meta-analysis. Psychology of Women Quarterly, 14, 299-324.

Hunter, J. E. (1986). Cognitive ability, cognitive aptitudes, job knowledge, and job performance. Journal of Vocational Behavior, 29, 340-362.

Jensen, A. R. (1984). Test validity: g versus the specificity doctrine. Journal of Social and Biological structures, 29, 93-118.

Jensen, A. R. (1987). The g beyond factor analysis. In J. C. Conoley, J. A. Glover, & R. R. Ronning (Eds.), The influence of cognitive psychology on testing and measurement. Hillsdale, NJ: Erlbaum.

Langenfeld, T. E., & Pajares. M. F. (1992). [Relationship of math self-efficacy to math-related constructs and internal consistency of MSES with 5-point Likert scale]. Unpublished raw data.

Lapan. R. T., Boggs. K. R., & Morrill, W. H. (1989). Self-efficacy as a mediator of investigative and realistic general occupational themes on the Strong-Campbell Interest Inventory. Journal of Counseling Psychology, 36. 176-182.

Lent, R. W., Lopez. F. G., & Bieschke. K. J. (1991). Mathematics self-efficacy: Sources and relation to science-based career choice. Journal of Counseling Psychology, 38. 424-430.

Llabre. M. M., & Suarez. E. (1985). Predicting math anxiety and course performance in college women and men. Journal of Counseling Psychology, 32, 283-287.

Maddux, J. E., Norton, L. W., & Stoltenberg, C. D. (1986). Self-efficacy expectancy, outcome expectancy, and outcome value: Relative effects on behavioral intentions. Journal of Personality and Social Psychology, 51, 783-789.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment and performance in mathematics. Journal of Educational Psychology, 82, 60-70.

Multon, K. D., Brown, S. D., & Lent, R. W. (1991). Relation of self-efficacy beliefs to academic outcomes: A meta-analytic investigation. Journal of Counseling Psychology, 38, 30-38.

Nicholls, J. G. (1990). What is ability and why are we mindful of it? A developmental perspective. In R. Sternberg & J. Kolligian (Eds.). Competence considered. New Haven, CT: Yale University Press.

Pajares, F., & Johnson, M. J. (1994). Confidence and competence in writing: The role of self-efficacy, outcome expectancy, and apprehension. Research in the Teaching of English, 28, 313-331.

Pajares, F., & Miller, M. D. (1994). The role of self-efficacy and self-concept beliefs in mathematical problem-solving: A path analysis. Journal of Educational Psychology, 86, 193-203.

Pajares, F., & Miller, M. D. (in press). Mathematics self-efficacy and mathematics outcomes: The need for specificity of assessment. Journal of Counseling Psychology.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. Journal of Educational Psychology, 82, 33-40.

Pressley, M., Borkowski, J. G., & Schneider, W. (1987). Cognitive strategies: Good strategy users coordinate metacognition and knowledge. In R. Vasta & G. Whitehurst (Eds.), Annals of Child Development (Vol. 5, pp. 89-129). New York: JAI Press.

Raven, J. C., Court, J. H., & Raven J. (1983). Manual for Raven's progressive matrices and vocabulary scales. New York: Psychological Corp.

Reyes, L. H. (1984). Affective variables and mathematics education. The Elementary School Journal, 84, 558-581.

Schunk, D. H. (1989). Self-efficacy and achievement behaviors. Educational Psychology Review, 1, 173-208.

Schunk, D. H. (1991). Self-efficacy and academic motivation. Educational Psychologist, 26, 207-231.

Schwarzer, R., Seipp, B., & Schwarzer, C. (1989). Mathematics performance and anxiety: A meta-analysis. In R. Schwarzer, H. M. Van Der Ploeg, & C. D. Spielberger (Eds.), Advances in test anxiety research (Vol. 6, pp. 105-119). Berwyn, PA: Swets North America.

Siegel, R. G., Galassi, J. P., & Ware, W. B. (1985). A comparison of two models for predicting mathematics performance: Social learning versus math aptitude-anxiety. Journal of Counseling Psychology, 32, 531-538.

Thorndike, R. L. (1984). Intelligence as information processing: The mind and the computer. Bloomington, IL: Center on Evaluation, Development, and Research.

Thorndike, R. L. (1986). The role of general ability in prediction. Journal of Vocational Behavior, 29, 332-329.

Zimmerman, B. J., Bandura, A., & Martinez-Pons, M. (1992). Self-motivation for academic attainments: The role of self-efficacy beliefs and personal goal setting. American Educational Research Journal, 29, 663-676.

Author Note

This research was supported by a grant from the Florida Educational Research Council, Inc The authors wish to thank Dr. Wes Corbett and Gio Valiante for their kind assistance with data gathering.

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Table 1

Means, Standard Deviations, and Zero-order Correlations for Variables in the Study.

Variable	M	SD	1	2	3	4	5	6	7
1. GENDER	--	--	--						
2. RACE	--	--	.02	--					
3. RAVEN	19.0	5.9	.04	.31***	--				
4. HSL	5.0	1.4	-.03	.15**	.40***	--			
5. MAS	25.8	8.7	-.28***	-.12*	-.43***	-.16**	--		
6. MSE	81.5	16.3	.10	.21**	.48***	.35***	-.53***	--	
7. PERF	9.4	4.3	.02	.33***	.63***	.52***	-.46***	.64***	--
8. CAL	11.2	3.1	.04	.25***	.42***	.38***	-.26***	.17**	.67***

Note: RAVEN = General Mental Ability; HSL = High school level; MAS = Math anxiety; MSE = Math self-efficacy; PERF = Math problem-solving performance; CAL = Calibration score

* $p < .05$. ** $p < .001$. *** $p < .0001$. N = 329

Table 2

Decomposition of Effects from the Path Analysis

Effect	(Intercept) Parameter Estimate	Standardized Estimate	t	R^2
On general mental ability	(18.75)			.00
of gender	0.49	.041	0.75	
On high school level	(3.29)			.16
of gender	-0.13	-.048	-0.95	
of general mental ability	0.09	.398	7.82***	
On math self-efficacy	(47.58)			.27
of gender	2.95	.090	1.90	
of general mental ability	1.09	.398	7.69***	
of high school level	0.32	.198	3.83**	
On math anxiety	(50.56)			.38
of gender	-3.95	-.226	-5.11***	
of general mental ability	-0.37	-.253	-4.85**	
of high school level	0.50	.079	1.62	
of math self-efficacy	-0.22	-.411	-8.01***	
On math performance	(-4.73)			.61
of gender	-0.49	-.056	-1.54	
of general mental ability	0.23	.321	7.48***	
of high school level	0.77	.247	6.31***	
of math self-efficacy	0.09	.348	7.78***	
of math anxiety	-0.06	-.113	-2.56**	

* $p < .05$, ** $p < .01$, *** $p < .0001$, N = 329

Figure Caption

Figure 1. Hypothesized path model representing relationships among variables predicting mathematics problem-solving performance.

Table 3.

Direct and Indirect Effects on Math Self-Efficacy and Mathematics Problem-solving Performance

Effect	r	Direct Effect	Indirect Effect	Total Effect	Noncausal Covariation
On math performance					
of math anxiety	-.460*	-.113*	.000	-.113*	-.347
of math self-efficacy	.643*	.348*	.046	.394*	.249
of high school level	.516*	.247*	.069	.316*	.200
of general mental ability	.632*	.321*	.306*	.627*	.005
of gender	.016	-.056	.083	.027	-.010
On math self-efficacy					
of high school level	.352*	.198*	.000	.198*	.154
of general mental ability	.480*	.398*	.079	.477*	.003
of gender	.101	.090	.010	.100*	.001

* $p < .05$

Table 4

Overconfidence, Underconfidence, and Congruence

	Girls	%	Boys	%	Total	%
Overconfidence	130	86	153	85	283	86
Underconfidence	11	7	18	10	29	9
Congruence--no errors	1	1	3	2	4	1
Congruence--equal errors	8	5	5	3	13	4

	Girls		Boys		Total	
	M	SD	M	SD	M	SD
Calibration score	11.01	2.75	11.30	3.33	11.19	3.08
Items overconfident	5.50	2.88	5.58	3.36	5.54	3.15
Items underconfident	1.43	1.39	1.13	1.39	1.27	1.40
Items confident and correct	7.88	4.26	8.33	4.98	8.13	4.66

Note: No differences were significant

Table 5

Mean Race differences in Math-Related Constructs

Variable	African American	Non-Hispanic White	Diff	*t*	Prob>|*t*|
High School Level	4.6	5.1	0.5	-2.30	.0234
Math Anxiety	27.9	25.3	-2.6	2.59	.0108
Math Self-Efficacy	74.9	83.2	8.3	-4.09*	.0001
Math Performance	6.5	10.1	3.6	-7.14*	.0001
Calibration	9.8	11.6	1.8	-4.19*	.0001
Items Underconfident	1.6	1.2	-0.4	2.05	.0435
Items Overconfident	6.8	5.2	-1.6	3.37*	.0010
Items Confident and Correct	5.0	8.9	3.9	-7.43*	.0001

$p < .005$

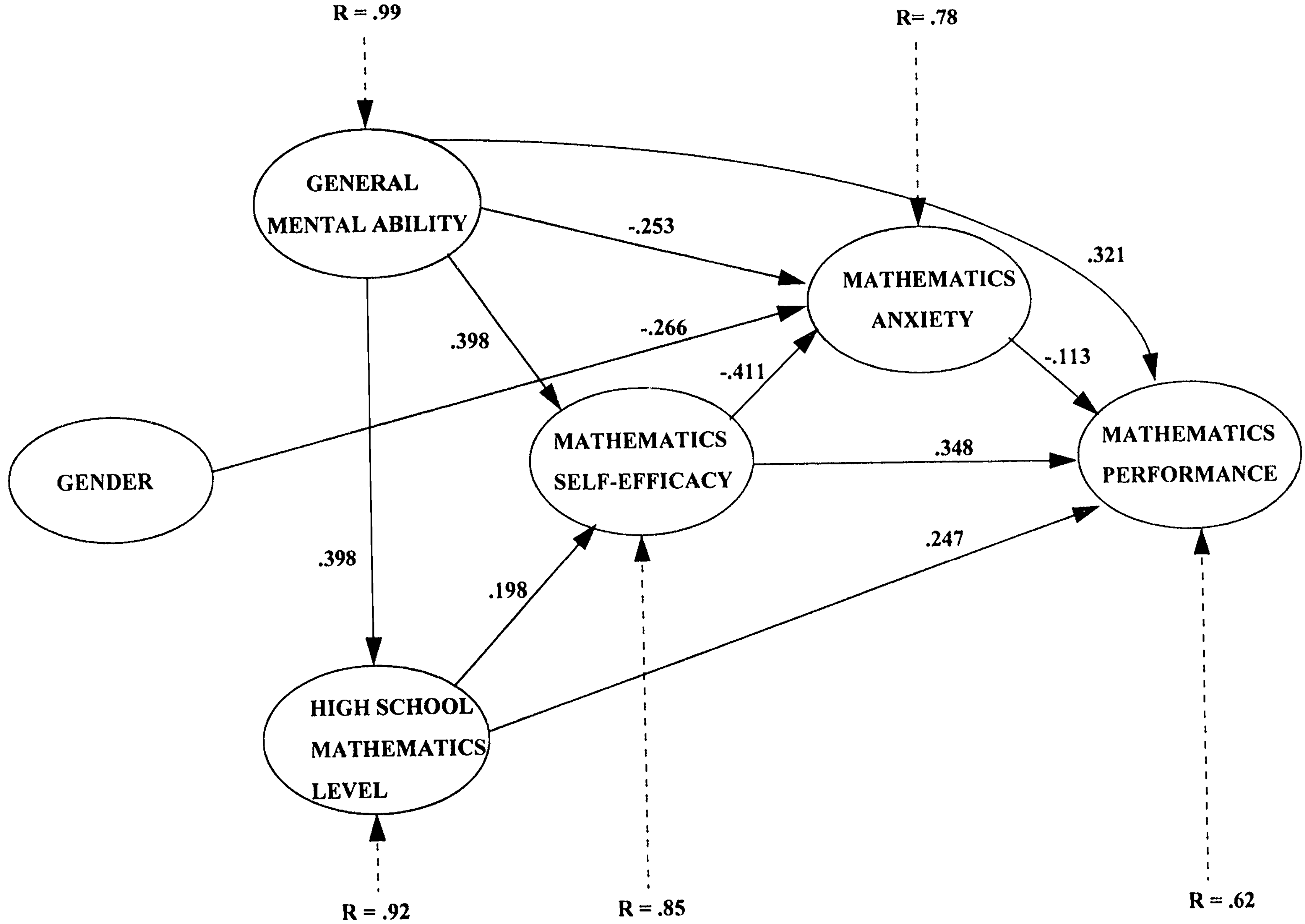
R = .99
R= .78
GENERAL
MENTAL ABILITY
-.253
.321
MATHEMATICS
ANXIETY
.398
-.266
-.411
-.113
GENDER
MATHEMATICS
SELF-EFFICACY
.348
MATHEMATICS
PERFORMANCE
.398
.198
.247
HIGH SCHOOL
MATHEMATICS
LEVEL
R = .92
R = .85
R = .62

Table 1.

Means, Standard Deviations, and Zero-order Correlations of Variables in the Path Analysis

Variable	M	SD	GENDER	HSL	CC	USE	MSC	MAS	MSE
HSL	4.9	1.2	.11*						
CC	10.3	6.0	-.07	.15**					
USE	50.9	15.2	.05	.12*	.06				
MSC	49.7	16.6	.13*	.48***	.25***	.40***			
MAS	31.8	10.9	.15**	.44***	.20***	.32***	.87***		
MSE	73.6	10.5	.24***	.47***	.23***	.19***	.61***	.56***	
PERF	14.1	2.8	.17***	.44***	.23***	.14**	.54***	.51***	.70***

HSL = High school level; CC = College credits earned; MSE = Math self-efficacy; MSC = Math self-concept; MAS = Math anxiety; USE = Perceived usefulness of mathematics; PERF = Math problem-solving performance.

* $p < .05$ ** $p < .01$ *** $p < .001$