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ABSTRACT

This volume is a teacher's edition in a series of books that contain open-ended exploration activities and experiments. These activities allow and encourage students to set their own goals, use their own creativity and ideas, investigate the wonders of nature, learn about the workings of real businesses, and draw conclusions from their investigations of these real-life situations. Students participate in many explorations by first making things such as an abacus or a three-dimensional model and then using their creations to complete the exploration or investigation. The 40 activities are grouped under the National Council of Teachers of Mathematics' (NCTM) Curriculum Standards of: communications, spatial sense, measurement, number sense, connections, estimation, and reasoning. Some activities include: Fingerprint Fun: Comparing Characteristics; The Ambitious Architect: Building with Different Shapes; Boat Races: Finding the Formula for Speed; I Forgot Your Birthday: Using the Calculator; Patterns in Art and Other Places: Exploring Tessellations; Twinkle, Twinkle, Many Stars: Estimating How Many Stars Are in the Sky; and It Is, Or Isn't It? Identifying Figures from Characteristics. Each activity contains a teacher's guide that lists: goal, student objectives, guide to the investigation, and vocabulary, along with a description of the activity that includes: an introduction, purpose, materials needed, procedures, observations, conclusions, and suggestions for further study. (MKR)

GREAT EXPLORATIONS IN MATHEMATICS

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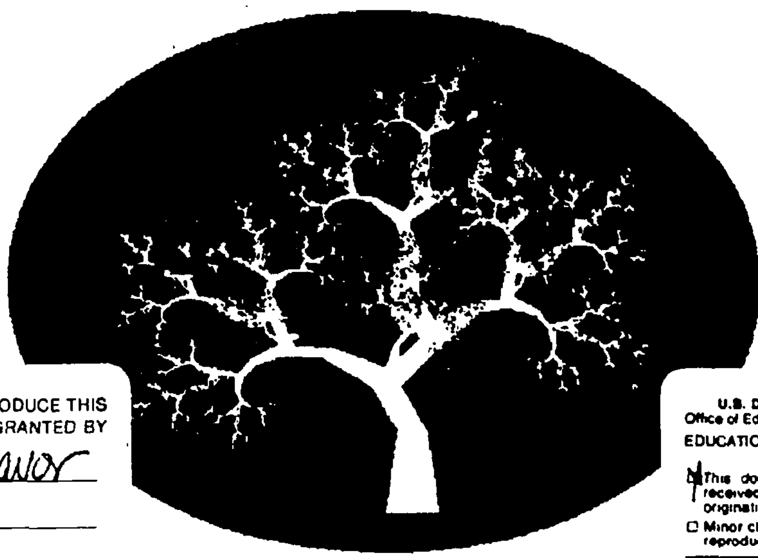

GRADES 5-8

TEACHER'S EDITION

Great Explorations in Mathematics

Grades 5-8
Teacher's Edition

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TABLE OF CONTENTS

MEASUREMENT

NUMBER SENSE

CONNECTIONS

ESTIMATION

REASONING

Introduction to
Great Explorations in Mathematics
Grades 5-8

How to Use The Book

For each of the forty explorations in GEM 5-8, there is a Teacher's Guide, two or more Student Activity pages, and almost always, a Response Sheet.

Teacher's Guides include a statement of overall goals of the exploration, student objectives, necessary vocabulary, and suggestions for ways to introduce the exploration. You will also find suggestions about implementing the exploration by having the students work alone, in pairs, or in small groups, and there are some suggestions for further study. In addition, a list of materials needed for the exploration is included. For the most part these materials are inexpensive and readily available. You can, of course, make substitutions.

Student activity pages include a short introduction which is intended to capture the students' attention. The purposes of the activity are stated in the form of questions and are restatements of the student objectives from the Teacher's Guide. Materials are listed before the Procedures Section, where the students will find step-by-step procedures for carrying out the exploration. When they have completed the procedures and recorded results on the Response Sheet(s), the students are asked questions which allow them to describe their observations. Then they record any generalizations they reach or conclusions they draw. Finally, students will be offered one or more suggestions for further study. These activities either extend and reinforce what they have experienced in the exploration, ask them to create their own exploration, or suggest ways to experience the concepts they have learned in other content areas.

Great Explorations in Mathematics is just that - **Explorations!** This is one of a series of books which cover all grades from K through 12. Each of the forty activities in this volume, Grades 5-8, contains open-ended explorations and experiments. These are not activities with right and wrong answers — virtually every student can experience the joy of exploration and discovery without fear of getting "the wrong answer" and being embarrassed.

Activities in this book allow and encourage students to set their own goals, to use their own creativity and ideas, to investigate the wonders of nature, to learn about the workings of real businesses, and to draw conclusions from their investigations of these real life situations. Students explore such phenomena as determining the room temperature by counting the number of times a cricket makes its distinctive noises. They "race boats" to find which "fuels" cause boats to travel faster and for longer distances. And they get a feeling of accomplishment by making almost all of the materials they use to do these activities.

You are involved in the investigations to provide the basic materials and a supportive classroom where students can investigate freely these fascinating concepts. Some explorations are to be completed in one class period, while others require several. Some cannot be done in school or during traditional school hours. Some explorations require observations made and recorded over a two-week period.

The discussions you lead or moderate after each exploration provide excellent opportunities to evaluate the effectiveness of each project as well as help your class develop a great spirit of cooperative achievement. There is no intent that a formal evaluation be made in the sense that a student would be assigned a letter or numerical grade.

An examination of the Table of Contents reflects the focus of the *Curriculum Evaluation Standards for School Mathematics* (1989) of the National Council of Teachers of Mathematics: Each "chapter" title - Communications, Spatial Sense, Measurement, Number Sense, Connections, Estimation, and Reasoning - is the main focus of one of the K-8 Standards.

About the Authors

Richard W. Dyches is director of Test Development for National Computer Systems in Minneapolis, Minnesota where he is involved nationally with authentic assessment. A former elementary teacher and college professor, Dr. Dyches is actively involved in the National Council of Teachers of Mathematics and is a frequent speaker at workshops and conferences. Dr. Dyches attended the University of Alabama at Birmingham and Rutgers University and holds a doctoral degree in education

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COMMUNICATIONS

Teacher's Guide
Man in the Moon

GOAL: To introduce students to the concept of a bell-shaped curve through exploring the variance of human eyesight.

STUDENT OBJECTIVES:

✓ To measure the variance of human eyesight.

✓ To plot measurements on a graph.

GUIDE TO THE INVESTIGATION: This investigation can be conducted by pairs of students or in small groups of 3 or 4 students. Each student will need paper, string or yarn, and a metric ruler.

Show a model or a picture which illustrates the parts of the eye. Discuss the eyelids, tears, eyeball, white of the eye, cornea, pupil, and iris. You may want to have references available for students who express an interest in further study of the eye.

Students should follow the directions in the Procedures section of their activities sheets. Pairs of students may use the string or yarn to find the distance from the eyes to where the man "jumps" into the moon. Demonstrate what "tally marks" are and how they are used. At the end of the exploration, discuss the Observations and Conclusions.

VOCABULARY: variance, data, tally marks, bar graph, bell-shaped curve, images, optical illusion

Man in the Moon

INTRODUCTION: Human eyesight varies. Have you ever seen something at a distance before your friend or parent could see it? In this experiment you will explore the variance in human eyesight and illustrate your findings on a bar graph. You will decide whether the data you collect forms any sort of pattern when placed on a graph.

PURPOSE:

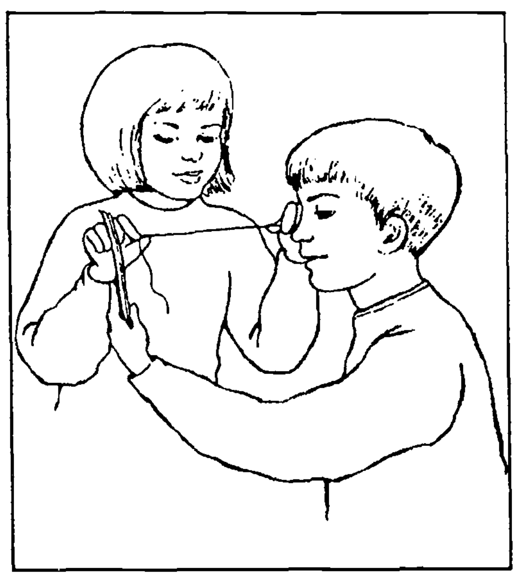
- ✓ Do we each see images differently with our eyes?
- ✓ As distance is increased or decreased, how do pictures we see blend together?
- ✓ What is an optical illusion?

MATERIALS:

paper
metric ruler
string or yarn

PROCEDURES:

1. Work with a partner. Each of you should trace or make a copy of the illustration shown on the Response Sheet.
2. Sit up straight and hold your copy at arm's length. Focus on the black dot between the "face" and the "moon", and stare at it.
3. Slowly bring the paper toward your eyes as you concentrate on the black dot. At a certain point it will seem that the face is inside the moon.
4. Stop right there, and do not move your eyes or your hand with the picture in it.
5. Have a friend use a piece of string to measure how far your eyes are from the "man in the moon". (See illustration 1-1.)


1 - 1

6. Once the distance is marked with the string, you can measure it with the metric ruler. Record this distance on your Response Sheet.
7. Have your partner repeat the same procedures. This is an example of an optical illusion. Even though you know that the face is not really inside the moon, for a moment it looked as though it were. The exact distance at which the face appears to jump inside the moon varies from person to person.
8. Get at least forty-nine other students to do this experiment. For each one of them, measure and record the distance at which they first say that the man "jumps into" the moon. You need not keep a record of each student's name—just the distance.
9. Record your observations on the table on the Response Sheet using tally marks. You should have your own and forty-nine others, for a total of fifty.
10. Use the data you recorded to make a bar graph that shows the number of students who saw the illusion at a distance "less than average", "at average", and "greater than average".

OBSERVATIONS:

1. What is the shortest distance you measured? *Answers vary.*
2. What is the longest distance? *Answers vary.*
3. What is the average distance? *Answers vary.*
4. How many students saw the illusion at the average distance? *Answers vary.*
5. What is the range between the shortest distance and the longest distance?
   Answers vary.
6. Does your data suggest a bell-shaped curve?
   Answers vary. It should be close to a bell-shaped curve.

CONCLUSIONS:

1. What happens to an image when it is moved closer to the eye?
   Answers vary. Possible answers may be: It appears to blur; change.
2. Did any students see the man jump into the moon exactly at the average distance? If so, how many? *Answers vary.*
3. How can you explain an optical illusion? *Answers vary.*

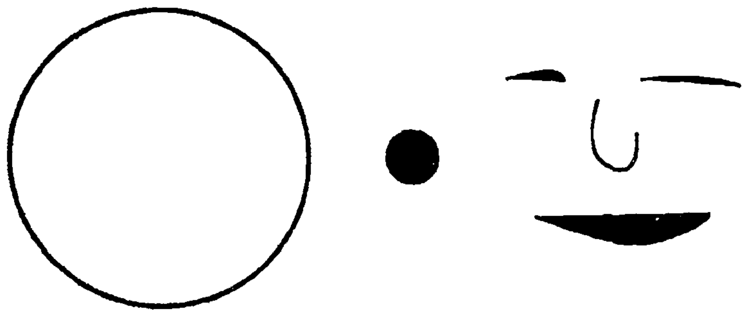
4. What conclusions or generalizations can you make about the variance in the ranges of eyesight in large groups of individuals?

   Answers vary. Possible answer may be: the more people "tested", the closer to a bell-shaped curve the results will be.

SUGGESTIONS FOR FURTHER STUDY

- Bar graphs may be used to show other variances which occur in nature. Gather leaves from a safe, non toxic plant. Measure the length of each leaf. Record the numbers on a table and then graph the results. Does the resulting graph resemble the "eyesight" graph?
- Try this same activity with sea shells. How were the results similar or different?

Response Sheet
Man in the Moon



1. Use tally marks to record for yourself and 49 other students the distance (from the face to the paper) where the face appears to "jump" inside the moon.

	Answers vary.			

2. Write the number of centimeters. Count the tally marks and record the frequency on the table.

Distance in centimeters	Number at the distance
________ cm	*All answers vary.*
________ cm	
________ cm	
________ cm	
________ cm	
________ cm	
________ cm	
________ cm	
________ cm	
________ cm	

3. Complete a bar graph here that illustrates your results.

Bar graph will vary depending on information on the frequency table.

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Teacher's Guide
Bubble, Bubble, Toil-But No Trouble

GOAL: To have students explore a way to plot the growth of yeast which are visible only with a microscope and to learn to record data in the form of a line graph.

STUDENT OBJECTIVES:

- ✓ To explore the growth of yeast plants and the conditions which effect their growth.
- ✓ To make a line graph to show the growth of yeast plants over a period of one or two days.

GUIDE TO THE INVESTIGATION: This activity is best conducted by small groups of four to six students. Each student will need three packages of yeast, a quart bottle, one-fourth cup of syrup or grape juice, a tape measure, a balloon, warm tap water, and a rubber band.

Explain to students that yeast plants are tiny one-celled fungi and that they are too small to be seen by the naked eye. When conditions are favorable, they usually reproduce by a way called budding. Since they are microscopic in size, their growth activity cannot be seen by the unaided eye. You may also want to discuss that yeast is used in baking bread and is a valuable source of Vitamin B in the human diet.

Students should follow the instructions in the Procedures section of their activity sheet. Each group should record the diameter of the balloon at half-hour intervals and record the data in a table from which they can plot the points on a line graph. After one or two days of recording and plotting growth, discuss the results. Have different groups compare their results and suggest reasons for any differences observed. When the exploration is completed, discuss the Observations and Conclusions.

VOCABULARY: diameter, yeast, line graph, solution

Bubble, Bubble, Toil-But No Trouble

INTRODUCTION: A package of dry yeast actually contains thousands of living plants. They are not the kind of green plants with leaves and flowers which you may first think about when you hear the word "plant". These plants are far too tiny to be seen by the human eye without a powerful microscope. As these plants live and grow, they give off a gas. You are going to "plant" some yeast plants in bottles and find a way to measure their growth even though the yeast plants will be too small for you to see.

PURPOSE:

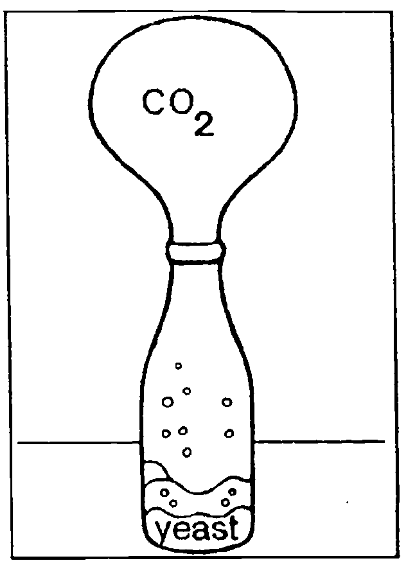
- ✓ How can you tell that your yeast plants are growing?
- ✓ What scientific principles are involved in making your observations?
- ✓ Why is a line graph a good way to display information about the growth of yeast plants?

MATERIALS:

dry yeast (three packages)
a quart bottle
tape measure
syrup or grape juice (one-fourth cup)
warm tap water
balloon
rubber band

PROCEDURES:

1. Begin this activity early in the day because you will have to take measurements and record over a period of time.
2. Make a line graph similar to the one shown on the Response Sheet.
3. Add three packages of dry yeast to one cup of warm tap water. Gently and thoroughly mix the warm water and yeast. Add the "plant food" - syrup or juice - and gently swirl the mixture again.
4. Put the yeast, water, and syrup solution in the bottle, and stretch the balloon over the mouth of the bottle. Use the rubber band to secure the balloon in place. Measure the



2-1

diameter of the balloon and write the time and measurement on the Response Sheet.

5. Place the bottle in a warm area. Every half-hour, measure the diameter of the balloon and record the measurement on your table. After each measurement gently shake the bottle to distribute the "plant food".
6. Record your measurements on a line graph on the Response Sheet. Observe and graph the data at half-hour intervals for ten hours.
7. Continue observing the bottle every few hours for a day or two.

OBSERVATIONS:

1. According to the graph, what were the peak hours of growth for the yeast plants? *Answers vary.*
2. After how long did the growth taper off? *Answers vary.*
3. What happened to the diameter of the balloon after a few days had passed? *It became greater.*

CONCLUSIONS:

1. If you had continued your graph even longer, how do you think the line would have looked?
It would have gone up as the yeast grew.
2. How does measuring the diameter of the balloon help you measure the growth of the yeast plants?
As the yeast plants grow, so does the diameter of the balloon.
3. What factors contribute to the growth of the yeast?
The syrup, temperature of the room, time, etc.
4. What scientific principles did you see when undertaking this exploration?
Answers vary.

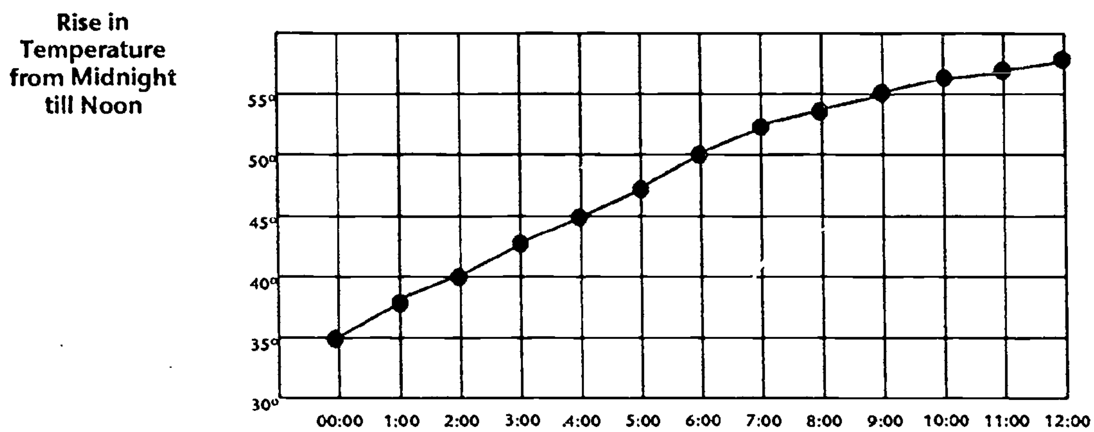
SUGGESTIONS FOR FURTHER STUDY

- Set up two bottles following the directions of this investigation. Place one of these in a warm location, but place the other in a very cool location. Follow the directions of this activity with both bottles. Use a double-line graph to record the data you get. Perhaps you can use a red line for the warm bottle and a blue one for the cool bottle. Do you see any differences? If you do, can you explain them? What does your graph indicate about the importance of warmth to the growth of yeast plants?

Response Sheet
Bubble, Bubble, Toil - But No Trouble

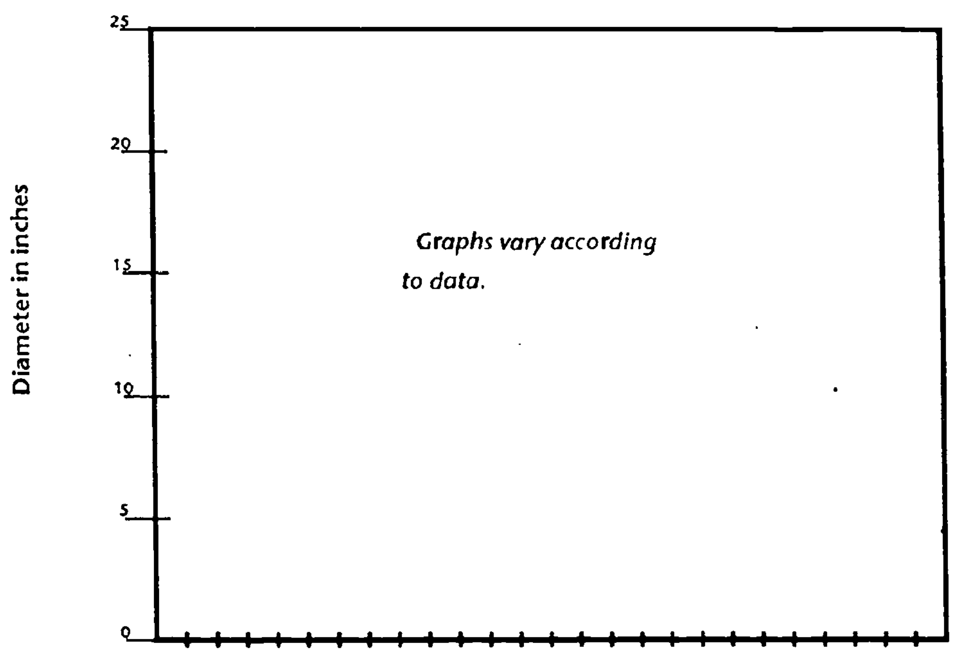
A Line Graph

Temperature at the North Pole on July 4



The Line Graph of Your Data

How Yeast Grows



Record your measurements here:

Time	Measurement	Time	Measurement	Time	Measurement
All vary.	*All vary.*				

Balloon Diameter

Teacher's Guide
Fingerprint Fun

GOAL: To have students explore fingerprint patterns and to learn to compare them using a Venn Diagram.

STUDENT OBJECTIVES:

- ✓ To identify three categories of fingerprint patterns.
- ✓ To compare sets of fingerprints.

GUIDE TO THE INVESTIGATION: This activity can be done by pairs of students or by small groups of 3 to 6 students. Each student will need fresh, soft carbon paper, index cards, and a magnifying glass.

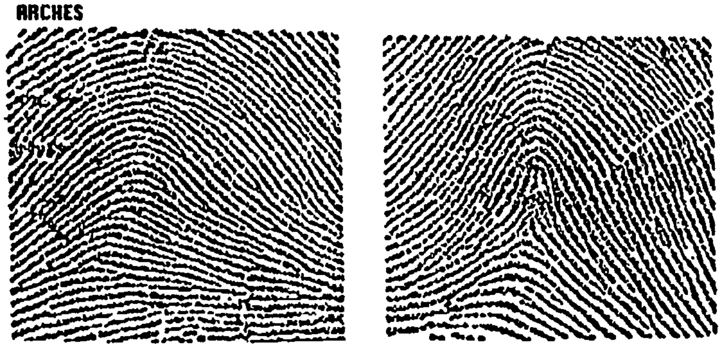
Point out to students that there are many ways of identifying people. No two people have the same fingerprints, so matching fingerprints with the person whose fingers made them is one way of identifying a person. Call students' attention to the fingerprint guide that identifies arches, loops, and whorls. If possible make a transparency and show on an overhead projector or make a poster of the pattern of arches, loops, and whorls to show the class.

Have the students follow the directions in the Procedures section of their activity sheets. Each pair or small group should complete the activities, after which the entire class should compare their findings and discuss them. Have each group or pair of students place their Venn Diagrams on the chalk board or draw them on newsprint or poster board which can be displayed. When the exploration is completed, discuss the Observations and Conclusions.

VOCABULARY: Venn Diagram, arches, loops, whorls

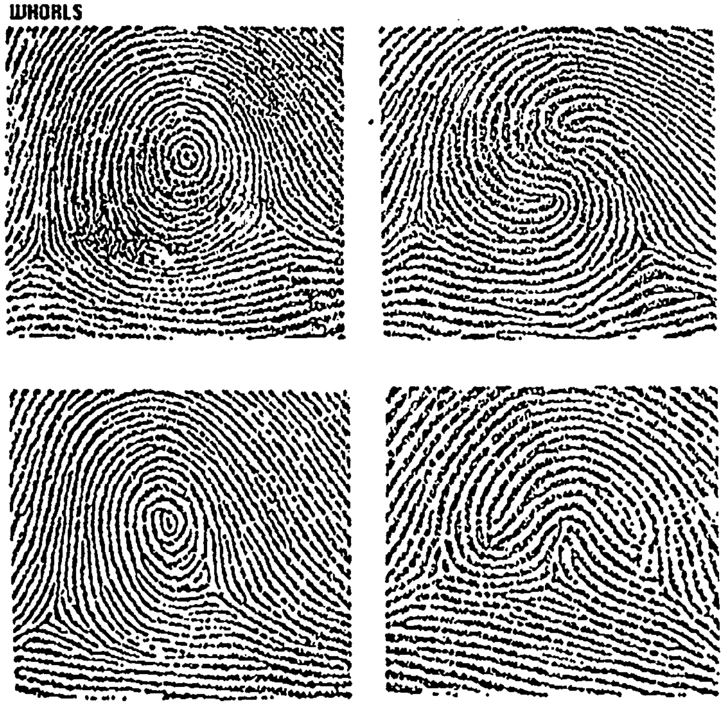
Fingerprint Patterns

ARCHES



LOOPS

WHORLS



Fingerprint Fun

INTRODUCTION: No two people have the same fingerprints, but there are similar features to be found on every one's prints. Experts use three general categories to classify and identify fingerprints. They are arches, loops, and whorls.

PURPOSE:

✓ What are the three categories of fingerprint patterns?

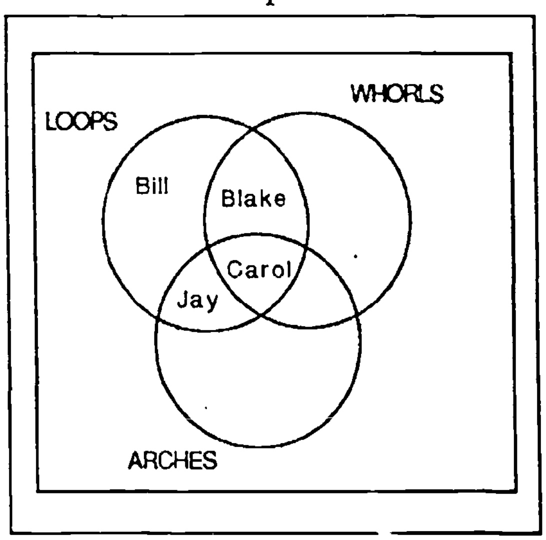
✓ What kind of diagram can be used to compare sets of fingerprints?

MATERIALS:

fresh carbon paper
index cards
magnifying glass

PROCEDURES:

1. Work with several other students. Each student should press the fingers and thumb of his or her right hand onto the carbon paper, and then make clear fingerprints of their fingers and thumbs on an index card. Use one card for each student, and write his or her name on the back of the card.
2. Compare the fingerprints to the patterns shown on the illustration, the transparency, or the poster. See if any of the sets you made have only one of the three patterns. Do any sets show all three patterns?
3. Record for each student which of the three patterns his or her prints show. Be careful to look for more than one! Record the results on the Response Sheet.
4. Make a Venn Diagram similar to illustration 3-1 to show how your friends' sets of fingerprints are alike and different.



3 - 1 *Venn Diagram*

OBSERVATIONS:

1. Whose sets of fingerprints are the most alike?
   Answers vary.

2. Whose sets of fingerprints are the most different?
   Answers vary.

3. Are there differences between the fingerprints of males and females?
   Answers vary depending on class data.

CONCLUSIONS:

1. Fingerprint experts use elaborate computers to compare fingerprints. How is a Venn Diagram like a small computer?
   Answers vary. It shows two or more sets of information and their relationship to each other.

2. Which category of fingerprints do you think are the most common?
   Answers vary.

3. Do you think that this is true for all people or just for your class? How can you find out?
   Opinions vary. Can gather data from entire class, but can only generalize for all people.

SUGGESTIONS FOR FURTHER STUDY:

- Go to a local post office and see if you can make copies of a few of the "Most Wanted" posters. Use a magnifying glass to classify the fingerprints you see on the posters.
- Borrow a copy of the *Boy Scouts of America Merit Badge Series No. 3287* from a friend or from the library. Prepare a poster illustrating what you have learned about fingerprint types and what you can learn from the book. Make a presentation to a lower grade about what you have learned. Explain the three main types of prints to them, and help them tell which types they have.

Response Sheet
Fingerprint Fun

1. Record what you observed about the fingerprints of the classmates in your small group.

Student's Name	Arches	Loops	Whorls
Observations vary.			

2. Make a Venn Diagram showing the fingerprint categories of students.

Diagrams vary.

Teacher's Guide
Marble Races

GOAL: To have students explore the effects of rolling marbles or other spheres down inclined planes of various slants.

STUDENT OBJECTIVES:

✓ To estimate distances.

✓ To measure distances.

✓ To formulate generalizations about the effect of the length and the slope of an inclined plane.

GUIDE TO THE INVESTIGATION: This investigation can be conducted by individual students, pairs, or small groups of students. Each student will need poster board, pencil, ruler, meter stick, scissors, paper, grid paper, and marbles.

Explain to students that an inclined plane is a slanting surface that connects a higher level with a lower level. Demonstrate its parts by showing an example or a photograph of an inclined plane. Have students name examples of inclined planes. Explain to the students that we use a trough rather than a flat surface for this exploration because marbles will roll off the side of a "real" inclined plane.

Students should follow the directions in the Procedures section of their activity sheets. Each group should prepare a chute for the marbles. The exploration requires a large clear area on the floor or a big table top. After the "races" are over, have the students compare their group's results and draw conclusions about the effect of the length and height on the marble race. Discuss the Observations and Conclusions.

VOCABULARY: incline, inclined plane, chute, trough, predict, slope

Marble Races

INTRODUCTION: Marbles are examples of spherical objects. When they roll down different inclines, they travel different distances and at different speeds. In this exploration you will estimate the distances marbles roll and then measure the actual distances.

PURPOSE:

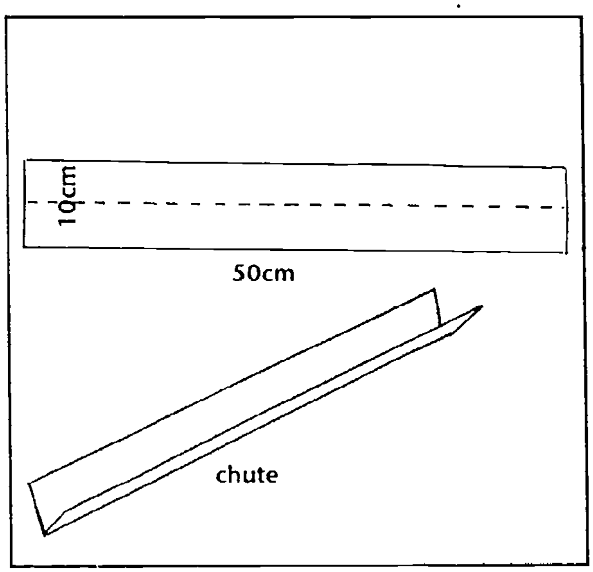
- ✓ Do all the marbles roll the same distance?
- ✓ How does changing the slant of the inclined plane effect the distance the marble will roll?
- ✓ How does changing the length of the inclined plane effect the distance the marble will roll?

MATERIALS:

poster board
pencil
ruler
meter stick
scissors
paper
graph paper
marbles
rope

PROCEDURES:

1. Make a chute for your marbles. Make a rectangle of poster board that measures 10 cm x 50 cm.
2. Draw a line down the middle of the chute. Crease the chute along the line to make a long trough as in illustration 4-1.
3. On the inside of the chute, measure and make marks at 10, 20, 30, and 40cm from the bottom (See illustration 4-3).

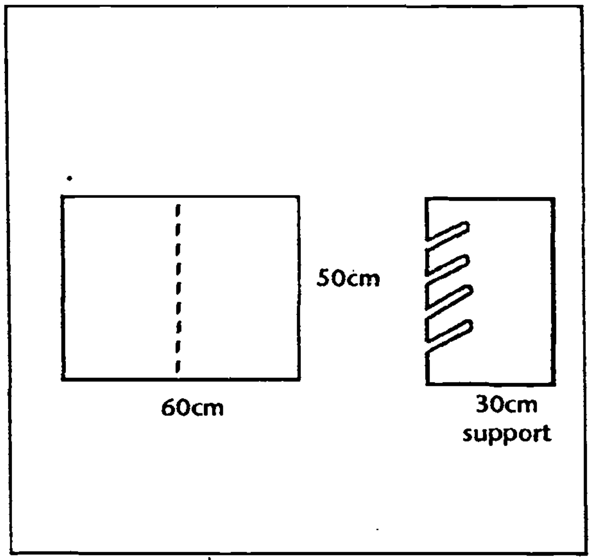


4-1

4. Make a support for the chute using a 60 cm x 50 cm rectangle of poster board. Draw a line down the middle.

5. Fold the cardboard as shown in illustration 4-2 and cut slits for bracing the chute. Be sure the chute fits into the holes as in illustration 4-3.

6. Place the chute in the lowest slit. Estimate how far the marble will roll when it is released from the upper end of the chute (the 40cm mark).



4-2

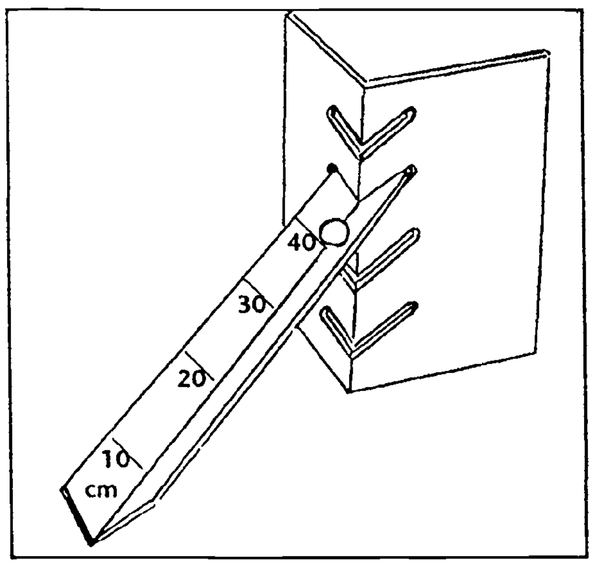
7. Write your estimate on the chart on the Response Sheet.

8. Then roll the marble down the chute.

9. Use the meter stick or the rope to measure the distance the marble rolled.

10. Roll the marble from this point several times, and record the distance it rolls each time.

11. Repeat the experiment rolling the marble from the 30 cm mark. Record your estimate and the actual distance the marble rolls.



4-3

12. Repeat the experiment with the chute in the higher slot rolling the marble from the 20 cm mark.

13. Finally, repeat when the chute is in the highest slot and the marble rolls from the 10 cm mark.

14. Record all your estimates and the actual distances on the Response Sheet.

OBSERVATIONS:

1. If you keep the trough at the same height and release the marbles from the same point on the trough, will they all roll the same distance?
Yes, if the speed is constant.

2. If you keep the trough at the same height and release the marbles from lower points on the trough, will they roll the same distance as they did when you rolled from higher up on the trough?
No.

3. If you lower the trough but continue to roll the marbles from the same point on the trough - higher up - as you did for No. 1, what happens?
Answers vary. Distance should decrease.

4. If you lower the trough and roll the marbles from a lower point on the trough, what happens?
Answers vary. Distance should decrease.

CONCLUSIONS:

1. When the marble is rolled from higher up, what happens to the speed with which it exits the trough?
Answers vary. It should increase.

2. How does the steepness of the incline effect the speed of the marble?
Answers vary. It should increase.

3. What generalizations can you make about the length and the incline of the chute?
Answers vary. They effect the speed and the distance traveled.

4. What generalizations can you make about the height above the table top or floor from which the marble is released?
Answers vary. It effects the speed and distance of the marble.

SUGGESTIONS FOR FURTHER STUDY

- Try this activity keeping the incline and length of the chute constant but varying the size of the marble. You should be able to get some larger marbles from a toy store. Does the size of the marble matter?
- Does the weight of the material matter? Ball bearings are heavier than marbles of the same size. Rubber balls weigh less then marbles. Repeat the experiment with a ball bearing and a rubber ball, and check out the effect of weight.

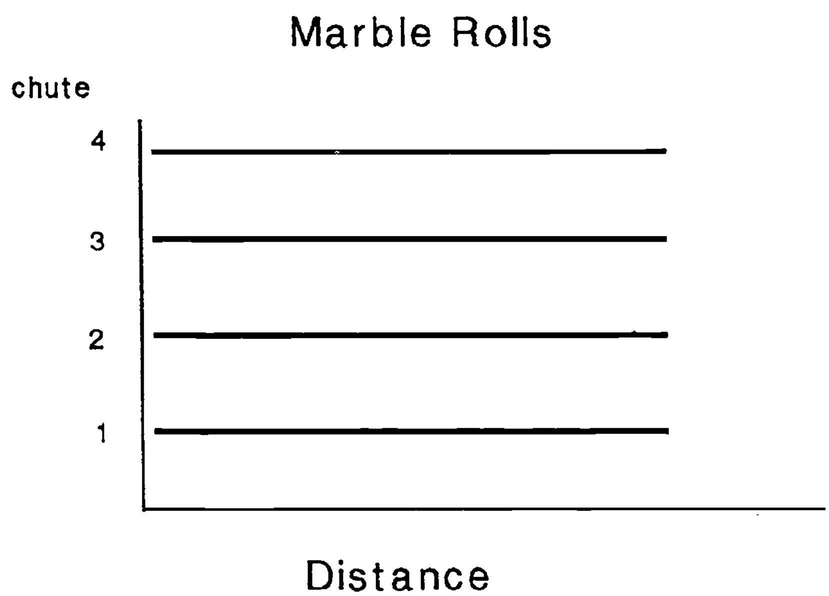
- Have a marble "race" with a friend. Predict and record how many rolls it will take you to have your marble roll a total of 100 meters. Have your friend do the same. Keep running totals of the distances you roll your marbles. Who made the better prediction? Whose marble rolled 100 meters first?

Response Sheet
Marble Races

1. Estimates and Actual Results ***Answers vary.***

Incline Position	Estimate	Actual
Roll from upper end of chute	_____ cm	_____ cm
Roll from lower end of chute	_____ cm	_____ cm

2. Distance marble rolled from each chute. Shade the line beside the numbers to represent the distance the marble rolled in each chute.



Teacher's Guide
Probable Cause

GOAL: To introduce students to variables in an experiment and to allow them to explore three kinds of variables.

STUDENT OBJECTIVES:

✓ To conduct an experiment in which variables are controlled.

✓ To draw conclusions from an experiment about the significance of the results.

GUIDE TO THE INVESTIGATION: This experiment will take approximately four weeks.

This investigation could be conducted by individual students, but because there are three plants involved, you might divide the class into groups of from three to six students. The students will need three clear plastic or glass containers, potting soil, vermiculite, an egg, liquid dish washing detergent, a cola drink, three potting plants of the same species, a measuring cup, a measuring spoon, a fork, tape, a ruler, and paper.

Before beginning the investigation, discuss the meaning of variable. The manipulated variable in this experiment is the fertilizer - we can vary the fertilizer and look at what effect our manipulations have on the result of the experiment. The responding variables - those which respond to the manipulated variable - are the height and general appearance of the plant. There are also constants in this experiment: the vermiculite medium in which the plants are planted, the watering regimen, and so forth. Discuss other variables which may contribute to the results: light source and direction, type of water, temperature, etc.

Have students follow directions in the Procedures section. At the end of the investigation, discuss the Observations and Conclusions.

VOCABULARY: variable, manipulated variable, responding variable, constant

Probable Cause

INTRODUCTION: People feed their house plants various kinds of plant food or fertilizer in order to keep them healthy and make them grow. Some kinds of feedings also help blooming plants to have more and better blossoms. Sometimes people use very unusual things for fertilizer, such as egg shells or aspirins. In this experiment you will grow three plants and feed each one of them different "food" to see what effect the different "plant foods" have on the plants.

PURPOSE:

- ✓ How can you control (manipulate) a variable in an experiment?
- ✓ What effect does each of the variables have on the growth of a plant?

MATERIALS:

clear glass or plastic containers (three)
vermiculite
potting soil
liquid dish washing detergent
soft drink (cola)
paper
tape
ruler
an egg
cup
plants (three alike: coleus, geranium, tomato, etc.)
fork
measuring cup
measuring spoons

PROCEDURES:

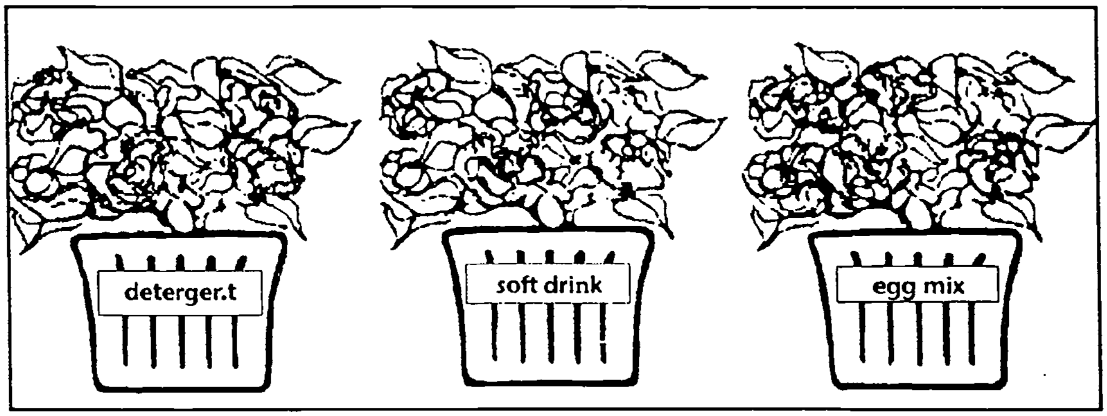
1. Measure and place two cups of potting soil into each of your three clear glass or plastic containers.
2. Into each container also put four tablespoons of vermiculite.
3. Mix the soil and vermiculite well with the fork.
4. Put one-fourth teaspoon of liquid dish washing detergent and one-half cup of water into a large measuring cup. Stir with the fork until the detergent is "mixed" with the water.

5. Add the mixture to one of the containers. Tape a small piece of paper to the container and write "liquid detergent" on it.

6. Next, measure one-half cup of a soft drink (cola) into the measuring cup, and pour it into a second pot. Make a label for this container on which you write "cola" or "soft drink".

7. Finally, measure one-third cup of water into the measuring cup, and break an egg into it. Use the fork to beat the egg-water mixture until the water, white, and yolk are blended. Put this mixture into the third container, and label the container "egg mixture".

8. Choose three identical potting plants as close to the same size as you can. Plant one each in the three containers, taking care to cover the roots with the soil mixture. (See illustration 5-1.)



5-1

9. Set the plants in a brightly lit place but not in direct sunlight.

10. Give each plant one-fourth cup of tap water once a week.

11. Measure and record the height of each plant on the day you planted them and on each of the days of the week you water them. Record the measures on the Response Sheet. On each watering day, write a short description of each plant on the Response Sheet.

OBSERVATIONS:

1. What happened to each of the three plants over the four-week period?
   Answers vary.

2. Which plants grew taller, and by how much?
   Answers vary.

3. Did any of the plants die?
   Answers vary, however sometimes one of the plants will die.

CONCLUSIONS:

1. Which variable did you manipulate in this experiment?
   The "fertlizer"
2. Which variables were effected by what you did?
   The growth of the plant — appearance and size
3. Which variables did not actually vary, that is, what were the constants of this experiment?
   The potting soil, vermiculite, the container, sunlight, watering
4. Did you think this was a "good" experiment? What are the reasons for the answer you give to this question?
   Answers vary — however, only one variable was used and all other factors were held constant.
5. What other variables could you have manipulated?
   Soil, with and without vermiculite; water; sunlight
6. What conclusions do you draw from this experiment?
   Answers vary, however look for a statement about which fertilizer is better and why.

SUGGESTIONS FOR FURTHER STUDY

- Fill one container with one cup of dampened vermiculite and another with an equal amount of paper confetti. Plant the same number of the same type of flower seeds in each container, and keep the "soil" moistened. Observe the growth of the plants. What differences do you observe? *Answers vary.*
- Read in science activities books about other experiments in which you manipulate one or more of the variables to have an effect on another variable. Are there also constants in these experiments? With two or three other students, select an experiment which your group can carry out, and keep records of what you do. Describe this experiment, the manipulated variables, the responding variables, and the constants to the class through a poster report.

Response Sheet
Probable Cause

Responding Variable Observation Tables

Plant 1 – liquid detergent

Date:	Height (in.)	Condition
Date planted		*All answers vary.*
Week 1		
Week 2		
Week 3		
Week 4		

Plant 2 – soft drink

Date:	Height (in.)	Condition
Date planted		
Week 1		
Week 2		
Week 3		
Week 4		

Plant 3 – egg mixture

Date:	Height (in.)	Condition
Date planted		
Week 1		
Week 2		
Week 3		
Week 4		

Teacher's Guide
Who Will Win - And Why?

GOAL: To help students investigate several game strategies.

STUDENT OBJECTIVES:

✓ To play several games.

✓ To explore strategies for winning games.

GUIDE TO THE INVESTIGATION: Students should complete the parking lot game individually, or in pairs. Have them compare the results in pairs or small groups so that they can share their strategies and understand what happened when some took fewer or more "moves" to drive away. For tic-tac-toe and checkers, students must work in pairs. The spinner exercise will probably work better if students work in pairs to get all hundred spins done. One can spin while the other records the results, and then they can change places.

Students will need pencil, paper, construction paper, scissors, checkers, and a spinner. Spinners can be made with a cardboard circle and a thumb tack.

Before beginning the activities in the Procedures section, have students discuss their favorite games and tell why they enjoy them. After students complete the investigation, discuss their observations and ask them to discuss what they have learned about strategies. Ask if anyone learned anything which will help them win more times. In your discussion include the questions in the Observations and Conclusions sections.

VOCABULARY: strategy

Who Will Win - And Why?

INTRODUCTION: Everyone likes to play games, especially when they win! Do you usually win card or board games? Or are you often frustrated because you never seem to win? In this activity, you will play several games and try to identify why people win or lose so that you can come up with strategies which may help you win more often.

PURPOSE:

✓ Why do some people seem always to win board games or card games?

✓ What strategies do the winners use to win?

MATERIALS:

paper
pencil
construction paper
circular spinner (with 6 colors)
checkers and checkerboard
scissors
markers

PROCEDURES:

You will complete four separate activities during this investigation.

Activity One

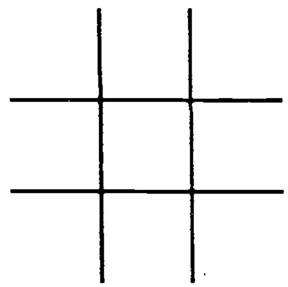
The first activity will be to get the car out of the parking lot. The diagram for this activity is on the last page of this exploration.

1. Follow the directions to make a model of the cars and the parking lot. You have to figure out which cars you will have to move first to get your car out of the parking lot. The cars can only be driven into empty spaces. They cannot jump over another car. The cars in the parking lot must be the same size and in the same places as they are in the diagram.
2. Model the cars in different colors. All of the cars' ends and sides must be exactly as they are shown. Make your model cars from different colors of construction paper, trace the parking lot on to paper, and "park" your cars as they are in the diagram of the parking lot.
3. Number all the cars except yours just as they are on the diagram.

4. Move the cars around until you can "drive" your car out of the lot onto the street.
5. Keep track of which cars you had to move and how many times you have to move each one of them.
6. When you have driven your car out of the parking lot, work with a partner or two or three other students. Compare the strategies each of you used including the number of cars you had to move to get your car out.

Activity Two

For the second activity, make four tic-tac-toe diagrams on a sheet of paper. Play four tic-tac-toe games with a partner. Each of you go first two times. Keep a record of who wins more games.

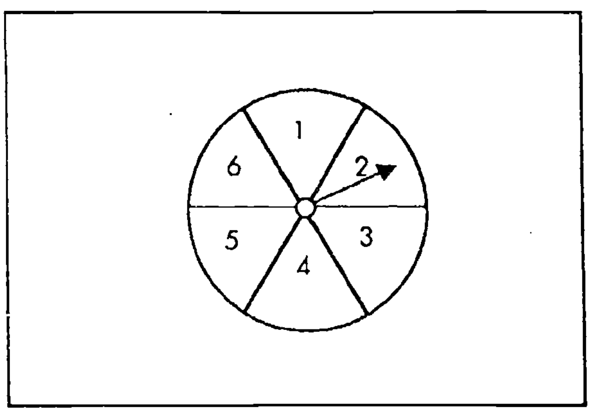


Activity Three

Use a checkerboard and checkers. Play this game with a new partner. Play four games of checkers. Each of you go first two times. Keep a record of who wins more games.

Activity Four

Use a spinner which is divided into six equal parts, and numbered 1, 2, 3, 4, 5, and 6 for this game. With a partner, spin the spinner a hundred times. Record the number of times the spinner stops on each number. (If you have to make your spinner, use scissors, cardboard or poster board, a thumb tack, and a pencil.)



6-1

OBSERVATIONS:

1. How many moves did it take you to get your car out of the parking lot?
Answers vary.

2. What strategy did you use to get your car out of the parking lot?
Answers vary.

3. Did you learn an easier way from anyone else's strategy?
Answers vary.

4. Who won the most tic-tac-toe games? Why?
Answers vary; however if both players play to block and do not make errors, the games always end in ties.

5. Who won the most checker games? Why?
Answers vary.

6. On which number did the spinner stop the most often?
Answers vary. If the spinner is "fair" each number will be landed on the same number of times.

CONCLUSIONS:

1. Some students took more moves to get their cars out of the parking lot than others did. Why did this happen?
Answers vary. They may have used more trial and error than planned strategies.

2. Are there students who took fewer moves to get out of the parking lot? How did they do it?
Answers vary. Yes; they studied the "parking lot" and the cars and then planned a strategy.

3. What strategy did the tic-tac-toe winners use to win and why did it work?
Answers v- v.

4. What strategy did the checkers winners use to win and why did it work?
Answers vary.

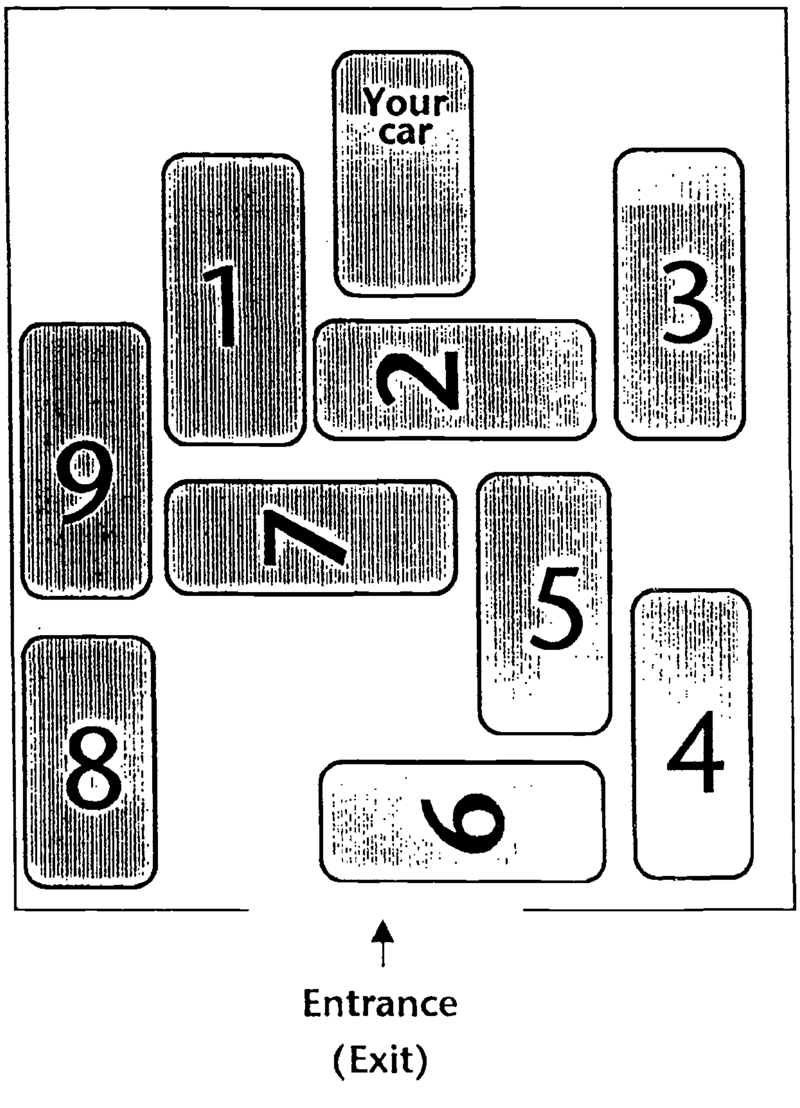
5. How could you use the results about the spinner to help you win more games?
Answers vary. Example: make one section larger.

SUGGESTIONS FOR FURTHER STUDY

- With two or three friends, play a board game such as Monopoly, the NIM game, Clue, Pay Day or the Game of Life. Try to create and describe a strategy to win the game you are playing.
- Create a game which uses the spinner. Make the cards, boards, playing pieces, or whatever you need to play the game. Develop the game using one of the strategies you discovered when completing this investigation.

Parking Lot
Who Will Win - And Why?

There is a fence around the lot, except at the gate.



SPATIAL SENSE

Teacher's Guide
Pythagoras 'n Me

GOAL: To have students test the Theorem of Pythagoras by using the body to make various measurements.

STUDENT OBJECTIVES:

- ✓ To form a right triangle using some string, one arm, the torso, and both legs.
- ✓ To measure the right triangle formed with the body.
- ✓ To calculate the length of the hypotenuse by applying the Pythagorean Theorem.

GUIDE TO THE INVESTIGATION: This activity requires cooperation between students in pairs. You may want to have them compare and discuss their results in small groups of three or four pairs and then report to the whole class for further discussion. The activity requires string, paper, pencil, two markers of different colors, a yardstick, and a calculator with a square root function or a square root table.

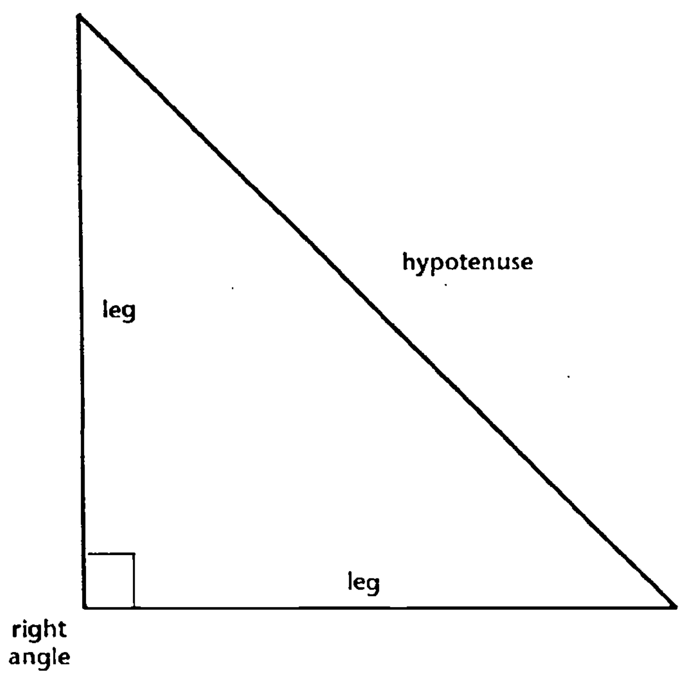
Before you begin the exploration, tell students a bit about Pythagoras and the Pythagorean Theorem. Explain right angles and the sides and hypotenuse of a right triangle. Demonstrate the use of the square root function on the calculator or the square root table.

Direct students to the Procedures section of the activity sheet. After they have completed the exploration, have students discuss Observations and Conclusions. Make a list of the ways the Pythagorean Theorem is used in the "real world", such as in carpentry and furniture making. Write the list on the chalkboard or on a poster.

VOCABULARY: theorem, Pythagorean Theorem, hypotenuse, diagonal, square root, torso

Pythagoras 'n Me

INTRODUCTION: Pythagoras, who was a Greek geometrician of the sixth century BC., studied the properties of triangles. It is believed that he developed a theorem relating the measures of the sides of a right triangle. We call it the Theorem of Pythagoras, or the Pythagorean Theorem. This theorem states that the square of the measure of one leg of a right triangle added to the square of the measure of the second leg is equal to the square of the measure of the hypotenuse of the right triangle. The hypotenuse is the side opposite the right angle, and the legs are the other two sides. They are opposite the two angles which are not the right angle.



The Parts of a Right Triangle

In this investigation, you will use your body and some string to form a right triangle. Then you will calculate the approximate length of the hypotenuse and compare your result with what Pythagoras tells you the answer should be!

PURPOSE:

- ✓ What are some ways to find the hypotenuse of a right triangle?
- ✓ When you form a right triangle with your body and some string, how can you use it to measure the length of the hypotenuse?
- ✓ How is the Theorem of Pythagoras useful to us?

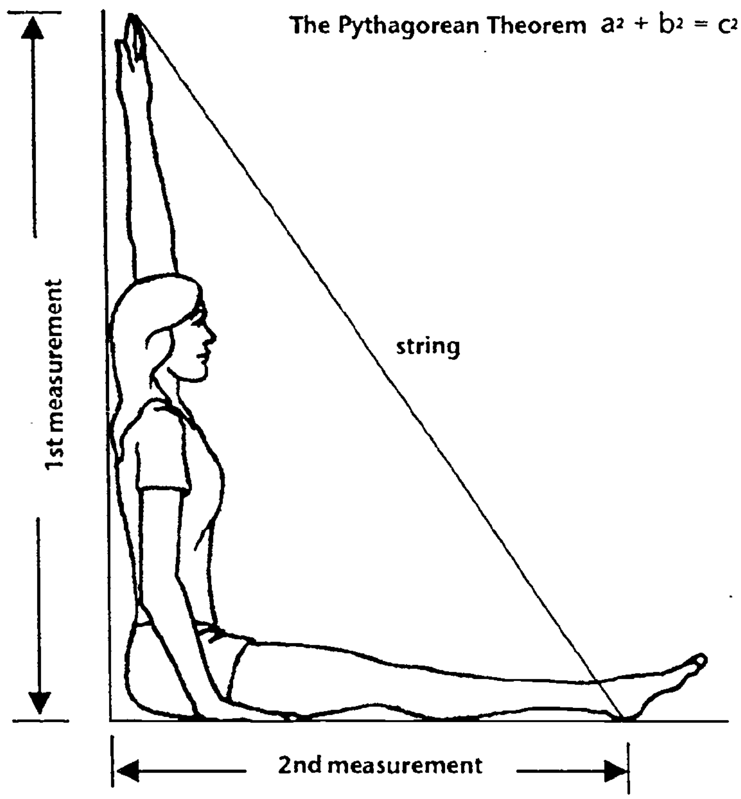
MATERIALS:

string (or yarn)
yardstick
calculator with a square root function
markers (two colors)

PROCEDURES:

1. Work with a partner. One of you sit on the floor with your back against the wall (as close to the wall as possible).
2. Position your legs as straight in front of you as you can. Now your legs and your torso form a right angle - just as the wall and the floor do.
3. Raise one arm as high and as flat against the wall as you can.
4. Your partner should now wrap a piece of string gently around the end of your middle finger so you can hold it down your back and to the floor. Measure and record this measurement on the Response Sheet.
5. Then your partner should measure the distance from the wall behind your back to the tip of your heel on the floor. This, too, is to be recorded.
6. Finally, have your partner use the string whose end is on your finger and mark the distance from your finger to where your heel touches the floor. Do not measure it yet - just use the marker to mark where the string leaves your finger and where it touches your heel and the floor.
7. Now exchange roles. You measure your partner, and use the other colored marker to mark the string so that you will not confuse the two sets of marks - one set for you, and the other for your partner. You can use a fresh piece of string, too, but the colors should still be different.

8. Use the measurements along your back and legs and the Theorem of Pythagoras to calculate what the length of the string (hypotenuse) should be for you.



9. Use the Theorem of Pythagoras (The Pythagorean Theorem $a^2 + b^2 = c^2$) and the measurements you made of your partner to calculate what the length of the string (hypotenuse) should be for your partner. Record these on the Response Sheet.

10. Now using the marks you and your partner made on the strings, measure the length of the hypotenuse for each of you, record these numbers, and compare them with what Pythagoras told you they should be.

OBSERVATIONS:

1. How close was your calculation of the measurement of the length of the hypotenuse to Pythagoras' calculation of what it should be?
   Answers vary, depending on the accuracy of measurement.

2. How close was your partner's calculation of the measurement of the length of the hypotenuse to Pythagoras' calculation of what it should be?
   Answers vary, depending on the accuracy of measurement.

3. Was one of you closer than the other? If so, give a reason this might have happened.

   Answers vary.

4. Why might your actual measurement be at all different from what Pythagoras tells you it should be?

   Measurement round-off error; maybe the measurements made with the string were inaccurate.

5. Is it more accurate to measure the string to find the length of the hypotenuse or to use the Theorem of Pythagoras? Give a reason for your answer.

   Answers vary. If the string is measured to the nearest cm and the answer using the Pythagorean Theorem is rounded to the nearest cm, the answers should be the same.

CONCLUSIONS:

1. Can we use Pythagoras' Theorem in the real world to help us build or make something? If so, what? Can you point to any examples in your home or school?

   Yes. Answers vary.

2. What conclusions can you draw about the triangle you formed with your body and the string?

   Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- Measure the triangle drawn as an example at the beginning of this activity. Do you find anything interesting? If you do, can you find other numbers like these?
- Try to use your body and string to form an isosceles triangle. Then try to form an equilateral triangle. Is it as easy as forming a right triangle? Why or why not?

- Measure the length and width of your television screen at home. Use the Theorem of Pythagoras to calculate the length of the diagonal of the screen. Make a record of the measurements and your calculation. Then turn the television on and repeat the measurements and your calculation with Pythagoras' Theorem. This time measure the length and width of the actual picture. Is it the same or smaller than the set? Now that you have written two diagonal measures, the screen and the picture, ask your parents what the size of your television is. (Televisions are sold by the size of the diagonal. A 27-inch television should have a diagonal which measures 27 inches). Record these numbers on the Response Sheet. Is your television as large as the manufacturer and store say it is? Did they tell the truth? Discuss this with your parents, and be prepared to tell your class what you found out. Talk with other students about their television investigations. What did you find out?

Response Sheet
Pythagoras 'n Me

Measurement Records

Measurements of Your Triangle	Inches
Finger to Floor	*Answers vary.*
Wall to Heel	
Pythagorean Theorem Hypotenuse Calculation	
Actual Hypotenuse Measurement	

The Measurements Your Partner Made of You

Measurements of Your Triangle	Inches
Finger to Floor	*Answers vary.*
Wall to Heel	
Pythagorean Theorem Hypotenuse Calculation	
Actual Hypotenuse Measurement	

The Measurements You Made of Your Partner

Measurements of Your Home Television Set	Screen (inches)	Picture (inches)
The Distance Across	*Answers vary.*	*Answers vary.*
The Distance from Top to Bottom		
Pythagorean Theorem Diagonal Calculation		
What the Manufacturer Says the Size is		

Your Television

Teacher's Guide
The Ambitious Architect

GOAL: To have students explore creating various shapes which can be incorporated into houses and other buildings.

STUDENT OBJECTIVES:

- ✓ To make various polygonal shapes.
- ✓ To construct a model building using polygonal shapes.

GUIDE TO THE INVESTIGATION: This exploration can be completed by individual students, but if they use straws rather than toothpicks, working in pairs may be easier for them. Students will need construction paper, string, glue, straws, toothpicks, pipe cleaners, scissors, a protractor, and a pencil.

Show students pictures of various types of architecture. Show pictures of geodesic domes and Japanese architecture. Both provide many examples of the incorporation of polygonal shapes. Discuss architectural designs used in this country and countries such as China, Japan, France, etc. Photographs of various types of bridges are rich sources of geometric patterns.

Students should then follow the directions in the Procedures Section. When they have finished the construction phase, invite them to share with the class which of the shapes were easier than others to construct and incorporate into buildings, and why they were easier. Which were more difficult? Why? Discuss with them how structures can be stabilized to stand alone and bear weight. Why are some shapes commonly used and others not? Discuss the Observations and Conclusions.

VOCABULARY: polygons, stabilize

The Ambitious Architect

INTRODUCTION: Have you ever seen pictures of houses or buildings designed by the famous architects Frank Lloyd Wright or R. Buckminster Fuller? Their houses use many polygonal shapes which give them unique character and style. In this activity you will become an architect and construct a building using polygonal shapes.

PURPOSE:

✓ Which polygonal shapes can be used to construct a building?

✓ How can various polygonal shapes be joined to create unusual and appealing designs?

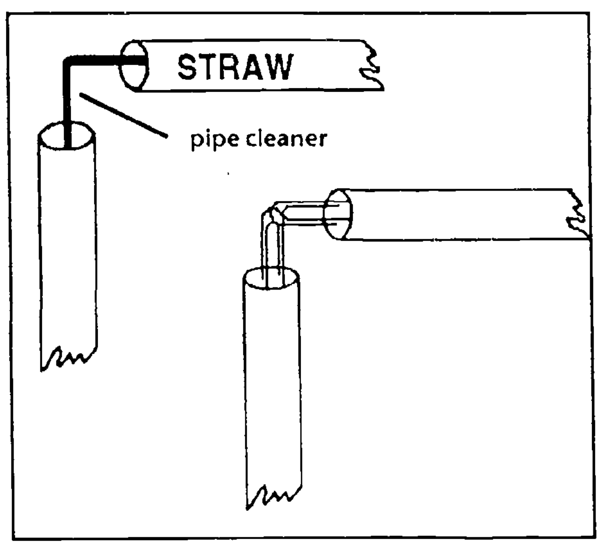
MATERIALS:

construction paper
glue
paper clips
rubber bands
straws
toothpicks
scissors
protractor
pipe cleaners
string

PROCEDURES:

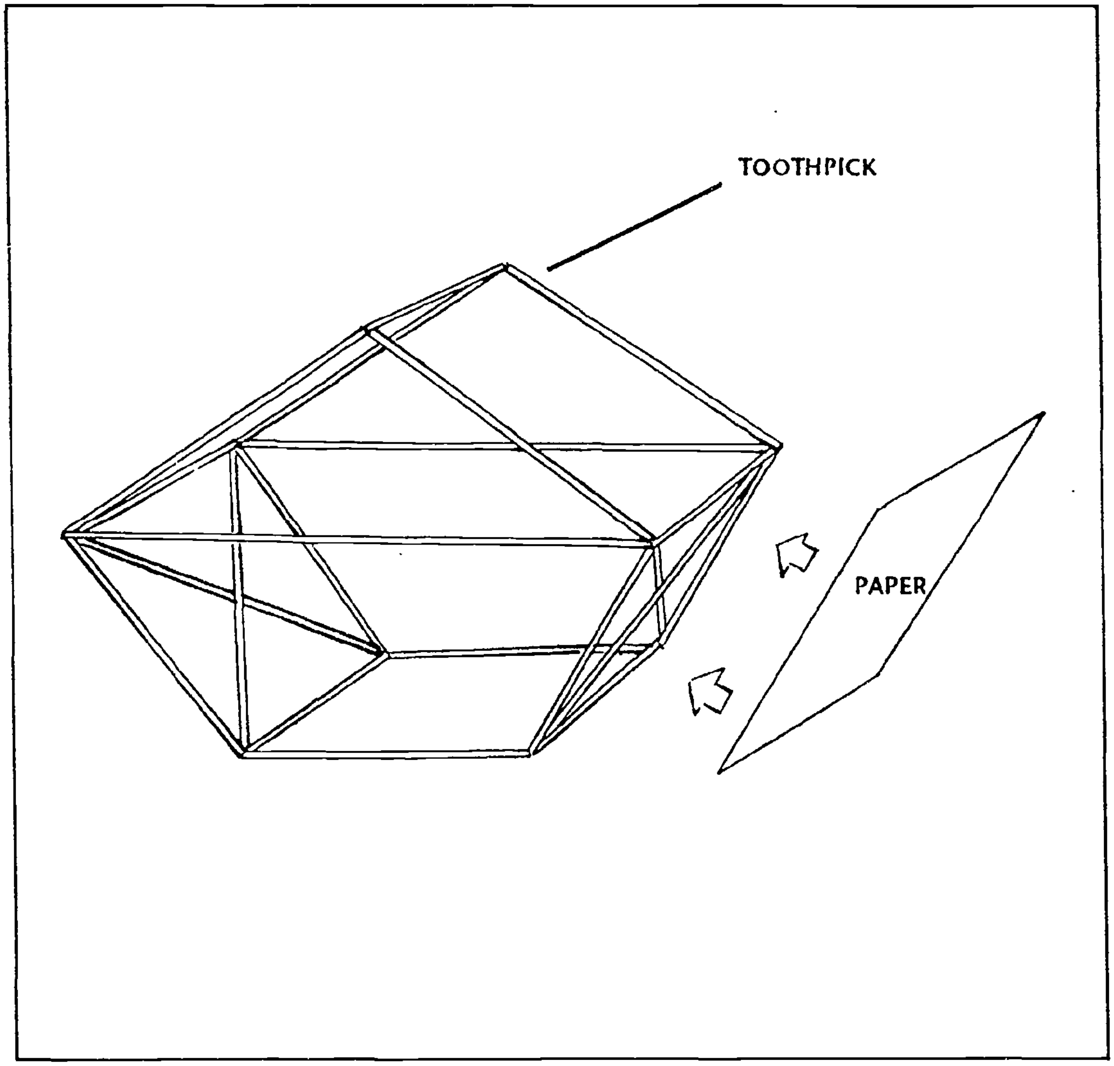
Make a list of common polygons which you think could be used in building design and construction. Once you have completed the list, use some of the materials listed to construct some of the shapes you have listed.

1. Cut straws to any length. Fasten them together by passing string through them and tying the string off at the two ends of the straws which you have "strung". You may also fasten them by linking two paper clips and putting the paper clips into the ends of two straws. A third way to join them is to insert a pipe cleaner into each of the two straws to be joined. The pipe cleaner should be bent in the middle at the desired angle. See illustration 8–1.



8-1

2. Glued toothpicks make a fine frame for a model building. Toothpicks may be glued together to be a part of your construction. (You will have to wait for the glued joints to dry.) Use a protractor to measure the angles as you construct with toothpicks. Once the frame is completed, attach various polygonals to extend the frame. Next cover the frame and shapes with cut-out pieces of construction paper to form the "walls". (See illustration 8–2.)



8 - 2

OBSERVATIONS:

1. What polygonal shapes were you able to incorporate into your building?
   Answers vary.

2. Were some of the shapes easier to build than others? Which ones?
   Answers vary.

3. Were some of the shapes easier to incorporate into the building's frame than others? Which ones?

   Answers vary.

4. Is your structure stable enough to stand alone? If not, what steps can you take to make it so?

   Answers vary.

CONCLUSIONS:

1. Why are some shapes, such as triangles and squares, used more often in construction than other shapes?

   Answers vary. They have a base and may be stronger.

2. What steps must you take if you wish to include triangular, trapezoidal, and hexagonal shapes in your architectural building design?

   Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- Work with a partner, and use the same kinds of materials as you used in this investigation to build a model boat, bridge, and car. Try to build the bridge big and strong enough so that the boat you build can "sail under" and the car you build can "drive over".
- Work with three or four other students to design a city block. Plan out your design on a large piece of construction paper. Then use toothpicks, glue, and construction paper to construct the buildings your group has planned. Use various colors for the different buildings, and name each of them when you have finished it. Write the name over the building's "door" or on its "cornerstone".

Teacher's Guide
I've Got It Covered

GOAL: To have the student explore the surface area of cubes and rectangular solids.

STUDENT OBJECTIVES:

✓ To make a "jacket" for a cube and other rectangular solids.

✓ To investigate the surface area of cubes and other rectangular solids.

GUIDE TO THE INVESTIGATION: This exploration can be conducted by individual students, by partners or in small groups.

Students need several two centimeter cubes, scissors, and tape for the activity. In addition, they need several sheets of two-centimeter grid paper.

Discuss surface area, volume, and rectangular solids with students before they begin the activity. Explain that in the activity students will be using two centimeter cubes and that each square on the two centimeter grid paper will cover one face of the two centimeter cube.

Students should complete the Procedures section of the activity. After they complete the procedures, discuss the Observations and Conclusions and the generalizations that have been made.

VOCABULARY: cube, area, surface area, volume, rectangular solid, centimeter, face, jacket

I've Got It Covered

INTRODUCTION: In mathematics class you learned that the area of an object does not change, no matter how the object or its parts are arranged. Is this true for the surface area of an object? In this activity you will explore making jackets for two centimeter cubes and find if the surface area of the cubes changes when you rearrange the cubes.

PURPOSE:

✓ How can a "jacket" be made for a cube to help you measure the area of its surface?

✓ Given an arrangement of cubes, how does rearranging the cubes change the surface area of the resulting construction?

MATERIALS:

several two centimeter cubes or two centimeter cubes glued together
2-centimeter grid paper
scissors
tape

PROCEDURES:

1. Select a two centimeter cube. Measure its length, width, and height and record these measures on the Response Sheet.
2. Take two-centimeter grid paper and, using the scissors, see if you can cut out a "jacket" which will cover every face of the cube. The jacket must be one piece! Cut only along grid lines on the centimeter grid paper.
3. When you have test-fitted the "jacket" to the cube and are satisfied that it is a good fit, tape it carefully in place.
4. Count the centimeter grids that cover the surface of the cube.
5. Record your answer on the Response Sheet.
6. Make a drawing of the jacket you made.
7. Make a second jacket for an identical cube, but make it a different way.
8. Repeat the measurements, as above.
9. Record the measurements, and draw the jacket on the Response Sheet.
10. Calculate and record the volume of the cubes on the Response Sheet.

11. See how many other jackets you can make without repeating the same pattern.
12. Now tape together two 2-centimeter cubes. Tape them carefully, side by side.
13. Make a jacket to cover this rectangular solid, and repeat all the measurements you made above.
14. Make a second, different jacket for the new rectangular solid, and record these measurements, too.
15. Calculate and record the volume of the rectangular prism on the Response Sheet.
16. Continue this process for 3 cubes and 4 cubes.

OBSERVATIONS:

1. What was the surface area of the first cube you used?
   6 square units, where 1 unit = $(2cm)^2$ or 24 cm^2
2. Use the formula for the area of a square, A = l x w. Compute the surface area by finding the area of one of the six faces of the cube and multiply the area by six.
   2x2 = 4; 6x4 = 24 square centimeters
3. How many different jackets did you make for the cube?
   Answers vary.
4. What happens to the surface area of a rectangular solid when the cubes are arranged differently?
   The surface area may change, because different arrangements have different surface areas — for example for 4 cubes there are 2 arrangements.
5. What generalization can you make about the volume of a rectangular solid made by joining cubes in different arrangements?
   The volume of the rectangular solid remains the same, regardless of the arrangements.

SUGGESTIONS FOR FURTHER STUDY

- Use four cubes of the same size as those used above, and find several ways to tape them together. Make jackets for each of the groupings you have taped together, and measure the surface area of each. They will all "hold" the same amount of something, but do they all have the same surface area?

  No.

- Explain why an exercise of this type is important to someone whose business it is to make boxes to package popcorn or sugar or pancake mix.

  Answers vary. Example: May need to pack more in a smaller box to lower the cost of shipping.

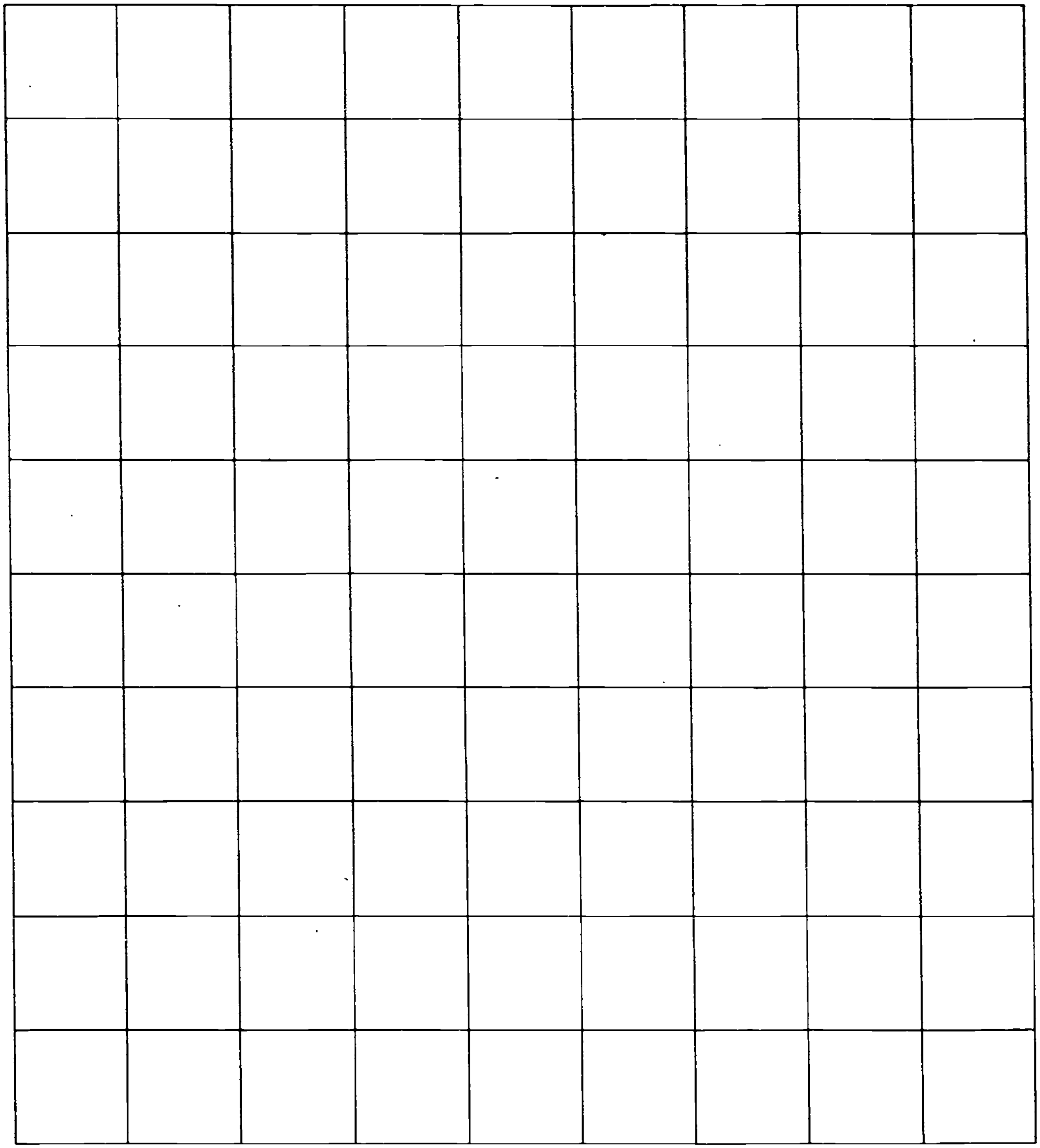
- Find a cylinder, and make a jacket for it. Measure its surface area. Use the formulas for the area of circles and rectangles and the formula for the circumference of a circle to compute the surface area. Develop a formula for the surface area of a cylinder.

  $2\pi r^2 + 2\pi rh$

Response Sheet
I've Got It Covered!

Number of 2 cm cubes	Volume of cube(s)	Drawing of the Jackets	Surface Area (Number of squares to cover the cube(s))
1	8 cm^3	*Answers vary.*	*6 units square or 24 cm^2*
1	*8 cm^3*		*6 units square or 24 cm^2*
2	*16 cm^3*		*10 units square or 40 cm^2*
2	*16 cm^3*		*10 units square or 40 cm^2*
3	*24 cm^3*		*14 units square or 56 cm^2*
3	*24 cm^3*		*14 units square or 56 cm^2*
4	*32 cm^3*		*16 units square or 64 cm^2* *Or* *18 units square or 72 cm^2*

Two Centimeter Grid Paper



Teacher's Guide
Cut It Out!

GOAL: To have students explore coordinate planes and gain an understanding of how a construction can be reflected on a plane.

STUDENT OBJECTIVES:

✓ To construct a figure on a coordinate plane.

✓ To reflect the figure through (across) the x-axis and the y-axis.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students or pairs of students. Small group discussions of the results will be helpful for the students to understand their findings. The "light show" will require a dark room.

Students will need grid paper, construction paper, glue or tape, a pencil, a ruler or straight edge, scissors, and two flashlights for the exploration.

Discuss the Cartesian coordinate plane, ordered pairs, how to graph points, and what the axes are before beginning the exploration. Show students a figure plotted in one quadrant which you then reflect for them in a mirror after you fold the graph paper along an axis. This will help them understand reflections through the axes. After students have completed the exploration, discuss the Observations and Conclusions.

VOCABULARY: Cartesian coordinate system, axes, reflect, ordered pairs, Cartesian coordinate plane

Cut It Out!

INTRODUCTION: Maps are often divided into squares or rectangles, called grids, to make it easier to locate a city or lake or other geographic feature. Have you ever used a grid on a map to find something? The grid on a map is similar to the coordinate system. The coordinate system consists of vertical and horizontal lines known as axes. Each point on an axis is associated with a number which specifies the location of that point, and each point in the plane is associated with an ordered pair of points which specifies its location.

In this exploration you will learn to draw a figure in the coordinate plane and to draw another figure which is the reflection of your first one!

PURPOSE:

✓ How can you construct a figure in the coordinate plane?

✓ How can you reflect a figure across an axis in the coordinate plane and obtain another figure which is the reflection of the first?

MATERIALS:

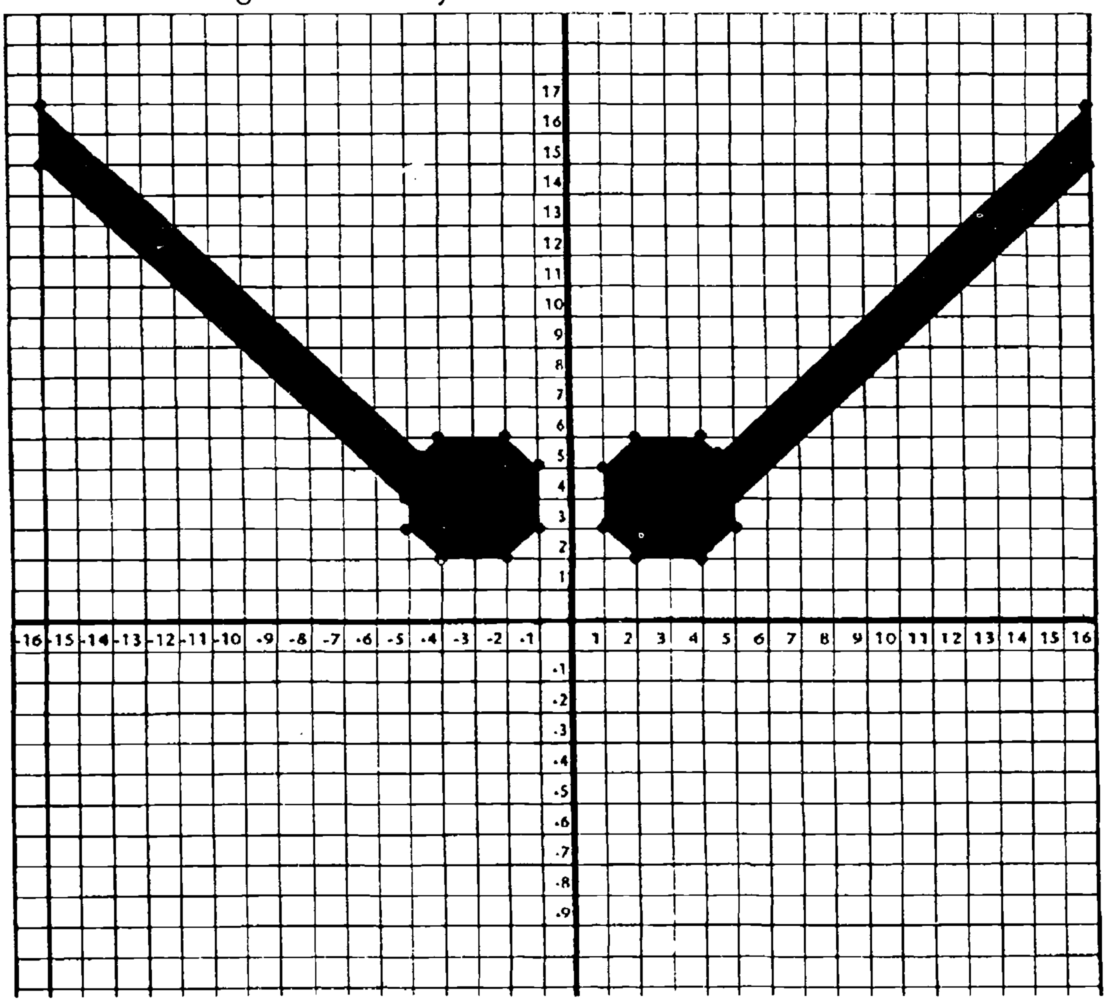
grid paper
construction paper
scissors or art knife (single-blade)
glue or tape
two flashlights
ruler or straight edge

PROCEDURES:

1. Set up a pair of coordinate axes on a piece of grid paper. (Note that you must plot negative numbers for the x axis. See illustration 10-1.)
2. Plot the points with the coordinates listed on the following page.

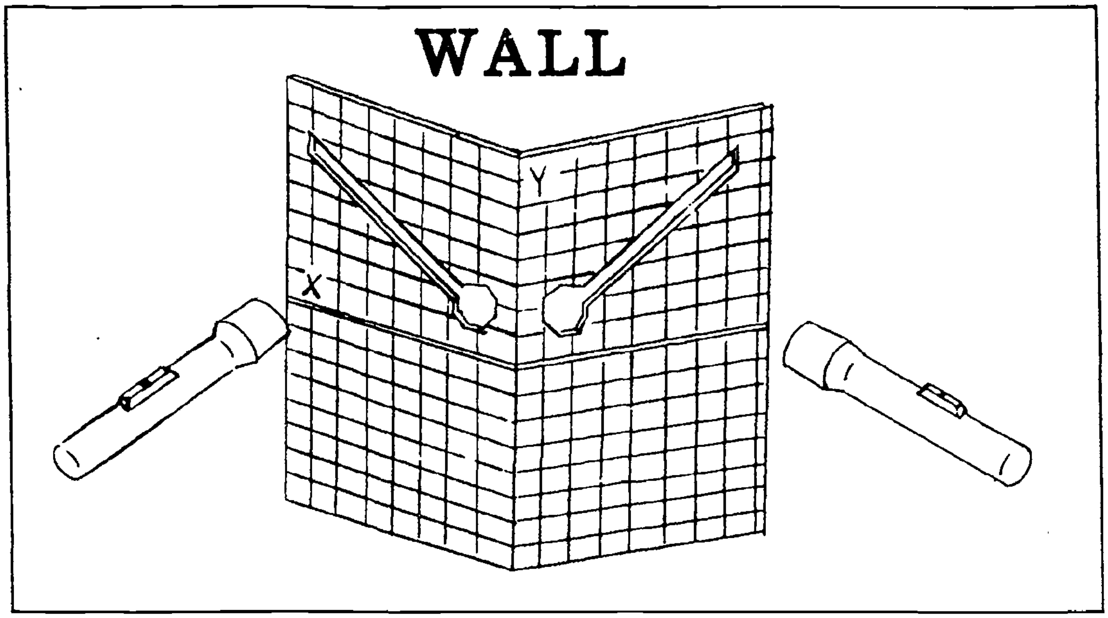
x	y
-16	17
-16	15
-5	4
-5	3
-4	2
-2	2
-1	3
-1	5
-2	6
-4	6
-4.5	5.5

3. Connect the points in the order in which the coordinates are listed.
4. Reflect the figure over the y-axis as in Illustration 10–1.



10 - 1

5. Tape or glue the graph paper to the construction paper and carefully cut the figures out. Fold the paper along the y-axis as in illustration 10–2.



10 - 2

6. Go into a dark room. Stand the paper upright near a wall as illustration 10–2 shows. Shine the flash lights toward the paper and at an angle to one another. Move the flash lights around until a familiar image shows on the wall.

OBSERVATIONS:

1. Make a list of the new points you got when you reflected the figure through the y-axis.
   16,17; 16,15; 5,4; 5,3; 4,2; 2,2; 1,3; 1,5; 2,6; 4, 6; 4.5,5.5 .

2. What is the sign of the x-coordinate?
   When reflected about the y-axis the sign of the x-coordinate changes.

3. What is the sign of the y-coordinate?
   When reflected about the y-axis the sign of the y-coordinate stays the same.

4. What figure do you get when you connect all the dots (points)?
   Answers vary—Circle with a handle; eleven-sided concave polygon.

5. What "picture" did you see on the wall?
   Answers vary.

CONCLUSIONS:

1. What would be the points you would get if you reflected the original figure through the x-axis instead of the y-axis?

   -16,-17; -16,-15; -5,-4; -5,-3; -4,-2; -2,-2; -1,-3; -1,-5; -2,-6; -4,-6; -4.5,-5.5.

2. What is the sign of the x-coordinate?

   When reflected about the x-axis, the sign of the x-coordinate stays the same.

3. What is the sign of the y-coordinate?

   When reflected about the x-axis, the sign of the y-coordinate changes

4. What "picture" would you see on the wall?

   Answers vary — eyes, glasses.

SUGGESTIONS FOR FURTHER STUDY

- Try creating your own figure and reflecting it first through the x-axis and then through the y-axis. Draw a picture of your figure and write the list of points for the original figure and for the reflected figures. Exchange the list of ordered pairs with another student. Each of you plot the others' ordered pairs on a graph to see if you can discover the figure created by the other.

Teacher's Guide
Lah-Di-Dough

GOAL: To allow students to investigate patterns that exist in solids by making solids from dough and slicing them across various planes.

STUDENT OBJECTIVES:

✓ To create three-dimensional solids from dough.

✓ To investigate plane figures made when the dough shapes are sliced across various planes.

GUIDE TO THE INVESTIGATION: The exploration can be carried out by individual students or by students working in pairs or small groups. Students should be reminded to take care with the knife they use to slice their solids. They need flour, salt, powdered alum, food coloring, cooking oil, hot water, a knife, a spoon, a bowl, a set of metric measuring spoons and cups, plastic wrap or a plastic bag, an unopened can of food, a small rectangular box, and a paper cup shaped like a cone. You could substitute modeling clay for the salt dough, but you would need a lot of it. Students may enjoy the activity much more if they make their own materials.

Have students follow the directions given in the Procedures Section and record their findings by making drawings on the Response Sheet. You will probably need to discuss with them the vocabulary used to describe a section – the plane figure one gets by slicing a solid and edges, faces and vertices. When students have completed the investigation, have them share their various findings with the entire class. Have a discussion of the Observations and Conclusions.

VOCABULARY: rectangular prism, edges, faces, vertices, plane, section

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Lah-Di-Dough

INTRODUCTION: In this investigation you will first make salt dough, and then make several shapes from the dough. When you have made the solids, you will carefully slice through them at several different angles and make a sketch of the flat shape you get in the plane of the cut.

PURPOSE:

- ✓ How can you form various three-dimensional shapes from dough?
- ✓ When you slice through three-dimensional shapes at various angles, what patterns or shapes are visible?

MATERIALS:

flour, salt, hot water, food coloring, powdered alum
plastic knife
bowl
spoon
metric measuring cups
metric measuring spoons
plastic wrap or plastic bag
a can of food (unopened)
a small box (rectangular)
a paper cup shaped like a cone

PROCEDURES:

Making the Dough:

1. Work with two or three other students. Follow the recipe, and make some salt dough.

Salt Dough

In a bowl mix:

250 mL flour
100 mL salt
15 mL powdered alum

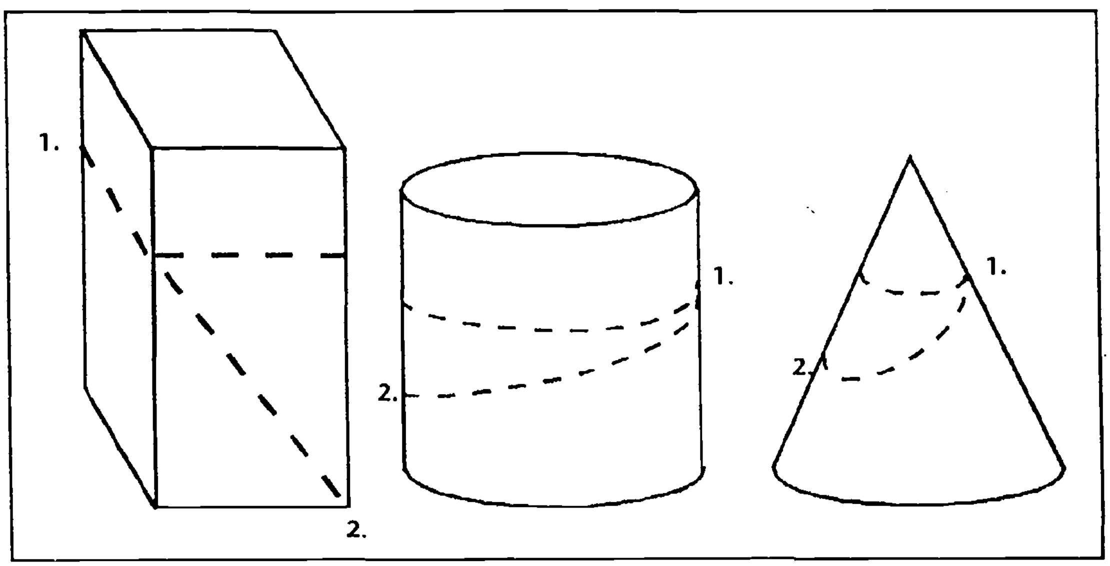
Add:

30 mL cooking oil
45 mL very hot water

2. Stir the dough well. Add a little more water or flour if needed.
3. Knead the dough and then shape into a flat piece.
4. Drop 3 or 4 drops of food coloring onto the dough.
5. Knead the dough again until the food coloring is absorbed.
6. Store the dough in an air tight container until you are ready to use it.

Making solids:

1. Use a small box, shaped like a rectangular prism, as a mold or as a model. If the box is empty, fill it tightly with dough, then tear away the box. Or you can shape the dough by hand. Use the plastic knife, the table top, and your fingers to make the sides of the dough prism smooth. Try to form sharp edges and corners (vertices).
2. Make a second solid from your dough in the shape of a cylinder. You may use a can (of food) as a model. Roll the dough on the table top to get a cylinder, and use the knife and table top to try to make the top and bottom flat and smooth.
3. Use a paper cup in the shape of a cone to form a third solid. Stuff it full of dough, starting with a little bit in the tip. Pack it hard. When it is full, carefully tear the cup away leaving a cone shaped figure of dough.
4. Place each shape on the table as shown in illustration 11-1: the rectangular prism should be stood up on its base; the cylinder on its bottom; the cone on its flat end.



11-1

5. Use the knife now to slice through your shapes about one-third of the way from the top. Make your cuts parallel to the bottom of the shape.
6. Draw on the Response Sheet a picture of what the top of your new figure looks like.
7. When you have finished making a record, slice through each figure again, this time about half-way down from the top of what was left. Instead of making your new cuts parallel to the bottom, this time make them at a 45° angle, (a diagonal cut as shown in illustration 11-1).
8. Draw on the Response Sheet what the new tops of the figures look like.

OBSERVATIONS:

1. What different shapes did you get when you made the first cut?

   prism *rectangle*

   cylinder *circle*

   cone *circle*

2. What different shapes did you get when you made the second cut?

   prism *parallelogram*

   cylinder *ellipse*

   cone *ellipse*

CONCLUSIONS:

1. Why do you get different shapes when you cut at different angles?
   Answers vary, however a cut at 45° produces more surface area in the cross section.

SUGGESTIONS FOR FURTHER STUDY

- If you cut the solids at still other angles, what would happen to the shapes? Make some more dough solids and try it.
- Make a salt dough sphere. See what shapes you get when you repeat the steps of this investigation.

Response Sheet
Lah-Di-Dough

Drawings of Cuts:

Drawings vary.

Rectangular prism

Cylinder

Cone

Teacher's Guide
A Show of Strength

GOAL: To help students explore various shapes, gain an understanding of the use of geometric shapes in construction, and to appreciate the great strength inherent in shapes.

STUDENT OBJECTIVES:

✓ To make a triangular construction which appears not all that stable but which will actually bear great weight.

✓ To explore ways to combine simple materials and shapes to form weight-bearing elements in the construction of buildings, bridges, etc.

GUIDE TO THE INVESTIGATION: This exploration can be conducted by pairs of students or by small groups. (Individual students may have difficulty placing and balancing the knives and cans.)

Each group of students needs three identical table knives, three identical plastic tumblers and several cans of food (beans, carrots, etc.), unopened.

Students should follow the directions given in the Procedures section of their activity sheet.

After students complete the exploration, discuss the results as recorded in Observations and Conclusions. Have students demonstrate any new structural designs they have found that would support more weights (cans). They may draw an illustration of their design and display it for other students to try.

VOCABULARY: horizontal, vertical, intersecting, hilt

A Show of Strength

INTRODUCTION: Architects choose particular shapes for use when designing their buildings because these shapes are strong and can bear the weight of the building. In this investigation you will construct a structure with geometric shapes which do not appear to be very strong, and you will test the materials to try to design a different structure to support more weight.

PURPOSE:

✓ Can you construct a structure with three knives (beams) and three water glasses (cylinders) which will be strong enough to support a number of cans of food (weight)?

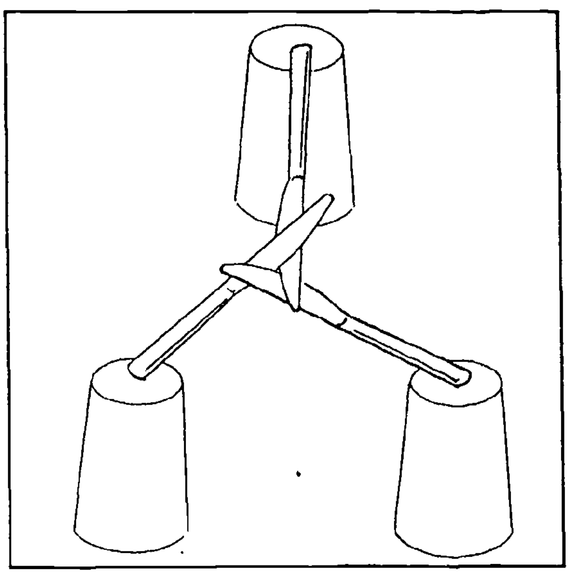
✓ Can you use the same materials to make another structure which supports even more weight?

MATERIALS:

three identical table knives
three identical heavy plastic water glasses
several cans of food (unopened)

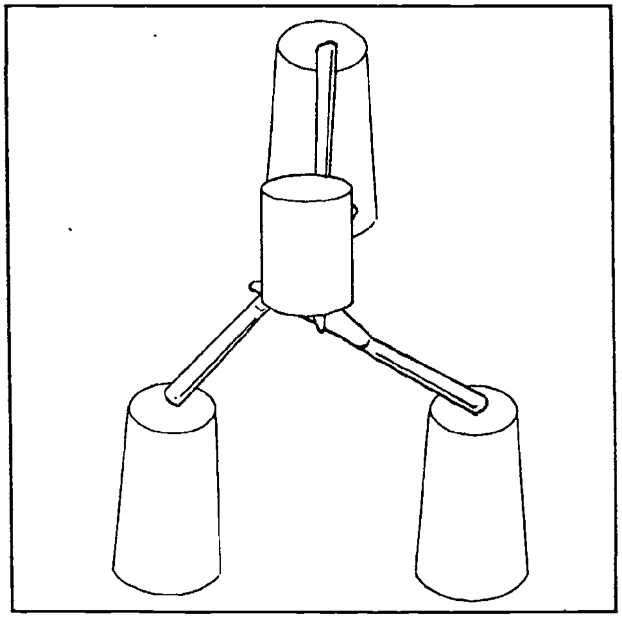
PROCEDURES:

1. Work with several other students to build the construction shown in illustration 12-1 with knives and glasses.
2. Begin by arranging the glasses, bottom up, in a triangle on a flat surface.
3. Overlap and interlock the knife blades as shown to form the inner triangle. The cutting edges of the blades should be on the inside of the triangle.
4. Suspend the knives by balancing the knife handles on the three glasses. You may need to adjust the positions of the glasses to make the construction stable.



12-1

5. Take one can of food and carefully place it in the center of the structure, as shown in illustration 12-2.
6. Then place a second can of food on top of the first can.
7. If the structure supported the first two cans, try to add a third can.
8. Using the three tumblers and three knives, try to make another structural design that will support more cans of food.


12 - 2

OBSERVATIONS:

1. How many cans of food did your first structure hold before it collapsed, if it did?

   Answers vary.

2. Were your results different when you rebuilt the structure? Why or why not?

   Answers vary. The more the students vary the structure, the more variability they will see in the ability of the structure to bear weight.

CONCLUSIONS:

1. What geometric shape was formed by the intersecting knife blades?

   Triangle—accept other answers.

2. Can you explain how your structure gets its strength? Does it have to do with triangles?

   Answers vary. It may have to do with the "strength" of the triangles. There is a postulate in plane geometry which states that 3 points determine a plane, thus using a triangle assures the bottom of the can is resting in a single plane.

SUGGESTIONS FOR FURTHER STUDY

- Use the knives and water glasses to see if you can make a different structure, and test it.
- Experiment with four knives and four water glasses to see what structural support you can build, and test it. Draw a diagram of your structure.

MEASUREMENT

Teacher's Guide
In the Middle

GOAL: To have students explore a way to locate the center of a circle without the use of a compass or any measuring tools.

STUDENT OBJECTIVES:

✓ To explore ways to find the center of a circle.

✓ To check the accuracy of a method of using paper to find the center of a circle using a compass.

GUIDE TO THE INVESTIGATION: This exploration can be conducted by individuals or pairs of students. Seat the students around tables, several to a table, so that they can compare work and discuss what they see as a group. They will need two sheets of construction paper, pencil, a cylindrical object or two, and a compass.

Begin by explaining that a circle is the "picture" of a set of points in a plane that are a fixed distance (radius) from a fixed point (the center). Ask students to use a compass to draw a circle, locate the center, and see how the definition is fulfilled.

Students should follow the directions in the Procedures section of their activity sheet and should record their findings on the Response Sheet.

When students complete the exploration, have small groups of students compare their circles to see if they were able to find the center of the circles. If any pair had difficulty, a successful pair can offer help. Discuss the Observations and Conclusions.

VOCABULARY: circumference, line segment, intersect

In the Middle

INTRODUCTION: Sometimes you trace around the top of a glass or some other solid to draw a circle which you need for some drawing or a circle graph. Usually you need to know where the center of the circle is. If you do not have a compass or any other measuring tool, you can still find the center. In this investigation, you will learn how to find the center of a circle without using a compass.

PURPOSE:

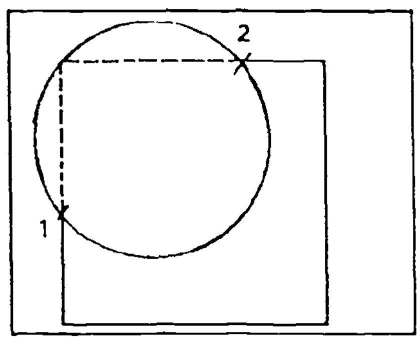
✓ How can you find the center of a circle when you do not have a compass or any other of the usual means of measuring?

✓ How can you check to see if using construction paper circles and paper is an accurate way to find the center of a circle?

MATERIALS:

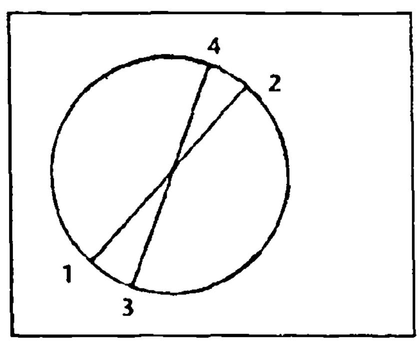
construction paper
a cylindrical object
compass
pencil

PROCEDURES:



13 - 1

1. Draw the outline of the circular object on one of the sheets of construction paper.
2. Place the other sheet of paper so that one of its corners lies on the circumference of the circle.
3. Mark the points on the circle where the edges of the paper cross the circumference of the circle (see illustration 13–1).
4. Use the edge of the paper to draw a line segment from one point to another.
5. Rotate the circle somewhat and repeat the activity. The point where the two lines intersect is the circle's center. (See illustration 13–2.)



13 - 2

6. Then use the compass to see if you were accurate in finding the center of the circle.
7. Repeat the exploration beginning with a larger or smaller circle.
8. Draw another circle, and try to find the center using the compass alone.

OBSERVATIONS:

1. Did the compass verify that you had found the center of the circle?
   Yes.
2. Which method of finding the center was easier? Why?
   Answers vary. Probably the paper method because it was less trial and error.

CONCLUSIONS:

1. Does the corner-of-the-paper method work with any size circle? Why, or why not?
   The circle must have a radius that is less than the measure of the shortest side of the rectangular sheet of paper.
2. Was one of the methods more accurate than the other? Which would you prefer using? Why?
   Answers vary, both methods are equally accurate, but with the compass you must guess the center and set the radius and see if it works, then adjust your guess. With paper and pencil no trial and error is required.

SUGGESTIONS FOR FURTHER STUDY

- Use scissors to cut out a circle from poster board. Place the center point of the circle on the tip of a sharpened pencil. Try to find the circle's center of gravity by balancing the "disc".
- Investigate the circumference, diameter, center, and radius of a circle. What are they? How do you find the measure of the radius? Of the circumference? What is the relationship between the radius and the circumference? *Circumference is the distance around the circle. All points on the circumference are equidistant from the center of a circle. The radius is a line from the center of the circle to a point on the circumference—the radius is half the diameter. The circumference is π times the diameter ($C = \pi d$), or about 3.14 times the diameter.*

- Draw a circle, and locate its center. Draw a line segment from a point on the circle through the center to the other side of the circle. Now mark any other point on the circle, and connect it with one line segment to one end of the diameter and with a second line segment to the other end of the diameter. What kind of triangle did you form? Will this always work? ***A right triangle. Yes.***

Teacher's Guide
An "Areal" View

GOAL: To help gain an understanding of the scale on a map and that a scale is not completely accurate.

STUDENT OBJECTIVES:

✓ To investigate the accuracy of a map's scale.

✓ To compare the area of regions on a map calculated using the map's scale with the information found in an atlas or encyclopedia.

GUIDE TO THE INVESTIGATION: This activity can be conducted by individuals or students can be divided into small groups for completing the investigation, sharing results, and discussion. Each student should individually undertake the investigation. Each will need a ruler, pencil, paper, and maps.

Before students begin, review with them the basic concept of the measure of the area of plane figures such as circles, triangles, and rectangles. Identify the scale of a map, and discuss its use.

Students should then follow the directions in the Procedures section. When the exploration is completed, discuss the Observations and Conclusions.

Note: Make sure students have inch rulers for maps using an inch scale and centimeter rulers for maps using a metric scale.

VOCABULARY: scale, area, accuracy

An "Areal" View

INTRODUCTION: Sometimes you have to find the measure of a particular area. If, for example, you want to shop for new carpet for a room in your house, you will need to measure the area so that you will know how much carpet you need and can figure its cost. It could also happen that you wish to know the measure of an area on a map - if you are comparing the size of states or countries. In this investigation you will learn to use the map's scale to measure land area, and you will learn the limitations of the method.

PURPOSE:

✓ How can you find the measure of an area on a map?

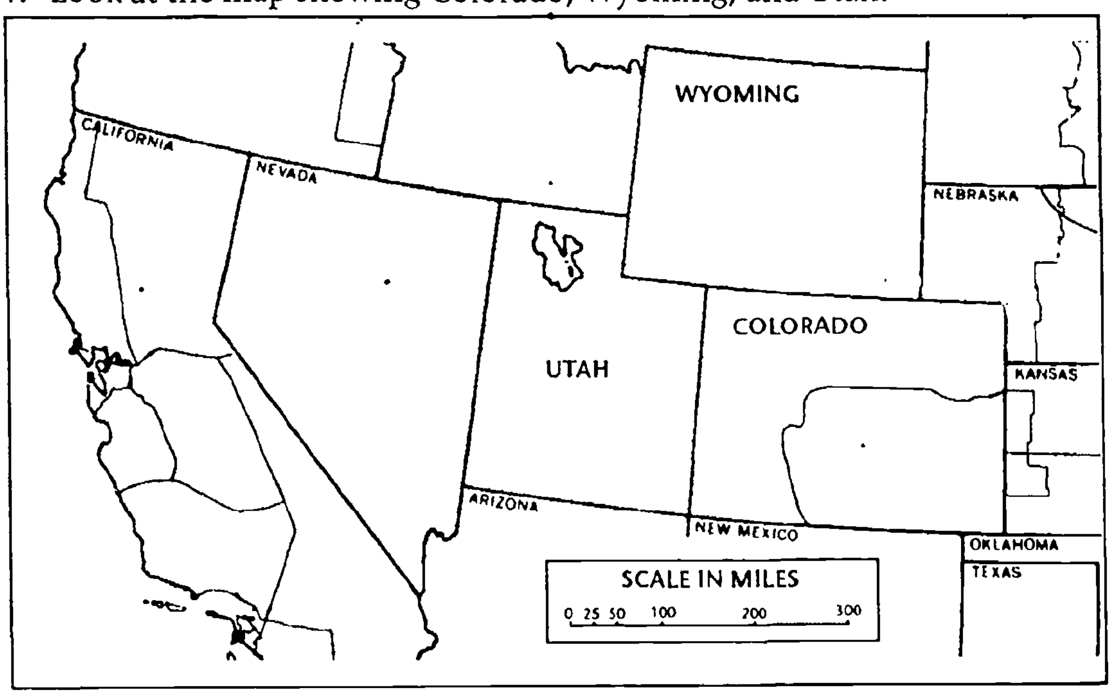
✓ How accurate are map scales?

MATERIALS:

ruler
pencil and paper
maps
atlas, world almanac, or encyclopedia

PROCEDURES:

1. Look at the map showing Colorado, Wyoming, and Utah.



2. Use the ruler to measure the length and width of the states of Colorado, Wyoming, and Utah, individually.
3. Record the length and width in centimeters or inches on the Response Sheet.
4. Use the scale that accompanies the map to convert the measurements into miles.
5. Then use the formula **Area = length x width** to find the area of each state.
6. After figuring the area, look in an encyclopedia, atlas, or other source, to find the areas of these states.
7. Record the areas on the Response Sheet.
8. From a United States map select five other states that are in the shape of a square or rectangle (or close to those shapes).
9. Measure the lengths and widths of the states and find the areas.
10. Place this information on the Response Sheet.

OBSERVATIONS:

1. Were your calculations close to those in the reference book? Note which measure was closest to the one in the reference book.

   Colorado *Answers vary.*

   Wyoming *Answers vary.*

   Utah *Answers vary.*

2. Why are some of your measures closer than others? (There may be more than one reason.)

   Answers vary. One possible reason is that the shape of the state is not really rectangular. Another possible reason is error in measurement.

CONCLUSIONS:

1. Which calculation is probably the more accurate, yours or the one in the reference book? Why?

The one in the reference book. Answers as to why, vary—Example: maps are distorted; many maps use measurements made from scaled maps.

2. Explain why map's scales cannot be 100% accurate. (Ask your teacher to see a large globe.)

A map is a globe that has been flattened, therefore it is distorted. On a flat map, the curvature of the earth's surface is not represented.

3. Which figures are most likely to be more accurate? Why?

Answers vary. Those in an atlas or encyclopedia are more likely to be accurate. Because of the distortion in maps and mistakes by the person measuring on a map.

SUGGESTIONS FOR FURTHER STUDY

- See if you can locate a huge world globe. Repeat the exercise using the globe. Make a Response Sheet and compare the results of using the globe with what you did here. Do you see any differences? Can you explain them? *Answers vary.*
- Choose four or five other land areas on the map whose shapes are roughly similar to squares or rectangles. Calculate the area and compare your results with those in a reference book.
- Plan a vacation trip across the country for your parents. Use an atlas to mark which roads they will travel, and use the map's scale to calculate how far they will travel. Choose the towns where they will spend the night by allowing them between 400 and 500 miles of driving each day. Calculate the total mileage.
- If you are actually going on a trip, perform the suggestion above before you go, and make a Response Sheet with your calculation in one column. When you take the trip, enter the actual miles you drive in the second column. At the end of the trip, compare all the results. How accurate were your estimates?

Response Sheet
An "Areal" View

State	Length	Width	Area (length x width)*	Area (from printed source)
Colorado	*Answers vary.*			*Check an encyclopedia or atlas.*
Wyoming				
Utah				
Others:				

** Multiplication compounds measurement error exponentially.*

83

Teacher's Guide
Hand-y Measurements

GOAL: To help students explore metric and customary units of measure and to learn to use the hands, forearms, and feet to estimate measures.

STUDENT OBJECTIVES:

- ✓ To investigate the usefulness of hands, feet, and forearms as "portable" measuring tools.
- ✓ To make accurate estimates of measures.

GUIDE TO THE INVESTIGATION: This exploration should be completed by pairs of students or by groups of up to six students. Students should be paired with partners whose heights are several inches different from their own, if possible. Students need a pencil and a metric and customary ruler for the exploration.

To introduce the exploration, discuss with students the need to make accurate estimates. Ask if any students have seen their parents or grandparents using a body part to measure length, width, or height. For example, they may have seen someone measure from the tip of the nose to the end of an outstretched arm for a yard or a "giant" step for a yard.

Have students work in pairs and use a centimeter tape to measure each student's height and arm span (from middle fingertip to middle fingertip with both arms stretched out). Have students discuss the relationship they found between height and arm span.

Have students follow directions in the Procedures section. When they have completed the exploration, discuss the Observations and Conclusions reached by each pair or group of students.

VOCABULARY: hand-span, forearm, customary measure, metric measure

Hand-y Measurements

INTRODUCTION: You may find yourself in a situation in which you will need a measurement and do not have any of the usual tools for measuring. In this exploration, you will learn to use your hands, arms, and feet as measuring tools.

PURPOSE:

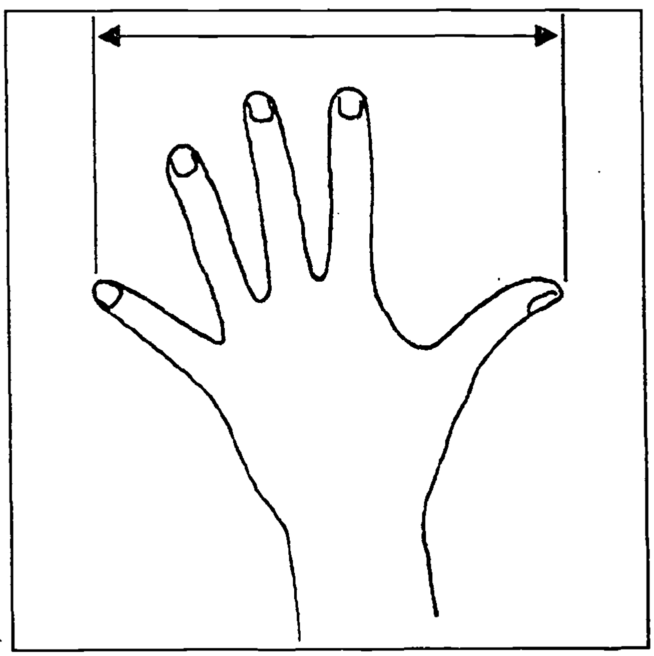
- ✓ How accurately can you measure with the measuring tools you always have on you— your hands, arms, and feet?
- ✓ Is the accuracy of your measure affected when you use metric measurements rather than customary measurements?
- ✓ How can you increase the accuracy of your estimates?

MATERIALS:

metric and customary rulers

PROCEDURES:

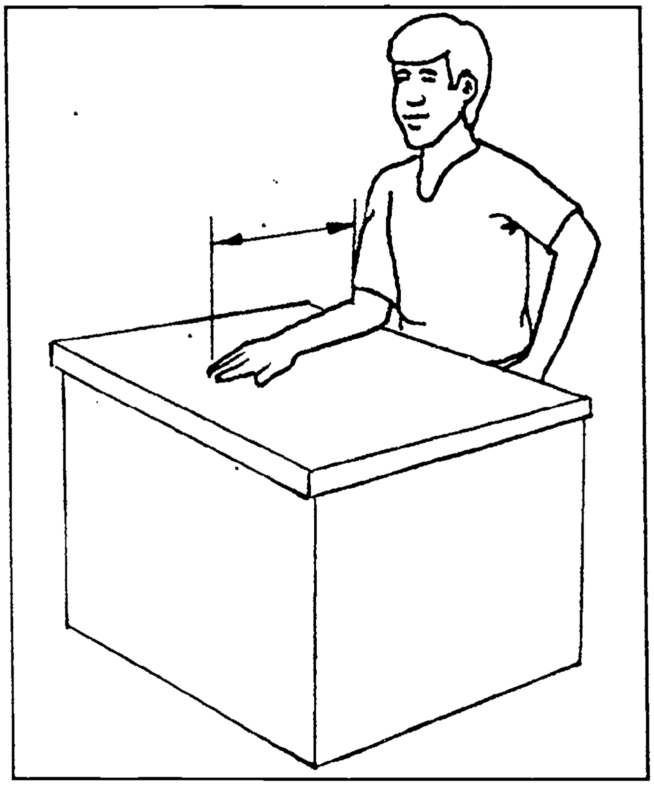
1. Spread your fingers and measure the distance between the tip of your thumb and the tip of your little finger, as in illustration 15–1. This is called your hand-span.
2. Record the measure of your hand-span on the Response Sheet.
3. Then measure your hand-span with the 12-inch ruler and with a metric ruler. Record on the Response Sheet.
4. Make the same careful measurements of the length of your foot.



15 - 1 *Hand-span*

5. Record the measure on the Response Sheet.

6. Sit facing a table. Lay your forearm on the table top with your elbow near you and your arm, hand, and fingers stretched straight out as shown in illustration 15-2.
7. Have a friend measure from the point of your elbow to the tip of your longest finger.
8. Record the measure on the Response Sheet.
9. Select a table in your classroom. First guess the measure of its length, width, and height in terms of your portable tools, that is, how many hand-spans and feet.


15-2

10. After you record the guesses on the table on the Response Sheet, measure the length, width, and height of the table with your portable tools.
11. Record these measures on the Response Sheet.
12. Estimate the width of a hallway in your own "foot" length.
13. Enter the estimate on the table.
14. Then measure the width of the hallway with your foot, and enter the actual measurements into the table.
15. Work with your partner. Estimate and then measure other items, this time using a tape measure or ruler.
16. Record these measures on the table on the Response Sheet and compare them with the figures you got with the personal measuring devices.

OBSERVATIONS:

1. Which of the "portable tools" gave the most accurate measurements?
Answers vary. Usually hand span and foot are more accurate because they are smaller units.

2. If you used more than one of your portable tools for a job, which was easier to use? Was it too hard to use one on one job and another on another? Why?

   Answers vary. Probably the foot is easiest to use, however, the tool to use depends on the relative size of the tool and the length to be measured.

3. Were your personal tools more accurate in meters and centimeters or in feet and inches? Explain your answer.

   Answers vary. Probably more accurate in feet and inches because centimeters are smaller measures than inches.

CONCLUSIONS:

1. Did one of your personal tools seem more accurate than the other? If so, which? Why do you think this happened? If not, why do you think not?

   Answers vary. Hand span and foot may be more accurate because they are the smallest "units".

2. Did one of your personal tools seem less accurate than the others? If so, which? Why do you think this happened? If not, why do you think not?

   Answers vary. The longest "unit" may be the least accurate.

3. Can you think of some ways to increase your accuracy when you use your personal tools?

   Answers vary. A possible answer is that the "unit" be calibrated into smaller "sub-units".

4. Can you think of some times when you will need to use your personal, portable tools?

   Answers vary. For example, farmers often "step-off" measurements in the fields.

SUGGESTIONS FOR FURTHER STUDY

- In England a person's weight was sometimes given in stones in the past. Find a reference in the library which explains the British unit of measurement of weight called a stone. Find an equivalent for it which you understand. How many stones do you weigh? Where did this unit of measurement come from? *Answers vary.*
- Measure the length and width of the room in which you sleep, and report your findings to your class. Do not use commercial measuring tools.
- Choose a large object to measure - your yard, your school's swimming pool, or tennis courts, for example - and you and a classmate measure it with your personal tools. Then use a yardstick or tape measure to measure the objects. Report your results in "body parts" and in feet and inches. Compare your results.
- Find a reference book which explains the customary system of measurement (feet, inches) and write a short paper on its history.

Response Sheet
Hand-y Measurements

Portable Measuring Tool	Customary Measure	Metric Measure
Hand Span	*Answers vary.*	
Foot Length		
Forearm Length		

Your Personal Measuring Equivalent

Item	Measurement	Handspan	Forearm Length	Foot Lengths	Customary Units	Metric Units
Table Length	Estimated:	*Answers vary.*				
	Actual:					
Table Width	Estimated:					
	Actual:					
Table Height	Estimated:					
	Actual:					
Doorway height	Estimated:					
	Actual:					
Doorway width	Estimated:					
	Actual:					
Hallway Length	Estimated:					
	Actual:					
Hallway Width	Estimated:					
	Actual:					

Estimated and Actual Measures

Teacher's Guide
Seeing Is Believing!

GOAL: To help students develop an understanding of the concept of volume and investigate ways to determine the volume of a cylinder.

STUDENT OBJECTIVES:

- ✓ To construct two different cylinders.
- ✓ To explore and compare two ways to calculate the volume of the cylinders.

GUIDE TO THE INVESTIGATION: This investigation can be conducted by individuals or small groups of students. Each student or group needs corrugated cardboard, construction paper, flour, scissors, centimeter ruler, metric measuring spoons, and the pattern sheet.

Students should follow the directions given in the Procedures Section of their activity sheet. Each group of students should construct the large cylinder and the small cylinder using the material listed. They should then compute the volume of each cylinder using the formula, $V=\pi r^2h$. After students have constructed the cylinder and measured the flour, compute the volumes and discuss the results. Have each group report their findings and share their explanations as to the most reliable way to determine the volume of a cylinder. Discuss the Observations and Conclusions.

VOCABULARY: cylinder, volume, formula, radius

Seeing Is Believing

INTRODUCTION: There are many ways to find the volume of a container. In this investigation, you will construct two cylindrical containers of different sizes. Then you will use two different methods to find the amount that each cylinder will hold. Finally you will be asked to judge between the two methods to choose the one you think to be the most reliable.

PURPOSE:

- ✓ What are some ways to find the volume of a cylinder?
- ✓ Are the results the same when you use more than one way?
- ✓ Is one way of finding the volume of a cylinder more reliable than another?

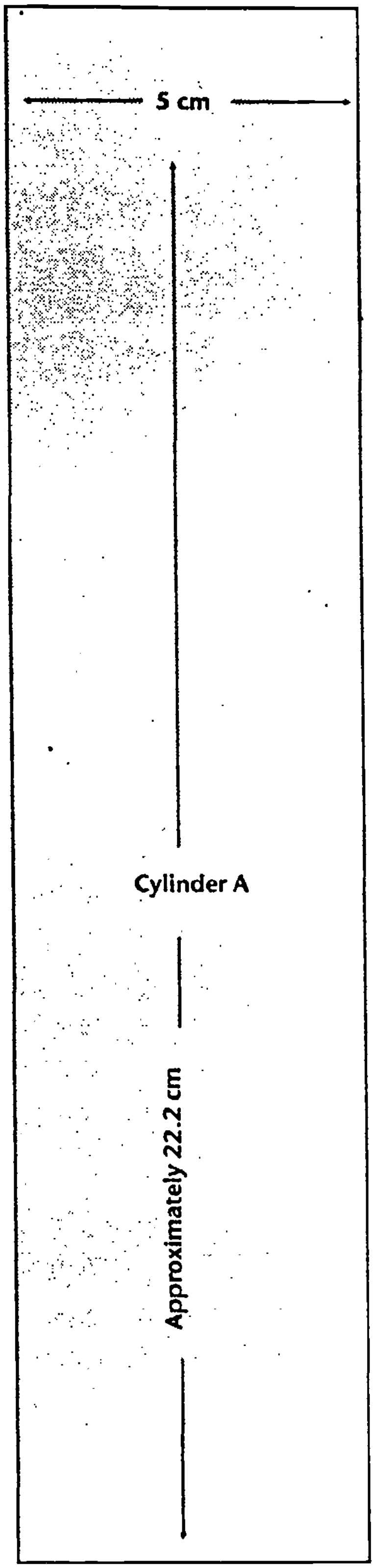
MATERIALS:

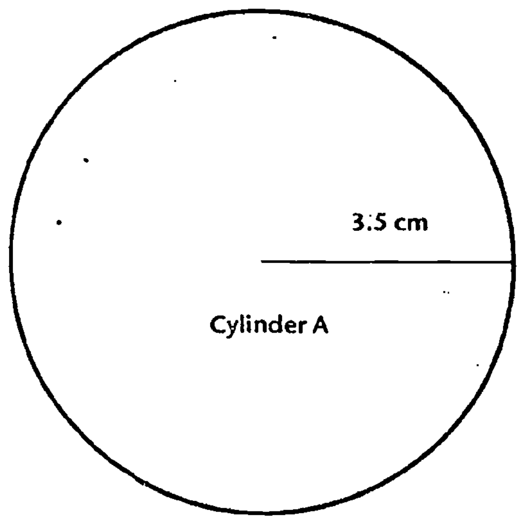
corrugated cardboard
construction paper
scissors
tape
centimeter ruler
metric measuring spoons
flour

PROCEDURES:

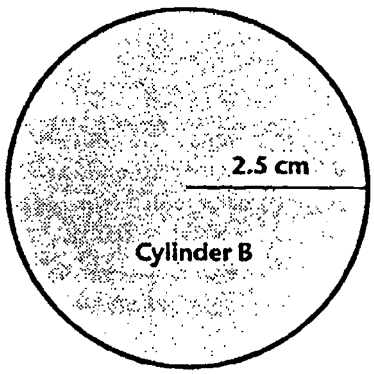
1. Use the rectangles and circles shown on the materials sheet as patterns.
2. Cut two bases (circles) for the cylinders from the corrugated cardboard.
3. Cut the strips from construction paper.
4. Assemble the bases and sides of the cylinder as indicated on the pattern.
5. Tape the sides to the base.
6. Fill each cylinder with flour, using the 10 mL spoon.
7. Count and record the number of milliliters of flour it took to fill each cylinder.
8. Use the formula $V=\pi r^2h$ to find the volume of each cylinder. (V stands for volume; r stands for radius; h stands for height.) Use 3.14 for π. Record your answer on the Response Sheet.

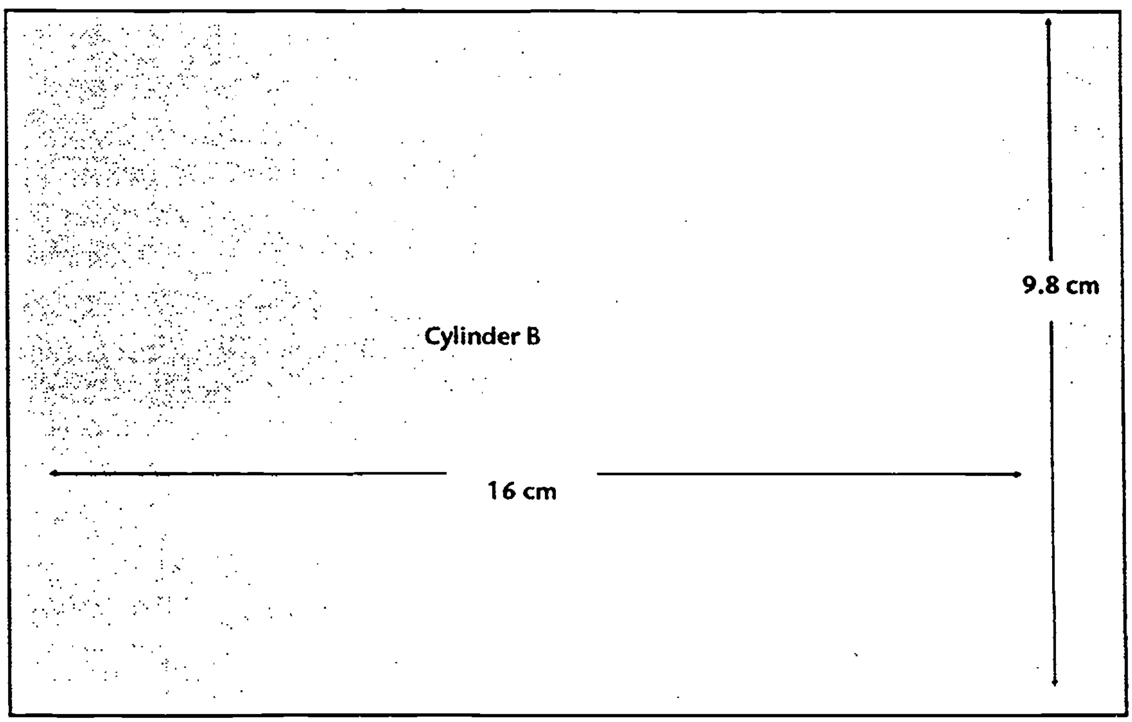
Materials Sheet 1
Seeing Is Believing!





Materials Sheet 2
Seeing Is Believing!





OBSERVATIONS:

1. Which cylinder held more flour and how much more?

   They hold about the same, however, the cylinder with a radius of 3.5 cm holds about 500 mL more.

2. When you used the formula for volume, which cylinder had the greater volume and by how much?

   They hold about the same, however, the cylinder with a radius of 3.5 cm holds about 539 mL more.

3. Did your calculations verify what you found with the flour? If so, why? If not, why not?

   Yes. Answers vary. Calculated volumes are the same and volumes found by filling are approximately the same.

CONCLUSIONS:

1. Which is a more reliable way to determine the volume of a cylinder? Why do you think so?

   Using the formula. Answers vary — you can be fooled by how the container looks.

2. If you used customary units of measure to complete this investigation, do you think you would get the same results as when you used metric units? Why?

   Yes. Volume would still be the same, that is the same number of units would be required to fill the two cylinders. Counted volume would not compare as well with calculated volume.

   Note: 1 mL = 1 cm^3 of water.

SUGGESTIONS FOR FURTHER STUDY

- Take several glasses, cups, and/or empty food cans at home, and calculate their volumes using a ruler and the formula for the volume of a cylinder. Calculate volumes also by measuring water poured into each. Compare the results. Do they confirm what you found in this investigation or not? Do any of the containers with different shapes have almost the same volume? *Yes; answers vary.*

Response Sheet
Seeing Is Believing!

Cylinder	Number of Milliliters needed to fill the cylinder
A	*About 850*
B	*About 300*

The Amount of Flour Used to Fill Each Cylinder

Cylinder	Height	Radius	Volume ($V=\pi r^2 h$)
A	*22.2 cm*	*3.5 cm*	*853.923 cm^3*
B	*16 cm*	*2.5 cm*	*314 cm^3*

Cylinder Volumes

Teacher's Guide
Boat Races

GOAL: To help students explore a method to determine the speed of a boat.

STUDENT OBJECTIVES:

✓ To construct several boats and a racing lane in which to float them.

✓ To use the boats and racing lane to learn to measure the speed of a boat.

GUIDE TO THE INVESTIGATION: This exploration can be conducted by students in pairs or small groups. Each should have a turn at each activity, and all should be involved in the calculations and discussion of the significance of the findings. Students will need light cardboard or posterboard, scissors, metric ruler, string, a rectangular shallow pan or cookie sheet with a lip, water, tape, a marker, cooking oil, an eye dropper, a measuring cup, and a stop watch or a watch with a second hand.

Have students follow directions given in the Procedures section of their activity sheet. Each pair of students should construct 3 or 4 boats to "run" in the race. Caution students to make sure that the boats are as identical as they can make them. Instruct students about how to operate the stop watches.

Discuss boat races, such as the America's Cup, with students. Explain that in this activity they will make several boats and a racing lane and will time the boats from start to finish in their racing lanes. Ask students to suggest some things that make a difference in the speed of a boat. After the "races" discuss the Observations and Conclusions.

VOCABULARY: speed, distance, rate, time, surface tension

Boat Races

INTRODUCTION: The average speed of anything which is moving at a fairly constant rate can be calculated by dividing the distance it travels by the length of time it takes to travel the distance. You can time your family's car over a distance of a mile, for example, and calculate its speed because you know the distance the car went and how long it took to go that distance. In this exploration, you will make some boats and a racing lane and apply that principle to calculate the speed of a moving object.

PURPOSE:

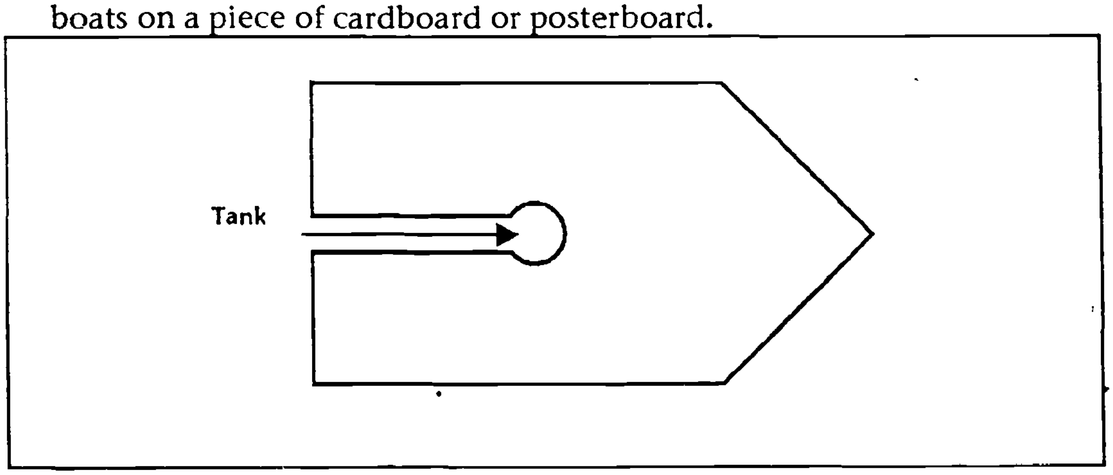
- ✓ How can you find the speed of a boat?
- ✓ Does the size of the boat make any difference?
- ✓ Does the liquid the boat is floating in make any difference?

MATERIALS:

cardboard or posterboard
scissors
eye dropper
cooking oil
marker
stopwatch or watch with a second hand
rectangular pan or tray with a lip
string
tape
water
metric ruler
measuring cup

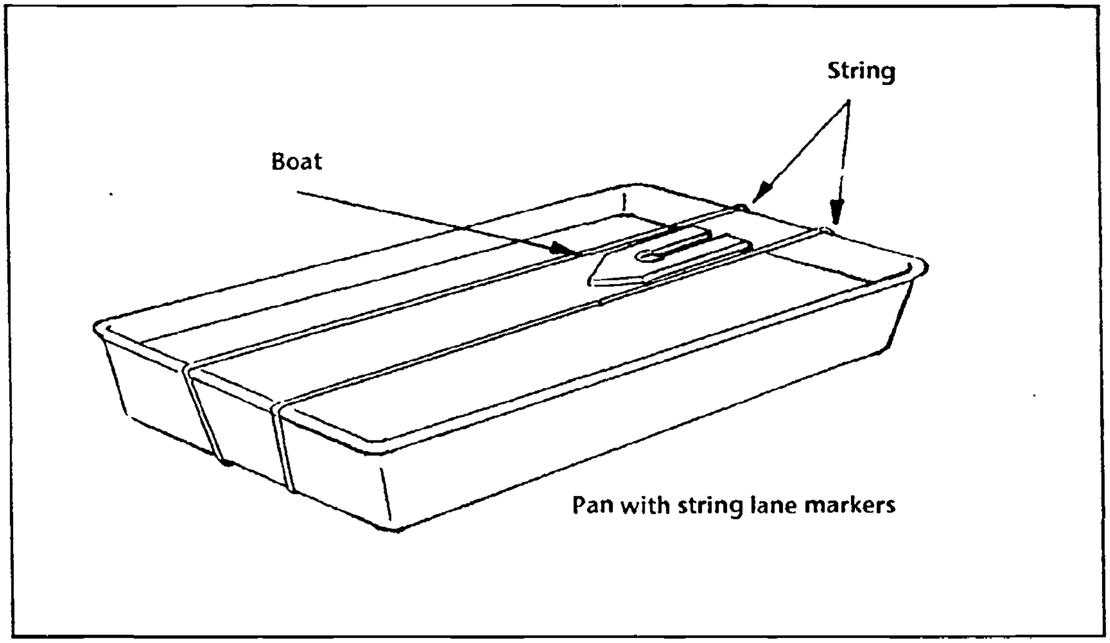
PROCEDURES:

1. Use the pattern in illustration 17-1 as a guide, and carefully draw similar boats on a piece of cardboard or posterboard.



17 - 1 ***Boat Pattern***

2. Cut the patterns out with scissors as carefully and evenly as you can. If your cuts are jagged or rough, the boat may not "race" in a straight line.
3. Make several identical boats.
4. Use a marker to put a different racing number on each boat.
5. Measure the length of your boat in millimeters and record the number on the Response Sheet.
6. Measure the length of your pan in millimeters and record the measure on the Response Sheet.
7. Tightly tie two pieces of string lengthwise around the pan. Be sure that



17 • 2 *Tape the ends of the strings to the pan.*

the strings are parallel to one another and that the distance between them is just wider than your boats. These pieces of string will form the racing lane for the boats, as shown in illustration 17-2.

8. Fill the pan with water so that the water is just below or on the string. You can carefully add the last bit of water from a measuring cup to avoid overflowing the pan.
9. Gently place one of your boats between the strings so that its stern is against the end of the pan and its bow points toward the other end of the race course. Be careful not to break the surface tension of the water so that the boat remains afloat and does not sink into the water at all, not even at a corner. If you slip up, take the boat out, wait until the water is

calm, and gently try with another boat. (It is a good idea to make extra boats.)

10. Once the boat is in position at the starting line, use the eyedropper to put a single drop of cooking oil (your fuel) into the boat's fuel tank.
11. The moment you drop the cooking oil, your partner must start the stopwatch.
12. When the point of the boat's bow touches the other side of the pan, your partner should stop the stopwatch and record the elapsed time in seconds on the Response Sheet. If the boat stops before it reaches the other end, your partner must stop the stopwatch when the boat stops, and you must measure the distance it travels. Be careful to take the length of the boat into account!
13. Repeat the time trial with three other boats, each one dry and unused.
14. Divide the distance traveled by each boat by the time it took to travel the distance, and record all these data on the Response Sheet.
15. Now make four boats about 0.5 cm longer than the ones you just raced. Race them, and record the results on the Response Sheet.

OBSERVATIONS:

1. Did each of the smaller boats take the same time to complete the race? Why, or why not?

   Answers vary. If the size of the boats are uniform, times should be approximately the same. If the size of the oil drops vary significantly it may cause variations in speed.

2. What was the average speed of the smaller boats?

   Answers vary.

3. Did each of the larger boats take the same amount of time to complete the race? Why, or why not?

   Answers vary. If the size of the boats are uniform, times should be approximately the same. If the size of the oil drops vary significantly it may cause variations in speed.

4. What was the average speed of the larger boats?

   Answers vary.

CONCLUSIONS:

1. What factors do you think influenced the speed of the boats?
   Length of the boat, construction material, kind of fuel, size of oil drops, etc.

2. Can you determine the speed of other vehicles the same way? Give some examples to support your answer. Tell how you would do it for each.
   Yes} run model cars down ramps; answers and methods vary— the mass of the vehicle is one of the factors effecting speed.

3. Does the same method work for vehicles on land? Why or why not?
   Yes. Reasons vary, but size and power of motor is more of a factor than vehicle length.

4. How did you determine the average speed of the boats?
   Add the total time and total distance. Divide the total distance by the total time. Then find the average for the 4 trials.

5. Which boats were faster, the small boats or the large boats? Why do you think this was true?
   Answers vary. Probably the smaller boat, but accept any reasonable answer.

SUGGESTIONS FOR FURTHER STUDY

- Repeat the investigation but try using different fuels such as rubbing alcohol and nail polish remover, and see what results you get. Are the boats faster or slower when they run on other fuels? Also try ice water and hot water. Will these propel the boats? *Answers depend on fuel.*
- Try floating the boat in a different liquid, such as half water and half apple juice. Does that make a difference? *Answers vary.*
- Try widening the racing lane. Does that make a difference? *Answers vary.*

Response Sheet
Boat Races

1. Be careful when you measure the distance traveled. The length of the pan is not the distance the boat travels. *All answers vary.*

   The length of the boat is ____________ millimeters.

   The length of the pan is ____________ millimeters.

Boat	Distance Traveled	Time	Speed (Distance ÷ Time)
A			
B			
C			
D			

2. The length of a large boat is ____________ millimeters.

   The length of the pan is ____________ millimeters.

Boat	Distance Traveled	Time	Speed (Distance ÷ Time)
A			
B			
C			
D			

Average speed of the 4 boats ________.

Teacher's Guide
My Cup Runneth Over Not!

GOAL: To help students develop an understanding that solid objects all have volume even when they are not, themselves, containers.

STUDENT OBJECTIVES:

- ✓ To find a way to compute the volume of a solid object which, itself, cannot hold anything.
- ✓ To determine the accuracy of the methods of computing.

GUIDE TO THE INVESTIGATION: This investigation is probably better carried out by students in pairs; however, it is also suitable for either individuals or small groups. Each student or group will need several beakers, preferably 100, 250, 500, and 1000 mL, some water, a metric tape measure, various sized marbles, some rocks, a marker, and a pencil.

Students should follow the directions in the Procedures section of their activity sheet. When they have completed the investigation, have each group report their results. Have each group compare the results they obtained with the rocks and marbles. Discuss the Observations and Conclusions

VOCABULARY: volume, relationship, overflow

My Cup Runneth Over Not!

INTRODUCTION: Have you ever placed ice cubes into a glass of tea or a soft drink and had your drink spill over? In this activity you will do something very similar by placing marbles in a glass. You will discover how to find the volume of objects which are not containers.

PURPOSE:

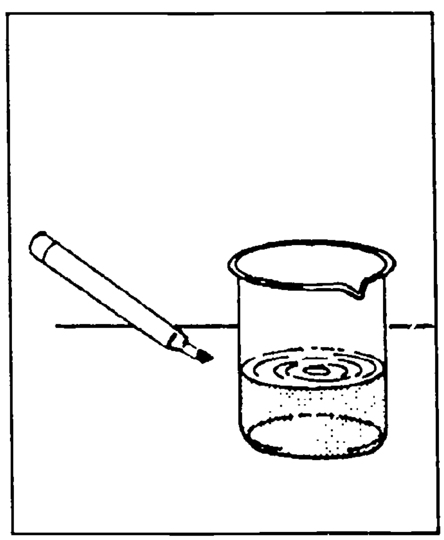
- ✓ How can you use a glass of water to find the volume of a rock or other solid object?
- ✓ Which has the greater volume, a rock, or a marble the same size as the rock?
- ✓ Can you make a rule about the relationship between the size of an object and its volume? Why, or why not?

MATERIALS:

several clear glass beakers, 100 through 1000 mL
water
marbles and rocks of various sizes
a marker
a metric tape measure

PROCEDURES:

1. Fill the 100 mL beaker half full of water. Use a marker to indicate the "half full" point on the beaker.
2. Place a marble in the water. Use the marker to indicate the new level of the water.
3. Use the tape measure to find how many millimeters or centimeters the water rose when the marble was added.
4. Add another marble of the same size. Did the water rise the same amount as when the first marble was added?
5. Mark the level of the water with the marker.



6. Now add a rock that is a little larger than the marbles. Measure and record the height of the water.
7. Repeat this experiment using a 250 mL beaker; repeat again using a 500 mL beaker, and then again using a 1000 mL beaker.
8. Repeat the entire experiment using larger marbles and a larger rock.

OBSERVATIONS:

1. When you added first one and then a second small marble to the 100 mL beaker, did the water level rise the same amount each time? Explain why, or why not.

   Answers vary. Yes— if the marbles are all the same size and made from the same material.

2. When you added first one and then another large marble to the 1000 mL beaker, did the water level rise the same amount each time? Explain why, or why not?

   Answers vary. Yes— if the marbles are all the same size and made from the same material.

3. Did you seem to get the same results when you used larger beakers? Why, or why not?

   Answers vary. The change in the height of the water is less, but if the objects are the same size and material then changes are proportional.

4. In the first experiment, with the smaller marbles and rock, which had the larger volume, a marble or the rock?

   Answers vary, depending on the size and type of rock.

5. In the second experiment, with the larger marbles and rock, which had the larger volume, a marble or the rock?

   Answers vary, depending on the size and type of rock.

CONCLUSIONS:

1. Which of the beakers seems to have been easiest to use and to have given the most accurate results? Why?
   Answers vary, however, the 100mL beaker will provide the greatest variation in height.

2. Did the use of the larger items have any effect on the accuracy of the experiment?
   Answers vary, depending on density of items.

3. Can you make any generalizations about the sizes and/or shapes of objects and their volumes?
   Yes—but size and shape are not the primary factors influencing volume. The density of the object is the primary factor.

SUGGESTIONS FOR FURTHER STUDY

- Fill a beaker to the very edge with water, and place it carefully in a pie tin. Then place a marble into it, and allow the water to overflow into the pie tin. Carefully remove the beaker without spilling any more water. Use a graduated cylinder to measure the water from the pie tin. Repeat with a second marble, and finally add a rock. Do your answers correspond with those you got in this investigation? Which method seems the more accurate? *Answers vary.*

1.0

- Fill a pie tin to the brim with water, and set it in your bathtub. Put one drop of liquid soap into the pie tin. This will break the surface tension, and some water may overflow the pie tin. Let it run down the drain, but the pie tin needs to stay full to the brim. After the water has settled down, as gently and carefully as you possibly can, float an empty tuna can on the water in the pie tin. Notice these things: (i) some water overflows the pie tin and runs down the drain, and (ii) part of the tuna can is below the water line. The volume of the part of the tuna can which is below the water level is equal to the volume of the water which ran out of the pie tin. Can you explain why, when you know that the metal in the can is heavier than water, the can still floats? *Answers vary.*

Response Sheet
My Cup Runneth Over Not!

WILL THE CUP RUN OVER?

Container	Height of water	Height of water with one marble	Height of water with two marbles	Height of water with two marbles and a rock
100 mL beaker	*All answers vary.*			
250 mL beaker				
500 mL beaker				
1000 mL beaker				

NUMBER SENSE

Teacher's Guide
Picture That

GOAL: To have students explore various number systems such as the Hindu-Arabic System, Chinese, Egyptian, and Babylonian systems, and to develop a working way of counting.

STUDENT OBJECTIVES:

✓ To study the history of counting.

✓ To develop an original counting system

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students, pairs of students, or students working in small groups.

Each will need a pencil or marker and one or more reference books: dictionary, encyclopedia, or an encyclopedia of mathematics.

Students follow the directions in the Procedures section of the activity sheets.

After the students have completed the exploration, discuss the results, using the Observations and Conclusions as a guide. If they have worked in groups, have each group present the group's counting systems to the class. Following the presentations, discuss the similarities and differences in the various systems. Have groups prepare posters to illustrate their systems.

VOCABULARY: symbol, idea, counting system

Picture That

INTRODUCTION: Look up the word, "count" in your dictionary. Read the definition which has to do with numbers. Counting is an idea. Ideas are not things which can be touched, but they can be represented by symbols. The word "count" is represented by the symbols c, o, u, n, and t, (which are letters of our alphabet) when we put them together in the proper order. Marks on paper such as 1, 2, 3, help us visualize what the idea of counting is all about. In this exercise you will develop your own symbols for numbers and develop your own system for counting. Symbols are in a sense, pictures. In this activity you will develop your own counting system and make symbols for the numbers.

PURPOSE:

- ✓ What are some number systems of ancient times and how did they work?
- ✓ Why, if they worked, do we no longer use them?
- ✓ What important characteristics must number systems have to make them effective?

MATERIALS:

dictionary
encyclopedia (or math encyclopedia)
pencils and/or markers

PROCEDURES:

1. Find a definition for counting in a dictionary.
2. Have a partner find a definition in another dictionary.
3. Compare the definitions. Are they similar to the one in the introduction?
4. In an "M" encyclopedia or in a math encyclopedia read about the counting systems of the Chinese, ancient Egyptians, ancient Babylonians, and the Mayans.
5. "Draw" some symbols used in these systems on the Response Sheet.
6. Answer questions 2 and 3 on the Response Sheet.
7. Next plan your own number system.
8. Decide what your numbers will look like.

9. Decide what base you will use.
10. How will you arrange your symbols - left to right or top to bottom?
11. Once your system is developed, draw the symbols on the Response Sheet and try it out.
12. Write some numbers, such as your birth date, or the current date.
13. Do some addition and subtraction problems in your new system.
14. Does the system work?

OBSERVATIONS:

1. When you made up your own system, were you able to make it work very well, or did you have to change it to make it work better?
   Answers vary.
2. What are some advantages of your own system?
   Answers vary.
3. What are the disadvantages of your system?
   Answers vary.

CONCLUSIONS:

1. Why do you think the system of counting we use today is in use all over the world?

   Answers vary. Can make infinite numbers; uses place value; has only ten symbols; uniformity; efficiency, etc.

2. What improvements would you suggest for the number system we use?

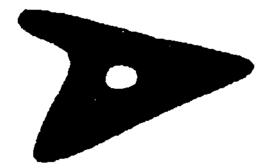
   Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- Find a reference book about counting systems which use a base other than ten in your library. Two and sixteen are common. Try to learn a new system and do some addition and subtraction in it. Make a poster to illustrate the system. Share the poster with your class.
- Find a math encyclopedia or other book that describes the base five system, a non decimal numeration system. Examine the structure of a base five system and explore how the base five system works. What are some advantages of a base five system? What are some disadvantages of a base five system? Try to do some addition, subtraction, and multiplication problems using the base five system.

Response Sheet
Picture That

1. Here are some symbols for ten in other systems:

Greek	Egyptian	Babylonian
I		

Draw the symbols for 1 and for 100 in the Chinese, Egyptian, and Babylonian systems if they had one.

Number	Ancient Greece	Ancient Egypt	Ancient Babylon
1	A	|	
100	P		

2. What was one unique feature of the Mayan System?
   It was one of the first systems to have a symbol for zero.

3. Why do you suppose that there was a need for the Hindu-Arabic System?
   Answers vary. The need for place value; or simplicity of symbols—in other systems it is difficult to write very large numbers.

4. Use the remaining space to write and explain your counting system. Write your explanation as if you were telling a friend how to count!

Answers vary.

Use more paper if you need to!

Teacher's Guide
Nature's Thermometer

GOAL: To help students gain an understanding that there are ways of approximating temperature, some of which were used in the past when people did not have modern thermometers.

STUDENT OBJECTIVES:

✓ To see if you can use a cricket to approximate room temperature.

GUIDE TO THE INVESTIGATION: This experiment is easier to undertake in warmer months when crickets are more abundant. Crickets are also available at stores which sell bait for fishing.

Students will need to find a cricket to complete this exploration. They should prepare a suitable temporary "home" for the cricket by placing barely damp grasses in a cardboard box which is closed, but very well ventilated with holes too small to allow the cricket to escape or you can cover the holes with screen.

Students may complete this exploration at home or you may want them to bring their crickets to class. The exploration can be carried out by individual students, pairs of students, or students working in small groups.

Refer the students to the activity sheet and have them follow the directions in the Procedures Section. If some crickets are brought to school, designate students to use the stop watches to count the number of times a cricket chirrs in fifteen seconds.

Have students record the results of their exploration on the Response Sheet.

After the students have listened to the crickets chirr and recorded the number of chirrs made by their crickets, discuss their findings. If they have worked in groups, ask that each group present it's results to the class as a whole. Following the presentations, have a discussion of why some calculations were closer to the thermometer readings than others. Include the Observations and Conclusions in your discussion.

VOCABULARY: chirr, approximate

Nature's Thermometer

INTRODUCTION: Did you know that it is possible to approximate the temperature in degrees Fahrenheit by listening to a cricket chirr? It is, and in this exploration you will try the idea!

PURPOSE:

✓ How can you use a cricket to approximate the temperature of a room where the cricket is located?

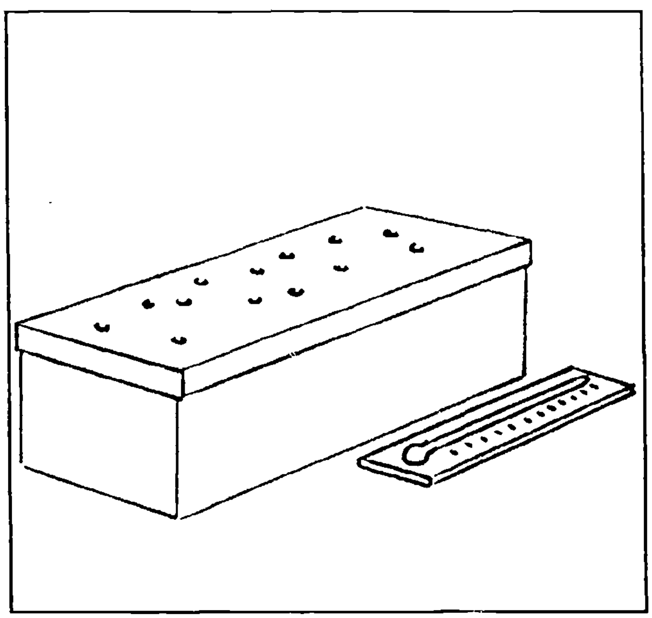
MATERIALS:

a box with moist grasses
a cricket
a watch with a second hand or a stop watch
a thermometer

PROCEDURES:

1. Prepare a small box (no larger than a shoe box) with moist grasses. You will need an enclosed box. The box must also have adequate ventilation so that the cricket can breathe, but the holes must be very, very small so that the cricket cannot get through them. Holes punched with a sharp pencil should be adequate.

2. Crickets are more abundantly available in warmer months. Listen for the cricket's sound which some people call a chirp, but which sounds more like chirr. Follow the sound until you find a cricket. Take care to move quietly because if the cricket senses your presence before you see it, it may stop chirring. You can usually catch a cricket between your two cupped hands or by placing a drinking glass upside-down over it. Be very care-



20 · 1

ful not to injure or squeeze the cricket. You need a healthy, happy cricket for this exploration!

3. When you have caught the cricket and made its home, set the box aside in a quiet place, and do not move it again for a good while. This will allow the cricket to become adjusted to its new, temporary home. Choose a place which will also be handy to conduct the investigation, and place a good thermometer beside the box. Arrange it so that you can easily read the temperature without moving it or the box. The location you choose should not be near a bright light or a breeze from a window. Either one might cause the temperature to vary too much from one place to another or from one time to another. If the thermometer is under a light, for instance, it may be hotter than the cricket's "house".

4. Listen for the cricket to chirr. When the cricket has been chirring for a few minutes, use the stop watch to count the number of times it chirrs in a fifteen-second period. Record the number on the Response Sheet. Add 40 to your result. Now read the temperature in degrees Fahrenheit from the thermometer and record that on the Response Sheet. Compare the temperature with 40 plus the number of chirrs. Are they close?

5. Wait until afternoon, and repeat the experiment. Record the date on the Response Sheet. Are the numbers close this time?

6. At night, just before your bedtime, the temperature may be lower. If you have the cricket at home repeat the procedure. Record your results on the Response Sheet. Are the temperatures close at night?

7. Repeat these observations and recordings for two more days, making note of the relationship between the temperature read from the thermometer and the number of chirrs in a fifteen second period plus 40.

8. When you have completed counting chirrs, release your cricket into some weeds.

OBSERVATIONS:

1. How accurate were your calculations for Day 1?
   Answers vary.

2. How accurate were your calculations for Day 2?
   Answers vary.

3. How accurate were your calculations for Day 3?
   Answers vary.

4. How accurate were your calculations for mornings?
   Answers vary.

5. How accurate were your calculations for afternoons?
   Answers vary.

6. How accurate were your calculations for evenings?
   Answers vary.

7. Was the cricket better at telling the temperature on one of the three days than the others? Or was it better at telling the temperature at one time of day rather than the others?
   Answers vary.

CONCLUSIONS:

1. Write a mathematical equation which illustrates the method for determining the temperature by counting cricket chirrs.
   Number of chirrs plus 40 equals Fahrenheit temperature or $n + 40 = F^{\circ}$

2. What conditions do you think might affect the cricket's chirring?
   Answers vary. Temperature, amount of light, amount of moisture, etc.

3. What can you do to count the cricket's chirrs more accurately?
   Answers vary. Count for 15 seconds, then repeat 2 or three times and average the counts.

SUGGESTIONS FOR FURTHER STUDY

- Locate a farmers' almanac. In it you will find different ways which have been used for years to find out things about nature. For example, how did farmers predict rain? How do they predict a "bad winter"? Prepare a poster or report to share what you found with others.
- Research and read stories about how Native Americans (American Indians and Eskimos or Inuits) made predictions about nature to help them survive. What natural medications (plants, herbs, etc.) did they use? Prepare a poster or report to share what you found with others.

Response Sheet
Nature's Thermometer

DAY ONE	Number of Chirrs in Fifteen Seconds	Number of Chirrs + 40	Thermometer Reading in Degrees Fahrenheit
Morning	*All answers vary.*		
Afternoon			
Evening			

DAY TWO	Number of Chirrs in Fifteen Seconds	Number of Chirrs + 40	Thermometer Reading in Degrees Fahrenheit
Morning	*All answers vary.*		
Afternoon			
Evening			

DAY THREE	Number of Chirrs in Fifteen Seconds	Number of Chirrs + 40	Thermometer Reading in Degrees Fahrenheit
Morning	*All answers vary.*		
Afternoon			
Evening			

Teacher's Guide
I Forgot Your Birthday

GOAL: To have students explore numerical operations on a calculator and to discover why "number tricks" work.

STUDENT OBJECTIVES:

- ✓ To find the month and day of some one's birth by performing certain operations on a calculator.
- ✓ To discover why calculator number tricks work.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by pairs of students or students working in small groups. It cannot be done by individuals because one student must call out steps to be performed on a calculator while another performs the operations.

Review the clearing operation, the operations symbols, and any other instructions for use of a calculator that students may need.

Refer students to the activity sheet, and have them follow the directions in the Procedures section.

Have students record the results of their exploration on the Response Sheet.

After the students have completed the exploration, lead them in a discussion of their findings. If they have worked in groups, ask that each group select a spokesperson to present the group's results to the class as a whole. Use the Observations and Conclusions as a guide for discussion.

VOCABULARY: calculator, display, clear

I Forgot Your Birthday

INTRODUCTION: Have you ever forgotten the date of your Mom or Dad's birthday? In this exploration, you will use a calculator to help you find out when birthdays are, and you will explore why this calculator "number trick" works.

PURPOSE:

- ✓ How can you use a calculator to find the day and month in which someone was born?
- ✓ Why does this calculator birthday "number trick" work?

MATERIALS:

a calculator

PROCEDURES:

1. Pair off with one other student, or work with more than one other. One student works the calculator, and another calls out the operations to be performed.
2. Have your partner clear the calculator and get a clear display.
3. Then have your partner enter a number for the month in which he or she was born: 1 for January, 2 for February, 3 for March, 6 for June, 9 for August, 12 for December, and so on.
4. Then have your partner multiply the number by 5.
5. Next add 6.
6. Then multiply by 4.
7. Next add 9.
8. Next, multiply by 5.
9. Then add the day of the month of your partner's birth to the above result.
10. Subtract 165 from the number and what appears on the screen will be the month and date of your partner's birthday.
11. Reverse roles, and see if your partner correctly gets your birthday.

12. If there are more than two in your group, make sure that everyone has a chance to calculate someone else's birthday and everyone's birthday is found.

13. Examine the steps in this "trick". Then discuss with your group why the steps work. Record your ideas and findings on the Response Sheet.

OBSERVATIONS:

1. Did the calculator always display each person's birthday correctly?
   Yes, if the steps are followed accurately.

CONCLUSIONS:

1. Why does this calculator "number trick" work?
   Because you have input, through multiplication, 100. That moves months to the hundred's place. Then after the day is entered, 165 is subtracted. 165 is the result of: 6 times 4 = 24; 24 + 9 = 33; 5 x 33 =165.
   Let x = month, let y = day—x, 5x, 5x+6, 20x +24, 20x +33, 100x + 165, 100x + 165 + y, 100x + y

2. Will this trick work with all birthdays? Why, or why not?
   Yes, because you are moving month to the hundreds place and subtracting out all excess numbers entered.

3. What things about our number system did you have to keep in mind when you figured out how the trick worked?
   Answers vary. Place value was important. You entered 5x4x5 which moved the month from the ones or ones and tens to the hundreds place. Also place value, distribution property, additive inverse, etc.

SUGGESTIONS FOR FURTHER STUDY

- Use a calculator for this "number trick". Make sure your calculator's display is cleared. Choose a three digit number, for example, 125. Put the number into your calculator two times (125125). Then divide by 7, then 11, and then 13. What is the surprising result. Now, your challenge, why does this work? Work with a friend if you need to do so to figure out why this activity works. Write an explanation of what you found. *7x11x13 = 1001; 1000x125 + 1x125 = 125125; this works for all 3 digit numbers.*

- Use a calculator and the concepts you learned in the birthday activity to create an original "number trick" of your own. You may work with a group, if you wish. Write the steps on a card. Swap cards with another person or group who has worked out another original "number trick". Figure out why the other group's trick works.

Response Sheet
I Forgot Your Birthday

1. Things I considered when figuring out how the birthday "number trick" worked.

   Answers vary. How to move the month to the hundreds place; why I subtracted 165, place value, maybe using a variable, etc.

2. My explanation of how the birthday "number trick" worked.

   Answers vary. Should be similar to explanation for #1.

Teacher's Guide
Abacus-Dabra

GOAL: To have the students construct an abacus and discover how to use it for addition.

STUDENT OBJECTIVES:

- ✓ To construct a simple abacus.
- ✓ To discover how to add using an abacus.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students, pairs of students, or students working in small groups. Each individual pair, or group will need a sturdy piece of cardboard or thin plywood, twelve tacks, some Cheerios or other donut-shaped pieces of cereal or beads, string, and scissors.

Have students follow the directions in the Procedures section to construct an abacus. The process of construction will probably require several class periods.

Have students record the result of their exploration on the Response Sheet.

After students have completed the construction of the abacus and performed the activities which use it, discuss their activities. Ask that each group present the abacus and explain its construction to the class. Following the presentations, discuss the Observations and Conclusions.

VOCABULARY: abacus, crossbar, heaven counters, earth counters

Abacus-Dabra

INTRODUCTION: The abacus is a very ancient form of a computer which probably originated in the Orient. Many Oriental cultures still make extensive use of the abacus to add bills and to make change in stores and to keep financial records.

PURPOSE:

✓ What is an abacus?

✓ What do the counters on an abacus represent?

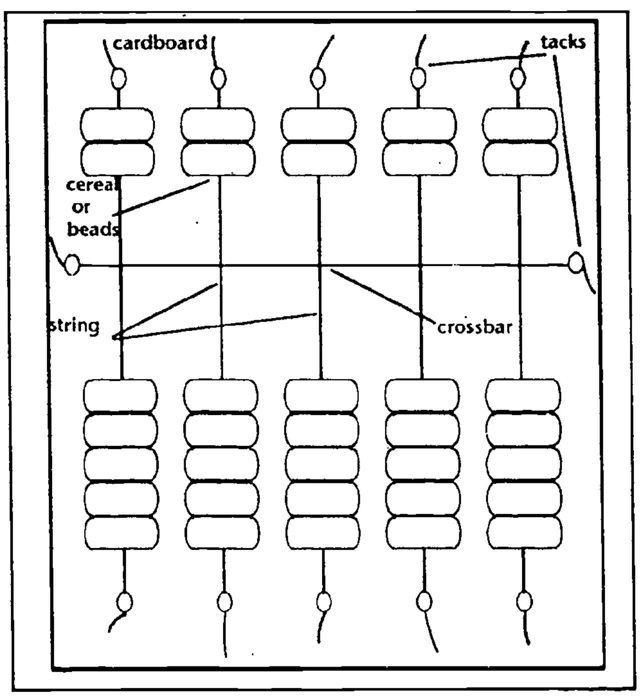
✓ How do you add on an abacus?

MATERIALS:

a piece of sturdy cardboard or a piece of thin plywood
twelve tacks
some Cheerios or other similarly shaped cereal or some beads
some string
scissors

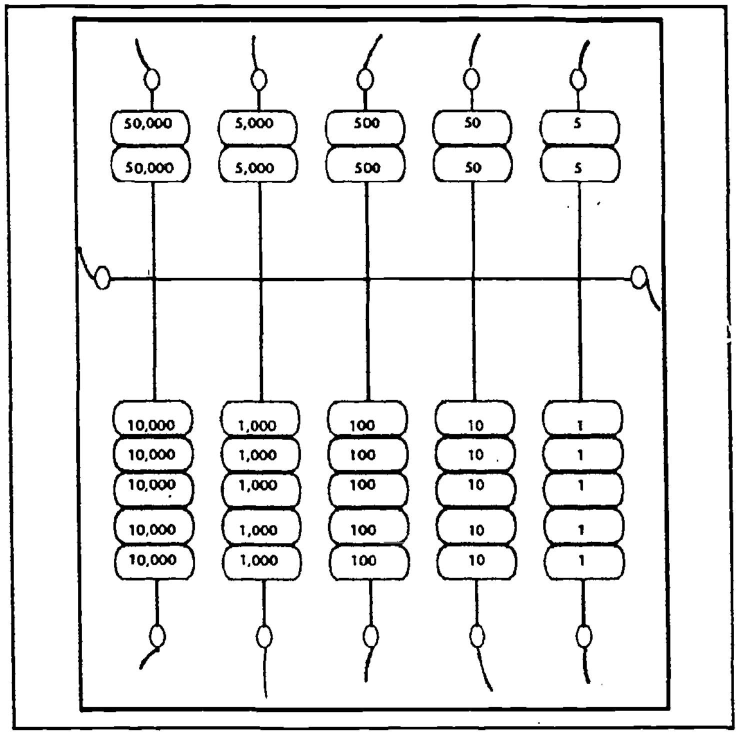
PROCEDURES:

1. Assemble the abacus as shown in illustration 22–1.
2. Each cereal piece or bead (counter) has a number value as shown in illustration 22–2.
3. To enter a number on the abacus, lay the abacus down on a flat surface. Begin with all of the counters pushed away from the crossbar. (This is the "clear" position, as shown in illustration 22–2.) All of the counters above the crossbar are called "heaven counters". Those below the crossbar are called "earth counters".

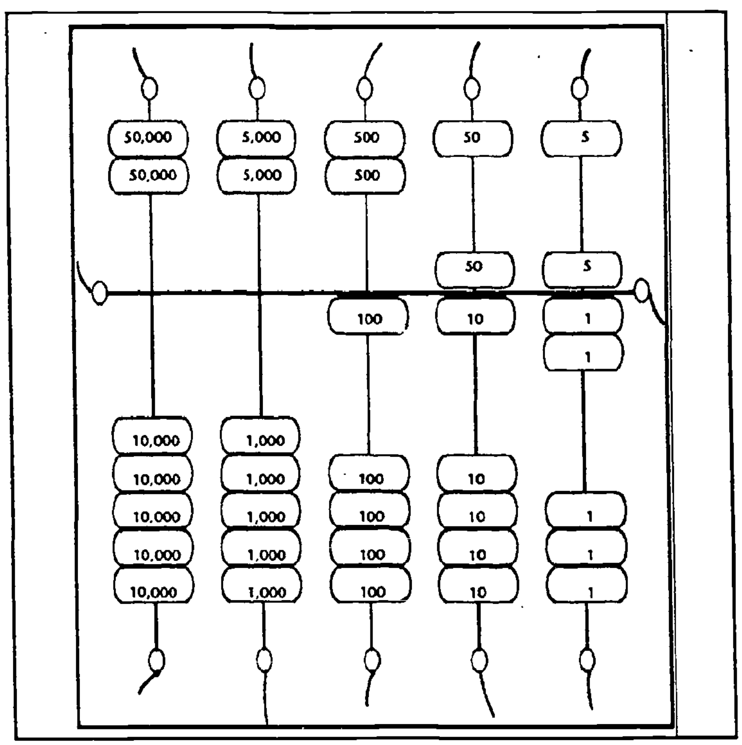


22 - 1 *Constructing an abacus*

4. To enter the number 167 on the abacus, begin at the far right of the abacus and slide one of the heaven counters (value of 5) down to the crossbar.
5. Move two of the earth counters up the crossbar to make a total of 7 on that column.
6. Move one column to the left.
7. Slide one heaven counter (value of 50) down to the crossbar and one earth counter (value of 10) up to the crossbar to make a total of 60 on that column.
8. Finally, move left to the next column. Each earth counter in this column has a value of 100, so slide one earth counter up to the crossbar. The number 167 is entered on the abacus (illustration 22-3).
9. Now add 532 to the 167.
10. Begin at the far right, and slide two earth counters up to the bar.

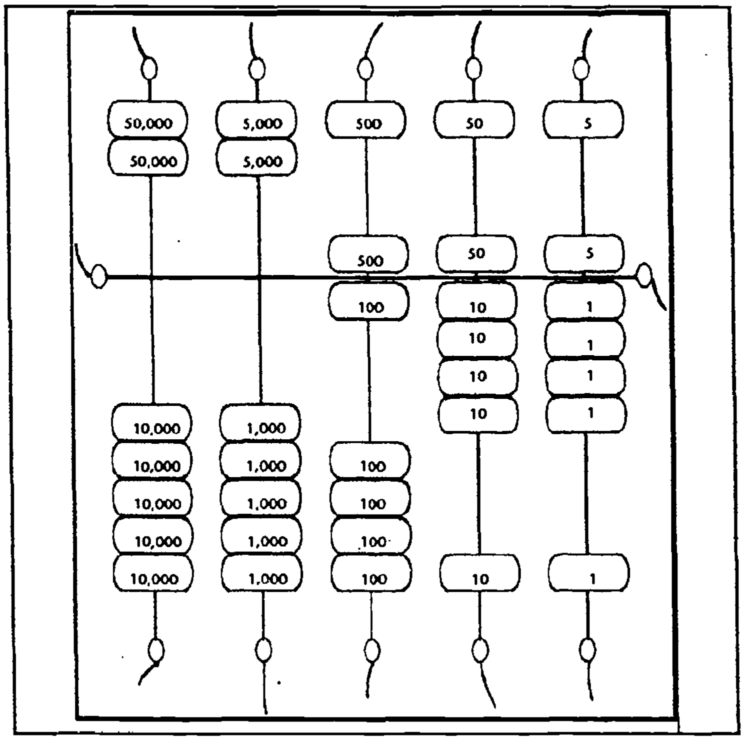


22 - 2 *Clear position*



22 - 3 *Abacus showing 167*

11. Move to the left one column and slide three earth counters to the bar.
12. Move to the left one column and slide one heaven counter down to the bar.
13. Then read the sum left to right (illustration 22-4).



22 • 4

If, in your addition, all of the earth counters are moved to the crossbar, slide them back to the clear position and slide (carry) a heaven counter down. Similarly, if both of the heaven counters are moved to the crossbar, move them up to the clear position and carry the value over to the next column to the left.

OBSERVATIONS:

1. What numbers are represented above the crossbar on the abacus?
   "Heaven" counters} multiples of 5 and 10— 5, 50, 500, 5,000, 50,000

2. What numbers are represented below the crossbar on the abacus?
   "Earth" counters— multiples of 10— 1, 10, 100, 1,000, 10,000

3. What is the largest number which you can represent on the abacus?
   159,999

CONCLUSIONS:

1. Upon what system is the abacus based?

   Base ten, with modification for five.

2. How would you explain to a friend how to add with the abacus?

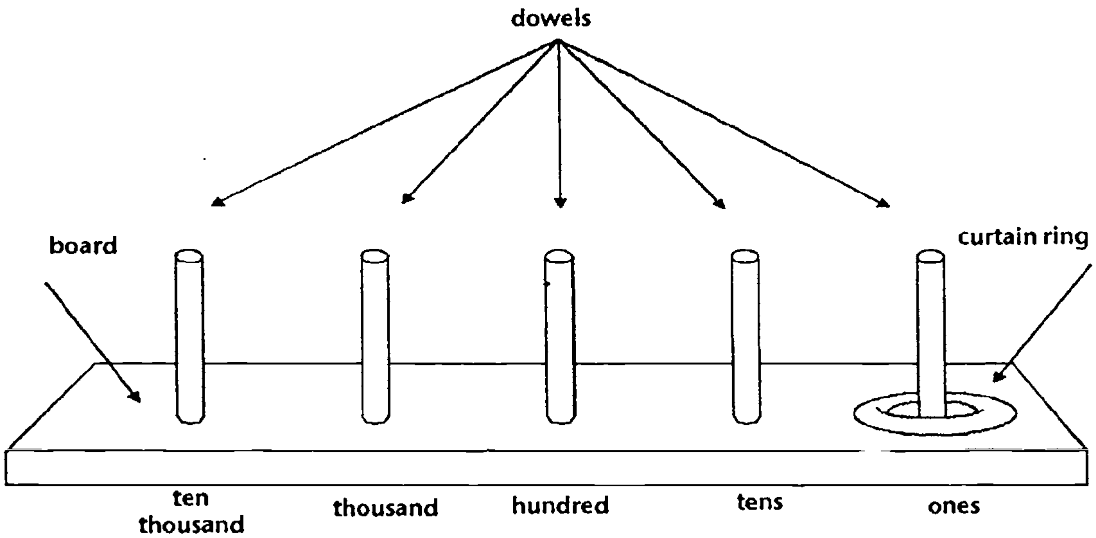
   Answers vary. Explain how to move the counters to the center bar — slide the largest amount first; subtract from the total; continue until you have "made" all the number.

3. Subtract 532 from 699 on the abacus. Write an explanation of how you did it.

   Make 699 by moving 500 and 100, 50 and 4 tens, a 5 and 4 ones to the center. Move 500 back to its starting position, move 3 tens and 2 ones away from the center bar,

SUGGESTIONS FOR FURTHER STUDY

- Construct an open-ended abacus with place values through ten thousands. You will need a board approximately eighteen inches long and four inches wide, five dowels six to eight inches tall, glue, and plastic curtain rings. Have someone drill five holes in the board which are the size of the dowels. Put a small amount of glue into each hole and stand the dowel up straight in the hole. The illustration will help:



Label the places on the board (ones through ten thousand). Use plastic curtain rings  to "make" numbers on the abacus. To show 2,349, place 2 curtain rings on the thousands dowel, 3 on the hundreds dowel, 4 on the tens, and 9 on the ones. How can you add on this abacus?

Explain how to add on this abacus. Is it easier to use than the one you built in this investigation?

Teacher's Guide
Tall As A Millionaire!

GOAL: To have students explore the height of one million stacked dollar bills.

STUDENT OBJECTIVES:

- ✓ To find the thickness of a dollar bill.
- ✓ To use multiplication to calculate the height of one million dollars.

GUIDE TO THE INVESTIGATION: This exploration should be carried out by students working in groups of five or six. Each group will need about twenty play dollar bills. In addition, each group will need a metric ruler marked in millimeters, rubber bands, and a calculator.

Discuss with students what a million is, that is, how many thousands, how many hundreds, and how many tens make a million.

Students should complete the Procedures section of the activity sheet.

After the students have completed the exploration, discuss how they were able to calculate the height of a million dollar bills using the measurements recorded in Part 1 on the Response Sheet. Discuss Observations and Conclusions, and discuss why different groups may have different observations.

VOCABULARY: place value, one million

Tall as a Millionaire!

INTRODUCTION: Have you ever dreamed of becoming a millionaire? Almost everybody has imagined what it would be like to have a million dollars. If you really had a million dollars, and you went to the bank and asked them to change all your money into dollar bills, you would have a million one-dollar bills. How high would your stack of money be? In this activity, you will explore ways to find the height of a million dollar bills.

PURPOSE:

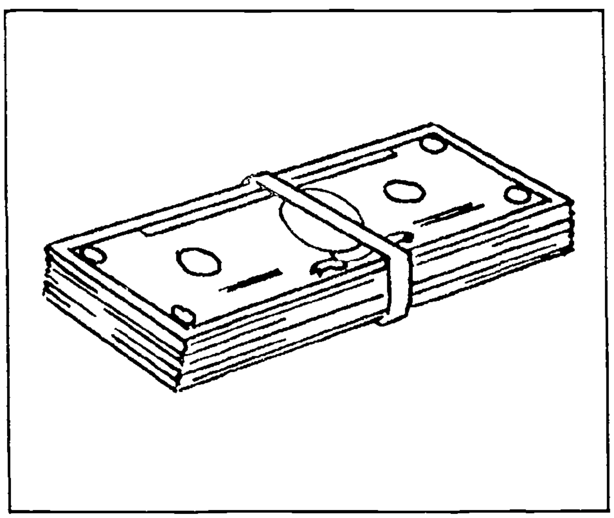
- ✓ How can you find the thickness of one dollar bill?
- ✓ What can you do to find the height of one million dollar bills?

MATERIALS:

about twenty play dollar bills for each group
ruler with millimeters
rubber bands
calculator

PROCEDURES:

1. You will need to work in a group with five or six students. Discuss how you can measure the height of a single dollar bill.
2. Once you have thought of a way, use it to measure the height of one of the dollar bills you have been given for this exploration.
3. Record your group's answer on your Response Sheet.
4. Next, make a stack of dollar bills that measures one millimeter high. Hold them tightly together with a rubber band and make your measurement. You may need to remove the rubber band several times to add and subtract dollar bills from the stack until you get as close to one millimeter in thickness as possible.



23 - 1

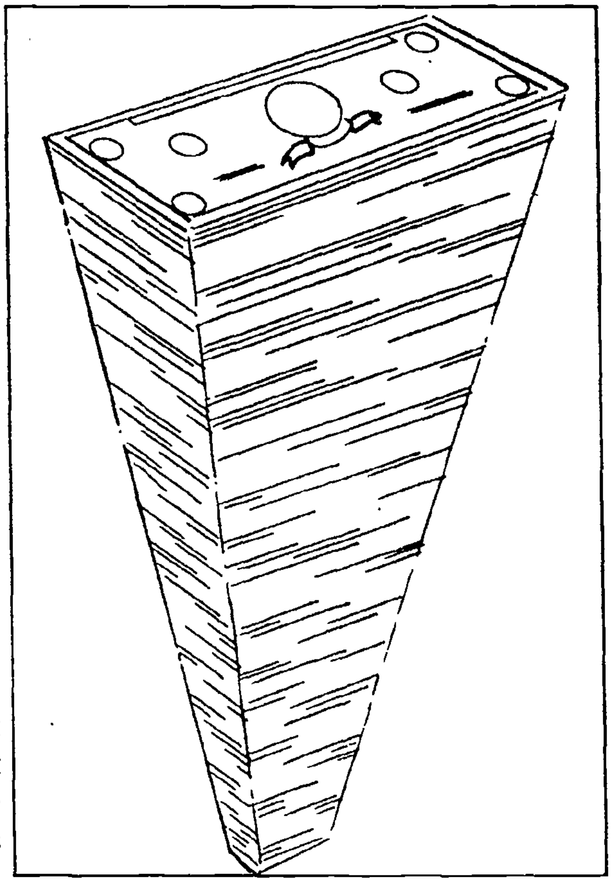
5. Count the number of bills in the stack, and record your group's answer on the Response Sheet.

6. Once you know how many dollar bills it takes to measure one millimeter, discuss how you would find the number it takes to measure one centimeter. Would you repeat the procedure above, or can you think of an easier way?

7. When your group has reached a conclusion about how to find the number of dollar bills it takes to measure one centimeter, write your answer on the Response Sheet. Then make a calculation of how high a million dollars would be!

8. Then discuss with members of your group how you and they found the height of a million dollar bills. If you learn a better way than you had thought of, write about it on the Response Sheet,

9. Then reach an agreement within your group about what your group thinks that the height of a million dollar bills will be.



23 - 2

10. Write that number on the Response Sheet.

OBSERVATIONS:

1. What is the thickness of a dollar bill? *Answers vary. About 0.1 millimeter.*

   Of ten dollar bills? *About 1 millimeter*

   Of a hundred dollar bills? *About 1 centimeter*

2. How many dollar bills stacked does it take to measure one millimeter? *About 10.*

   One centimeter? *About 100.*

3. What operations did you use to find the height of one million dollar bills?
   Multiplication and/or division

4. How high will a million dollar bills be?
   Answers vary. Approximately 10,000 cm or 100 m.

CONCLUSIONS:

1. What method seems to be the best to use to find the height of a million dollar bills?
   Answers vary. Find the height of 10 dollar bills and then 100 dollar bills. Divide 1 million by 100. Multiply the result by the height of 100 dollar bills, or by using ratio and proportion.
   $\frac{\#\ bills}{1 cm}$ $\frac{1000\ bills}{x\ cm}$

2. Why did some groups of students get different heights when they calculated the height of a million dollar bills?
   Answers vary. They got different numbers of bills for 1mm and 1cm. Variables include)new or old bills, some groups packed the bills tighter' etc. It is difficult to accurately measure the bills needed to be one millimeter high.

3. What can you do to make sure that your measurements are as accurate as possible?
   Answers vary. Measure carefully, do several measurements, average the measurements.

SUGGESTIONS FOR FURTHER STUDY

- Work with a partner and a calculator to "solve" this problem. If you got a job which paid you 1¢ the first day, 2¢ the second day, 4¢ the third day, 8¢ the fourth day 16¢ the fifth day, and so on, how much money would you make in all if the first thirty days pay was added together? On which day would you finally make more than one million dollars for that single day? ***You would make over a million on the 28th day and over three million in all.***
- Make a list of ten jobs you would like to have. Ask a librarian to help you investigate the yearly salary which you might make in each of these ten jobs. For each job, assuming that your yearly salary stays the same, how many years would you have to work to make a million dollars? Are you surprised by the answers? Prepare a poster to share what you found with your class. ***Answers vary.***
- One million is a very large number, but is it the largest? Consult a math encyclopedia to see what is the largest place value you can find. Write that number and its name on a sheet of paper. Share your findings with other students.
- Do you think it is likely that you will live a million days? If you did, how many years old would you be? ***Not likely; 1 million days = 2,739 years, 73 days.***

Response Sheet
Tall as a Millionaire!

1. Height of a dollar bill *Approximately 0.1 millimeter.*

   Number of dollar bills in a millimeter *Approximately 10.*

   Number of dollar bills in a centimeter *Approximately 100.*

2. Using the data recorded in part 1, how will I be able to find the height of a million dollar bills?

   Find the number of 100's in one million; that tells you the height in centimeters; divide that number by 100 to find the number of meters.

3. Height of a million dollars *Answers vary. Approximately 100 meters.*

4. After comparing my measurements with that of other members of my group, the misconceptions I had or the mistakes I made:

   Answers vary.

5. Height of a million dollars as determined by my group *Answers vary.*

Teacher's Guide
How Long Does It Take To Count To A Billion?

GOAL: To have students explore ways to count to one billion, and to figure out how long it takes to count to one billion.

STUDENT OBJECTIVES:

✓ To find a reasonable way to determine how long it will take to count to one billion.

GUIDE TO THE INVESTIGATION: Students need to work in pairs to complete this exploration. While one student counts, the other will use a stop watch to time the counting. For this exploration, each pair will need a calculator and either a stopwatch or a watch with a second hand.

Before you begin this exploration with the students, check the type calculator students will be using to see if they have the constant-for-addition feature. If, when you press one plus one equals, equals (1+1=,=), the calculator counts up 1, 2, 3, 4 etc. then the calculator does have this feature. If your class is not familiar with how to use this feature, demonstrate it before they begin.

Discuss with the students how much a billion is. Try to use images they can grasp, such as, if they had a billion dollars in the bank, they could have $27,397.26 a day for one hundred years, $19.03 each minute for every minute of every day for the next hundred years. Ask students how long they think it would take to count to a billion.

Direct students to the activity sheet, and ask them to follow the directions on the Procedures section.

After the student pairs have completed the exploration and estimated the time they think it will take to count to a billion, discuss their findings. Have each pair write on the chalkboard their best time estimate and explain the methods they used to get it. Discuss what factors may contribute to differences in results obtained by different pairs. Go over the Observations and Conclusions as part of your discussion.

VOCABULARY: billion

How Long Does It Take to Count to a Billion?

INTRODUCTION: A billion is a huge number. If you had a billion dollars, you could use $19.00 every minute of every day for the next one hundred years and still have some money left over. Have you ever thought about how long it would take you to count to a number that big? Would it take all day? All week? Longer than a week? In this activity, you will explore how long it would take to count from one to one billion using a calculator.

PURPOSE:

- ✓ How long does it take to count from one to one billion?
- ✓ What is a reasonable way to make this calculation?

MATERIALS:

a calculator with "constant addition"
a stopwatch or a watch with a second hand

PROCEDURES:

Your calculator will not show a number as large as one billion - and very few show one million - on its displays, so you will probably need to use pencil and paper with your stopwatch.

1. Work with a partner. Start counting orally, beginning at one and count, at a reasonable rate, while your partner times you using a stop watch or a watch with a second hand.
2. Count for 30 seconds.
3. Record how far you counted on the Response Sheet.
4. Repeat the activity using a calculator, one with a constant addition.
5. Record how far you counted using the calculator.
6. Exchange roles with your partner. Have your partner count both orally and using a calculator while you use the stopwatch.
7. With the information you obtained from actually counting, discuss ways to calculate how high you could count orally in one minute, two minutes, five minutes, thirty minutes, an hour, a day, five days, ten days, and thirty days.

8. When you and your partner have decided how to calculate these numbers, then each of you fill in Part II of your Response Sheet.
9. Now brainstorm how you could figure out how long it would take for each of you to count to a billion! Write on Part III of the Response Sheet how you would do this.
10. Then record on the Response Sheet how long you would take to count from one to one billion orally and with the help of a calculator. Are you surprised? Are you ready to try it?

OBSERVATIONS:

1. How long will it take you to count orally to one billion?
   Answers vary — between 3 and 4 years.
2. How long will it take to count to one billion with a calculator?
   Answers vary — probably a little faster, depending on button pushing speed.

CONCLUSIONS:

1. Why is there a difference between the time it will take you to count to a billion orally and the time it will take using a calculator?
   The calculator with a constant can "count" a lot faster than you can orally.
2. What things have you learned about a billion? How large is it? How many thousands are in a billion? What else?
   Answers vary — It is very large. There are a million thousands in a billion.

SUGGESTIONS FOR FURTHER STUDY

- Look in newspapers and magazines to find things which are measured in billions. Cut out articles that report data in billions. Prepare a poster and a report for the class. Use a highlighting marker to show where billions are mentioned. See if you can explain why the numbers in the articles are so large.

- Locate a list of the richest people in the world. Your librarian can help you find such a list. Write down the names of some billionaires and how much money they have.
- Is there anything which costs a billion dollars? See if your librarian can help you find some things which are that expensive. Share your findings with a partner.

Response Sheet
How Long Does It Take to Count to a Billion?

Part I: Counting in Thirty Seconds

1. In thirty seconds, I counted orally to *Answers vary.*
2. In thirty seconds, counting with a calculator, I counted to *Answers vary.*

Part II: How High Can I Count?

How high in...	Counting Orally	Using a Calculator
3. one minute	*All answers vary.*	*All answers vary.*
4. two minutes		
5. five minutes		
6. thirty minutes		
7. one hour		
8. one day		
9. five days		
10. ten days		
11. thirty days		

Part III: Counting from One to One Billion

1. Here is how I calculated my answers to Questions Nos. 3 and 4:
   In #3, the answers to # 1 and #2 are doubled, in #4 the answers to #4 are doubled.

2. How long will it take me to count orally from one to one billion if I did not stop counting until I reached one billion:
   Answers vary. The 1 day answer divided into a billion will give this amount in days.

3. How long will it take me to count with a calculator from one to billion if I did not stop counting until I reached one billion:
   Answers vary. Th 1 day answer divided into a billion will give this amount in days.

CONNECTIONS

Teacher's Guide
Fun With Femurs

GOAL: To allow students to explore the ratio of the lengths of various body parts to other body parts and to compare various students' heights using ratios.

STUDENT OBJECTIVES:

- ✓ To use ratios and percents to determine body height.
- ✓ To compare the measures of various students' body parts.

GUIDE TO THE INVESTIGATION: This exploration should be carried out by three or more students. Select the students by their heights. There should be two or more inches difference in the height of each student selected for each group.

Each group of students will need a firm chair, a yardstick or measuring tape, and a calculator.

Introduce this exploration by discussing the fact that a person's adult height can be predicted at a very early age based on their height at about age two.

Students should follow the directions in the Procedures section of their activity sheet and record their responses on the Response Sheet.

After students have completed the exploration, discuss the results. Have each group compare the results they obtained with ratios and prediction. Discuss the Observations and Conclusions.

VOCABULARY: femur, percentage, ratio

Fun with Femurs

INTRODUCTION: You have two femurs. The femur is the upper leg or thigh-bone. Sometimes the femur is referred to as the "bone of strer ¸th". The measurement of some parts of the body such as the femur will let you predict the measurement of other parts of the body. In this activity you will measure lengths on various sections of the body and calculate percentages and ratios to find if you can make predictions about other measurements.

PURPOSE:

✓ How can ratio and percents of various body measurements be used to determine a person's height?

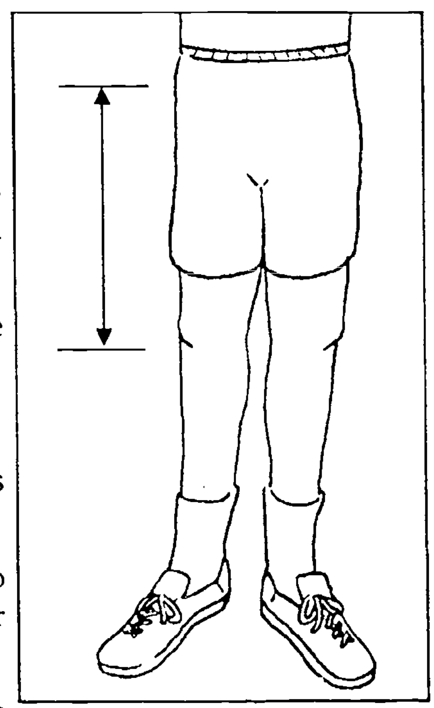
✓ If you know the size of one's body parts, can you predict a person's height?

✓ Are the ratios of height and body-part measurement the same among all persons?

MATERIALS:

a firm chair
a yardstick or measuring tape
a calculator

PROCEDURES:



25.1

1. Use a yardstick or measuring tape to measure from your hip joint to your knee to find the length of your femur.
2. Record the measurements on the Response Sheet.
3. Measure your height in your bare feet.
4. What percentage of your total height is represented by the length of your femur?
5. To find the percentag. ıse a calculator to divide the length of your femur by your total height.
6. Move the decimal point two places to the right.

11.

7. Record the percentage on the Response Sheet.
8. If you know the length of the femur, can you predict one's height?
9. Make the same measurements for three other students.
10. Record the measures on the Response Sheet. Did you find a pattern that lets you predict one's height?
11. Express the relationship of each student's height to the height of the other students as a ratio in simplest form.

$$\frac{\text{height of the shorter student in inches}}{\text{height of the taller student in inches}}$$

12. Record the ratios on the Response Sheet.
13. Next have a student sit in a chair.
14. Measure the distance from the top of the student's head to the seat.
15. Measure the student from his or her knee to the heel.
16. Add the two measurements.
17. Then make the same measurements for yourself and the other two students in your group.
18. Express these measurements in ratios in simplest form as you did for the heights.
19. Record the ratios on the Response Sheet.

OBSERVATIONS:

1. Did the length of the femur help you predict the person's height? If so, how?

   Yes; a prediction equation was developed which found that the total height is about 4 times the length of the femur.

2. What relationship exists between the length of measure from the head to the "seat" and measure from the knee to the heel?

   The ratio is about 1 to 2, however, it may vary slightly from group to group.

CONCLUSIONS:

1. How do the ratios comparing students heights relate to ratios comparing sums?

   Answers vary. Ratios comparing sums are more uniform than ratios comparing heights.

2. What made the difference?

   Length of the femur.

3. Why is the femur called the "bone of strength"?

   Answers vary. It supports the body; is a large bone, etc.

4. What predictions can you make about the length of the femur and a person's height?

   Answers vary. The length of the femur is about one-fourth of a person's height — the femur has a ratio of 1 to 4 to height.

SUGGESTIONS FOR FURTHER STUDY

- Measure a piece of yarn about thirty-six inches long. Wrap an end of the yard around the knuckle of one of your thumbs two times. Mark the length of yarn it took to go around your thumb twice. Wrap this length of yarn around your wrist. What do you discover? Next wrap the yarn around your wrist twice, and mark the length. Then wrap this length of yarn around your neck. What do you discover? Finally wrap the yarn twice around your neck, and mark the length of yarn it takes. Wrap this length about your waist. What do you discover? Repeat this activity with at least ten of your friends. What generalizations can you make? Are these generalizations true in all cases?

  We are "put together" in many ways that illustrate relationships or ratios.

Response Sheet
Fun with Femurs

Part I: Initial Data

Student	Length of Femur	Height in Bare Feet	Length of Femur is What Percent of Height
A (yourself)	*All answers vary.*	*All answers vary.*	*About 25%*
B			
C			
D			

Part II: Ratios of Heights of Students

Ratio of Height of Student A to Height of Student B	*All answers vary.*
Ratio of Height of Student A to Height of Student C	
Ratio of Height of Student A to Height of Student D	
Ratio of Height of Student B to Height of Student C	
Ratio of Height of Student B to Height of Student D	
Ratio of Height of Student C to Height of Student D	

Part III: Seated Measurements

Student	Head to Seat	Knee to Heel	Sum
A (yourself)	*All answers vary.*	*All answers vary.*	
B			
C			
D			

Part IV: Ratios of Seated Data for Students

Ratio of Sum for Student A to Sum for Student B	*All answers vary.*
Ratio of Sum for Student A to Sum for Student C	
Ratio of Sum of Student A to Sum for Student D	
Ratio of Sum for Student B to Sum for Student C	
Ratio of Sum for Student B to Sum for Student D	
Ratio of Sum for Student C to Sum for Student D	

Teacher's Guide
And A Weigh We Go...!

GOAL: To have students weigh objects using a pan balance and to explore ways that objects of different weights can be weighed using only three weights.

STUDENT OBJECTIVES:

- ✓ To weigh using a pan balance.
- ✓ To find ways to weigh objects that cannot be weighed directly with available weights.

GUIDE TO THE INVESTIGATION: Each student or group will need a pan balance and three weights: one gram, five grams, and ten grams. The exploration can be carried out by an individual student, but the discussion of ways to weigh various objects will require two or more students. If there is an insufficient number of pan balances to allow students to work individually, divide the class into groups. Before students begin, demonstrate how to use the pan balance.

Then direct students to follow the directions in the Procedures section.

When students complete the Procedures section of the activity sheet, conduct a group discussion of the Observations and Conclusions. Try to solicit as many ways as possible that your students used to weigh things using only three weights.

VOCABULARY: assayer, pan balance, gram

And A Weigh We Go...!

INTRODUCTION: In the Old West, prospectors took their gold ore, dust, and nuggets to the assayer's office where the assayer evaluated the purity of the ore and weighed the nuggets and dust in a pan balance. Since gold mines were often in the wilderness and very far apart, many assayers traveled on horseback from place to place to weigh the prospector's gold dust and nuggets. The assayer did not carry a lot of weights because there was not much room in his sack and he did not want to weigh down his horse with anything he did not need. In this exploration you will learn to use a pan balance and only three weights to weigh many things.

PURPOSE:

- ✓ How can you find the weight of an object using a pan balance and weights?
- ✓ How can you find the weight of an object if it is not the same as one of your weights or a combination of them?

MATERIALS:

pan balance
three weights: one gram, five grams, and ten grams

PROCEDURES:

1. Discuss with a partner how you can find the weight of objects that weigh 6 grams, 11 grams, or 16 grams using the 4, 5 and 10 gram weights.
2. Gather 10 light objects that cannot be weighed directly (that is, they are not 1 gram, 5 grams, 6 grams, 11 grams, or 16 grams), with the three weights you have. Record the names of the objects on the Response Sheet.
3. Use the pan balance, the 3 weights (1 gram, 5 grams, and 10 grams) and the objects gathered to find the weight of the 10 objects you gathered.
4. Record the weight of each object on the Response Sheet.
5. Write a brief explanation on the Response Sheet of exactly how you found the weight of each object.
6. Now with a partner, discuss the different ways you used to find the weight of the ten objects.

OBSERVATIONS:

1. How much did each of the 10 objects weigh?
   Answers vary.

2. Which ones weighed more than 16 grams?
   Answers vary.

3. What did you do to find the weight of objects that weighed more than 16 grams?
   Answers vary. Combined the 10 gram weight and other smaller weights.

CONCLUSIONS:

1. What things did you consider when you had to weigh objects which weighed more than five grams but less than ten grams?
   Answers vary. Does the object weight more than 6 grams? Have I weighed other objects that weigh less than 10 grams? Could I use other objects to find this weight?

2. How were you able to find the weights of objects which weighed more than 16 grams?
   Answers vary. "Make" other weights by balancing them with weights I have: use nickels for grams or find other objects that weigh 5 grams or 10 grams and use them.

3. How were you able to get accurate weights of things which weighed 2, 3, or 4 grams?
   Answers vary. Find several objects that balance with the one gram weight. Tape 2 together to become a 2 gram weight, tape 3 together for a 3 gram weight, etc.

4. How were you able to find the weights of objects that weighed 6, 7, 8, or 9 grams and objects that weighed more than 10 but less than 15 grams?
   Answers vary. Use techniques similar to responses 2 and 3.

SUGGESTIONS FOR FURTHER STUDY

- Work with a friend to solve this problem.

  You have eight baseballs. Seven of them weigh the same, but one of them weighs more than each of the other seven. You have a pan balance, but the only weights you have are the balls themselves. If you are allowed only two weighing, can you find which ball is heavier than the other seven? Explain how it can be done, and demonstrate it to your class.

  (Hint: You may want to use models for the baseballs, that is - 8 objects, 7 that weigh the same and 1 heavier - and a pan balance, to carry out this investigation. Remember, you can only make two weightings!)

  Place 3 balls on each side of the balance. If the sides balance then one of the remaining two is the heavy ball. Place one on each side of the balance to discover the heavier one. If one side holding 3 balls is heavier than the other, remove all the balls. Place 2 from the heavier side – one on each side of the balance. If these two balance, the third ball is heavier. If these two do not balance the heavier ball tips the balance and it will be obvious.

Response Sheet
And A Weigh We Go...!

Object Name	Weight of Object	How I Found the Weight
1.		*All answers vary.*
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Teacher's Guide
"I" Am the Most Popular!

GOAL: To have students explore articles in newspapers, magazines, and stories to find which vowel is the most used.

STUDENT OBJECTIVES:

- ✓ To read newspaper stories, magazine articles, and short stories, and to count the numbers of vowels in each.
- ✓ To determine from a newspaper story, a magazine article, and a short story the most used vowel.
- ✓ To predict which vowel is used more than any of the others.

GUIDE TO THE INVESTIGATION: Students need one page of a newspaper, a magazine article, and a short story or book. Students should read the stories and articles individually, and record the frequency of the vowels on the Response Sheet. After each student has completed the Procedures section of the activity sheet, he/she should make a prediction about which is the "most popular" vowel. Then groups of 4 to 8 students should combine their data and predict which is the "most popular" vowel. Finally, combine data from all groups. Have students discuss whether or not their individual findings and that of the total group were the same. Also discuss whether they believe that their findings will be proven correct when they read other stories or articles. Go over the Observations and Conclusions as part of your discussion of findings.

VOCABULARY: tally, frequency, sample, data

"I" Am the Most Popular!

INTRODUCTION: In language arts classes you have learned about consonants and vowels, what they are, and how they are used. Words are made of both consonants and vowels. Have you ever wondered which vowel is used the most in the English language? Is it a? Or is it e, i, o, or u? In this activity, you will tally the numbers of times each is used in written materials so that you can make a prediction as to which is the most popular vowel.

PURPOSE:

- ✓ Which vowel is the one most used in a newspaper story, a magazine article, and a short story?
- ✓ How can you decide which vowel is used in English more than any other?

MATERIALS:

a newspaper story
a magazine article
a short story

PROCEDURES:

1. Choose a short article from a newspaper. Read the article.
2. As you read the article, make a tally on the Response Sheet indicating the number of vowels (a, e, i, o, u) that are in the article.
3. Count the tallies and record the frequency for each vowel.
4. Read a magazine article and repeat the procedures above.
5. Read a short story and again repeat the procedures above.
6. Make a prediction based on your data as to the "most popular" or most used vowel.
7. Combine your data with a partner.
8. Combine your data with a small group and then with your entire class.

OBSERVATIONS:

1. Which vowel appears most often in the newspaper story? *Answers vary.*
2. Which vowel appears most often in the magazine article? *Answers vary.*
3. Which vowel appears most often in the short story? *Answers vary.*

CONCLUSIONS:

1. Did the prediction you made about which would be the most popular vowel agree with the prediction when the data for the entire class was combined?
   Answers vary — however most will agree.
2. Did your group prediction about which would be the most popular vowel agree with the prediction made by the class?
   Answers vary — most likely the group prediction will match the class prediction.
3. What other types of reading materials do you think the class should check to find if your class prediction is correct? Would you have to read many books? Why or why not?
   Answers vary — books, encyclopedias, technical journals, dictionaries, etc.

SUGGESTIONS FOR FURTHER STUDY

- If someone in your class speaks Spanish, ask that person to perform the investigation for three articles written in Spanish. Is the result the same or different as for English? What is the most popular vowel in German and French? How can you find out?
- Complete a similar investigation about the different kinds of cars in your area. Choose one spot to gather data. Make a tally for the make of every car that passes the spot during one hour. Have three or four of your friends on other streets or roads, and have them make a tally of the number of cars of each kind which pass. Decide which car each of you thinks is the most popular. Combine your data, and make a group decision. Do you think that your conclusion will be valid for another city in your state? How about a city in a neighboring state? Why or why not?

- Find a book about samples and sampling. The technique of sampling enables a person to draw a wider conclusion from a narrow sample, just as you did when you predicted what the most popular vowel must be from counting the vowels in only a small part of what has been written in the English language. Make a list of ways business and industry use sampling to draw conclusions and make predictions.

Response Sheet
"I" Am the Most Popular!

Newspaper Article

Vowel	Tally	Frequency
a	*All answers vary.*	
e		
i		
o		
u		

Magazine Article

Vowel	Tally	Frequency
a	*All answers vary.*	
e		
i		
o		
u		

Short Story

Vowel	Tally	Frequency
a	*All answers vary.*	
e		
i		
o		
u		

Teacher's Guide
Stocking Up

GOAL: To have students gain an understanding of consumer economics, specifically the concept of common stocks, and to develop an appreciation for the role of mathematics in understanding the way a stock market functions.

STUDENT OBJECTIVES:

- ✓ To define common stocks and how they are bought and sold on a stock market.
- ✓ To identify two common stocks and "track" their gain or loss over a two-week period.

GUIDE TO THE INVESTIGATION: This investigation, which takes place over a two-week period, can be carried out by individuals or small groups of students.

Students need a resource (economics book, social studies book, or encyclopedia) to find out exactly what stocks are and how the stock market operates. In addition, students need the daily stock market report from a newspaper for two weeks.

After students have researched stocks and the stock market, conduct a discussion to make sure all understand the concepts.

When students complete the Procedures section, discuss the results through the Observations and Conclusions. Students can continue the investigation for a longer period of time.

If your class raises money, you could purchase a share of stock and "track" its value until the end of the year - and then sell it!

VOCABULARY: consumer economics, stocks, fourths, eighths, open, high, low, close, prices, loss, gain, profit

Stocking Up

INTRODUCTION: Have you seen the stock market report on television news? Have you seen them in the newspaper with many columns and row after row of abbreviated names and numbers? These are the stock pages. You may have heard the radio or the television say that the Dow Jones Average rose or fell - sometimes they just call it "the Dow" or "the Index". What does all of this mean? In this investigation, you will learn about stocks, how to read the stock market reports, and gain an understanding of what you see and hear about stocks. You will "buy" stocks and keep a record of their prices for two weeks to see if they gain or lose and if you would make a profit or suffer a loss.

PURPOSE:

- ✓ What is stock and the stock market?
- ✓ How do I tell the price of a share of stock and whether it is going up or down?
- ✓ Where and how can you buy and sell shares of stocks?

MATERIALS:

newspaper stock market pages daily for two weeks
reference books which explain stocks:
economics
social studies or
business books

PROCEDURES:

Read about stocks and the stock market in a reference book. There are two m: in types of stocks, common and preferred. Common stocks are the ones you will study. When you buy shares of common stocks, you own a share of the company. That is why they are called "shares of stock". Stock markets are places where stocks are offered for sale. The numbers in the stock report in the paper tell you many things about how a stock was bought and sold the last day the market was open before the paper was published. Saturday or Monday's paper has Friday's stock report, Tuesday's paper has Monday's report, and so on. Here is what a typical line looks like:

NorSou 2.12 37 $60\frac{3}{8}$ 59 $59\frac{5}{8}$

This line tells you that yesterday the common stock of Norfolk Southern Corporation paid a dividend of $2.12 a share and closed at $59.625 ($59\frac{5}{8}$) a share, the last number in the line. This was the last price for which the stock was bought and sold before the stock market closed for the day. The $60\frac{3}{8}$ means that the highest price paid for the stock yesterday was $60.375, and the 59 means that the lowest was $59.00 The 37 means that 37,000 shares of Norfolk Southern Stock were bought and sold yesterday. Some newspapers also show other numbers, but you will not study these in this exploration. What you will need to be able to find and record is the closing price of the stock at the end of each day's trading (buying and selling).

1. Look in the stock market section of a newspaper. Find two stocks you would like to purchase (if you had the money).
2. Record the name and the number of shares of stock you "purchased" on the Response Sheet.
3. Record the price you "paid" for each share and the total amount you "paid" for all the shares.
4. Read the stock market report each day for two weeks.
5. Record the closing price of your stock at the end of each day.
6. Figure the value of each share and for all your stocks each day.
7. At the end of two weeks, calculate how much you made or lost on each stock.

OBSERVATIONS:

1. Over a two-week period, what happened to the closing price of your Stock 1?
   Answers vary.
2. At the end of the two week period, did it end up or down?
   Answers vary.
3. How much did you make or lose on Stock 1? *Answers vary.*
4. Over a two week period, what happened to the closing price of your Stock 2?
   Answers vary.
5. At the end of the two week period, did it end up or down?
   Answers vary.
6. How much did you make or lose on Stock 2?
   Answers vary.

7. Do you think that two weeks is long enough to keep a stock you have bought to see if you can make money? Why yes, or why no?
Answers vary. In most cases great changes do not occur over short lengths of time.

CONCLUSIONS:

1. What are some of the things you should consider before you buy a company's stock?
Answers vary. Price to earnings ratio of the stock, reliability/stability of the company, prospects for the company and its industry, etc.

2. What are some of the things that influence whether a stock's price increases or decreases?
Answers vary. General health of the economy, profitability of the company, merger take-over potential, etc.

3. When you have been watching a stock's price and find that it is going up or down, what are some of the things you should think about before you buy the stock if you do not own it or what should you consider before you sell it?
Answers vary. Will the change in price continue in the same direction? What is causing the change? Are the causes permanent or temporary?

SUGGESTIONS FOR FURTHER STUDY

- Continue to "track" your stocks weekly over the next three months, keeping a record from Tuesday's paper of the closing stock price on Monday. Investigate whether your stocks pay dividends, and, if they do, when they pay them. At the end of the three months, add the amount of the dividends you would have received to your stock profit or loss.
- See if you can have class or school projects to raise money and buy some stock. Elect a group of students to decide which stock to buy. Then track the stock over the period of a year. Keep weekly records of the stock price and the dividends you are paid. Investigate what a dividend reinvestment program is and whether your stock has one. If it does, you can make even more money.
- Invite a stock broker to come to your class and bring materials from her or his work: annual reports, stock trade forms, quarterly state-

ments, and so forth. When he or she is visiting you, have someone ask what he or she would recommend that you buy right now! Brokers earn their money by advising others what stocks to buy and which ones they already own should be sold.

- Bonds are also sometimes good investments. They are very different from stocks. Do some research about bonds - municipal, tax-free, savings, etc. - there are many types. Find out what the differences between stocks and bonds are and explain how bonds are bought and sold. If you could own either stocks or bonds but not both, decide which you would want to buy, and write a paragraph explaining the differences and why you decided on the one you chose.
- In Germany they do not have dollars, they have Deutsche Marks, which are usually just called Marks. DM 5.25 means five Marks and twenty-five of their pennies. In Great Britain, they have pounds. £7.36 means seven pounds and 36 pence. Every day in large newspapers, there are reports about how many dollars there are to the DM and the pound, as well as how many DM and pounds a dollar is worth. Find in your newspaper or ask a local bank or broker how much the dollar is worth in comparison to the DM and the pound on a certain day. Research what money is named in Canada and find out about the value of their money, too. See if you can find out what things make the value of the dollar in another country's money rise or fall. Does this influence the stock market? Experts often disagree about these things, so do not be surprised if you get different answers when you ask different people!

Response Sheet
Stocking Up

Personal Stock record of: ______________________________

First Stock Company Name: ______________________________

Date Bought	Purchase Price per Share	Number of Shares	Total Price Paid
	Answers vary.		

Date Price Checked	Closing Price	Number of Shares	Total Value of Stock 1
	All answers vary.		

The amount of money I ( )gained or ( )lost on this stock is: $ ________

Second Stock Company Name: ______________________

Date Bought	Purchase Price per Share	Number of Shares	Total Price Paid
	Answers vary.		

Date Price Checked	Closing Price	Number of Shares	Total Value of Stock 2
	All answers vary.		

The amount of money I ( )gained or ( )lost on this stock is: $ ________

The total amount of money I ( )gained or ( )lost is: $ ______________

Teacher's Guide
Patterns in Art - and Other Places

GOAL: To have students explore tessellations as one way to create patterns in art and to experience the beauty which exists in geometric patterns.

STUDENT OBJECTIVES:

- ✓ To identify and create tessellations.
- ✓ To describe ways that geometric patterns are used in works of art.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students, pairs of students, or students working in small groups. Each individual or group will need a set of crayons or markers, construction paper, scissors, glue or paste, several templates, tracing paper, and white paper.

Before students begin the exploration, discuss with them what tessellations are and how both polygonal shapes and colors are used to repeat patterns and tiling to form tessellations. Direct students to the Procedures section, and ask them to follow the directions they will find there.

After the students have colored tessellations and have created their own using the templates, glue, and construction paper, they can display their works of art on poster paper or a bulletin board. Have an "art sharing" where students describe how they chose and designed the colors. Discuss the Observations and Conclusions.

For further information, you may want to read "The Art of Tessellation", by Paul Gignati, Jr., and Mary Jo Cittadina, *The Arithmetic Teacher*, (37) 7: 6-16.

VOCABULARY: tessellations, polygons, parallelograms, slide transformation, "nibble"

Patterns in Art - And Other Places

INTRODUCTION: A tessellation is a tiling made up of polygons repeated many times. You have probably seen examples of tessellations. Some kitchen or bathroom tiles, wall paper, and some fabrics have patterns which are tessellations. Some are simple; others not so simple. Many are very, very beautiful. Some have many colors; others are black and white. Some tessellations have curves and combinations of curved and polygonal shapes. All fill the entire plane with no gaps or overlapping. In this activity you will use parallelograms and other polygons to design your own work of art.

PURPOSE:

✓ What is a tessellation?

✓ How can tessellations be used in works of art?

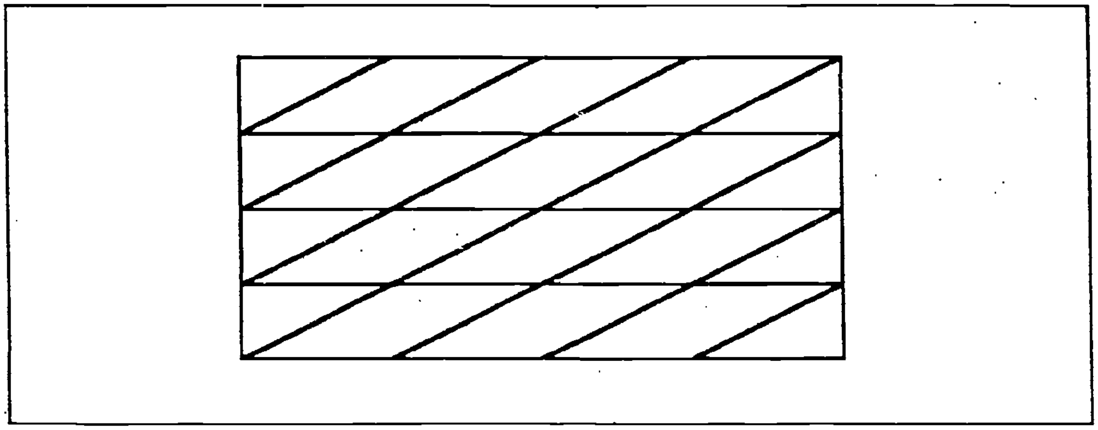
MATERIALS:

tracing paper
several sheets of colored construction paper
white paper
scissors
crayons or markers
templates:
  squares
  rectangles
  hexagons
  other parallelograms
glue or paste

PROCEDURES:

1. Trace several copies of the tessellation pattern shown on the Response Sheet. There are no gaps between shapes, and the shapes do not overlap.
2. Use several different colors to make each tessellation different from the others so that you create several different works of art. Use three or more colors for your tessellations by repeating your pattern.

An example of a simple tessellation which is "colored" two different "colors", white and shading is shown. Notice how the patterns and colors are repeated in the sample tessellation.



29 - 1 *A simple tessellation*

3. Use the template to make your own tessellation on white paper. Make each design about two cm long or wide. Cut "tiles" from the colored construction paper. Glue the colored paper tiles onto the tessellation.

OBSERVATIONS:

1. What patterns do you find in your tessellations?

   Answers vary.

2. How are the several tessellations you colored alike and how are they different?

   Answers vary.

3. Are other students' tessellations similar to yours? Why or why not?

   Answers vary.

CONCLUSIONS:

1. How many different works of art could you make from the template designs on the Response Sheet?

   Answers vary.

2. What are the common attributes or characteristics of each of the tessellations made using the Response Sheet designs?

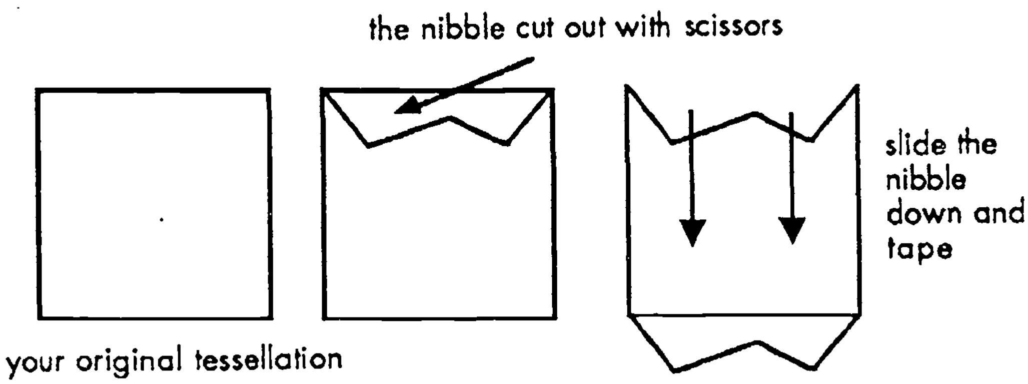
   Answers vary. Triangles and quadrilaterials are used.

3. Can you find some examples of tessellations in the real world? If so, make a list of them and tell where they are. Use a white sheet of paper to sketch one of them.

   Answers vary. Wallpaper, bathroom or kitchen flooring, etc.

SUGGESTIONS FOR FURTHER STUDY

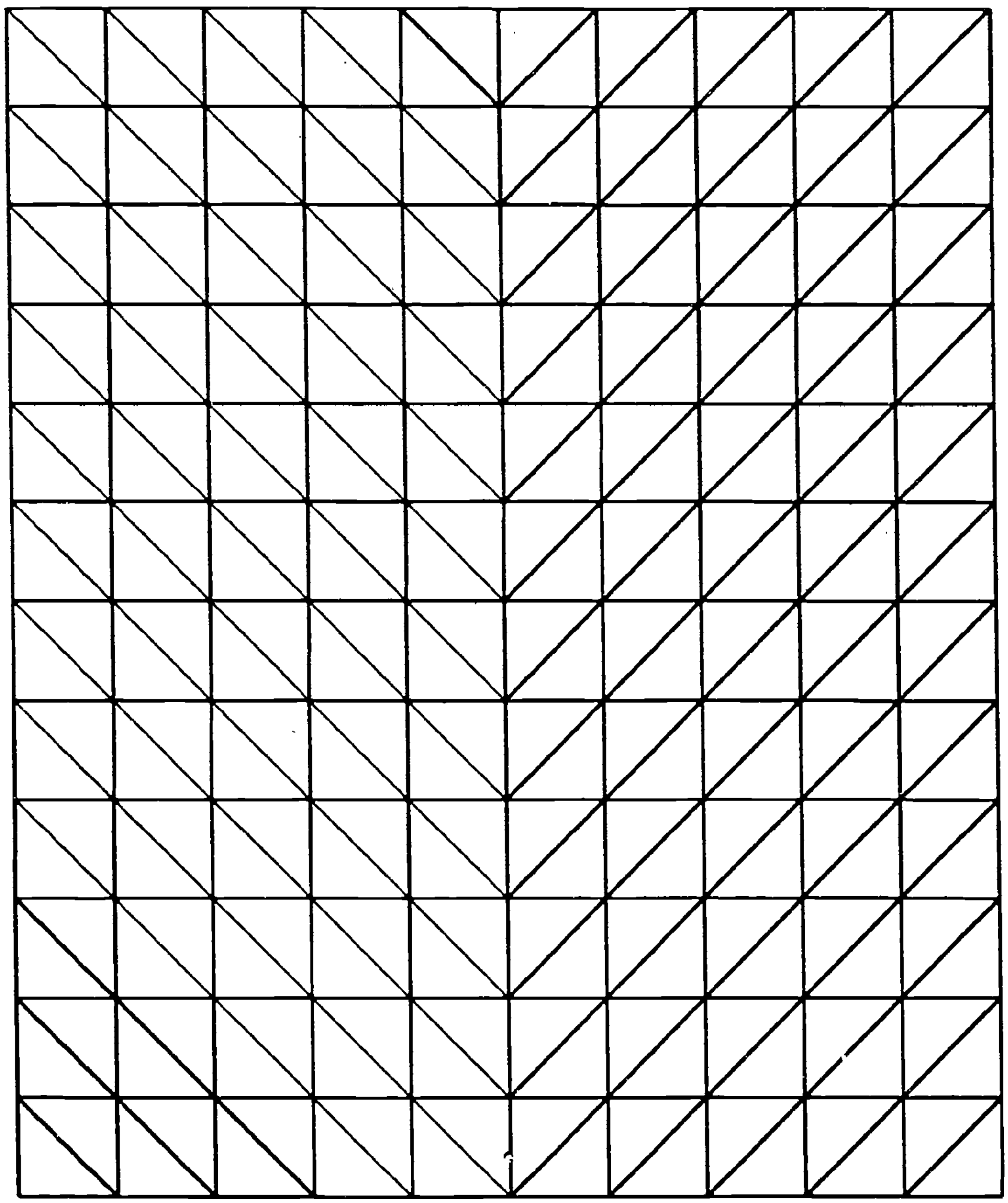
- Use only parallelograms and hexagons to draw a tessellation. Once you have covered a sheet of paper with them, either cover the shapes with colored construction paper tiles or color them. Next change the shape of the tessellation using a technique called "nibbling". Take a "nibble"' from one side of the tessellation, and slide it to the other, taping it on as shown below.



Sliding the nibble to another place on the tessellation and taping it there is called making a slide transformation. After you have made a transformation from top to bottom, make one by taking a nibble from the left side of the tessellation and sliding it to the right side and taping it there.

Response Sheet
Patterns in Art - and Other Places

Tessellation Template



ESTIMATION

Teacher's Guide
Discovering Pi

GOAL: To help students explore how Pi (π) was conceived and to discover its relationship to the ratio between the circumference and the diameter of a circle.

STUDENT OBJECTIVES:

✓ To explore the relationship between the diameter and the circumference of a circle.

✓ To divide the circumference of a circular object by its diameter.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students, pairs of students, or students working in small groups.

Students will need string, circular objects (coins, jar lids, cans, cups, clock faces, bicycle wheels, etc.), a calculator, a marker, a meter stick, and a centimeter ruler.

Give each group a selection of circular objects to measure. Students should record the measurements of circumferences and diameters on the Response Sheets. Calculators will be used to divide the circumference by the diameter.

After the students have completed the exploration, lead them in a discussion of their findings. If they have worked in groups, ask that each group select a spokesperson to present the group's results to the class as a whole. Include the Observations and Conclusions in your discussion. Following the presentation, lead the general discussion, giving attention to similarities and differences in the various results, and evoking from the students the reasons for any parallels or lack of parallels.

Finally ask each group to find the average of all numbers in the last column on the Response Sheet. Have them make a generalization of what they expect the ratio will be for any circle.

VOCABULARY: circumference, diameter, ratio, relationship, pi π

Discovering Pi

INTRODUCTION:

Pi is a letter of the Greek alphabet. It is written π and pronounced just like the word "pie". Both pi and pie have to do with round things. Have you ever heard of π before? Do you know where it comes from? What does it measure? Can you use it in everyday life? In this exploration you will measure the distance around and across circular objects and find the relationships that exist.

PURPOSE:

✓ How do you measure around a circle? What is this measure called?

✓ What is the name of the measure of a line segment from one side of a circle to the other side which is drawn through the center of the circle?

✓ What is the relationship of the circumference of a circle to its diameter?

✓ What is π?

MATERIALS:

string
meter stick and a centimeter ruler
round (circular) objects, such as coins, jar lids, bicycle tires, etc.
marker
calculator

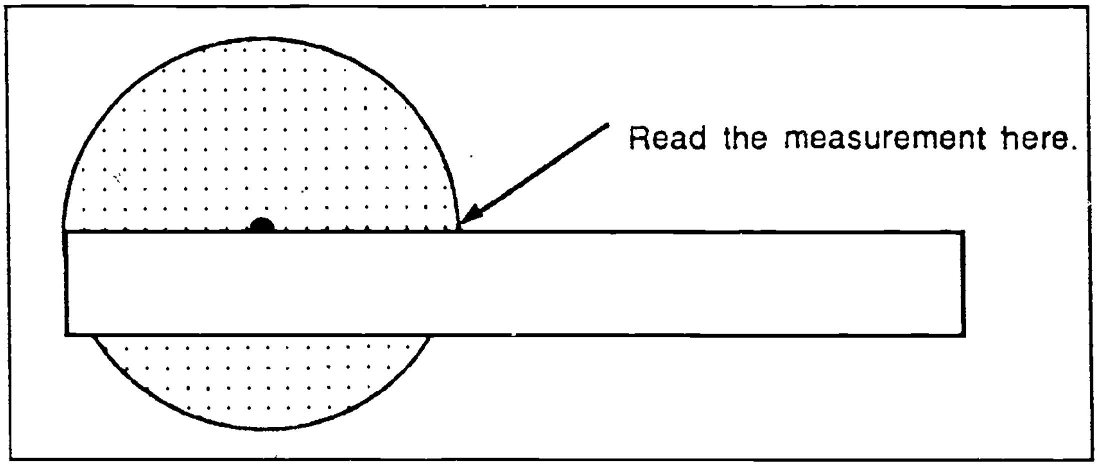
PROCEDURES:

1. Use the meter stick and a string to measure around the outside of each round object you or your group has been given. Do this by carefully holding the string tightly around the edge and marking the point on the string where it meets its own end. Hold the string as tightly and as close to the circular edge of the object as you can, but be careful not to stretch it.

2. Use the meter stick to measure the length of the string from its end to where you made your mark, and that will be the distance around the circular object. This distance is called the circumference.

3. For each object you have been asked to work with, write its circumference on the Response Sheet.

4. Next use the centimeter ruler or meter stick to measure the "width" of each circular object at its widest place. You can determine this by marking the center of each object as accurately as possible and holding the centimeter ruler or meter stick so that its end is on one edge of the object and its edge passes right by the center as shown in illustration 30-1.



30-1

5. If you read the measurement from the meter stick at the point indicated in illustration 30-1, then you will have measured the distance across the circular object at is widest point. This is called its diameter. Record each diameter on the Response Sheet.

6. For each object, use a calculator to compute the following ratio: $\frac{\text{circumference}}{\text{diameter}}$. Enter the ratio into the proper place on the Response Sheet.

7. Find the average of the numbers in the Ratio column, by adding and dividing by the number of objects. Record the result on the Response Sheet.

OBSERVATIONS:

1. When you calculated the ratios $\frac{\text{circumference}}{\text{diameter}}$, what did you find? Do you see a pattern?

In each case the ratio was about 3.14

2. When you calculated the average of these ratios, what was your answer? Compare it to the answers obtained by other students or pairs or groups of students.

   The average for the entire group is about 3.1 or 3.14

CONCLUSIONS:

1. What generalizations can you make about the quotient $\frac{\text{circumference}}{\text{diameter}}$ for all the circular objects you measure?

   No matter the size of the circle, the ratio of circumference to diameter is always the same.

2. Do you think the generalization is true for all circles? How can you prove this?

   Yes. Different plans of proof will be given. Discuss differences in deductive and inductive reasoning.

SUGGESTIONS FOR FURTHER STUDY

- Read about the history of π in a book on the history of mathematics. Prepare a report for the class on the subject. If you can find more than one book, see if there are any differences in what you can learn. More recent archaeological work suggests that the Egyptians knew about π when they built the pyramids. This would be earlier than many others have thought that π was "discovered". What did you find?
- Find a high school algebra or geometry book, and see what you can learn from it about how π can be used in various formulas. How is π used to find the area of a circle, the volume of a cylinder or cone? Can you explain why it is used? In other words, can you explain how the formulas work?

Response Sheet
Discovering Pi

Object Measured	Circumference	Diameter	$\frac{\text{circumference}}{\text{diameter}}$
Answers vary.			

The average of the ratios $\frac{\text{circumference}}{\text{diameter}}$ is *Answers vary.*

Teacher's Guide
How Tall Is Small?

GOAL: To enable students to discover a way to measure small, thin objects accurately.

STUDENT OBJECTIVES:

✓ To measure the thickness of a stack of objects.

✓ To devise a way to measure very thin objects.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individual students, pairs of students, or students working in small groups. Each student, pair, or group will need twenty pennies, ten nickels, ten dimes, and ten quarters. They will also need a millimeter tape or ruler and cellophane tape, to hold the coins in stacks while the students measure the height of the stack.

Before students begin, discuss with them the millimeter as a unit of measurement. Make sure that students understand that the marks closest together on the tape measure indicate millimeters.

Direct the students to the activity sheet and ask them to follow the directions in the Procedures section.

Have students record the results of their exploration on the Response Sheet.

After the students have completed the exploration, discuss their findings. If they have worked in groups, ask that each group present their group's results to the class as whole. Following the presentations, discuss any differences in the various results. Discuss the Observations and Conclusions.

If any group or student has discovered a particularly different or even unique way to measure thin objects with particular accuracy, ask for a description and explanation of the method.

VOCABULARY: thickness, millimeter, accuracy in measurement

How Tall is Small?

INTRODUCTION: Some things are so very thin that it is hard or almost impossible to measure how thick they are with the usual ruler or measuring tape. Scientists, engineers, and others whose daily work involves measuring very small objects have special, delicate, and expensive instruments to help them make accurate measurements of very small things. In this exploration you will develop a method by which you can make accurate measurements of some thin objects without using these scientific instruments, and you will learn about the millimeter, a small metric measure of length.

PURPOSE:

- ✓ How can thin objects be measured accurately without expensive instruments?
- ✓ Are there times when you would really need to measure thin objects with a great degree of accuracy?

MATERIALS:

twenty pennies
ten nickels
ten dimes
ten quarters
a millimeter tape or ruler
cellophane tape (optional)

PROCEDURES:

1. Make a stack of twenty pennies, and use the tape or ruler to measure its height as accurately as you can. You may find it easier to handle the stack of pennies if you use a small length of cellophane tape up one side of the stack to make it more stable. If you use tape, be careful not to use it - or much of it - on the top or bottom of the stack so that you do not make it impossible to measure the height of the stack of pennies. When you are ready, make your measurement, and record what you get on the Response Sheet. Divide the measure by 20 to get the thickness of a single penny.
2. Repeat Procedure No. 1 with a stack of ten nickels. This time divide by 10 to get the thickness of a single nickel.

3. Do the same for stacks of ten dimes and ten quarters. Record all your results on the Response Sheet, as directed.
4. Work with a partner or member of your group to brainstorm a list of situations in real life in which you would want or need to be able to measure the thickness of small and/or thin objects. Write your list on the Response Sheet.
5. Use what you learned in this experiment to find the thickness of 100 pennies, 50 nickels, 45 dimes and 86 quarters. Record your answers on the Response Sheet.

OBSERVATIONS:

1. What was the thickness of one penny? Hold one penny up to the millimeter tape, and see if you could have measured its thickness accurately that way. What do you think?

   Answers vary. Approximately 1 mm – it is fairly difficult to measure one penny because it is so thin.

2. What was the thickness of one nickel, one dime, and one quarter?

   Answers vary. Have various groups compare their answers. Nickel– approximately 1.5 mm; dime and quarter approximately 1 mm.

3. What real life situations can you think of in which you would need or want to measure the thickness of very thin objects?

   Answers vary. For example, calipers are used to measure the thickness of metal plates to assure quality control.

CONCLUSIONS:

1. Can you think of ways to get an even more accurate measure of the thickness of each coin?

   Answers vary.

2. In what real situation might the discovery made in this exploration be used?

   Answers vary. To identify counterfeit coins.

3. How can the concept in this activity be used to find the height of a stack of 500 pennies?

   Answers vary. The thickness of 1 penny times 500 or the thickness of 20 pennies times $\frac{500}{20}$.

SUGGESTIONS FOR FURTHER STUDY

- In this activity you have measured very thin objects. Now think of how you could measure very thick or very tall objects. How could you measure the height of a very tall building? What unit of measurement would you want to use? Work with at least one other student to think up a way to measure a very tall building. Explain your method to the rest of the class.
- Go to a bank and "buy" a roll of pennies and a roll of nickels. Use your measuring tape to find the height of each role. How high would $1000 in pennies be? How can you find out? What can you do to find the height of $5000 in nickels?

Response Sheet
How Tall is Small?

Record the "thickness" of each on the chart.

20 pennies *20* mm	*Answers are approximate.* one penny *1* mm	100 pennies *100* mm
10 nickels *15* mm	one nickel *1.5* mm	50 nickels *75* mm
10 dimes *10* mm	one dime *1* mm	45 dimes *45* mm
10 quarters *10* mm	one quarter *1* mm	86 quarters *86* mm

List times when you might need or want to measure thin things.

Answers vary.

Teacher's Guide
Twinkle, Twinkle, Many Stars

GOAL: To have students explore ways of finding the number of stars in the sky on a cold, clear night.

STUDENT OBJECTIVES:

- ✓ To estimate the number of visible stars by counting the stars in sample areas of the sky.
- ✓ To find the number of stars expected in the entire celestial hemisphere by multiplying.

GUIDE TO THE INVESTIGATION: Each student will need a large piece of poster board, a ruler, and a pair of scissors.

This exploration can be conducted by single students, in pairs, or small groups.

Students should carry out this exploration on a cold, clear night for it to be at all accurate. They should be encouraged to do so away from the bright concentration of city lights which may be in your area. The more stars they can see, the better.

Students should follow the directions given in the Procedures section of their activity sheets.

When all have completed their explorations, conduct a large group discussion to bring their findings together. Have each result (whether reached by an individual or group) reported, and ask students to compare the numbers of stars seen through the square with the total number calculated. Discuss any significant similarities and any significant differences in results. Have students speculate on the possible reasons for differences. Discuss the Observations and Conclusions.

VOCABULARY: celestial hemisphere

Twinkle, Twinkle, Many Stars!

INTRODUCTION: If you are standing where lights from your town do not make it hard to see when you look up at the sky on a cold, clear night, you can see what appears to be millions of stars. They appear as small dots of light because they are so far away, but almost all of them are actually as bright as our sun. When you look at all those stars, you may wonder how many you can really see. In this activity you will explore ways of answering that question even though it would be extremely difficult to count stars one-by-one.

The celestial hemisphere is the part of the sky you can see from where you are on Earth!

PURPOSE:

✓ How can you estimate the number of stars visible on a cold, clear night?

✓ How can you estimate the number of stars in the celestial hemisphere?

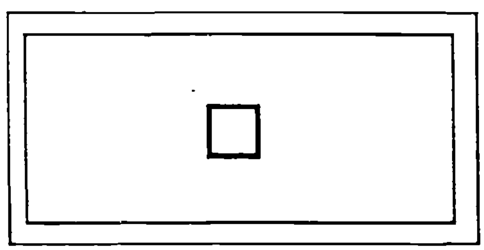
MATERIALS:

a large sheet of poster board or cardboard
a ruler
a pair of scissors

PROCEDURES:

You may work alone or with a partner to complete this exploration. Prepare your equipment during a class session.

1. Cut a square from the center of the large sheet of poster board or cardboard. Make the square four inches long and four inches wide. You want your poster board to look like illustration 32-1.



32 - 1

2. On a cold night when there are no clouds, try to find a place where the lights of your town are not too bright. Notice that you can see more stars if the night is cold and if you are as far away from ground lights as possible. Can you think why these things may be true?

3. Use the ruler to hold the poster board exactly one foot from your eyes. Hold your head and the poster board as still as you can, and, moving only your eyes, count all the stars you can see through the square hole you have cut. The area of the sky which you can see is about 1/57th of the entire visible sky.

4. Multiply the number of stars you counted by 57. The answer should be a good estimate of the total number of stars visible on that night in the entire area of the sky which you can see. This is called the celestial hemisphere.

OBSERVATIONS:

1. How many stars did you observe in the sample area which you could see through the square hole?

   Answers vary.

2. What method did you use to convert your observations into an estimate for the number of stars in the celestial hemisphere?

   Answers vary. Multiply the number seen by 57.

3. Why do you think that the number 57 is used to find the total number of stars visible?

   Answers vary. The area counted is approximately 1/57 of the entire area.

CONCLUSIONS:

1. Why is the area of the sky which you can view through the square hole in your poster board one-fifty-seventh of the area of the celestial hemisphere?

   Answers vary.

2. What do you think would happen if you repeated the exploration two more times, each time looking at a different area of the sky? Would you get nearly the same results or not? Explain the answer you give.

   Answers vary. You probably would get different numbers since stars are not uniformly positioned in the sky. You need to get an average of the observations.

3. What do you think would happen if you repeated the exploration two more times, each time looking in the same direction but on nights three months apart? Would you get nearly the same results or not? Explain the answer you give.

Answers vary. You may see different numbers because of different sky conditions–clouds, etc. Since the earth rotates around the sun different stars are visible.

4. If you used a telescope or binoculars to count the total number of stars, would you get the same answer as if you did it without the aid of either? Explain your answer.

Answers vary. You count more because a telescope or binoculars let you see things that are farther away.

5. If you repeated the experiment on a hazy night, would your answer be the same? Why or why not?

Answers vary. Probably not since the haze will obscure the view.

6. What do you think the results would be if you did the activity on a hot night?

Answers vary. Usually not as many stars can be seen on a hot night.

SUGGESTIONS FOR FURTHER STUDY

- Devise a technique to estimate the number of beans in a jar. Carry out the exploration.
- Devise a technique to estimate the number of leaves on a tree. Carry out the exploration.
- Devise a technique to estimate the number of blades of grass in a lawn. Carry out the exploration.
- Devise a technique to estimate the number of hot dogs eaten in the United States in one year. Carry out the exploration.
- Read a book about stars. Find out what double stars, star clusters, nebulae, and galaxies are. Collect pictures of stars and try to find an example of each of the different varieties of stars. Make a scrapbook or a poster with your pictures and what you learned in this exploration. Display your scrapbook or poster in the classroom or the school library.

Teacher's Guide
The Dog's Hind Leg

GOAL: To have students explore various ways of finding the approximate area of irregularly shaped surfaces.

STUDENT OBJECTIVES:

✓ To discover a way to approximate the area of an irregularly shaped surface.

✓ To identify real life situations in which it is necessary to be able to approximate the area of an irregularly shaped surface.

GUIDE TO THE INVESTIGATION: This exploration can be carried out by individuals or small groups. Each will need tracing paper and centimeter grid paper.

Direct the students to read through the Procedures section of their activity sheet and then to return to the beginning and follow the directions they find there. They may make their drawings on the grid paper and record their answers in the Observations section. Each student or group should select several irregularly shaped surfaces to approximate area.

When students complete the activities, have them discuss their findings and ask them to characterize the situations in which it is useful for them to be able to approximate areas of irregular surfaces. Have them report their findings for the dog's hind leg, and compare all the results. Lead them in a discussion about why the answers differed. Go over the Observations and Conclusions students recorded during the exploration.

VOCABULARY: irregular, area, approximate

The Dog's Hind Leg

INTRODUCTION: There are formulas for determining the measures of the area of certain polygonal regions such as rectangles, trapezoids, squares, circles, and triangles. To find the area of a rectangle, find the product of its length and width. It is not easy to find the area of an irregularly-shaped figure. In this exploration you will try to find ways to measure the area of irregularly-shaped figures.

PURPOSE:

✓ How can you approximate the area of an irregularly shaped surface?

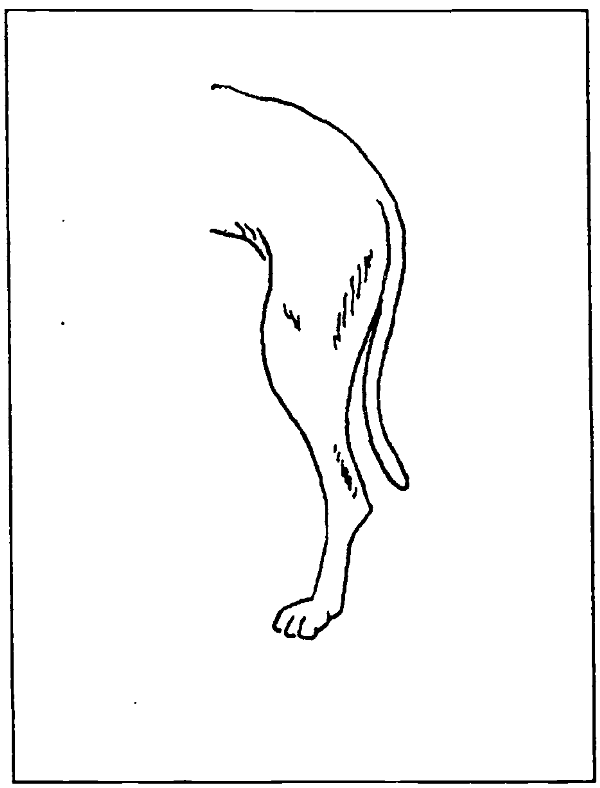
✓ How can you calculate the area of an irregularly shaped surface?

MATERIALS:

tracing paper
centimeter grid paper

PROCEDURES:

1. Use tracing paper to trace the outline of the dog's hind leg in illustration 33-1 onto the grid paper on the Response Sheet.
2. Count the number of squares that are completely enclosed within the outline and those that are partially outside.
3. Add the two numbers.
4. Divide this sum by 2.
5. The quotient is the approximate area of the outlined figure.
6. Choose another irregularly-shaped object. Make a drawing on the grid paper. Count to find the area of that object.



33-1

7. How can you find the area of an irregularly-shaped surface by calculating?

OBSERVATIONS:

1. How many squares are entirely inside the outline of the dog's hind leg?
   About 4.

   How many squares are partially inside the outline of the dog's hind leg?
   About 13.

   What is the total of the squares?
   About 17.

   When the total is divided by 2, the approximate area of the dog's hind leg is:
   $8\frac{1}{2}$ square units

CONCLUSIONS:

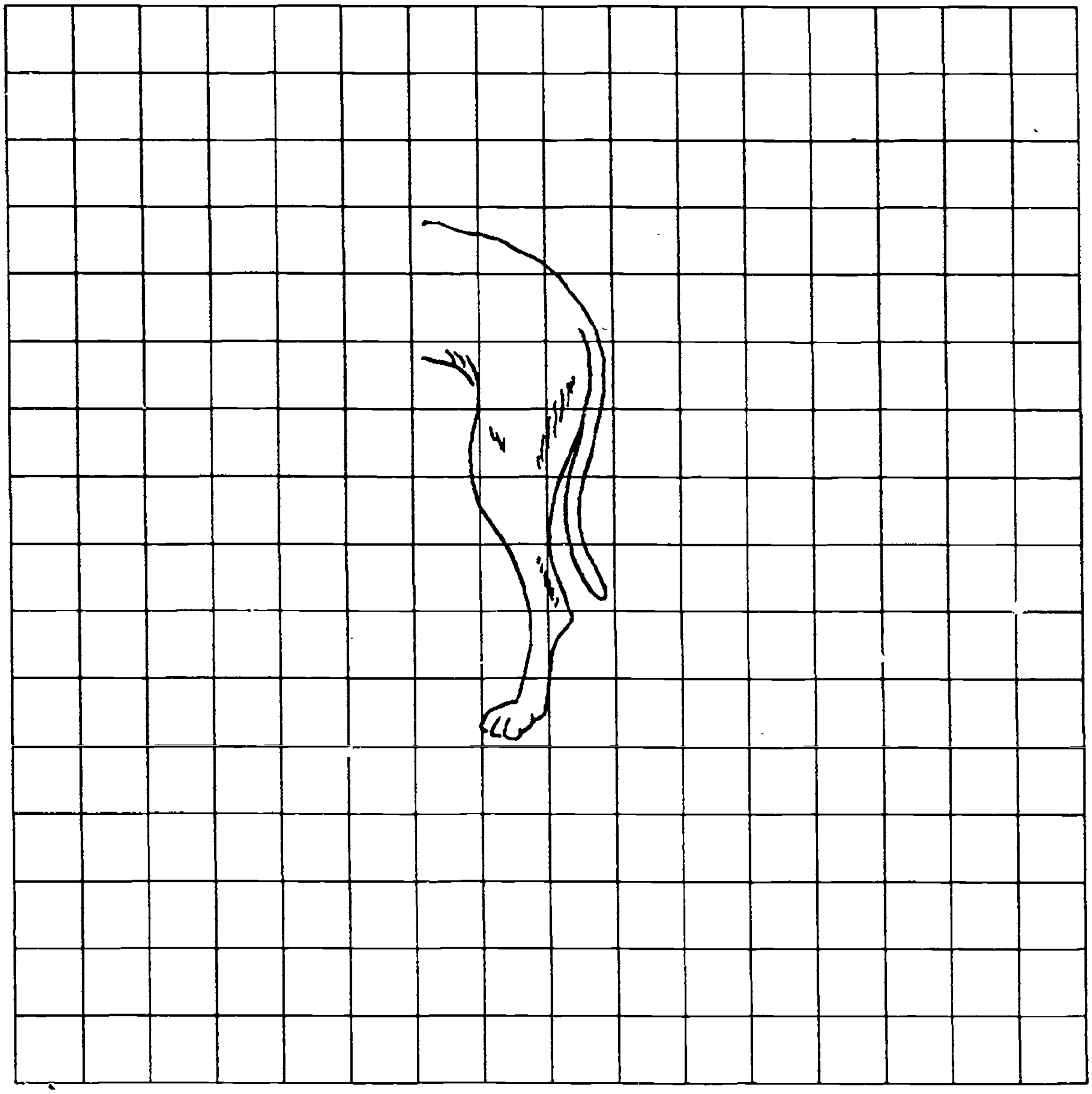
1. Why did different students get different areas for the drawing of the dog's hind leg?
   Answers vary. Some miscounted or positioned the picture on the grid differently.

2. How can you find the exact area of the dog's hind leg or any other irregular outline?
   Answers vary. Divide the figure into regular shapes if possible. Use Pick's Theorem for a better approximation.

SUGGESTIONS FOR FURTHER STUDY

- Trace your hand print and your footprint on centimeter grid paper. Find their areas.
- Trace the area of your city or county from a map onto centimeter grid paper. Find the area of the map in square centimeters. Can you think of a way to convert the area of the small map to the actual area of the city or town you might find reported in an atlas?

Response Sheet
The Dog's Hind Leg



1. Approximate area of the dog's hind leg: $8\frac{1}{2}$ *square units*
2. Approximate area of ____________ I have drawn: *Answers vary.*
3. How can I find the area of an irregularly-shaped surface by calculating?
   Answers vary but may include use of a grid and the method shown in the exploration..

Teacher's Guide
Dekameter Rope

GOAL: To have students explore the dekameter as a measure of length and to gain an understanding of estimation of "long" lengths.

STUDENT OBJECTIVES:

✓ To make a dekameter rope.

✓ To use the dekameter rope for estimating and measuring.

GUIDE TO THE INVESTIGATION: Students should work in pairs or in groups to complete the dekameter rope exploration. Each group will need a piece of clothesline or long, heavy rope at least 10.5 meters long, scissors, a meter stick, a marker, three meters of yarn, the cardboard tube from a roll of paper towels, and a pencil.

Before students begin this exploration, show them a meter stick and discuss the meter and kilometer as measures of length. Explain to students that deka means "ten", so dekameter means ten meters.

Students should follow the directions in the Procedures section of their activity sheet. When they complete the activities, discuss Observations and Conclusions and data recorded on the Response Sheet.

VOCABULARY: dekameter, kilometer, estimate, meter

Dekameter Rope

INTRODUCTION: A dekameter is a 10 meter distance. Meters and kilometers are used to measure distances more often than dekameters, but it is sometimes helpful to measure objects and distances in dekameters. In this activity you will make a dekameter rope and explore estimating and measuring using the dekameter rope.

PURPOSE:

✓ What is a dekameter?

✓ What is a dekameter rope?

✓ How do you use a dekameter rope to estimate and measure longer distances?

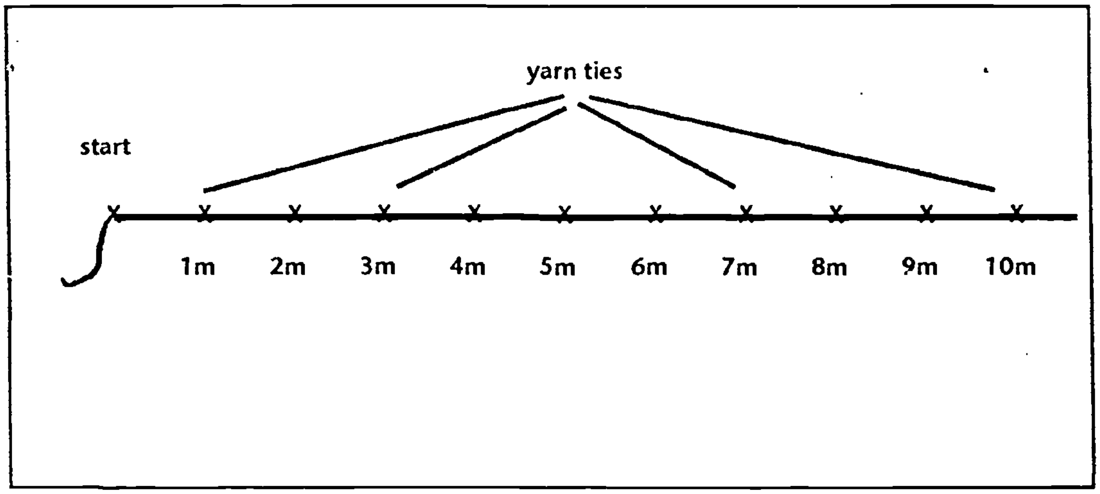
MATERIALS:

clothesline or heavy nylon rope about 10.5 meters long
scissors
meter stick
yarn (at least three meters)
marker(s)
tube from the center of paper towel roll
pencil

PROCEDURES:

1. Take the length of rope and mark about 30 cm from one end. This will be the "zero" or "start" mark, and the 30 cm end will be used to hold the rope securely in place.
2. Use the meter stick to measure carefully one meter down the rope from the "zero" mark, and make a mark for "one meter".
3. Repeat the process marking "two meters", "three meters", and so on until you have marked "ten meters".
4. When you mark "ten meters", leave another 30 cm at the other end and cut off the rest of the rope.
5. Cut eleven small pieces of yarn, and tie yarn markers at each mark you made, from zero through ten meters. (See illustration 34-1.)
6. Cut a slit into each end of the empty paper towel tube.

7. Put one end of the rope into the slit, and carefully wind the rope around the tube so that it does not become tangled. The second end can then fit into the second slit, and you have a practical and handy rope holder and dispenser.



34 - 1

8. With a partner, take your rope outside on a day when the weather is satisfactory and the ground is not damp. One of you hold the tube while the other walks away with one end of the rope. When the rope is fully extended, lay it on the ground in a straight line.

9. Each of you walk along the rope to get a good idea of how long a dekameter is. When you have reached the end, continue to walk in the same direction, stopping only when you think that you have walked another dekameter.

10. Mark the place at which you stop.

11. Hold the end of the rope nearest you while your partner brings the other end as close as possible to the point you marked. How far did you actually walk? Record the distance on the Response Sheet.

12. Place the rope in a different place and/or a different direction.

13. Perform this exercise three more times. Each time record how far you actually walked.

14. Identify four or five pairs of objects on the school grounds which you think to be approximately one dekameter apart.

15. Use your rope to measure the distance between each pair and record your answers on the Response Sheet.

OBSERVATIONS:

1. When you tried to walk one dekameter, did you get closer each time you tried? Why or why not.
   Answers vary. Probably yes, because they are comparing to counter examples.

2. Were you able to accurately select objects one dekameter long? Why or why not?
   Answers vary.

3. Did your estimations become better the more you estimated and then measured? *Answers vary. Probably yes.*

   Why do you think so?
   Answers vary. Adjusted estimates based on prior experience.

CONCLUSIONS:

1. Why do you think it useful to be able to measure in dekameters?
   Answers vary. When measuring long distances.

2. Why do you think people measure only in meters and kilometers?
   Answers vary. Possible answers–cm and mm are used for small distances; m for intermediate distances; km for long distances. One reason dm are not used very often is that a meter stick is used to make measurements so answers are given in meters.

3. What conclusion can you make about estimating and measuring, and then estimating and measuring?
   Answers vary. The more you estimate and measure, the more accurate your estimates.

4. Do your estimations become more accurate the more you estimate and measure? Explain
   Answers vary. We hope it would be yes.

SUGGESTIONS FOR FURTHER STUDY

- See if you can find some things that are at least one dekameter long or wide. First estimate the length and then measure them - one object at a time. Do you get better at estimating as you practice?
- What is the name of the distance which measures 100 meters? Can you think of things for which this would be a useful measure? Prepare a poster or a collage of things that might be 100 meters tall or 100 meters long. Write a paragraph to accompany the poster or collage to convince someone of the importance of measuring in lengths or distances of 100 meters
- Work with 2 or 3 other students. Prepare a board game on poster board with cards and game pieces that move around the board. The cards should direct the player to measure distances, such as 5 meters, 1 dekameter, etc. To move to the finish line, a player must measure the distances on the cards correctly. Exchange game boards with other groups and play the games.

Response Sheet
Dekameter Rope

Distances I Walked

Trial Number	Actual Distance
1	*All answers vary.*
2	
3	
4	

Write the names of pairs of objects and estimate how far apart they are in dekameters. Then use the dekameter rope to actually measure the distance.

Distances Between Pairs of Objects

Object 1	Object 2	Estimated Distance	Actual Distance
	All answers vary.	1 dekameter	
		1 dekameter	
		1 dekameter	
		1 dekameter	
		1 dekameter	
		1 dekameter	

Write names of several objects you think are about one dekameter long. Then measure the objects and record the actual measurements.

Object Estimated to be 1 dekameter	Actual measurement
All answers vary.	

REASONING

Teacher's Guide
A Bag 'O Gold

GOAL: To have students confront some problems whose solutions are not immediately obvious, to consider strategies for finding solutions to them, and finally, to solve them and reflect on the process.

STUDENT OBJECTIVES:

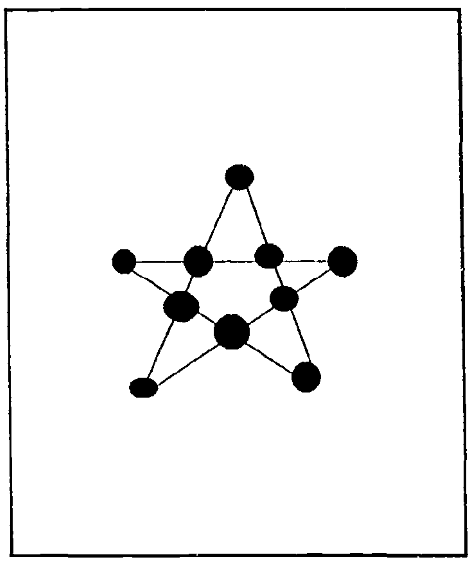
- ✓ To choose strategies for solving "brain teasers".
- ✓ To solve "brain teasers".

GUIDE TO THE INVESTIGATION: In this investigation, students will be asked to work together to solve problems whose solutions are not obvious.

Students will need to work together in small groups of about four. Students need pencil, paper, construction paper, scissors and encouragement to explore, share, and listen. Show them books from the school or public library in which they can find more brain teasers.

Before students try their hand at the "castle brain teaser", start them thinking with this one.

A farmer has ten apple trees to plant. She has decided to plant five rows with four trees per row. Draw a diagram to show how she planted the trees.



35 - 1 *Solution to the farmer's problem*

Allow individual students or small groups ample time to solve the problem. Once the problem is solved and discussed, direct students to the activity page. Read the castle problem with them.

Have students discuss the problem within their groups. Again, allow ample time for students to discuss and try many ways to solve the problem. After all groups have solved the problem, have a large group discussion of the strategies each has chosen and of the solution to the problem. Discuss their responses to Observations and Conclusions.

VOCABULARY: brain teaser

A Bag 'O Gold

INTRODUCTION: "Brain teaser" is a name given to the sort of problem which makes us think - one which we cannot solve right away but which "bends" our minds. Brain teasers are fun to work! Sometimes it takes many ideas and many trials before we find a way to solve a brain teaser and often we need to work with other people to get their ideas too. In this exercise, you will solve some brain teasers.

PURPOSE:

- ✓ What are some strategies for solving "brain teasers"?
- ✓ Which strategy is best for solving the "brain teaser" in the Procedures section?

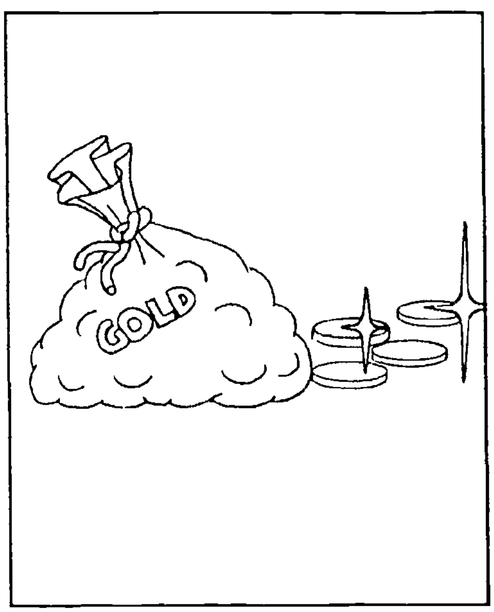
MATERIALS:

pencil
paper
construction paper
scissors

PROCEDURES:

1. Read the brain teaser.

There was once a castle in a land far away. One day the castle was attacked and taken over by a band of robbers. The leader of the robbers allowed Prince Nicholas to leave the castle and go into exile in a foreign land. He allowed the Prince to take several bags of gold with him. There were four doors which Prince Nicholas had to pass through as he left the castle to go into exile. Each door had a guard posted by it. The guard at the first door made Prince Nicholas give him one-half of the bags of gold he had plus one before he would let him pass. Prince Nicholas met his demands. The guards at the second,



35-2

third, and fourth doors made identical demands, which the Prince had to meet in order to be able to leave the castle. When he finally got out of the castle he had a single bag of gold remaining. How many bags did he have to begin with?

2. Discuss the problem with your group.
3. Brainstorm ways to solve the problem. Listen carefully to everyone else's thoughts.
4. Try each of the ways your group thinks of. See if any work, and, if they do, see which is easier, simpler, and quicker.
5. Making "bags of gold" from construction paper to act out the problem, may help you solve it.
6. Write the steps on the Response Sheet you used to solve the problem. If you made a diagram, picture, or drawing to help you, also include it on the Response Sheet.
7. Next, work with members of your group to create your own "brain teaser".
8. Write the "brain teaser" on an index card.
9. Swap cards with another group and try to solve the other original "brain teaser".

OBSERVATIONS:

1. How many bags of gold did Prince Nicholas take with him to begin with?
   46 bags
2. How many bags of gold did Prince Nicholas give the guard at the first door?
   24 bags
3. How many bags of gold did Prince Nicholas give the guard at the second door?
   12 bags
4. How many bags of gold did Prince Nicholas give the guard at the third door?
   6 bags
5. How many bags of gold did Prince Nicholas give the guard at the last door?
   3 bags

6. Describe the method you used to solve this problem.
Answers vary. Work backwards, trial and error, acting out the problem, writing number sentences, etc.

CONCLUSIONS:

1. Was there only one way to solve the castle brain teaser?
No.

2. Were there steps that the members of your group all used even though you may have solved the problem in different ways? If so, what were they?
Answers vary.

3. What suggestions would you give to others who are trying to solve brain teasers?
Answers vary. Try many strategies to find the one that seems to work best.

SUGGESTIONS FOR FURTHER STUDY

- There are many books of brain teasers. Find a book and select three interesting and challenging problems and write them on index cards. When you and your friends have written three problems each, rank each problem as easy, average, or difficult, and write an E, A, or D in the upper right corner of the problem's card. Have each of your friends do the same so that all of you classify every problem. Were there any disagreements? Why? Who was "right"? Why? Is it reasonable that some problems seem harder to one person than they do to another? Could it be that there is no "right" or "wrong" judgment about how hard or easy a problem is?
- With your group, create another brain teaser. Write out its solution. Remember to include any pictures you used to help you.

Response Sheet
A Bag 'O Gold

Possible ways to solve the castle problem:

1. *Work backwards.*
2. *Make a model.*
3. *Make a diagram.*
4. *Draw a picture.*
5. *Write an equation.*

Steps we used to solve the castle problem:

1. *Answers vary with strategy chosen.*
2.
3.
4.
5.

Drawings or models we used (if any) to solve the castle problem:

Drawings or models vary.

Our Original Brain Teaser, our Solution, and Drawings, if any:

Answers vary.

Teacher's Guide
Sweet or Sour? Thick or Thin?

GOAL: To have students explore oranges to see if they can find any relationship among the thickness of the skins, the number of sections, the number of seeds, the volume of juice, and the sweetness of the orange.

STUDENT OBJECTIVES:

- ✓ To explore several characteristics of oranges: thickness of the skin, the number of seeds, the number of sections, the amount of juice in the orange, and the sweetness of the fruit.
- ✓ To explore any relationships which might exist among these various characteristics.

GUIDE TO THE INVESTIGATION: Students should work in groups of five or six. Each group should have ten or twelve oranges with which to work. Each student should have two oranges which appear as identical as possible in size, color, and thickness of skin. The groups will each need a graduated cylinder or measuring cup which measures in milliliters, a paring knife, paper cups, and newspapers.

Direct the students to the activity sheet and ask them to follow the directions they find in the Procedures section.

Have students record the results of their exploration on the Response Sheet.

When the exploration and Response Sheet are completed, combine all the descriptions and findings. Discuss if and how the findings were changed when all the data were combined. Summarize what relationships were found among the various characteristics, and discuss whether the students think their findings can be generalized to all oranges. If they think not, have them name ways they can make generalizations about the relationships among characteristics of oranges. Include the Observations and Conclusions in your discussion.

VOCABULARY: relationships, sections (of an orange), "equator"

Sweet or Sour? Thick or Thin?

INTRODUCTION: Oranges were first grown by the Chinese many, many years ago, and now they are available and loved all over the world. You can peel and eat them; you can squeeze them to get their juice to drink; and you can cut them into pieces and put them into salads. You can even use ground orange peel in breads and cakes. There are many sizes and kinds of oranges. Some are quite large and others are small. Some have thick skins; some thin. Some are very sweet; some not so sweet. Some are full of seeds; others have very few seeds. In this activity you will explore various characteristics of oranges.

PURPOSE:

✓ What are some characteristics of oranges?

✓ Can you predict the "sweetness" or "sourness" of an orange by the thickness of its skin?

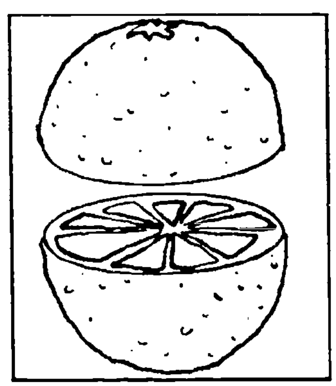
MATERIALS:

several oranges with different characteristics:
- thick and thin skins
- large and small

a paring knife
paper cups
a graduated cylinder marked in milliliters or a milliliter measuring cup
newspapers

PROCEDURES:

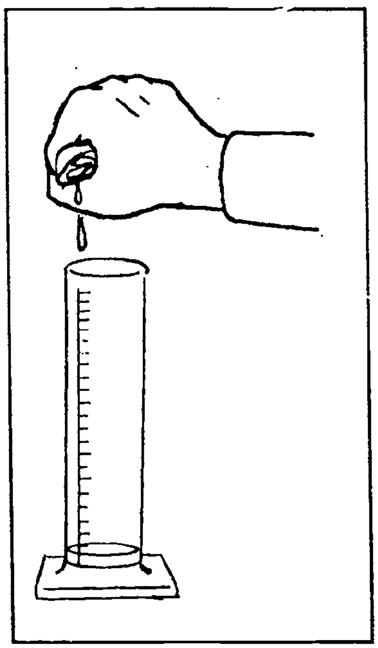
1. Work with 4 or 5 other students.
2. Spread newspapers over a table or other flat surface.
3. Divide the oranges among the members of the group so that each member gets two oranges which look as much alike as possible. If necessary, see if you can swap an orange or two with another group so that you can do this. When everyone has two oranges which seem to be alike, take one of them and make an "equator" cut. To do this hold the orange with the top up. The top



36 - 1

is where it was attached to the tree. Then cut smoothly through the skin of the orange where the orange is thickest (see illustration 36-1).

4. Count the number of sections into which the orange is divided, and record that number on the Response Sheet.
5. Record whether the skin is thick or thin.
6. Remove and count all the seeds, and enter the number on your Response Sheet.
7. Eat the orange, and record whether it was sweet of sour.
8. Repeat the above with the second orange, but this time, do not eat the orange.
9. After you have recorded the number of sections, the number of seeds, and the thickness of the skin, squeeze as much of the juice from the two halves as you can.
10. Measure the juice with the graduated cylinder, and record the volume of juice on the Response Sheet.
11. Then drink the juice, and record whether you found it to be sweet or sour.
12. When each student in your group has completed a study of each of the two oranges, each of you share what you found with your group.
13. When each person has shared individual results, then combine all these data by answering the questions on the Response Sheet.



36 - 2

14. Have someone read Question 1 from the Observations section, and then discuss it until your whole group agrees on an answer.
15. Then have another student read No. 2 and discuss it until your whole group agrees on an answer.
16. Do this with all six questions.
17. Then select a member of your group to present your group's conclusions to the whole class. See if the entire class can make any generalizations about the relationships (if there are any) among the skin thickness, sweetness, number of sections, size, juice volume, and number of seeds in an orange.

OBSERVATIONS:

1. Which oranges were the sweetest, those with thick skins or those with thin?
   Answers vary.

2. Which oranges were the sweetest, those with more seeds or those with fewer seeds?
   Answers vary.

3. Did the oranges with more sections also have more seeds than those with fewer sections?
   Answers vary.

4. Did the oranges with fewer sections have fewer seeds than those with more sections?
   Answers vary.

CONCLUSIONS:

1. How could you test the accuracy of the observations of one group?
   Answers vary. Compare with other groups.

2. Were there any generalizations (relationships) which were found by all the groups? If so, which ones?
   Answers vary. List all agreements between groups.

3. Were there any generalizations (relationships) which were found by almost all the groups? If so, which ones?
   Answers vary. List all generalizations found by multiple groups.

4. What advice would you give shoppers who are looking for the sweetest oranges?
   Answers depend on results of class findings.

SUGGESTIONS FOR FURTHER STUDY

- Duplicate the Procedures section of this investigation with another type of fruit, such as lemons, grapefruit, watermelons, pumpkins, etc. (In some cases, you will not have sections, and in other cases, it will be hard to count all of the seeds in the usual fashion. You may have to devise a counting method. Adapt the exploration to the type fruit you are using - for instance, with apples you may want to record skin color rather than skin thickness.) What kinds of relationships can you discover, and what generalizations can you make?
- For this study, you will need a farmer's almanac or obtain information from you state's local agricultural agent. Learn from one or both of these sources if there are ways to select the most desirable fruits and vegetables when you are shopping. For example, some people thump melons and buy the ones whose sounds "tell them" that the melon is ripe and tasty. Other people never buy a mushroom which has "opened up". Write a report and/or prepare a poster with what you learn. Share your findings with the class.

Response Sheet
Sweet of Sour? Thick or Thin?

My Individual Findings *Answers vary.*

Attributes	Orange One	Orange Two
Is the orange large or small?	Ring: Large Small	Ring: Large Small
Thickness of Skin	Ring: Thick Thin	Ring: Thick Thin
Number of Seeds		
Number of Sections		
Sweetness	Ring: Sweet Sour	Ring: Sweet Sour
Amount of Juice	_______ mL	_______ mL

Generalizations from My Group's Data *Answers vary.*

Yes	No	Place a check in the Yes or No Column for each of the following.
		1. As a group, we found a relationship between the thickness of the skin of an orange and the sweetness of the orange.
		2. As a group, we found a relationship between the thickness of the skin of an orange and the number of seeds of the orange.
		3. As a group, we found a relationship between the thickness of the skin of an orange and the number of sections in the orange.
		4. As a group, we found a relationship between the number of seeds in an orange and the sweetness of the orange.
		5. As a group, we found a relationship between the thickness of the skin of an orange and the amount (volume) of juice in the orange.
		6. As a group, we found a relationship between the number of seeds in an orange and the amount of juice in the orange.

Teacher's Guide
Loop the Loop

GOAL: To have students explore simple, closed curves (loops), and discover the inside and the outside of the region bounded by the closed curve.

STUDENT OBJECTIVES:

✓ To make various simple, closed curves (loops) with string.

✓ To investigate loops, and determine what is the inside and outside of the loop.

GUIDE TO THE INVESTIGATION: Students should work in pairs or in groups. Students will need six or eight feet of string or yarn, a nickel, and a penny.

Before students begin the exploration, explain the nature of a closed curve and a simple, closed curve, one which does not cross itself. Draw an example of each on the chalkboard and discuss the distance between various points on the curve "as the crow flies". Have students follow the directions in the Procedures section. Help them get started and make sure that when they alter the loop from its circular form to a squiggly one, they do not lift the string off the table or floor. If they do, they may change the position of a coin relative to the inside and outside of the loop.

When students complete the Procedure section, have them work in groups of 4 to 6 students to compare Observations. Then discuss Conclusions in the total group.

VOCABULARY: closed curve, simple closed curve, loop, interior, exterior

Loop the Loop

INTRODUCTION: A loop separates a plane into two regions, the inside and the outside. If the loop is at all complicated, then it my be hard to tell whether something is on the inside or the outside of the loop. In this activity you will make some complex loops and see what you can tell about the inside and outside.

PURPOSE:

- ✓ What configurations (loops) can you make with string or yarn?
- ✓ How can you distinguish between the inside and the outside of a loop?
- ✓ Can you always trust your own eyes to tell the difference between the inside and outside of a loop?

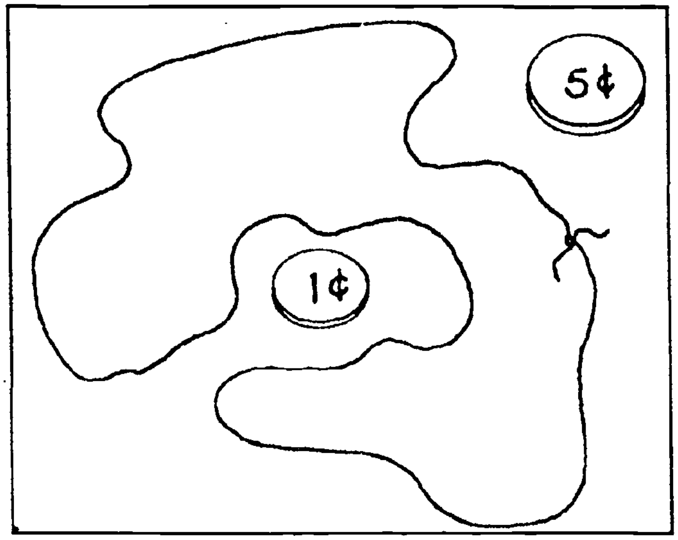
MATERIALS:

string or yarn
a nickel
a penny

PROCEDURES:

1. Work with a small group of students – or at least one other student.
2. Cut a piece of string or yarn about two yards long, and tie the ends together to form a loop.
3. Lay the loop on a large, flat surface, such as a large table or the floor, and arrange it nearly in a circle. You have formed two regions: one region is the inside of the loop, which is called the interior; the second, the outside, which is called the exterior.
4. Place a nickel in the exterior region and a penny in the interior region. If you drew a line from the penny to the nickel, how many times would it cross the loop?
5. Record your answer on the Response Sheet.
6. Now place both coins on the outside of the loop and rearrange the string in a squiggly pattern similar to illustration 37-1.
7. When you rearrange the string, move it around on the flat surface without lifting it over either coin so that you are certain that each coin remains

in its original region. Also be sure that the string does not cross itself. That is how you guarantee that it still divides the flat surface into two regions: the outside and the inside. If you let the string cross itself, you will have a figure where there is no longer one outside and one inside but three regions.



37 - 1 Loop

8. Close your eyes and have a member of your group arrange the string in a squiggly pattern without ever letting the string cross itself.
9. Ask someone to place the nickel and the penny somewhere on the table but not on top of the string. Can you tell if the penny is inside or outside?
10. Write your answer on the Response Sheet.
11. Can you tell if the nickel is inside or out? Can you tell whether they are both in the same region even if you cannot tell which it is? Record your answer on the Response Sheet.
12. If you drew a line from the nickel to the penny, how may times would this line cross the squiggly loop? Record your response.
13. Would the line cross the string an odd number or even number of times? How do you know?

OBSERVATIONS:

1. In which shape of the loop was it easiest to tell whether the coins were inside or outside?

   Circle or other simple closed curve

2. How could you tell whether the coins were inside a squiggly loop or outside?

   Can you hold it on its edge and roll it out without crossing the string?

3. How could you tell whether the coins were on the same side of a squiggly loop?

   If a line from one to the other crosses the yarn an even number of times, then they are on the same side.

CONCLUSIONS:

1. What is the relationship between the location of the coins and the number of times the line between them crosses the loop?

   If both coins are outside the loop, the line would have to cross the loop an even number of times. If one coin was inside, the line would cross an odd number of times.

2. How does the number of curves (squiggles) affect the number of times the "line" crosses the string?

   The more squiggles, the more times the "line" crosses the string.

SUGGESTIONS FOR FURTHER STUDY

- Repeat the activity, but this time have a member of the group cut a porthole from a large sheet of butcher paper which will allow you to see both coins and the space between. You will not be able to see the loop anywhere else, so you will not be able to get "the big picture" Have a partner position the paper so that both the penny and nickel can be seen through the porthole. Conceal the outer edges of the string with the edges of the paper. What can you tell about the positions of the coins relative to the loop? Is the penny inside or outside the loop? How can you tell? *Answers vary.*
- Make a complex, closed curve with a pencil on a sheet of paper. Complex, closed curves cross one another once or more. Use various colors to color the different regions into which you divided the piece of paper with your curve. How many were there? *Answers vary.*
- Use a geoboard and rubber bands. Make several "loop" or closed curves on the geoboard. How can you make squiggly patterns on the geoboard? *Answers vary.*

Response Sheet
Loop the Loop

1. How many times will the imaginary line between the nickel and penny cross the circular loop?
   2 times

2. When you were certain that both coins were on the outside of a squiggly loop, how many times would the imaginary line between them cross the loop? Record a separate answer for every squiggly loop your group tried.
   Answers vary. Depends on the squiggly loop.

3. Do you see a pattern here? If so, what is it?
   Yes. Both out = even number of crosses.
   One in, one out = odd number of crosses.

4. Can you tell by the number of times the "line" crosses the string whether a coin is inside or outside or whether they are both on the same side? Explain.
   Answers vary.

5. What did you discover about odd and even numbers when you repeated the activity several times?
   Same answer as # 3

Teacher's Guide
Aren't We Special?

GOAL: To have students explore patterns of squares and rectangles and discover some interesting properties of a special group of numbers.

STUDENT OBJECTIVES:

✓ To make rectangles by manipulating squares.

✓ To discover and characterize a special group of numbers.

GUIDE TO THE INVESTIGATION: Students will need construction paper, pencil, ruler, and scissors, a paper bag or cardboard box or some other container. They will also need table or desk space where they can arrange things undisturbed. This investigation can be carried out by pairs, individuals, or in small groups.

Have students follow the directions in the Procedures section.

When students complete the exploration, have them discuss in small groups what they found and the generalization they have made. See if they can offer some suggestions about how their discovery might help them in their future studies or even in their work outside the classroom. Discuss the Observations and Conclusions

Have students see what they can learn from reference or mathematics books from the school library about the special numbers they have discovered.

VOCABULARY: rectangle

Aren't We Special?

INTRODUCTION: Squares are rectangles in which all four sides have equal length. In this investigation, you will explore properties of a special group of numbers by using squares.

PURPOSE:

- ✓ How can you manipulate a number of squares to make a rectangle?
- ✓ Can any number of squares be manipulated in more than one way to form different rectangles?
- ✓ Which property of numbers did you discover when you made rectangles from different numbers of squares?

MATERIALS:

ruler
construction paper
scissors
a box (shoe box, gift box)

PROCEDURES:

1. Cut twenty one-inch squares from construction paper.
2. Put the squares into a box.
3. Draw two squares from the box.
4. Use them to make a rectangle.
5. Try to use them to form rectangles different from the first one you made. If you can make a rectangle in more than one way, make a check (✔) mark in Column B on the Response Sheet. If there is only one way to make a rectangle with two squares, put the ✔ in Column A. Be very sure that the rectangles are different sizes and not just rotated one way or another.
6. Draw a third square from the box, and make a rectangle with three squares.
7. Try to use the three squares to form a rectangle different from the one you did first. If you can make two or more different rectangles with three squares, put the ✔ in Column B. If you can make only one, put it in Column A.

8. Draw a fourth square from the box, and make a rectangle with four squares.
9. Try to use the four squares to form a rectangle different from the one you did first. (Hint: A square is also a rectangle.) If you can make two or more different rectangles with four squares, put the ✔ in Column B. If you can make only one, put it in Column A.
10. Repeat the activity with 5, 6, 7, 8..., 20 squares, and record your findings on the Response Sheet.

OBSERVATIONS:

1. Make a list of all the numbers with check marks in Column A.
   2, 3, 5, 7, 9, 11, 13, 15, 17, 19
2. Do you notice a pattern? Is there anything special about these numbers?
   Except for the 2 they are odd numbers. They are prime numbers; they have only 2 factors.
3. If you continued your investigation using 100 squares, would this special pattern continue? What about 1,000 squares? *Yes and yes.*

CONCLUSIONS:

1. State your findings above as a generalization about making rectangles from squares.
   If you can make only one rectangle from x squares, then x is a prime number.

2. Can you think of some ways this discovery might help you in science classes, future math classes or in the "real world"?

   Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- From the special group of numbers, choose any two and add them. Now do so for three of four more pairs. Do the sums appear to have anything in common? If so, what? *The sum of 2 odd numbers is an even number.*
- On a plain sheet of paper, write all the whole numbers from 1 to 100. Try to space them evenly, using the entire sheet of paper. You will need a set of colored pencils. Choose a color, and mark out every number on the page which has a factor of two (all even numbers). Choose another color, and mark a slash through every number which is a multiple of three. Take a third color, and do the same for multiples of five. With still other colors, mark through the multiples of seven. What numbers are unmarked? If you were to continue this process, which multiple would you do next? Describe these numbers: 2, 3, 5, 7, etc. Do they have a name? Write your own definition for the remaining numbers. *Prime numbers. The remaining numbers are composite—that is, they have factors other than one and the number itself.*

Response Sheet
Aren't We Special?

Results of Manipulating Squares to Form Rectangles

Number of squares	Column A (only 1 way)	Column B (more than 1 way)
2	✔	
3	✔	
4		✔
5	✔	
6		✔
7	✔	
8		✔
9	✔	
10		✔
11	✔	
12		✔
13	✔	
14		✔
15	✔	
16		✔
17	✔	
18		✔
19	✔	
20		✔

One or More Different Rectangles

Teacher's Guide
Is It, Or Isn't It?

GOAL: To explore the use of characteristics and attributes to identify figures.

STUDENT OBJECTIVES:

✓ To use characteristics or attributes to identify, sort, and classify figures.

✓ To use characteristics to develop clues which will help identify figures.

GUIDE TO THE INVESTIGATION: This activity can be completed by individuals or pairs of students. Each will need construction paper, scissors, and markers.

Before students begin the exploration, discuss how books are classified in the library or have students give directions on how to make something. Discuss the importance of being able to accurately describe characteristics and to give clear directions. Discuss how characteristics of figures and objects help one sort and classify into categories.

Then have the students follow the directions in the Procedures section. After they have completed the exploration, discuss the Observations and Conclusions. Then play "I'm Thinking Of..." using the figures made from construction paper. You may also want to expand the game to include favorite foods, sports, music groups, entertainment programs, etc.

VOCABULARY: characteristics, attributes, parallel, polygon, classify

Is It, Or Isn't It?

INTRODUCTION: Have you ever played the game, "I'm Thinking Of..."? Someone thinks of a thing,, and someone else asks questions to which the answer can be only, "yes" or "no". After a certain number of questions, the person asking the questions must be able to identify the thing of which the first person is thinking. In this activity you will play this game using clues to identify geometric figures and eliminate incorrect figures.

PURPOSE:

✓ How can characteristics or attributes be used to identify geometric figures?

✓ What kinds of clues are helpful in identifying geometric figures?

MATERIALS:

construction paper
scissors
markers

PROCEDURES:

1. Together with your partner, look at the figures on the Response Sheet and describe each of them.
2. Then trace and cut out each of the shapes on construction paper.
3. Use a marker to make stripes, or dots on each figure - or to color in the whole figure.
4. Mix the shapes together at random and lay them out on a table top so that you can see any markings you made on them.
5. Now use the characteristics to describe a figure, and see if you can identify the unique figure which satisfies all the elements of this description.

It is dotted. It is not circular. It is not large. It has three sides.

6. When you have agreed on which shape is characterized by those criteria, do the same for each of the following three sets of characteristics:

It is a polygon. It has no interior pattern. It is taller than it is wide. It is small.

It is plain. It is not small. It is not a polygon.

It has four equal sides. It does not have stripes. It is not large. It has dots.

OBSERVATIONS:

1. What are some of the characteristics which distinguish one figure from another?

   Answers vary. Size, shape, interior markings (such as none, stripes, dots), number of sides, etc.

2. Which figures on the Figure Sheet are alike? How are they alike?

   Many have the same shape, but different shading and different size; many are the same size, but have different shape and shading; many have same shading but are different size and/or shape; all are geometric shapes.

3. What are some of the ways in which figures are different?
Size, shape, shading

CONCLUSIONS:

1. How were you able to eliminate the incorrect figures?
By using "it is not" clues.

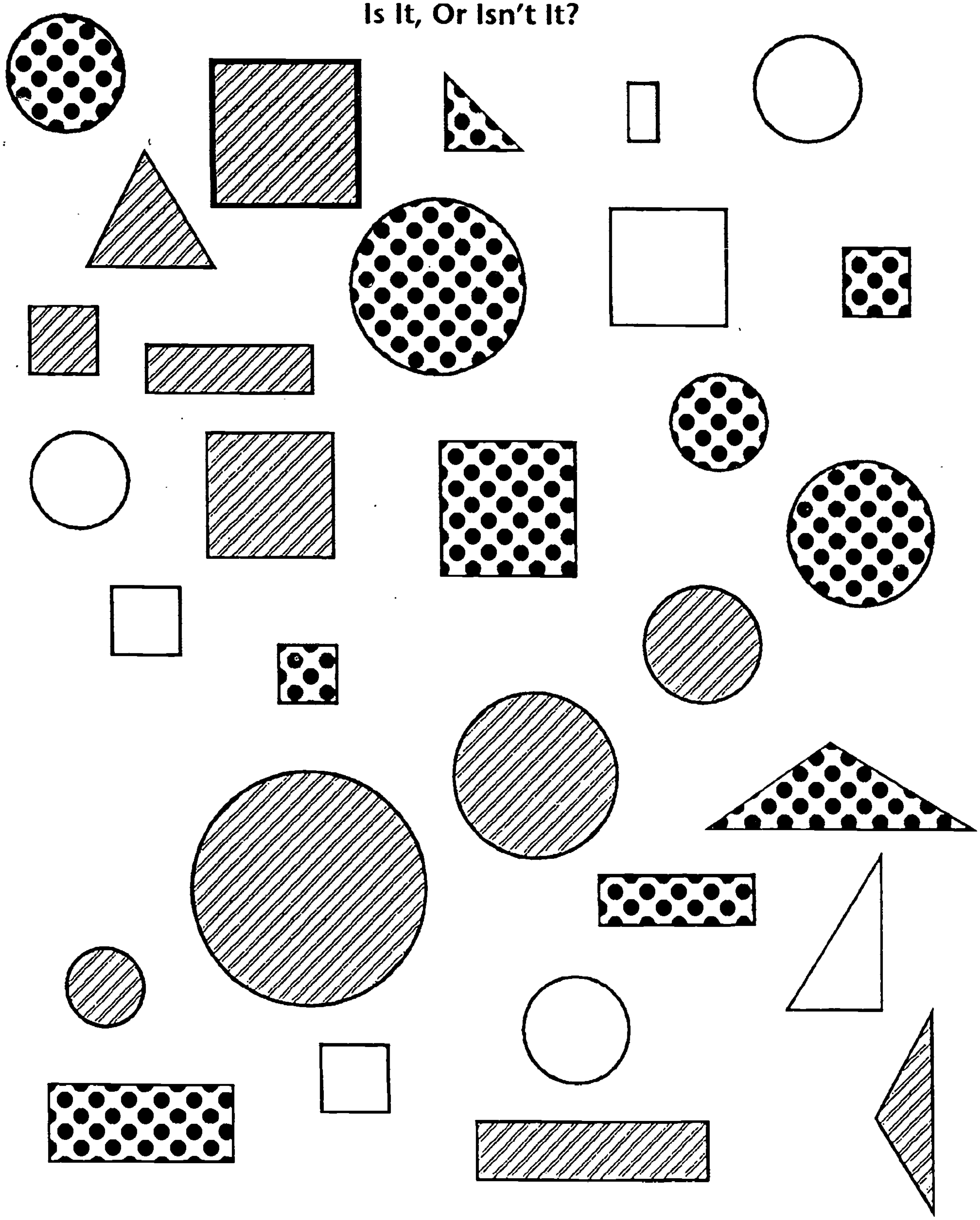
2. Why is it important to be able to describe figures or other things in detail?
Answers vary. Sometimes it is necessary to get directions with just spoken words.

3. How can the ability to describe attributes accurately be helpful to you in your everyday life?
Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- With a friend, take a walk around your neighborhood or school grounds. Observe things that there are many of - leaves, trees, birds, automobiles, buildings, etc. Observe what you see. One of you take notes while the other names things and describes them. Gather samples, if you like. Repeat the same walk with your roles reversed so that you get as complete a list as possible and really good descriptions. Write clues similar to the ones in this investigation. Have your partner use the clues to eliminate the incorrect objects and to identify the object you had in mind.
- Play a game of "I'm Thinking Of..." with your class.
- Make a list of things in the "real world" where classification or sorting is used and/or is necessary.

Response Sheet
Is It, Or Isn't It?



Teacher's Guide
Working Backwards

GOAL: To have students explore situations in which the most appropriate problem-solving strategy is to work backwards.

STUDENT OBJECTIVES:

✓ To apply the working-backwards strategy to problem-solving.

✓ To explore the "work backwards" strategy in several different kinds of situations.

GUIDE TO THE INVESTIGATION: This exploration is appropriate for pairs or small groups of students. Have the students work together so they can share and discuss problems and strategies with each other. Each pair or group will need six clear plastic tumblers, red food coloring, and water.

Before students begin the explorations, discuss the strategy of working backwards to solve a problem. Have students give two examples in their own experience where the strategy was useful and when it is an appropriate problem-solving technique. You might offer this as an example: Suppose you want to buy something which costs $32.95. If they know the rate at which they will be paid for work in the yard or neighborhood, how long would they have to work to save $32.95? Explain that they have to work backwards from the answer to the question, "How much did I earn?".

Have students complete the Procedures section on the activity sheet. Then discuss Observations and Conclusions reached by students.

VOCABULARY: working backwards, strategy, implications

Working Backwards

INTRODUCTION: Have you ever had to travel to a friend's house or to a party and needed directions? If so you have probably been given directions about how to get there. When it is time to go back home, you have to work backwards with the directions. There are many situations in which working backwards is the best strategy to solve a problem. In this activity you will use the working backwards strategy to complete a challenge.

PURPOSE:

✓ How can working backwards help to solve a problem?

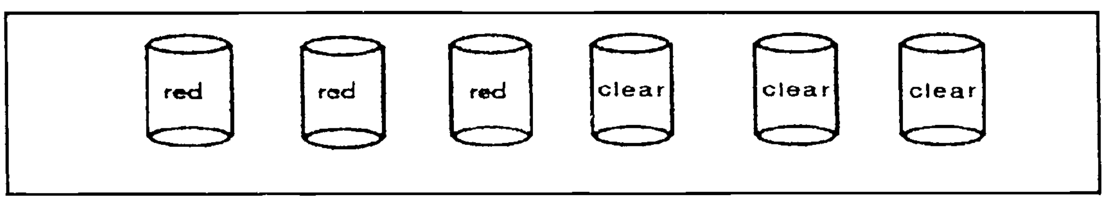
✓ In what kinds of situations is the strategy of working backwards helpful?

MATERIALS:

six clear plastic containers
red food color
water

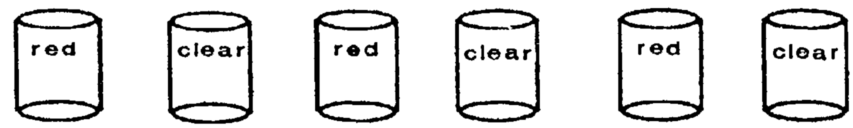
PROCEDURES:

1. Fill three of the containers a little less than half full with water.
2. Place three drops of red food coloring into the other three containers, and add water to fill them to the same level.
3. Place all the containers on the table in the following order:



40 - 1

4. Now consider this challenge. How could you get from the original arrangement to the one shown in illustration 40-2 if you must follow this rule: You may move or touch only one container. You may need six additional containers to duplicate the problem and to help in your exploration.



40 - 2

OBSERVATIONS:

1. How did you complete this challenge?
Answers vary. Pick up glass #2 and pour the liquid (or some of it) into glass #5.

2. Were you successful on the first try? If not, how may times did you try before you were successful?
Answers vary.

3. Was there a "hidden clue" you used to complete the activity? If so, what?
Answers vary. Possibly—nothing said you couldn't pour.

CONCLUSIONS:

1. How did you decide what you could and could not do to complete this experiment?
Answers vary.

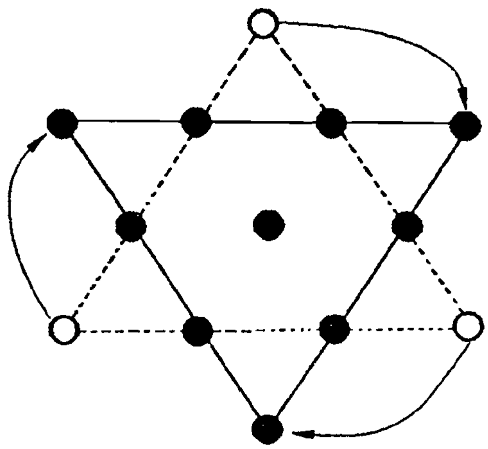
2. What things did you keep in mind when you worked on this activity?
Answers vary. You could only move or touch one container.

3. What implications does this investigation have for solving other kinds of problems?
Answers vary. Shows the power of working backwards. Read directions carefully. Use logical reasoning. Rule out "mental blocks".

4. What are some other types of problems which can be solved using this strategy?
Answers vary.

SUGGESTIONS FOR FURTHER STUDY

- Arrange ten coins in the pattern shown.



Dotted line shows original arrangement
Solid line shows new direction (arrangement)

Note that the coin on top seems to be pointing upward. Your mission is to change this pattern by moving only three coins. How can you do this? *Move the white coins as indicated by the arrows.*

- Find some mathematics problem situations that can be solved by working backwards. You may find some in your own book, or you may need to look at books in the library. Choose one or two, and demonstrate how the working-backwards strategy is used to solve them. Then write several original problems that can be solved by working backwards. Exchange the problems with another student. Solve the other student's problem.

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