Discriminant analysis (DA) is a multivariate technique concerned with either prediction/classification (predictive discriminant analysis) or distinguishing among groups (descriptive discriminant analysis). DA has many potential applications in gifted education research, including investigation of identification procedures in order to obtain evidence of validity (PYRT, 1986), determination of a piece of evidence of the discriminant validity of alternative assessments, a post hoc procedure after a significant multivariate analysis of variance result, and description of group differences based on longitudinal data (e.g., Arnold, 1993). Problems and issues related to the use of DA, including sample size issues and group distinction issues, are discussed. (Contains 19 references.)

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Application of Discriminant Analysis
in Research with Gifted Students

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Abstract

Discriminant analysis (DA) is a multivariate technique concerned with either prediction/classification (predictive discriminant analysis; PDA) or distinguishing among groups (descriptive discriminant analysis; DDA). DA has many potential applications in gifted education research, including investigation of identification procedures in order to obtain evidence of validity (Pyryt, 1986), determination of a piece of evidence of the discriminant validity of alternative assessments, a post hoc procedure after a significant MANOVA result, and description of group differences based upon longitudinal data (e.g., Arnold, 1993). Problems and issues related to the use of DA, including sample size issues and group distinction issues, are discussed.
Discriminant Analysis

Introduction

In reported research on gifted education and talent development, multivariate analyses are used much less often than univariate and bivariate analyses. Considering the considerable advantages of multivariate analyses in many situations encountered by researchers (Huberty, 1994b; Huberty & Morris, 1989; Moon, 1995), they should become familiar with the use and interpretation of multivariate techniques.

The purpose of this paper is to familiarize researchers concerned with gifted education and talent development with the use of discriminant analysis (DA). Huberty (1994a) differentiates between two types of DA: Predictive discriminant analysis, in which the researcher is concerned with predicting group membership given a set of variables; and descriptive discriminant analysis, in which the researcher is concerned with describing the difference between groups using a given set of variables. The following descriptions of both DA techniques are meant to be summative and illustrative -- the reader interested in the more complex statistical topics (e.g., derivation of linear functions, calculation of various classification statistics, development of DA techniques) are referred to Huberty (1994a) and Das Gupta (1973).

Predictive Discriminant Analysis (PDA)

Predictive discriminant analysis is used to answer the question, "How well can group membership be predicted using these variables?" The result of a PDA for k groups is k linear classification functions (LCF). These LCFs are then used to determine Group membership for each subject is then determined by calculating each of the k LCFs using the values of the predictor variables for that particular subject. The LCF with the largest value determines group prediction (Huberty, 1984).

"Hit rates" can be determined by calculating the percentage of cases correctly classified by the LCFs. Hit rates can be determined in two ways: an internal analysis, in which data used to arrive at the LCFs is used to determine correct and incorrect classification; and external analysis, in which data from different subjects is used to determine hit rates. Although internal analysis is frequently used, Huberty (1984; Huberty & Barton, 1989) recommends the use of external...
Discriminant analysis, especially if the sample size is less than five times the number of variables used in the DA. Another advantage of external analysis is that internal analysis will usually result in an acceptably large hit rate because the LCFs were created using the same data.

**Descriptive Discriminant Analysis (DDA)**

Used more frequently in educational research than PDA (Huberty, 1994a), descriptive discriminant analysis is used to answer the question, "How can we best distinguish among groups using these variables?" The result of a DDA for \( k \) groups with \( p \) variables is the minimum of \( p \) or \( k-1 \) linear discriminant functions (LDF). To determine whether these LDFs distinguish among groups better than chance, a variety of statistical tests can be used (e.g., Roy's Criterion, Wilk's Lambda, Hottelling's \( T^2 \), Hottelling's Trace).

Variable importance (i.e., how each variable contributes to the discrimination between groups) can be determined in several ways (see Huberty, 1984), but common practice is to interpret the correlation between the variable and the LDF (i.e., the structure coefficient; Tabachnick & Fidell, 1989). As with factor analysis, a cut-off value for interpretation of structure coefficients is not "set in stone" -- Tabachnick and Fidell (1989) interpret loadings greater than .50, but several researchers use .30 or .40. The factor analysis metaphor is appropriate: Loadings on each LDF can be taken collectively to reveal the underlying construct that is responsible for group differentiation. Classification of cases like that associated with PDA is not required in DDA, since the canonical correlation between the groups and predictor variables on each LDF can be interpreted as a measure of association (i.e., effect size; Tabachnick & Fidell, 1989). However, a classification table is usually constructed and reported as an additional measure of LDF effectiveness.

Another method for interpreting variable importance is the F-to-remove statistic (Huberty, 1984; Huberty & Morris, 1989). An example of its use is provided by Burns (1990), who determined variable importance by examining the F-to-remove statistic for each variable that was a result of the final step in a stepwise DDA.
DDA is mathematically identical to multivariate analysis of variance (MANOVA), and DDA is commonly used as a follow-up procedure when a MANOVA reveals significant differences among groups. Multiple univariate analyses (i.e., ANOVAS) are usually used as the MANOVA follow-up, but there are several statistical problems with this practice in most cases (Huberty & Morris, 1989). Use of DDA to interpret the significant MANOVA effects is recommended, especially considering the information that is provided regarding underlying constructs in the predictor variables.

Current and Recommended Uses of DA in Research with Gifted Students

Current Uses of DA

With one exception, published uses of DA in gifted education research have been restricted to DDA (e.g., Burns, 1990; Pyryt, 1993; Pyryt & Mendaglio, 1994). Arnold (1993) reports two descriptive analyses and one predictive analysis, but the reported classification analysis appears to be based on calculations of a new LDF score for each subject (as opposed to an LCF). Reported uses of DDA vary widely: Burns (1990) used DDA to discriminate between students who initiated and did not initiate creative investigations as part of the Enrichment Triad Model, Pyryt (1993) described the difference between successful and unsuccessful gifted adults, and Pyryt and Mendaglio (1994) use DDA as a post hoc procedure to aid in the interpretation of a significant MANOVA effect. DA was also used in several dissertations in gifted education, although determining the exact type of DA is not possible through a reading of the abstracts. Marquardt (1988) investigated the differences in personality variables between a group of gifted achievers and a group of gifted at-risk students, Hartley (1989) discriminated among groups of Caucasian and Navajo teachers and parents using their self-reported perceptions of giftedness and talent, Fellers (1991) discriminated among average and gifted males and females with a battery of teacher rating scales of behavioral characteristics, and Edelman (1992) used standardized tests and recommendations to discriminate between students selected for a school district's gifted program and students who were not selected. While these four studies are only a small sample of the
dissertation research using DA, they give a good picture of the many and varied applications of DDA for gifted education research, especially when considered with the few published studies.

**Recommended Uses of DA**

In addition to the applications mentioned above, DA has several applications to gifted education research. First, the use of DDA as the follow-up to significant MANOVA effects is highly recommended (Huberty, 1994a; Huberty & Morris, 1989). A second major use is the investigation of differences among specific populations of gifted students and/or between students classified as gifted and those not classified. For example, a colleague recently sought to answer the question "How are suburban and rural gifted students different with respect to access to extracurricular educational and mentoring opportunities?" He surveyed a relatively large sample of suburban and rural gifted students on the frequency of their participation in a few dozen activities. But rather than analyze the data with a multivariate technique such as MANOVA/DDA, my colleague performed an ANOVA or independent t-test for each activity. In the last few years, gender and racial differences among gifted and average ability students have been examined with increasing frequency. DDA is a statistically powerful tool for answering these questions.

**Identification and assessment.** Pyryt (1986) recommended the use of discriminant analysis to determine relative rates of effectiveness and efficiency in gifted screening processes. Examination of group differences among identified and non-identified students is a natural application of DDA. In a recent evaluation, the author discriminated among identified students, screened but not identified students, and students not recommended for screening utilizing a variety of demographic, achievement test, affective, and performance assessment variables.

Discriminant analysis may also be useful during the difficult process of establishing evidence of validity for performance and other alternative assessments. Does a battery of alternative assessments predict gifted program placement three years later better than standardized tests? Can alternative assessments differentiate between high- and low-creative students as identified by observation and/or creativity tests? Do performance assessments differentiate among high- and low-achieving students from specific ethnic groups better than standardized achievement
measures? These and additional questions can all be answered in a relatively straightforward manner with either PDA or DDA.

Problems and Issues Associated with the Use of DA

Huberty (1994a) and Klecka (1980) mention several problems and issues associated with the use of DA, including missing data, potential influence of outliers, highly correlated variables, unequal group sizes, violation of assumptions, sample size issues, group definition issues, and stepwise DA. Several of these issues (missing data, influence of outliers, lack of orthogonality, unequal group sizes) are rather intuitive for researchers familiar with multivariate analysis and will not be discussed here. Readers are referred to Huberty (1994a), Klecka (1980), Tabachnick and Fidell (1989) for discussions of the impact of these issues upon DA and techniques for addressing them.

Violations of Assumptions

The techniques suggested by Moon (1995) may lead the researcher to suspect that the assumptions of DA are violated. Klecka (1980) notes that DA is fairly robust, and that all of the assumptions are not applicable to all of the numerous aspects of DA (e.g., violation of multivariate normal distribution influences tests of significance, lack of homogeneity of group variance-covariance matrices effects classification functions and linear discriminant functions). If the violation is minor, researchers can use and interpret both PDA and DDA with little concern. Refer to Klecka (1980) and Huberty (1994a) for discussions of addressing more serious violations. Of course, other multivariate techniques (such as logistic regression in the 2-group case) may be more applicable if assumptions are violated and should be investigated.

Sample Size Issues

As with other multivariate techniques, large sample sizes are required to perform a discriminant analysis. Huberty (1994a) suggests that the size of the smallest group should be at least three times the number of variables \(p\) used in a PDA (\(5p\) for an internal classification analysis) and roughly \(4p\) or \(5p\) for DDA. Arnold (1993) used DA to describe the differences between two groups of female college valedictorians (high and low vocational aspiring) on a...
number of variables, including expected age at marriage, desire to work with people in career, ACT scores, socioeconomic status, and other demographic, test score, and self-report data. The resulting DDA used 14 variables with a sample size of 46 (1.6p in smallest group), and a follow-up DDA was performed with data collected six years later (1.3p in smallest group). Arnold also included a PDA by using the classification function from the first DA to predict group membership from the data collected six years later. Due to the small N/p ratio, the results of the three descriptive discriminant analyses may not accurate. For example, the hit rate for PDA tends to depart substantially from the true hit rate when the ration of subjects to variables falls below 5:1.

**Group Definition Issues**

One of the most important issues involved in the correct usage and interpretation of DA is group definition. Once a grouping variable is chosen, special care should be exercised to ensure that the independent variables included in the DA were not used to determine group membership. For example, Pyryt (1993) used discriminant analysis to reanalyze the data from Terman's Success Study. Terman divided his subjects at mid-life into three groups: A, the most successful; C, the least successful; and B, everyone else. Pyryt, using variables representing educational attainment, acceleration, and IQ, found a discriminant function that significantly differentiated between groups A and C. Educational attainment was the major discriminating variable, although structure coefficients for all three variables were in excess of .35. However, evidence exists that suggests that Terman was biased in his initial classification toward subjects that were well-educated (Shurkin, 1992). Considering this, the magnitude of the structure coefficient for educational attainment (.96) is not surprising.

Another example is provided by the work of the author. In a study attempting to describe differences between students recommended and not recommended by teachers for screening for possible entry into a gifted magnet school, standardized achievement test scores, performance assessment scores, teacher rating scales, attitudinal measures, and demographic data were used as predictor variables. The resulting discriminant function was significant, and interpretation of structure coefficients revealed that socio-economic status and standardized achievement test scores
contributed the most to the significant function. However, during discussions with the school
district's coordinator of gifted services, I realized that the achievement tests were the most heavily
weighted entry criterion and that teachers were asked to look especially for students from low SES
backgrounds. My results, unfortunately, could have been determined without the several hours of
data screening, data analysis, and computer time that I expended!

**Initial misclassification.** Ambiguous or incorrect group assignment will have an obviously
negative impact upon DA, both in determining significance of classification/discriminant functions and in determining hit rates (Huberty, 1994a). Huberty notes that ambiguity in group classification (e.g., "fence sitters" who could almost be classified as either group) will result in frequently misclassified cases. Pyryt (1993) wisely avoided this problem in his reanalysis of the Terman data by eliminating the cases in the B group from his analysis.

**Stepwise DA**

Several authors (e.g., Klecka, 1980; Tabachnick & Fidell, 1989) provide overviews of stepwise methods to answer questions such as "What is the best subset of predictors?" in PDA or "What is the best subset of discriminating variables?" in DDA. However, the problems that are usually associated with stepwise methods in multiple regression are also applicable to DA, including misinterpretation of results (Huberty & Barton, 1989) and letting the computer control your data analysis. Huberty (1994a) believes that stepwise methods during PDA are rarely advisable, and similar methods may be useful during DDA if group separation is the main concern -- and if the results are not used to determine variable-ordering incorrectly (see Huberty, 1984, for a detailed discussion of both stepwise DDA and variable ordering).

**Conclusion**

The application of discriminant analysis to research on gifted education has occurred with relative infrequency. Considering the frequency of educational research in which the primary research questions are concerned with group prediction or differentiation, DA should be utilized more often. This paper has reviewed the two major types of DA, predictive and descriptive.
discussed current and suggested uses of DA in gifted education research, and provided an overview of several significant issues and problems associated with the use of DA.
References


