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ABSTRACT

Graphing calculator use is often thought of in terms of pre-calculus or continuous topics in mathematics. This paper contains examples and activities that demonstrate useful, interesting, and easy ways to use a graphing calculator with discrete topics. Examples are given for each of the following topics: functions, mathematical induction and recursion, graph theory and matrices, and combinatorics and the Binomial Theorem. A list of discrete topics that do not require the programming of the graphing calculator includes also these topics: counting paths, equation solving, factorials, Fibonacci numbers, inequalities, iteration, sequences, and statistics. (MKR)

Teaching Discrete Mathematics with Graphing Calculators

by

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Teaching Discrete Mathematics with Graphing Calculators

Introduction

Graphing calculator use often is thought of in terms of pre-calculus or continuous topics. The following examples and activities, however, demonstrate useful, interesting, and easy ways to use a graphing calculator with discrete topics. As most students have used graphing calculators in other courses, I find that they welcome the opportunity to use them in discrete math.

Since the use of graphing calculators with discrete topics is not yet readily available from other sources, I set out to develop my own materials. With the aid of area secondary math teachers, I designed the following examples and activities to help students learn discrete math, as opposed to teaching them how to use a graphing calculator. Thus, along with explorations, the activities provide practice in discovery and in linking mathematics and technology. They also lead to and emphasize the important connection between studying algorithms and using them; a task made easier and more interesting when using a graphing calculator.

Where a Graphing Calculator is Most Useful

I have grouped the topics and activities by content area and essentially in the order in which they are presented and assigned.

Area 1. **Functions:** graphing functions (absolute value, exponential, linear, quadratic); domain and range; solving inequalities; solving equations; evaluating polynomials.

The activities for this area range from graphing functions and inequalities to building tables. I also include iterative problems such as

Use Horner's method (and iteration) to find P(3), where $P(x) = 3x^5 - 8x^3 + 2x - 17$.

This becomes easy and fast on a calculator. When students develop the solution, they usually end with the screen at the right. That is, they find that P(3) = 502.

```
Ans*X + 0
               57
Ans*X + 2
              173
Ans*X - 17
              502
```

<u>Area 2</u>. **Mathematical Induction and Recursion:** iterations and recursive calculations; generating and graphing sequences and progressions; the Fibonacci numbers; motivating generating terms; exercises that lead to induction.

For this area, I begin with problems such as

Find the value of $a_n = (-3)^{n-1}/n!$ as n grows large.

While such work can be thought of as doing continuous mathematics, the approach clearly is discrete. Thus a graphing calculator speeds up the process, making it routine, and making it easier for students to create patterns. A graphing calculator also makes problems like the following more manageable and much less daunting than pencil and paper work.

Use the function "Int" to rewrite the algorithm for the gcd(a,b) in the form $r = a - b*Int(a/b)$. Use this form of r and iteration to find the gcd(525, 90).

As students work through each step, they obtain the pattern at the right. Since the last non-zero value of r is 15, then 15 is the gcd(525, 90).

525 - 90Int(525/90)	
	75
90 - 75Int(90/75)	
	15
75 - 15Int(75/15)	
	0

I also use graphing calculators and recursive problems to motivate induction. For example, I use problems like:

Let $a_1 = 1$ and $a_n = 2a_{n-1} + 1$ for $n > 1$ and use iteration to develop values of a_n. Next, noting the pattern of answers on your calculator screen, develop and test a a formula for a_n as a function of n.

In terms of closure, such activities obviously lead directly to the notions of generating terms, closed forms, and of course mathematical induction. Based on our calculator explorations we formulate generalizations that we then prove inductively.

<u>Area 3</u>. **Graph Theory and Matrices:** representing graphs and digraphs with 0,1 or incidence matrices; matrix algebra and matrix properties; using matrices to count the number of paths

of given length in a graph.

While the following example can be done with paper and pencil, the use of a graphing calculator makes the work fast and easy. Moreover, the use of a graphing calculator allows me to use and analyze more complicated problems in a routine and efficient manner. For example,

Using $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -2 & 1 & 2 \end{bmatrix}$,

find: $A(BC)$, $(AB)C$, $A(B + C)$, $AB + AC$, $B + C$, $C + B$, BC, CB.
What does your work suggest about associative, distributive and commutative properties for matrix operations?

Obviously, we are looking for the non-commutativity of matrix multiplication, the distributive property, and so on.

A graphing calculator is especially nice for the following counting problem in graphs.

For the given graph, find the number of paths as indicated:

incidence table

X	Y,Z
Y	Z
Z	X

matrix you used

$$A = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

<u># of paths of length:</u>

From	1	2	3	4
X - X				
Y - X				
Z - X				

(Recall that if A is the incidence matrix of a graph, the number of paths of length n from V_i to V_j is the (i,j) entry of A^n.) In this example, the number of paths of length 4 between X and Z is 2 since the (1,3) entry of A^4 is 2.

<u>Area 4</u>. **Combinatorics and the Binomial Theorem:** using nCr to create tables; using the Binomial Theorem; Pascal's triangle; summation notation; approximating $(1 + x)^{1/K}$.

Most of the problems that I use here ask students to discover patterns. For example,

> After calculating the values of $\Sigma C(n,r)$ for $1 \le n \le 6$ and $0 \le r \le n$, what formula would you suggest for $\Sigma C(n,r)$?

Problems used for this area also provide an excellent opportunity to use mathematical induction and to experience connections between discrete and continuous mathematics. For example,

> Given that
> $(1 + x)^r = 1 + rx + r(r-1)x^2/2! + r(r-1)(r-2)x^3/3! + \cdots,$
> for $0 \le r < 1$, use this (and iteration) to find $\sqrt{1.3}$ to 3 decimal places. [Hint: let $x = .3$]

Discrete Topics and Suggestions for Their Use

The following, while not exhaustive, exhibits most of what can be done in discrete mathematics with a graphing calculator <u>without</u>

having to programming it:

Algorithms	Equation Solving	Iteration
Binomial Theorem	Factorials	Matrix Algebra
C(n,r) or nCr	Fibonacci Numbers	Matrix Applications
Counting Paths	Functions	P(n,r) or nPr
Directed Graphs	Generating Terms	Recursion
Domain/Range	Induction	Sequences
Evaluating P(x)	Inequalities	Statistics

Overall, the activities and problems I use are not solely for students in computer science and math, but also for students interested in business, biology, and social studies as well. In fact, while I use such activities extensively in my discrete math classes, I also use some of them in our freshman level survey course in Contemporary Mathematics.

I normally use calculator activities at the start of a topic for motivation: eg., counting techniques, algorithms, induction, recursion, matrix properties, or generating terms. I hand out the work, do similar problems with the class, begin the work, and make completing it an assignment.

I also assign calculator problems at the end of a topic in order to provide experiences that combine concepts: eg., using matrices to count the paths in a graph, using recursive algorithms to evaluate polynomials, or developing tables of values, such as those used with the Binomial Theorem and its applications in probability. I also include calculator problems on my exams. And

students really do say that they "enjoy" them!

Conclusions

Graphing calculator use helps students focus on essential patterns and algorithms in discrete math. This in turn aids them in terms of insight and generalization. In fact, the ease with which students can use graphing calculators makes them an excellent tool for investigating many areas of mathematics.

Using graphing calculators often leads to complex algorithms and the use of software. A useful and student-accessible software package is Kemeny-Kurtz's Discrete Math [1993, West Lebanon, NH 03784-9758 (800 872-2742). Includes truth tables, Venn diagrams, counting algorithms, combinatorics, recursion, sorting, equations, trees and binary trees, graphs, and algorithms.] More advanced students may want to write their own programs to investigate and extend the ideas presented here.

The applications and activities presented here are just the beginning of graphing calculator use in discrete math. The power, ease of use, and the graphics appeal of graphing calculators will continue to motivate students to experiment with discrete topics.

F. E. Masat, 1994

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