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ABSTRACT

This paper describes mathematical modeling activities from a secondary mathematics teacher education course taken by fourth-year university students. Experiences with mathematical modeling are viewed as important in helping teachers develop a more intuitive understanding of mathematics, generate and evaluate mathematical interpretations, and connect mathematics to real-world and applied situations. A four-step cycle of the modeling process 1s presented, involving: a real problem situation, the formulated problem, the mathematical model, and conclusions and answers to questions generated through analyzing the model. Three concepts were incorporated into the teacher education course: (1) the importance of developing and understanding one's conceptual model of the situation; (2) emphasis on reflective knowledge; and (3) the participants' evaluation of the entire activity identifying their goals and biases and evaluating their own thought processes. In the course, the teachers found examples of relationships in everyday language that can be described mathematically, drew graphs, and compared the relationships to familiar functions as possible models. They then classified functional relationships to build families of functions through sorting activities. For example, teachers analyzed U.S. census data, investigated problems involving compounded interest, and discussed possible connections between a new business's advertising expenditures and their sales, thereby using linear, quadratic, exponential, logarithmic, periodic, rational, and algebraic functions. Problems encountered in teaching the modeling process are discussed. (Contains 15 references.) (JDD)

REFLECTIVE MODELING IN TEACHER EDUCATION

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Paper presented at the Sixth International Conference on Teaching Mathematical Modeling and Applications, Newark, DE, August, 1993.

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Research on mathematics teachers' beliefs indicates that mathematics is presented by many teachers as a "cut and dried" subject in which a single correct number is the object of solving problems (Brown, Cooney, & Jones, 1990). The current reform movement in mathematics education addresses this problem and calls for a shift to a more open view of mathematics (NCTM, 1989). In order to promote this shift, preservice mathematics teachers need experiences with mathematic , which would help them develop a more flexible understanding of mathematics and teaching mathematics. This understanding includes an ability to recognize and interpret mathematics in situations and is promoted by the teachers improving their propensity to and skill at reflecting on mathematical problems and their own learning and thought processes. Modeling activities are important contexts for these experiences. The activities describe-1 in this paper are from a secondary mathematics teacher education course taken by fourth-year university students.* As part of the course, these preservice teachers were engaged in data collection and analysis activities that lead to modeling relationships.

Several mathematical content and pedagogical goals provide the general foundation for the course (Shealy & Cooney, 1992). In addressing mathematical content, we wanted the preservice teachers to develop a more informal, intuitive understanding of mathematics, to be able to construct and organize mathematical ideas and generate and evaluate mathematical interpretations. We wanted the, teachers to be able to connect the mathematics to real-world and applied situations--including connections between different areas of mathematics--and appreciate he historical development of mathematics and the role of mathematics in society. Our pedagogical goals include encouraging the teachers to become active and autonomous learners,

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promoting interaction and cooperative work among the teachers, placing the teachers in new learning situations, and promoting reflection on their experiences.

Modeling Process

Experiences with mathematical modeling of situations are an important means toward the goals described above. D'Ambroso (1989) said that the modeling process is the essence of creative, intelligent activity and gives a broad definition: Modeling is the process one goes through in which "one is faced with a situation in a real context subject to an undefinable number of parameters, some of them even unidentifiable" (p. 23). Most often the modeling process is outlined as a cycle with as few as three steps (Lambert, Steward, Manklelow, & Robson, 1989) or as many as ten (Niss, 1989). Figure 1 illustrates a summary model we used in designing our activities.

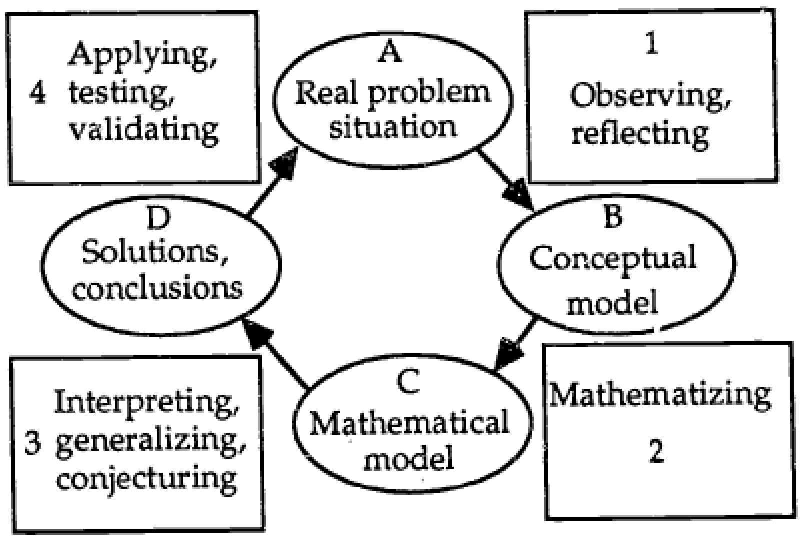


Figure 1. A model of the modeling process.

In Figure 1, A represents the real world situation of interest. *B* involves the formulated problem and the simplified version of the real world. *C* is the mathemmatical model that consists of such mathematical objects as relationships, equations, and patterns. *D* represents conclusions and answers to questions generated through analyzing the model. Then, these conclusions are applied back to the real world situation, *A*, and tested for reasonableness. The boxes 1, 2, 3, and 4 represent the processes between each product.

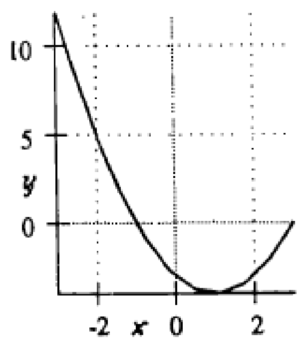
The step from the real world problem to the conceptual problem (A to *B)* often receives little explicit emphasis as some see the complete modeling process as moving from the real world to the model (A to C) (e.g., Moscardini, 1989; Swetz & Hartzler, 1991). Thus, modeling could consist of being familiar with developed models and applying the appropriate one to the situation in question (see the discussion of models versus modeling in Galbraith & Clatworthy, 1990). Lambert et al. (1989), emphasize the importance of viewing this step from a cognitive psychology perspective. They say that in order for the student to make the jump from the real-world to a conceptual model, he or she must have a strong understanding of the real world problem domain and the mathematical domain related to it. Further, their beliefs about the situation will affect how they bring these two domains together to formulate a conceptual model of the situation and how they develop the mathematical model. The step from the conceptual model to the mathematical model *(B* to *C)* is generally referred to as "mathematization." This step involves identifying variables, formulating equations or applying known models, (Moscardini, 1989) and looking for patterns and relationships and tends t(be more technically mathematical in emphasis.

Skovsmose (1989; 1990) emphasizes the importance of the final stage (C to *D)* with what he calls "reflective knowledge." In modeling, Skovsmose says that we need knowledge about mathematics, technological knowledge about the modeling process, and reflective knowledge--"general conceptual framework, or metaknowledge, for discussing the nature of models and the criteria used in their constructions, applications, and evaluations" (p. 767). In particular, he stresses the importance of evaluating our models and conclusions in light of our pre-understanding of the situation and presuppositions, guiding interests that may be present, and being cautious about creating and applying nonexistent objects.

In developing our activities, we placed a great emphasis on the first and last stages. This emphasis led to the incorporation of three important concepts into the activities. First, the importance of developing and understanding one's conceptual model of the situation (Lambert et al., 1989). Second, considering and analyzing one's pre-understanding, goals and interests, and the implications of the model--Skovsmose's (1990) reflective knowledge. In choosing situations, we included those that would stimulate concerns beyond mathematics, thus developing critical attitudes (de Lange, 1987). The teachers develop and analyze a mental model, consider data in light of this intuitive model, formulate and interpret a mathematical model, and reconsider the original real-world situation to validate the model. Finally, the participants evaluate the entire activity identifying their goals and biases and evaluating their own thought processes.

Modeling Activities

To help the teachers begin to recognize mathematical situations and practice developing conceptual models, we have the teachers consider the use of dependent-independent and correlational relationships in everyday language and in the media through vignettes that we provide (in one vignette, a doctor discusses the effect of a person's cholesterol level on their risk of heart disease) and by finding their own examples of relationships that may be described mathematically. The teachers describe the relationships, draw graphs, and compare the relationships to familiar functions as possible models. In the second stage of the course they classify functional relationships to build families of functions through sorting activities (see Figure 2 for an example).

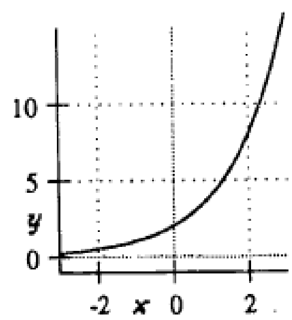


The 1990 census shows that Central City has a population of 40,000 people. Recent studies indicate that over the past 20 years the population has steadily increased at an annual rate of 2%. Social scientists predict Central City will experience this growth rate over the next 20 years. Ms. Callahan has been asked to predict Central City's population for each of the next 15 years.

x	-2	-1	0	1	2
y	1	0	1	4	9

Fred is considering which size pizza is the better buy. He wonders what happens to the area of the circular pizza when the diameter of the pizza is changed.

x	-2	-1	1	2	4
y	1	2	-2	-1	-1/4



Consider the six representations of functions given above. Group them into either two or three piles using whatever criteria you wish. How many different groupings did you identify and what criteria did you use?

Figure 2: Sample card sorting activity.

To build on this intuitive understanding of functional relationships, the students participate in several data collection and analysis activities. In one activity, the teachers investigate the relationship between the diameter of the area that can be seen on a wall through a paper tube and, first, the distance from the wall, second, the length of the tube, and third, the diameter of the tube. A second investigation compares the period of a pendulum to the mass, length, and initial displacement angle of the pendulum. The teachers first describe and discuss the characteristics they expect of the relationships. After collecting the data, the teachers give written descriptions of the relationships, provide tables, and sketch graphs. The teachers then describe the relationships and compare their results to their expectations.

After the data collection activities, the teachers analyze United States census data, investigate problems involving compounded interest, and discuss possible connections between a new business' advertising expenditures and their initial sales. By the end of the investigations the teachers have seen real-world relationships modeled by linear, quadratic, exponential, logarithmic, periodic, rational ($y = \frac{}{x}$), and algebraic ($y = k\sqrt{x}$) functions. We then have the teachers compare and contrast these relationships to strengthen their understanding of the characteristics of different types of functions.

Discussion

Davis (1991), quoting Berlinski, said that "mathematical descriptions tend to drive out all others" (p. 4). Davis followed this quote by saying that "once a mathematical description is in place it is harder to change than moving a grave yard" (p. 4). Research in mathematics teachers' beliefs (Thompson, 1992) provides evidence that teachers have preconceived mathematical ideas that are resistance to change. This difficulty was evident as we worked with the teachers. Consider what happened in one of the investigations (recall that these teachers have a strong university mathematics background).

When the students were analyzing and discussing United States population data and trying to determine types of functions that would model the data. All of the students expected an exponential function to provide the best model. It happens that from 1940 to 1990, a linear function or a logarithmic function is as good or better than an exponential model (see Figure 3). In the discussion that followed, one student strongly argued that any non-exponential trend would not continue and that over the long run we would see a continuation of exponential growth. "Population grows exponentially, that's the way it is." This commitment exemplifies the perseverance of mental models that did not match relationships well and the strength of commitment to prior convictions in light of evidence to the contrary.

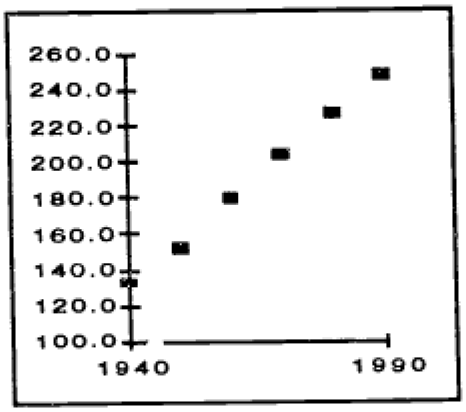


Figure 3. United States population, 1940-1990.

Another problem that arose over the course of the investigations was the tendency for the teachers to give superficial descriptions and expect most relationships to be linear. Also, some of the teachers questioned the value of bringing "non-mathematical" ideas into the mathematics classroom (e.g., issues surrounding the uses of the United States census). Some of the teachers were concerned that using data related to such areas as war and disease might offend or upset some students.

Overall the teachers were positive toward the activities and felt that they grew in their understanding of mathematics and teaching. Our subsequent research provides evidence that the teachers involved in these modeling activities have become more open and flexible in their understanding of mathematics and plan to use modeling activities in their classroom as they begin teaching (Shealy, Arvold, Zheng, & Cooney, 1993). Thus, modeling activities, particularly emphasizing the reflective nature of developing conceptual models and reconsidering the process and interpretation in the last stage of modeling, seem to provide the opportunities teachers need to develop more flexible and powerful understandings of mathematics and teaching mathematics.

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