At the University of Southwestern Louisiana, an innovative course in college mathematics, Math 107, was designed to place mathematics into the context of everyday life and to use concepts from several disciplines within mathematics including algebra, geometry, statistics, probability, and data analysis. Inherent within the teaching of Math 107 was the use of reform strategies such as extensive real-world problem solving and cooperative learning. A comparison of (n=140) students enrolled in either Math 105, a traditional college algebra course, or in Math 107 showed that students enrolled in Math 107 had a lower attrition rate and a higher rate of satisfactory course completion. Statistical analysis showed that Math 107 students showed greater gains in positive attitudes towards mathematics. There were no significant differences based on size of class or the gender of the student. (Contains 18 references.) (Author/MKR)
An Investigation into the Effectiveness of an Integrated Mathematics Course for College Freshmen Compared to Traditional Algebra for College Students

Abstract

A comparison of 140 students enrolled in either MATH 105 or MATH 107 showed that students enrolled in MATH 107 had a lower attrition rate and a higher rate of satisfactory course completion. Statistical analysis showed that MATH 107 students showed greater gains in positive attitudes towards mathematics. There were no significant differences based on size of class or the gender of the student.

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February, 1995
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PROBLEM STATEMENT

This research project concerns itself with the general improvement of mathematics education at the higher education level. Specifically there are seven sub-problems considered:

(1) Does the reorganization of college freshman mathematics content, methodology, and assessment along lines recommended by the National Council of Teachers of Mathematics (NCTM) produce superior growth in mathematics understanding?

(2) Does the reorganization of college freshman mathematics content, methodology, and assessment along lines recommended by the National Council of Teachers of Mathematics (NCTM) produce a positive shift in student attitude toward mathematics?

(3) Do students in relatively small class sizes demonstrate greater growth in understanding mathematics than students in larger class sizes?

(4) Do students in relatively small class sizes demonstrate a more positive change in attitudes toward mathematics than students in larger classes?

(5) Does gender have an impact on the growth of mathematical understanding among freshman level math students?

(6) Does gender have an impact on changes in student attitude toward mathematics among freshmen math students?
(7) Is there a relationship between student attitudes towards mathematics and their performance in mathematics?

**SIGNIFICANCE OF THE PROBLEM**

Mathematics is in the process of undergoing major changes, both in content and approach. Two major protagonists for this change have been the National Council of Teachers of Mathematics (NCTM) and the Mathematical Association of America (MAA). The Standards for school mathematics (NCTM, 1989) presented sweeping changes which were needed in mathematics content, instruction, and assessment at the K-12 levels. This was followed by standards relative to how mathematics teachers should be prepared (NCTM, 1991). The MAA issued *A Call For Change* (1991) which suggested that major re-structuring was needed to improve the preparation of mathematics educators. While these major documents considered primarily pre-college mathematics and the preparation of teachers, it is easy to extend the basic concepts on content, instruction, and assessment to the collegiate level, especially the initial course taken by all college or university freshman.

The state of Louisiana was successful in securing grant funds from the National Science Foundation (NSF) for the purpose of improving science and mathematics at the collegiate level. The project, called the Louisiana Collaborative for Excellence in the Preparation of Teachers (LaCEPT), allows for competitive grants to institutions of higher education which may be interested in fostering the goals of the reform movement and providing systemic change within the state, and beyond. This research and the course development reported here are some results of the LaCEPT initiative.
The size of class sections has been an active area of concern for decades, and no consistent conclusion has yet to be formed. Feldman (1984) conducted a study linking college students evaluation of instructors with the size of the class and found that there was a weak inverse relationship between instructor ratings and class size. Toby (1988), in a similar study, found that when classes become very large (> 100) instructors lose their effectiveness even though they are effective with smaller classes. Watkins (1990) studied 20,000 student ratings from the University of Canterbury (England) and found a positive relationship between, among other factors, smaller class size and higher ratings for instructors. In a longitudinal study conducted by Knight (1991) at Kent State University it was found that students enrolled in remedial college classes where the class size was 15 or fewer students performed at a significantly higher level when they were enrolled in the subsequent regular class than did students who had been in larger remedial classes. These types of positive effects have not, however, been consistent. Williams, David, et. al. (1984) studied class size as it effects achievement tests for 318 sections of students at Brigham Young University. Section size varied from a low of 13 to a high of 1,008. The results of the Williams' investigation suggest that class size is not a significant factor in student achievement as measured by achievement tests. However, the fact that the Williams study relied only on standard achievement tests may obscure other factors of performance and attitude which may be effected by class size.

Class size studies below the college level have, perhaps, shed more light on the issue. The Educational Service District 189 in Washington (1987) produced a research brief based on then recent publications on class size and found that smaller classes at the lower grade levels produce increased performance, attitudes, and behavior. The study goes on to note that 20 students in a class was necessary to produce these effects. Finn (1990)
completed a three year longitudinal study of Tennessee elementary school mathematics and reading students and found that smaller classes provided for increased performance in both mathematics and reading. It may be that children are more in the formative stage during the early years and because of this the effect of size is more pronounced. It also may be that there needs to be some research which truly uses small sections (N = 20) at the collegiate level.

The popular literature has been replete with stories on how gender effects student interest and performance in mathematics. Research studies have provided some interesting perspectives on the problem. Sax (1994) demonstrated that females showed less self confidence in mathematics than did males, except when females were in a largely female environment, in which case their self confidence improved. Surprisingly, that same study showed that female interaction with mathematics professors frequently lowered their self confidence. Another study dealing with higher ability students (Gallagher and De Lisi, 1994) also concluded that females had a poorer self confidence than do males, but also found that females actually were superior in solving conventional problems while males were superior in solving those which were unconventional or required an unconventional approach to solution of a problem.

Elizabeth Fennema and Laurie Hart (1994) reviewed twenty five years of the Journal for Research in Mathematics Education to synthesize what had been investigated with respect to gender differences. The investigators noted that there were differences based on the type of student group or school studied. They also suggest that the gender gap is narrowing. Another study (Thorndike-Christ, 1991) studied 722 male and 794 female students at the secondary level and found that female attitudes toward and self confidence in mathematics was higher than had been expected. A third study involving over
15,000 students (Coladuci and Lancaster, 1989) showed that by the senior year of high school gender had little or no effect on mathematics achievement. It would seem that although there may be gender differences early in academic life, that those differences may be significantly reduced by the time students reach the university level.

**HYPOTHESES**

Several hypotheses are being tested:

(1) There will be no significant difference in growth in mathematics understanding between students in the control group (MATH 105, Algebra for College Students) and the experimental group (MATH 107, Integrated Collegiate Mathematics).

(2) There will be no significant difference in attitudes toward mathematics between students in the control group and the experimental group.

(3) There will be no significant difference in growth of mathematics understanding and skill between the control group two (large classes) and the experimental group two (small classes).

(4) There will be no significant difference in student attitude toward mathematics between the control group two (large classes) and the experimental group (small classes).

(5) There will be no significant difference between growth in mathematical understanding based on the gender of the student.
(6) There will be no significant difference in attitudinal changes toward mathematics based on the gender of the student.

(7) There will be a positive correlation between mathematics attitudinal measures and mathematics performance measures.

DEFINITIONS AND DESCRIPTIONS

1) **MATH 105** is defined as the standard Algebra for College Students course whereby students learn what is commonly taught in a second year high school algebra course with content augmentation in the area of functions. Methodologies in this course are primarily traditional lecture.

2) **MATH 107** is defined as an innovative course in Integrated Collegiate Mathematics designed for freshmen college students. The course places mathematics into the context of everyday life and uses concepts from several disciplines within mathematics including algebra, geometry, statistics, probability, and data analysis. Inherent within the teaching of MATH 107 is the use of reform strategies such as extensive "real world" problem work and cooperative learning. The concept is that mathematics is not something you learn, but something you do (Clark and Lopez, 1994).

3) **Large class**, for purposes of this study, is defined as a section of either MATH 105 or MATH 107 with a maximum enrollment of 45 students. This represents the normal load in such classes at the university.
4) Small class, for purposes of this study, is defined as a section of MATH 105 or MATH 107 with a maximum enrollment of 35 students. It is recognized that this class size is not "small" except as it relates to the normal or large class.

5) Control group one is defined as all those students enrolled in MATH 105.

6) Control group two is defined as all those students enrolled in large classes.

7) Experimental group one is defined as all those students enrolled in MATH 107.

8) Experimental group two is defined as all those students enrolled in small classes.

9) Drop/Add period is defined as the first week of class when students are permitted to drop and add various classes or sections of classes.

10) Desirable attitudes towards mathematics were determined by the mathematicians involved in the research. These attitudes include:
   a) Believing that doing mathematics can be an enjoyable experience.
   b) Believing that everyone can learn mathematics.
   c) Believing that mathematics is useful and necessary in everyday life.
   d) Believing that calculators are a valuable tool in mathematics, but that it is still important to understand the mathematics concepts necessary to solve problems.
e) Believing that people can learn mathematics without always having an instructor.

f) Believing that estimations and approximations are valuable mathematical skills and that there may not always be an exact answer to a problem.

RESEARCH METHOD

Population

The initial population consisted of 140 students at the University of Southwestern Louisiana who were eligible to enroll in the first college level mathematics course. Eligibility was determined by students receiving either a score of 20 on the mathematics portion of the American College Test (ACT) or a grade of "C" in MATH 92, a remedial course designed for students who did not have the required ACT score.

Students were enrolled in the standard MATH 105 course or the experimental MATH 107 which stressed application of a variety of mathematics principles rather than a concentrated course in college level algebra. Preference for enrollment in MATH 107 was given to those persons who were majoring in education, except that science education and mathematics education majors were excluded.

Attitudinal Measurement

To measure student attitudes toward mathematics, and the change in those attitudes, a Likert-type scale was developed. The mathematicians involved in the development of the new course individually produced a list of...
desired attitudes based on their concept of the spirit of NCTM reform initiatives. A conference was then held involving the mathematicians and the professional educator for the purpose of developing a refined and delineated list of attitudinal attributes on which all could agree. A twenty-three item Likert-type scale was then constructed, twenty items of which related to the agreed upon attitudinal attributes with the other three items being of internal use and not related to attitude. The first draft of the instrument was then reviewed by the mathematicians which resulted in the modification of one item. Based on the final review, the instrument was considered to have face validity.

The inventory was administered to all 140 students involved in the study on the first day of class after the drop/add period, and again shortly before the final examination. For purposes of determining instrument reliability the first administration was scored on a split test format. A correlation was run for half test reliability with the results corrected for full test reliability using the Spearman-Brown prophecy formula as suggested by Mueller (1986). The resulting correlation of 0.815 was interpreted to mean the instrument was reliable. As an additional test the split test correlation was conducted using the Fisher r to z procedure to convert correlation to P - values as described by Sager (1992) with the result that the test was reliable (P < .0001).

To test the various hypotheses relating to attitudinal change the changes in scores from pre to post test were used. In determining significance within the control and experimental groups a two tailed paired t-test was used. To determine the significance of change between groups a two tailed unpaired t-test was employed.
control and experimental groups a two tailed paired t-test was used. To
determine the significance of change between groups a two tailed unpaired t-
test was employed.

Cognitive Growth

Separate tests were constructed for the experimental and control groups. However, although the actual content of each test was different, each test was structured based on NCTM standards. Further, the fact that each test measured the objectives of the appropriate course comparisons are valid. Each test contained real world word problems and opportunities for written responses on the process involved in problem solving. The pre-test was administered during the second class session after the drop/add period. The same test was again administrated shortly before the final examination.

While one may logically assume that there should be a significant growth in mathematical knowledge and skills within each group, a two tailed paired t-test was conducted to verify or refute the assumption. To ascertain if there was a significant difference between the experimental and control groups a two tailed unpaired t-test was used. The results of these tests were used to test the various hypotheses relative to cognitive growth.

Relationship Between Attitude and Performance

To ascertain if student changes in attitude and cognitive knowledge were related a correlation calculation was made.
ASSUMPTIONS

1) Because the requirements for entrance into either MATH 105 or MATH 107 are identical, it is assumed that all students have the capability to learn the material.

2) It is assumed that the mathematics faculty assigned to teaching the sections are all competent. The faculty members were selected based on involvement with the reform movement and having excellent student evaluations of instructor performance. Only full time members of the Department of Mathematics were involved.

Limitations

The project was limited to the 140 students who enrolled in the two sections of MATH 107 and the two selected sections of MATH 105 which were conducted during the fall, 1994 semester at the University of Southwestern Louisiana.

RESULTS

Hypothesis One

Of the 140 students originally enrolled in the two courses only 71 took both the pre and post tests. Of that total 28 students were in MATH 105 and 43 were enrolled in MATH 107. The difference in population is accounted for by (1) a number of students failed to complete the course and (2) a number of students were not present for both of the class sessions when the tests were administered.
Table one, below, shows that MATH 107 students had initial scores and ending scores which were higher than those students enrolled in MATH 105. Further, the mean gain in scores for MATH 105 was lower than for MATH 107 (21.179 vs. 25.814). However, while student cognitive growth was significant within each group (p < .0001) the difference between groups was not statistically significant (p = .1723).

Table One
Cognitive Gains by Course

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>28</td>
<td>17.107</td>
<td>38.286</td>
<td>21.179</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>107</td>
<td>43</td>
<td>29.070</td>
<td>54.884</td>
<td>25.814</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>105/07</td>
<td>71</td>
<td>Mean Difference.................4.635</td>
<td>.1723</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on these data, null hypothesis number one is accepted in that there is no statistically significant difference in growth in mathematics understanding between the control and experimental groups.

Hypothesis Two

Of the original 140 students there were 72 who were present for both the pre and post administrations of the attitudinal instrument. Of this number 29 were enrolled in MATH 105 and 43 were enrolled in MATH 107. As was the case with the content assessment, the numerical difference is accounted for by student attendance and course drops.

Table two, on the following page, shows that there was no significant change in attitude for students enrolled in MATH 105 (P = .2967) while those
enrolled in MATH 107 shows a significant positive change (P < .0001). A comparison of groups shows no significant difference (P = .1562). However, because the unpaired

Table Two
Attitudinal Changes by Course

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>29</td>
<td>59.082</td>
<td>60.979</td>
<td>1.897</td>
<td>.2967</td>
</tr>
<tr>
<td>107</td>
<td>46</td>
<td>58.512</td>
<td>63.093</td>
<td>4.581</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>105/107</td>
<td>72</td>
<td>Mean Difference</td>
<td>2.685</td>
<td>.1562</td>
<td></td>
</tr>
</tbody>
</table>

T-test results indicated a lack of significance, while the paired t-test showed such dramatic differences it was decided to conduct an F test as a means of analyzing the significance of the variances. When the groups were compared using the F test it was found that there was significance at the .0016 level.

Based on these data the null hypothesis is rejected in that there is a statistically significant difference in positive change in attitude toward mathematics.

Hypothesis Three

Of the original 140 students 87 were enrolled in large sections for an average of 43.5 students per section while 53 were enrolled in small sections having an average of 26.5 per section. Of this number 47 large section students and 25 small section students were present for the administration of both the pre and post cognitive instruments. Table three, on the following page, presents data that indicate each group made significant gains (P < .0001).
comparing the two groups there was no evidence of a significant difference ($P = .5439$).

**Table Three**

*Cognitive Growth by Class Size*

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>47</td>
<td>22.304</td>
<td>45.543</td>
<td>23.239</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Small</td>
<td>25</td>
<td>28.120</td>
<td>53.480</td>
<td>25.360</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Large/Small</td>
<td>72</td>
<td>Mean Difference..................2.121</td>
<td>.5439</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the data collected, the null hypothesis must be accepted in that there is no significant difference in cognitive growth between the small and large classes, regardless of course type.

**Hypothesis Four**

The relationship between class size and attitudinal responses is always of interest. The data indicates that class size does not make a difference. As can be seen in Table Four, on the following page, the pre score on the attitudinal instrument was nearly the same for students enrolled in the larger or smaller sections. Further, while the post test scores were somewhat higher for the smaller sections (64.560 compared to 60.979), there was no significance in the change ($P= .4820$).
Table Four
Attitudinal Changes by Size of Class

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>47</td>
<td>57.958</td>
<td>60.979</td>
<td>3.021</td>
<td>.0106</td>
</tr>
<tr>
<td>Small</td>
<td>24</td>
<td>60.160</td>
<td>64.560</td>
<td>4.400</td>
<td>.0119</td>
</tr>
<tr>
<td>Large/Small</td>
<td>71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis Five

Of those students who completed both pre and post tests in mathematics 52 were female and only 17 were male. In each case there was significant cognitive growth (P < .0001). In comparing growth table five, below, shows that there is no statistical difference between groups (P = .9421).

Table Five
Cognitive Growth by Gender

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>17</td>
<td>21.222</td>
<td>45.000</td>
<td>23.778</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Female</td>
<td>52</td>
<td>25.415</td>
<td>49.472</td>
<td>24.075</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>M/F</td>
<td>71</td>
<td></td>
<td>Mean Difference....</td>
<td>0.279</td>
<td>.9421</td>
</tr>
</tbody>
</table>

The null hypothesis is strongly accepted in that there is no significant difference in performance growth between genders of students.
Hypothesis Six

With respect to changes in attitudes only females exhibited a significant positive shift in attitude ($P = .001$). As can be observed in table six there was a non significant shift by males and the total shift was the same for each group. A comparison between groups shows no significant difference based on change in attitude.

Table Six
Attitudinal Changes by Gender

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN PRE TEST</th>
<th>MEAN POST TEST</th>
<th>MEAN GAIN</th>
<th>P VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>17</td>
<td>58.778</td>
<td>62.278</td>
<td>3.500</td>
<td>.1278</td>
</tr>
<tr>
<td>Female</td>
<td>54</td>
<td>58.704</td>
<td>62.204</td>
<td>3.500</td>
<td>.0010</td>
</tr>
<tr>
<td>M/F</td>
<td>71</td>
<td>Mean Difference</td>
<td>0</td>
<td></td>
<td>---</td>
</tr>
</tbody>
</table>

Again, the null hypothesis is strongly accepted in that there is no significant difference in attitudinal change between male and female students.

Hypothesis Seven

A correlation was conducted to ascertain if there was a relationship between attitudinal change and increased performance in mathematics. The resulting correlation of 0.61 was converted to a probability using the Fisher $r$ to $z$ method with a resulting $P$ value of .6245. Hence the null hypothesis that there is no relationship between attitudinal change and cognitive growth is upheld.
DISCUSSION

Problems

The fact that there was significant attrition of students reduces the effectiveness of this research. The loss of approximately fifty students is, however, not unusual for freshman at this university. None the less, had a larger number of students completed the course, we could have an increased level of confidence in the results.

The "small" class sizes were not really small. Comparing sections with 356 maximum to those with 45 maximum may not provide a sufficient difference in size. Research cited earlier suggested class sizes of 20 to be more realistic in terms of small classes. This size, however, is not realistic for freshman level courses at a research university.

The male / female balance in the study was not as desired. A probable cause for the imbalance was the registration preference given to education majors. This preference was necessary due to the nature of the grant which funded the project. However, since approximately 65% of entering freshman education students are intending to be elementary teachers, it is natural that there be more females than males enrolled in the freshman courses.

Results

The attitudinal change in students enrolled in MATH 107 was highly significant. The change in MATH 105 students was not significant. The statistically significant growth of MATH 107 students compared to MATH 105 students is viewed as an indicator that MATH 107 is most likely superior for cultivating positive attitudes toward mathematics.
The pre-test scores in mathematics for students enrolled in MATH 107 were higher than for those enrolled in MATH 105. The difference can most likely be accounted for by the different types of mathematics tested. The MATH 105 test was all algebra and functions while the MATH 107 was more integrated and contained data analysis. It is noteworthy, however, that although both groups showed statistical gains and the difference was not significant (P = .1723), there is room for encouragement that MATH 107 students did out gain MATH 105 students. Further, the fact that the successful completion rate for MATH 107 was 74.1% compared to 46.1% for MATH 105 suggests that MATH 107 is a preferred option.

The fact that female students showed a significant growth in attitude toward mathematics is encouraging. Male students also showed growth, though not at a statistically significant level. With respect to content mastery, however, the difference between male and female students was almost nonexistent (P = .942).

CONCLUSIONS

Based on this research it can be concluded that MATH 107 is a viable option for freshman students who do not have calculus in their program. The fact that some cognitive growth was superior, though not significantly so, and attitudinal change was statistically superior would suggest that MATH 107 is more effective than MATH 105 for a great many university freshman. This view is further supported by the lower attrition rate and higher grade distribution for students enrolled in MATH 107.
The study does not give any support to the desirability of small classes so long as small is defined as 35 students. This does not preclude, however, using even smaller classes (20 students) and repeating this research.

Finally, it is apparent that gender is not a factor in performance, though it may be in attitude changes. Both male and female students showed the same degree of attitudinal change, yet the small number of males resulted in non significance of the growth. An expanded study with more students would provide an improved data base upon which to form more realistic conclusions.
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