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ABSTRACT

The pre-algebra lexicon is a set of classroom exercises designed to teach the technical words and phrases of pre-algebra mathematics, and includes the terms most commonly found in related mathematics courses. The lexicon has three parts, each with its own introduction. The first introduces vocabulary items in three groups forming a learning hierarchy: numbers; operations; and applications. These groups are subdivided into 14 categories for more precise treatment. The second part contains annotations for terms from part one that are found especially interesting or problematic. In some cases, it is urged that a term be eliminated from pre-algebra. In addition, tips on improving classroom instruction are offered. The third part suggests diagnostic assessment techniques to test student mastery of the language of mathematics. In this section, guidance is offered in creation of such diagnostic measures, not yet available commercially. Many sample test items are included. Technical words and phrases are also indexed. (MSE)

PRE-ALGEBRA LEXICON

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PRE-ALGEBRA LEXICON

CONTENTS

PRE-ALGEBRA LEXICON

OVERVIEW

The Pre-Algebra Lexicon (PAL) consists of three parts, each having its own introduction, and contains technical words and phrases of pre-algebra mathematics. By "pre-algebra" we mean a typical course in basic mathematics (sometimes described as "General Math"), whether that be a course spread over several years at the primary level, or a more compact course at the secondary or (developmental) college level. Thus we include a number of words and phrases which one might consider to be arithmetic, geometric, algebraic, set-theoretic, etc. in nature.

The list is intended to be complete in this sense: it includes the terms most frequently found in pre-algebra mathematics courses. However, we exclude the names of individual numbers (except for pi), and we also exclude some of the less-used units of measure (e.g., deciliter, peck). Further, we include only a small number of terms from algebra and geometry (namely, the terms that occur in most pre-algebra courses).

In Part I, the items have been placed into three large groups: Numbers, Operations, and Applications. These groups form a learning hierarchy. The groups are subdivided into 14 categories. These categories make it possible to focus on particular parts of pre-algebra mathematics. For example, under Category 7 (Division) there are 19 items; and we consider these 19 items to be a complete technical vocabulary for the operation of division. The system of categories makes it easier to create diagnostic assessment instruments (cf. Part III).

In Part II, annotations on those terms that have been found to be especially interesting or problematic from the list in Part I are developed. In some cases we urge that a term be eliminated from the vocabulary of pre-algebra. In many cases we give tips on improving classroom instruction.

Part III suggests diagnostic assessment techniques. To improve student achievement, teachers need to assess in detail how much of the language of mathematics each student has mastered. Unfortunately, tests are not yet available for that type of assessment. So in Part III we provide guidance for teachers to create their own diagnostic measures. Many examples of assessment items are included.

An index contains an alphabetical listing of the technical words and phrases of pre-algebra mathematics.

PART I

INTRODUCTION

In PART I of the Pre-Algebra Lexicon, the terms are arranged into the following categories:

NUMBERS

1. Sets of numbers
2. Relationships among numbers
3. Other vocabulary

OPERATIONS

4. Addition
5. Subtraction
6. Multiplication
7. Division
8. Powers and roots
9. Other

APPLICATIONS

10. Expressions and parts of expressions
11. Measurement
12. Geometry
13. Algebra and set theory
14. Comparatives

Our rationale for these categories is as follows. The fundament of arithmetic is numbers (Categories 1 to 3). After learning names of individual numbers (not included in this lexicon), the student must learn the names of important types of numbers (*e.g.*, PRIME NUMBER), important relationships between numbers (*e.g.*, MULTIPLE OF), and other vocabulary relating to parts of numbers (*e.g.*, DENOMINATOR).

Once students have mastered some of the vocabulary relating to numbers, they learn operations (Categories 4 to 9), and each operation involves a short list of words. So the building blocks are numbers and operations. Then the student is ready to move into applications.

As students learn numbers and operations, they begin writing expressions (collections of symbols) of greater and greater complexity, and these expressions and their parts require further vocabulary (*e.g.*, PARENTHESES, EQUATION) - see Category 10. Proficiency in writing and manipulating these expressions is a prerequisite for problem solving. But problem solving also requires that students learn some of the vocabulary associated with measurement (Category 11), geometry (Category 12), and algebra and set theory (Category 13). Finally, some problems include a special type of phrase called a comparative (*e.g.*, AS LARGE AS) - see Category 14.

There are three reasons for arranging the words into categories. First, we think teachers will find it enlightening to see how the words (for the most part) fall easily into closely related groups. Second, we wish teachers to take hope when they see that the amount of language to be mastered is really quite limited -- *e.g.*, twelve words (or fewer) pertaining to subtraction. Third, although a single diagnostic test based on all the terms in the lexicon would be far too time-consuming, several short diagnostic tests, each based on a few categories, are entirely feasible. Diagnostic assessment techniques are discussed in PART III.

Note that we have left some blank space at the end of each category. We encourage you to write down other words and phrases to which, in your experience, attention should be paid.

NUMBERS

1. SETS OF NUMBERS

complex fraction
composite number
counting number
decimal
decimal fraction
even number
fraction
improper fraction
integer
mixed number
natural number
negative number
non-terminating decimal
odd number
pi
positive number
prime number
proper fraction
repeating decimal
sign
terminating decimal
whole number

2. RELATIONSHIPS AMONG NUMBERS

absolute value
additive inverse
approximately
as . . . as
between
common denominator
common factor
common multiple
consecutive
conversion (decimal/fraction/percent)
convert
cross multiply
cross products
different
divisible by
equals
equivalent
estimation
exceeds
greater than
greater than or equal to
greatest common divisor
greatest common factor
is
least (lowest) common denominator
least (lowest) common multiple
less than
less than or equal to
lowest terms

(continued)

multiple of
multiplicative inverse
negative of
not equal to
opposite of
prime factorization
rationalize
reciprocal
reduce
regroup
rename (2 5/8 = 1 13/8)
same
terms (to higher or lower terms)
transform
twice as large as

3. OTHER VOCABULARY

affix
annex
connoted, connotation
denominator
denoted, denotation
digit
expanded notation
factor (noun)
for example
numerator
operation
over
pair
place value
such that
that is

OPERATIONS

4. ADDITION

add
addend
addition
altogether
and (= added to)
carry
horizontal (arrangement of numbers to be added)
in all
increased by
more than
plus (= add)
regroup
sum
together
vertical (arrangement of numbers to be added)

5. SUBTRACTION

borrow
decreased by
difference
diminished by
left
less (= subtract)
minuend
minus (= subtract)
regroup
subtract (from)
subtraction
subtrahend
take away

9

6. MULTIPLICATION

all together
cancel
factor
in all
increased by a factor of
multiplicand
multiplication
multiplication table
multiplier
multiply
of
partial product
product
times
together

7. DIVISION

borrow
divide by
divide equally among
divide into
divided by
divided into
dividend
divisible by
division
divisor
into
invert
long division
over (as in "3 over 4")
per
quotient
ratio
reciprocal
regroup
remainder

8. POWERS AND ROOTS

base
cube
cube root
cubed
exponent
exponentiation
power
radical
radicand
raising to a power
square (= 2nd power)
square root
squared

9. OTHER

answer
arrange
associative property
average
checking
commutative property
compute
distributive property
estimate
identity element
inverse
mean (= average)
order of operations
percent
percentage
plus or minus
property
round (a number)
rule
simplify
substitute (a number for a variable)

10. EXPRESSIONS AND PARTS OF EXPRESSIONS

and (for reading a decimal point)
braces
brackets
comma
compound statements
constant
decimal point
denoted
equal sign
equation
expression
extremes (in a proportion)
formula
grouping symbols
inequality
means (in a proportion)
notation
open sentence
parentheses
proportion
radical
term (in an expression)
variable

11. MEASUREMENT

A.M.
acre
age
approximate number
area
bill (currency)
block
Celsius
cent
centimeter
change (currency)
conversion (metric, English)
cubic ______ (unit of measure)
cup
degree (of temperature)
depth
dime
distance
dollar
English (system)
enough
Fahrenheit
foot
gallon
gram
half dollar
half past
hectare
height
hour
how many
how much
inch
kilogram
kilometer
length
liter
meter
metric (system)
mile
milligram
milliliter
millimeter
minute
nickel
ounce
P.M.
penny
pint
pound
precision
quart
quarter (currency)
quarter after (past)
quarter to (of, until)
rate
second
significant digits
speed
square ______ (unit of measurement)
time
ton
volume
weight
width
yard

12. GEOMETRY

acute angle
adjacent angle
circle
circumference
coordinates
corresponding parts
cube
cylinder
diameter
diagonal
edge
equilateral triangle
graph
hypotenuse
isosceles triangle
legs (of a right triangle)
number line
obtuse angle
parallel
parallelogram
perimeter
perpendicular
Pythagorean Theorem
quadrilateral
radius
rectangle
right angle
right triangle
sides (of a polygon)
sphere
square
triangle
vertex
vertices

13. ALGEBRA AND SET THEORY

- both
- determine
- empty set
- evaluate an expression
- intersection
- like terms
- literal factor
- member(s) (of a set)
- null set
- one-to-one correspondence
- root (of an equation)
- set
- sides (of an equation)
- solution (of an equation)
- solve
- subset
- term
- union
- universal
- unknown

14. COMPARATIVES

A) WITH SPECIFIC REFERENCE

- farther (& further) vs. nearer (distance)
- faster vs. slower (speed)
- heavier vs. lighter (weight)
- older vs. younger (age)
- taller (& longer) vs. shorter (length)

B) WITH GENERAL REFERENCE

- as . . . as
- greater (& more) vs. less (& fewer)
- larger (& bigger) vs. smaller
- than

PART II

INTRODUCTION

From a literary point of view, the richness of the English language is one of its glories. From a technical point of view, that richness can be a source of ambiguity and is usually a burden to those for whom English is a second language. For example, a teacher in the United States may say the symbol "5^2" in any of three different ways ("5 squared" or "5 to the second power" or "5 to the second"). Students in US schools must therefore learn multiple readings for the same symbol.

Teachers of math are hardly in a position to reform the English language. However, if math teachers and language teachers act in concert, they can improve the way that teachers and students communicate with each other. If teachers communicate better, students are likely to learn better. If students communicate better, teachers can assess their progress more accurately. As we are reminded in NCTM's Curriculum and Evaluation Standards for School Mathematics (1989, p. 78), "The ability to ...communicate about problems will develop and deepen students' understanding of mathematics." In the annotations below, we make some suggestions about such improvements.

In these annotations we speak primarily to math and language teachers. We offer them some very succinct analyses of various words, together with some ideas about instructional strategies. If the annotations prove helpful in the classroom, they will serve their purpose.

Our suggestions that certain words be eliminated from the language of mathematics are based on two general principles. The first is Occam's razor: "If it is a hindrance rather than a help, cut it out!" One application of this principle concerns generic vs. specific terms. We believe that one should use a generic term unless a specific term has a specific mathematical content and brings clear benefits to the user. Thus we prefer to CHANGE a fraction to lowest common terms rather than to CONVERT it (see CONVERT, p. 21).

A second general principle concerns the distinction between a technical and a descriptive term. In the sentence,"We went to a wonderful party last night." we recognize that WONDERFUL is a descriptive term. We don't ask for a definition of it, though we might, as a matter of curiosity, ask for more details. On the other hand, in the sentence,"While you are at the store, buy me a duplex outlet," we recognize that DUPLEX OUTLET is a technical term, and that we had better know its definition or we may buy the wrong thing. This distinction - far from airtight - is useful in examining the language of pre-algebra mathematics.

A technical term has several properties. I has a precise definition. Once created and generally accepted, its usage tends to be stable over a considerable period of time. And it has few or no synonyms. A descriptive term, on the other hand, may achieve precise communication, but it tends to have a shorter lifespan and to have quite a few equally good synonyms. Here are two examples. PARALLEL is precise; it has been used for several centuries, and it is hard to think of any single word that can be called a synonym (true, "non-intersecting" comes close). Now consider BORROW, as in, "We want to subtract 8 from 43; but 8 units is larger than 3 units; so we BORROW one of the tens..." Once the procedure is clearly explained, BORROW has precise content. But the word has not been used for very long--before World War I, textbooks told students to "take one of the tens," not to borrow it. Most important is that many different words could be chosen as the name of the procedure: TAKE one of the tens, LOP OFF one of the tens, PUSH one of the tens, etc. Some books use REGROUP instead of BORROW. In short, teachers should not feel compelled to use, nor compel students to use, a descriptive term that is easily replaced by other descriptive terms. If you want to use BORROW, fine; but if a student does not understand BORROW, try using synonyms until you find one that the student does understand. In some cases, however, we judge that certain descriptive terms are inappropriate and we advocate their elimination (*e.g.*, cf. ANNEX).

Readers will of course have different opinions about the questions raised in the annotations. The authors hope that these differences will lead to discussion and debate--and ultimately to improvements in math instruction.

ANNOTATIONS

1. ACRE, HECTARE

Many students, especially in urban areas, have no sense of how large an acre or a hectare is. Perhaps the best thing you can do for your students is to give them a visual image: an acre is about the size of a football field. Or more precisely, make the field 90 yards long instead of 100; and that's quite close to an acre. A hectare is about two and one-fourth football fields. If you want students to do computational problems, give them the formula connecting acres and square feet so that students need not memorize it. For example:

> A new housing subdivision advertises rectangular quarter-acre lots. (Each lot is one-fourth of an acre). An acre is 43,560 sq. ft. If the front of the lot measures 120 ft., how deep is the lot?

2. ACUTE ANGLE

ACUTE comes from the Latin word, *Acuere*, meaning "to sharpen to a point." An acute angle looks like a knife point. The word *Cute* used to mean something sharp or knifelike, *e.g.*, a very fine distinction, a tightly reasoned argument, a clever strategem. Only in the mid 20th century did it take on the meaning of physically attractive.

3. ADDEND, etc.

Over the centuries, math people have given names to the numbers that are operated on. You have your dividend, your quotient, your divisor, etc. Those who know Latin may well enjoy some of these words because they are so appropriate: dividend (based on the Latin gerundive) = that which is to be divided. Unfortunately, few students (not to mention teachers) know any Latin; so few people enjoy the appropriateness of some of these terms. The question arises whether all these terms are needed. The tendency in recent decades has been to de-emphasize and eventually abandon all those gerundive words (ADDEND, DIVIDEND, MINUEND, MULTIPLICAND, RADICAND, and SUBTRAHEND). The other names (BASE, DIFFERENCE, DIVISOR, EXPONENT, MULTIPLIER, PRODUCT, QUOTIENT, REMAINDER, and SUM) are easier for most people to say, to spell, and to remember.

You may be surprised to know that math terminology, which has the appearance of approximating Divine Immutability, does change over time. For example, you will find that 19th and early 20th century textbooks describe the answer to a subtraction problem as a "REMAINDER or difference". Note that REMAINDER is listed first and emphasized. By the mid 20th century, American textbook writers had abandoned the word REMAINDER (except for the last step in a division problem) in favor of DIFFERENCE.

4. ADDITIVE INVERSE, NEGATIVE OF

To comprehend the concept of INVERSE, one must go through three levels of understanding. At the first or factual level, one notices that 3 and -3 add up to zero, 4 and -4 add up to zero, and that every real number is matched with another (called its additive inverse, its opposite, or its negative) such that the two numbers add up to zero. One also notices that every real number except zero is also matched with another (called its reciprocal or multiplicative inverse--cf. MULTIPLICATIVE INVERSE) such that their product is one. At the second level one sees that if there is an identity element (like zero and one) for any binary operation on real numbers (binary means that you operate on two numbers), then one may inquire whether each real number has an inverse for that operation. At the third level one realizes that questions on identity elements and inverses can and should be posed about any set on which a binary operation has been defined.

In courses that precede algebra, students are normally at the first level of understanding (at best!). Given two numbers that are inverses, the most important thing at this level is fast and accurate computation. The student who is confronted with (3/4) (4/3) and who reaches for a calculator is not in good shape. Mastery of computation and of terminology should go hand in hand.

5. ALL TOGETHER, TOGETHER, IN ALL

"Susie has four skirts, Ann has six, and Marie has twice as many as Susie. How many skirts do they have all together?" ESL students may have more trouble with ALL TOGETHER than with the calculations. It may help to give students a visual image. The terms ALL TOGETHER and TOGETHER come from the same root as GATHER. ALL TOGETHER means that you GATHER all the skirts into one pile. Mathematically, you are ADDING; or, from a set-theoretic point of view, you are finding the number of the UNION of three sets.

Some problems containing ALL TOGETHER concern rectangular arrays of objects. *e.g.*, "A classroom contains seven rows of desks, and each row has three desks. How many desks are there all together?" This problem is essentially the same type as the problem about skirts. In the skirt problem we had a pile of four, a pile of six, and a pile of eight. In the desk problem we have one row of three, a second row of three, etc. In both cases we add the numbers for all the piles (or rows). The only difference is that in the desk problem, it is more efficient to multiply, since multiplication (in this context) is repeated addition and is faster. Note that the switch from addition to multiplication is merely for the sake of efficiency.

6. ANNEX, AFFIX

In the 19th century one of the hot topics of conversation was whether or not the US should annex this or that piece of territory. Perhaps that is what is behind the curious phrase (still found in some pre-algebra books) to ANNEX zeros. There are indeed situations in which writing some zeros at the end of a decimal number is a good idea. However, there is no need for spuriously technical terms such as ANNEX and AFFIX. Just WRITE the zeros.

7. APPROXIMATE NUMBER

APPROXIMATE NUMBER is a perfect example of how mathematical language, which we like to think of as utterly pure, can be misleading. In the English language, nouns denote sets; and an adjective modifying a noun selects a subset. Thus EYES denotes a set of objects; GREEN EYES denotes a subset of the set of EYES. So the phrase APPROXIMATE NUMBER, by its very structure, suggests that there is a set of objects called numbers; and that a subset of them have the special quality of being APPROXIMATE. Nothing could be further from the truth. Each number is precisely itself: numbers do not have identity crises. In the sentence, "Its length is approximately ten inches," note that the adverb APPROXIMATELY modifies IS, not TEN. There is nothing intrinsically approximate about the number ten. The question raised by the word APPROXIMATELY is whether ten is the best choice for describing the length of the object. That question has to be answered by a discussion of the nature of measurement, the purpose of this measurement, and the accuracy of the measuring instrument. In other words, APPROXIMATENESS is not a property of numbers but of the application of numbers. In short, one should not use the phrase APPROXIMATE NUMBER. Rather, one should discuss the appropriateness of

using this or that number in a particular situation. One would say things like, "According to this ruler, the length of the pencil is about six and three-quarters inches," or "These calipers tells us that the length of the rod is approximately 2.3 centimeters."

8. AS . . . AS

Textbook authors assume that a sentence such as "Jill is as old as John" will immediately be translated in the student's head into "Jill's age is equal to John's age." Would that it were so! Students with limited English language skills will need help with this. Usually, the structure "AS . . . AS" means that two things are equal with respect to whatever category comes between the two AS's; or that someone wishes for equality ("If only I were as thin as you.") Occasionally, however, the phrase has a somewhat different meaning. *e.g.*,

--- As strong as you are, you will not be able to...

--- ... as long as I have you...

Also note that NOT AS . . . AS is a denial of equality. The construction tells us that two things are not equal, but gives no further information about how they are related.

9. ASSOCIATIVE, COMMUTATIVE, DISTRIBUTIVE PROPERTIES

Many pre-algebra books take pains to define these three properties. One wonders if it is worth the trouble. We believe that students absorb a knowledge of the first two of these properties from their experience in math; and that until they get to matrix multiplication or abstract algebra, they profit little from memorizing these names. On the other hand, if you like to have students practice mental calculation, then the DISTRIBUTIVE property will be helpful: *e. g.*, $9\ (21) = 9\ (20 + 1) = 180 + 9 = 189$. Note: in some states (*e.g.*, Florida) you <u>must</u> teach these names because they are used on statewide tests.

10. AVERAGE, MEAN

Our impression is that pre-algebra books for children (and also for older students) do not treat the concept of average very creatively. Half of the exercises seem to be about a student's average test score. Since many students do not like to think about tests (real or fictitious), these questions are not much fun. We urge teachers to dream up some applications that are more imaginative. We

also urge you to find applications that are more thought provoking - that is, they lead the student to think more deeply about the concept of AVERAGE. For example, in the first suggestion below, students can be encouraged to think about how much variation from the average is possible (Could the lake be 50 ft. deep in places?). In the last suggestion below, students can discuss in what ways a day (or a person) might be described as average. Here are some suggestions:

-- The average depth of a certain lake is 2.5 ft. If you are a poor swimmer, would you feel comfortable about swimming in that lake?

-- Billy went for a walk in a spooky forest. His average speed was 4 mph. What was the fastest that he moved? What was the slowest? Did he ever walk exactly 4 mph?

-- A roller coaster averages 50 mph on the 2 miles of straightaway, 60 mph on the 1/4 mile steep hill, and 45 mph on the 1 mile of curves. What is the average speed for the entire ride?

-- "On an average day, our letter carrier leaves us ten pieces of mail." What is an average day? Is there such a thing as an average letter carrier?

11. BETWEEN

The word BETWEEN provides an occasion to explain to students the difference between everyday language and mathematical language. In everyday language we can say on Monday that we are between the devil and the deep blue sea, implying that we are not yet in the grip of either; and on Tuesday we can say that the temperature that day was between 52 and 70, implying that it was 52 at one point and 70 at another point. In short, in everyday language we don't bother to say whether the boundary markers are to be included or not--either we don't care or we tell from context. In mathematical language, on the other hand, we try to be as explicit and accurate as possible (though sometimes we fail--cf. APPROXIMATE NUMBER). Unfortunately, there are no words in everyday English that will help us make the distinction in question without tiresome circumlocution. So we can write " $3 < x < 7$ ", for example, with perfect clarity; but when we say it, we are forced to say "x is greater than 3 but less than 7" or "x is between 3 and 7" and hope that the written symbol conveys our meaning precisely. Tragedy? No. Something to think about? Yes.

12. CANCEL

The good thing about CANCEL is its brevity. "In 3x/3y, you cancel the 3's." The bad thing about CANCEL is that students find endless ways to misuse the idea (see Blubaugh, "Why Cancel?," in *Mathematics Teacher*, April 1988, pp.300-302). Especially in complicated fractions and in rational expressions, such as

$$\frac{5+3-7}{2+3+5} \quad \text{and} \quad \frac{5+3x+y}{2+3x+t},$$

many students will happily cross out every 3 in sight. We suggest that the price of brevity is too high. If CANCEL were eliminated from mathematical language, we think that other more suitable language would not be hard to find. For example, instead of saying:

"In $3x + y - 3x$, the $3x$ and $-3x$ cancel, so you end up with y,"

one might say:

"In $3x + y - 3x$, the $3x$ and $-3x$ add up to zero, and $0 + y$ equals y."

In the same way, one might say:

$\frac{3x}{3y}$ equals $\frac{3}{3}\frac{x}{y}$, which equals $\frac{x}{y}$.

To say, further, that 3/3 equals 1, and that 1 is the identity element for multiplication, is a good idea initially; but after the students catch on, you don't need to say that every time. Remember that the eighth cardinal sin is to be boring. (See also the annotation on LOWEST TERMS.)

13. COMMON DENOMINATOR, COMMON FACTOR, COMMON MULTIPLE

As an exercise in language analysis, ask your students to talk about the word COMMON--to give examples of the ways in which it is used and to express the similarity that runs through the examples. Some examples might be: (1) What Joe and Jill have in common are blue eyes and blond hair; (2) The House of Commons; (3) Walking across the commons; (4) Common Law; and

(5) Common sense. The similarity, of course, is that various individuals share in some one thing. Thus two fractions have a COMMON DENOMINATOR when their denominators are the same number. When we say that two numbers have a COMMON FACTOR, we mean that the same number divides them both evenly. A COMMON MULTIPLE of two numbers is a single number that they both divide evenly.

14. COMPARATIVES

Students need to be told that some comparatives refer to a specific situation--*e.g.*, OLDER THAN refers to age and to nothing else. Other comparatives (*e.g.*, LARGER THAN) are used in a variety of contexts. Unfortunately, (for ESL students), these more general comparatives are not universal in their reference: There are definite patterns of usage. Consider these examples:

He scored MORE points than José.
She got a BIGGER piece of cake than Saba.
Her share will be LARGER because her investment is larger.
He's BIGGER than Maria.
The GREATER the setback, the greater your opportunity.

In some of these examples you cannot substitute one comparative for another without sounding off key. Finally, at the most abstract level (*e.g.*, in $x + 5 > 3$), mathematicians prefer to use consistently the terms GREATER THAN and LESS THAN. One might argue (with some merit) that GREATER THAN is not more precise or appropriate than LARGER THAN. But such is life.

Finally, remember that THAN and THEN are easy to confuse. Remind students that THAN is part of the comparative structure, whereas THEN refers to a sequence (in time, or in logic).

If you find that some of your students confuse THAN and THEN, you can easily make up exercises in which they must insert the correct word. For example:

"I am faster . . . . you."
"I will go first and . . . . you will follow."
"If you are right, ... I am wrong."
"Don't lock the door if you get home earlier . . . . me."

15. CONSTANT

Be careful not to complicate the meaning of CONSTANT. In the 1960's and 1970's some books described constants as symbols whose replacement set contained only one number. Such mind games should be avoided until college or graduate level courses in the foundations of mathematics. In pre-algebra mathematics and algebra, constants are numbers. The only reason for using the term CONSTANT is to distinguish numbers from variables; and we could easily live without it. Incidentally, there are some sloppy speech patterns in mathematics that we have learned to live with. Some algebra books contain sentences like this:

> " A linear function is defined by an equation $y = Ax + B$, where A and B are constants."

Strictly speaking, the symbols A and B are not constants: they are parameters (a species of variable) that stand for numbers and will be replaced by numbers. The expression "$y = Ax + B$" is on a different level of abstraction compared to an individual linear function, such as the one defined by $y = 2x + 3$. The expression containing A and B is not a linear function and does not define a linear function until the A and B have been replaced. Thus a better phrased definition of function would be this: "A linear function is defined by an equation of the form $y = Ax + B$, where A and B stand for constants; for example, $y = 3x + 4$."

16. CONVERT, CONVERSION

Occam's razor implies that you should use a generic term unless a specific term has clear benefits. For example, the specific term INTEGER is beneficial: it enables us to refer easily to an important subset of the real numbers. We see no such benefit in the apparently specific terms CONVERT and CONVERSION, which in fact have no specific mathematical content. Why not just CHANGE fractions to decimals, or CHANGE feet to meters? The words CONVERT and CONVERSION have a long and interesting history in religion; but in mathematics, we think they have no future.

17. COMPLEX FRACTION

Concerning CONVERT we said that one should use a generic term unless there is a clear benefit to using a specific term. That principle applies equally to COMPLEX FRACTION. This term is used to call student attention to fractions, such as:

$$\frac{1/2 + 5}{1/3 + 2}$$

which are rather complicated and could be made simpler. Now, there are many things that can complicate a mathematical expression, and having a fraction as part of a numerator or denominator is only one of them. The fraction above does not need a special name. Just say to students: "This fraction is kind of complicated, so let's make it simpler." Some pre-algebra books have already taken this approach by omitting the term COMPLEX FRACTION.

18. CORRESPONDING PARTS

Imagine that English is your second language and that your math teacher suddenly starts talking about corresponding parts of two triangles. What thoughts would go through your head? "Do the triangles send letters to each other? How does one know which part corresponds to which other part?"

Bright students seem to catch on to the concept of corresponding parts with little guidance. Many students, however, need to be led through a sequence of steps.

a) By CORRESPONDING (a one-to-one correspondence) we mean that each element in one finite set is matched with one element in a second finite set, so that each item in either set has exactly one "mate" with which it is matched. (In this context do not worry about infinite sets.)

b) If each of the two sets contains n members, such matchings can be done in n! ways ($n! = 1 \cdot 2 \cdot 3 \cdot \cdot \cdot n$).

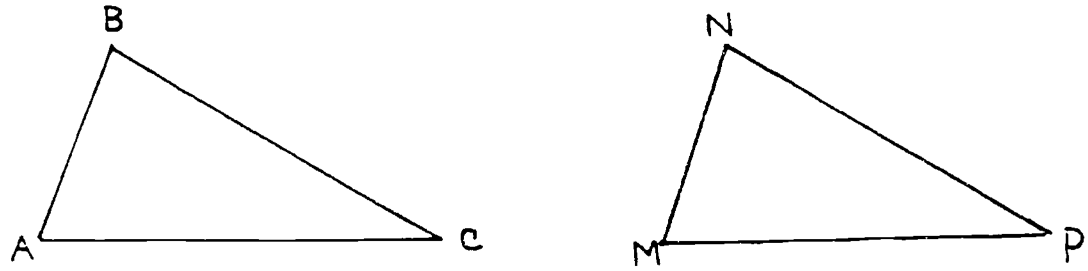
c) If we think of a triangle as a set containing 6 members (3 angles and 3 sides), then we can match it with another triangle in 6! or 720 ways.

d) Suppose we impose these two conditions on the matchings:

   i) angles may be matched only with angles

   ii) when two angles are matched, then the sides opposite these angles must also be matched.

Then two triangles can be matched in only 6 ways:



A with M,	B with N,	C with P
A with M,	B with P,	C with N
A with N,	B with M,	C with P
A with N,	B with P,	C with M
A with P,	B with M,	C with N
A with P,	B with N,	C with M

Once angles have been matched, sides are automatically matched. Note: Each teacher must decide whether to include or omit Steps (b) and (c) in explanations to their students.

In short, the expression, CORRESPONDING PARTS, is not self-explanatory. You need to explain, give examples, and have students practice this concept, just as you would with any other important concept.

19. CROSS MULTIPLY, CROSS PRODUCTS

People who work in a specialty (computer programmers, plumbers, etc.) invariably develop a large arcane vocabulary. That is okay; it enables them to communicate with each other with great brevity. That same vocabulary is deadly for beginners, however, for it hinders rather than helps communication. For some users of math language CROSS MULTIPLY, and CROSS PRODUCTS are efficient terms, but keep these terms away from beginners. Students indeed need

to be shown how to tell when two fractions are equal: $a/b = c/d$, if and only if $ad = bc$; but there is no good reason why students should memorize special terminology for this criterion. Further, too much emphasis on this simple definition can later lead to trouble in algebra where students are apt to assume (incorrectly) that the following equations necessarily have the same solution set:

$$f/g = h/k \quad \text{and} \quad fk = gh$$

when f, g, h, and k are functions.

20. DECIMAL FRACTION

Imagine that you are learning English. Imagine that you now know that a fraction looks like this: 3/4; and that a decimal looks like this: 2.345. And then your teacher starts talking about DECIMAL FRACTIONS! Rampant confusion! (Also called cognitive dissonance.) It may well be excellent pedagogy to say: "You want to change 3/5 into a decimal? Fine. First change it into a fraction whose denominator is 10 or a multiple of 10: namely, 6/10." However, there is no need to call this new form a DECIMAL FRACTION.

21. DENOTED, DENOTATION (CONNOTED, CONNOTATION)

These words, denoted and denotation, refer to a one-to-one correspondence between a concept and a term, or a term and a symbol. For example, every definition says that a certain term (being defined) will denote (stand for) a certain concept. DENOTATION is distinguished from CONNOTATION in that the latter is not one-to-one: a thing may have many connotations. For instance, FIRE has one denotation (per context), but it has almost endless connotations: purification, cleansing, end of the world, pain, forest renewal, romance, guns, etc.

22. DEPTH, HEIGHT

Students need to be warned that DEPTH is used in two quite distinct ways. First, it denotes a vertical measurement. In this sense it differs from HEIGHT only in its perspective. If you imagine yourself looking upward toward the top of a tree, you naturally (if you are a native English speaker) use HEIGHT. If you imagine yourself looking downward to the bottom of a gorge, you naturally select the word DEPTH. The difference is thus one of imagination, not mathematical content. Second, DEPTH is sometimes used for one of the horizontal measurements of a box. In many math problems, one uses the terms length/width/height for the dimensions of a box, and one

does not really care which measurement goes in which direction. In some applied problems, however, the word DEPTH is used for the horizontal measurement that goes away from the observer (and NOT left to right). There is no way to avoid this difficulty, because in real life we need the word DEPTH in this last sense. If a refrigerator is 3 ft. by 2.5 ft., we need to know whether the 3 ft. is parallel to the wall or perpendicular to it. Depth needs to be explained. (Also see the annotation on WIDTH, which contains a suggestion for a classroom exercise.)

23. DIGIT

To define a number system, one needs to specify (a) the set of written symbols (called numerals) and (b) how these symbols are used. In English the numerals in our arabic numbering system are also called DIGITS. Most children find it easy to memorize the list of digits. The hard part--and the thing that prepares them for polynomials in algebra--is mastering the concept of positional notation. Thus in the symbol "33", the two digits are identical, but they have different meanings: the first "3" (from the left) means 3 tens; the second means 3 units.

Mention to your students the connection DIGIT has with related phrases, such as DIGITAL WATCH, DIGITAL COMPUTER, ETC.

24. DIVIDED BY, DIVIDED INTO

In non-commutative operations such as division, we need to identify carefully which number is which. In traditional language, we need to identify which number is the dividend and which is the divisor. However, in the US, we have three different ways of symbolizing division:

$$a/b \qquad a \div b \qquad b\overline{)\,a}$$

In the first two ways we list the dividend first, but in the last way we list the divisor first. In some languages (*e.g.*, French) all three symbolisms are read in exactly the same words, but in English we have two distinct readings: "a DIVIDED BY b "(for the first two), and "b DIVIDED INTO a "(for the third). Teachers who deal with ESL students will recognize these two readings as a familiar source of difficulty. In our view, having two readings provides no benefit and is a stumbling block for many students. We therefore advocate a single reading for all three symbols:

"a DIVIDED BY b".

25. EQUIVALENT

The word EQUIVALENT has been with us for over 500 years; and for more than a century it has appeared in pre-algebra mathematics textbooks, especially in reference to fractions (such as 1/2 and 3/6). Further, we suspect (but cannot prove) that the word's reputation was upgraded in the 1960's and the 1970's, when math teachers who had not been math majors learned from articles and workshops about the formal construction of rational numbers. The article or lecture went like this: the set containing numbers of the form a/b is created; and then an equivalence relation is defined on that set, resulting in equivalence classes; and thus each rational number is actually an equivalence class! EQUIVALENT thus has a long pedigree and also good standing in abstract mathematics. On the other hand, we doubt that a student who never heard the word EQUIVALENT (in a math context) before reaching college could aptly be described as deprived. There's absolutely nothing wrong with saying that two fractions are EQUAL. At the very least, do not spend time on EQUIVALENT with students whose mathematical language is very limited.

Incidentally, some textbooks speak of EQUIVALENT SETS, meaning sets that have the same number of members. This is an unfortunate choice of terms. Students often confuse this with EQUAL SETS (sets having exactly the same members). Instead of saying "Sets A and B are equivalent," one can simply say that "Sets A and B have the same number of members." Later on, one can use functional notation: $n(A) = n(B)$; or talk about a one-to-one correspondence between A and B.

26. ESTIMATE, ESTIMATION

One of the headaches we now face is caused by the student who hands in a crazy answer and then says: "But I did it on my calculator!" Pushing the wrong button may produce wondrous results. NCTM and other groups and individuals have stressed that ESTIMATION is an important part of problem solving. Now in some cases a checking procedure is available (*e.g.*, long division), so that a check can substitute for an ESTIMATION. And in some cases there are so many steps to solution that ESTIMATION is very difficult or impossible. But in all other cases an ESTIMATION provides protection against pushing the wrong button (and against other procedural errors).

By the way, if you listen to students explain how they got their estimates, you may get insight into how well they understand a concept or procedure. For example, given:

$$(13 \times 21)/2$$

suppose a student says: "21 is almost 20, and 20/2 is 10, and 13 times 10 is 130; so the answer should be close to 130." Such a student clearly understands (a) how to round 21 to 20; (b) that division is not distributive with respect to multiplication (that is, $(xy)/z$ equals $x(y/z)$ but does not equal $(x/z)(y/z)$). The student also understands that the order of operations can be changed from multiplication first (*i.e.*, $(13 \times 21)/2$) to division first (*i.e.*, $13(21/2)$). On the other hand, given

$$\frac{147 + 21}{2}$$

suppose a student says: "21 is almost 20, and 20/2 is 10, and 147 plus 10 is 157; so the answer should be close to 157." Such a student clearly does not understand that division is distributive with respect to addition, and that therefore, 2 must divide both 147 and 21.

27. FARTHER THAN, FURTHER THAN

Many of us were trained in school to make a rigid distinction between FARTHER (= a greater distance) and FURTHER (in all other cases). American textbook writers by and large follow that policy. Yet in Fowler's Modern English Usage, we read: "... hardly anyone uses the two words for different occasions; most people prefer one or the other for all purposes, and the preference of the majority is for FURTHER." The rigid distinction between the two words does not have a long history and its passing need not be mourned. Speak as you prefer, but do not give your students a hard time if their preference is different from yours.

28. FEWER, LESS

The distinction between FEWER and LESS is a matter of good usage, not a matter of mathematics. Students who have not mastered the distinction will probably not be hindered in learning math. Yet we want our students to be literate as well as numerate; so if the occasion arises, math teachers should not hesitate to explain the distinction. William Safire has some interesting comments on fine points (see *N.Y. Times Magazine*, March 27, 1988, pp.22-24). However, the basic distinction is as follows:

-- When the reference is to objects that can be counted using counting numbers (what language teachers refer to as count nouns), we use FEWER and a plural noun (fewer cookies, fewer voters, fewer children).

-- When the reference is to an object that is measured continuously using real numbers (what language teachers refer to as mass nouns or non-count nouns), we use LESS and a singular noun (less butter, less power, less time).

29. FORMULA

Have you noticed that pre-algebra mathematics and algebra textbooks never or hardly ever define the concept of FORMULA? That's probably a good thing. In grade school, after students have learned a few formulas (such as the area of a rectangle or circle), they seem to intuit the notion of FORMULA without great difficulty. Thus there is no need to pose any question about the nature of a FORMULA. If a pre-algebra student raises such a question, a simple answer is appropriate: Given some information (*e.g.*, the length and width of a rectangle), a FORMULA tells you how to compute some more information (*e.g.*, the area). If a high school student in AP calculus raises the question, a more sophisticated answer would be appropriate: A FORMULA is an equation that defines a function of one or more variables; and the variables have specific interpretations in applied math.

30. GALLON

In case one of your students looks up GALLON in a dictionary and then asks about the Imperial Gallon, tell the student that the Imperial Gallon is no longer in use in Great Britain and Canada. Gasoline used to be sold there by the Imperial Gallon (about 20% larger than the US gallon), but now it is sold by the liter.

31. IDENTITY ELEMENT

We doubt that this term appeared often (and maybe not at all) in primary and secondary textbooks before the mid 1950's. IDENTITY ELEMENT is a very classy term from abstract algebra. We believe that students at the primary and secondary levels should have their attention brought time

and time again to how zero and one behave in addition and multiplication respectively. Whether they should memorize the technical name IDENTITY ELEMENT is open to question. Knowing the behavior is of great importance. Knowing the technical name is in our opinion of little importance at these levels. Naturally, it could be part of an enrichment unit (*e.g.*, on finite algebras). Note: in some states (*e.g.*, Florida) you must teach this term because it appears on statewide tests.

32. INCREASED BY, INCREASED BY A FACTOR OF

The word INCREASED in general means that something gets bigger. "Increase by n" (where "n" stands for some number) has come to be a signal that one is to add, whereas "Increased by a factor of n" is a signal that one is to multiply by "n". This distinction should be taught, not as a logical imperative, but simply as a language fact: that is the way English speakers use the terms.

33. INEQUALITY

You may be able to help students who have poor language skills to make more sense of English if you give them a little background for the prefix IN-. In Latin, IN- is used in two quite opposite ways: 1) to deny or negate something, and 2) to increase or intensify something. These two uses came over into English. During this transfer, changes sometimes occurred. The I sometimes became U ("untold" instead of "intold"), and the N sometimes changed to match a following consonant ("illegible" instead of "inlegible"). In mathematics there are a variety of words that have the IN- prefix: inequality, infinite, irreducible, incommensurate, irrational, etc. You might ask your students to look in their textbooks for such words and to identify those in which the IN- serves the purpose of negating something.

34. INVERT

In the next census it would be fun to ask citizens whether they remember the phrase "invert and multiply." We suspect that the responses would be fairly positive, but that many people would have little or no idea what the phrase means. The moral is that drilling phrases into students' heads is of little value unless the students make connections among the concepts and procedures. Since we are already using terms like "reciprocal" and "inverse for multiplication," it is unnecessary to add INVERT to the list of terms to be memorized.

35. LEFT

In past generations it may not have occurred to some teachers that they should explain the word LEFT (in the sense of LEFT OVER or REMAINING). Now that so many students have limited English skills, the need is greater and also more obvious. LEFT vs. RIGHT is something that many ESL students learn early; but LEFT as LEFT OVER may be acquired much later. Math teachers must therefore check to see if all students grasp (1) LEFT as LEFT OVER in everyday language, and (2) LEFT in math as a cue for subtraction. While they are doing this, they might as well do English teachers a favor and cover the whole gamut: LEAVE, LEAVING, LEFT, etc.

36. LESS

Less and less often, one hears a sentence such as "eight less five is three," and few textbooks now even mention this use of LESS. If you are in part of the country where this usage is still common, please keep it out of your classroom. We don't need it. The statement:

$$8 - 5 = 3$$

can be read as "8 minus 5 is 3" or "8 subtract 5 equals 3."

37. LOWEST TERMS, RATIONALIZE, REDUCE, RENAME, SIMPLIFY, TRANSFORM

At every level, both students and professional users of mathematics spend a fair amount of time replacing expressions with other expressions that are described as equal or equivalent (cf. EQUIVALENT) to the original expression. Examples: RENAME the number of (2 5/8) as (1 13/8); SIMPLIFY the expression $(x + 5 + 2x)$ as $(3x + 5)$; REDUCE the fraction (10/4) to LOWEST TERMS and get (5/2); RATIONALIZE the denominator in $(1/\sqrt{2})$ into $(\sqrt{2/2})$. From one point of view, all these activities are merely specific instances of the generic action of changing the appearance or form without changing the value of the expression. A word that might be used for this genus is TRANSFORM, both as a verb and as a noun. (Admittedly, we also use TRANSFORM, to mean a substantial change, as in "her love transformed his whole life.") From another point of view, some of the words indicating transformation do have specific content. To REDUCE a fraction, for example, means to write an equal fraction that has smaller numbers in it.

The problem here is primarily one of intellectual clarity and only secondarily one of language. Teachers need to be clear (and to make it clear to students) that every mathematical expression can be written in many, many forms; that no one of these forms is good or bad; that no one of these forms is intrinsically better or worse than another; that the selection of one form rather than another is based on convenience, and that convenience depends on one's situation and goal. Incidentally, there is a nice analogy here with English prose: any sentence can be paraphrased and the paraphrase that you choose depends on your goal-- what effect you wish to achieve.

Here are some examples. If you want to add $\sqrt{8}$ and $3\sqrt{2}$, it's convenient to replace $\sqrt{8}$ with $2\sqrt{2}$. If you want a decimal for $\sqrt{8}$, it is not convenient to replace it with $2\sqrt{2}$ because the latter requires more steps on a calculator. If you are doing rough carpentry, 3/8 is more convenient that 24/64; but if you intend to add 24/64 to 1/64, then 3/8 is less convenient. In short, we urge teachers not to treat the six words in the title of this note as important technical terms, because they are not. You could omit all of them and lose very little. For example, instead of saying "Rationalize the numerator," you could say "Make the numerator simpler." (See also CONVERT and COMPLEX FRACTION.)

38. METRIC SYSTEM

Many ESL students have grown up with the metric system, so metric terminology in English should pose no special difficulty. For all students, the key is to implant visual and tactile images of metric numbers. For example, 1 centimeter. is about the length of the nail on your little finger. Show your students a pair of dice: each is about 1 cubic centimeter; and if it were made of ice, it would weigh about 1 gram.

39. MINUS

Some people have, at times, rather smugly portrayed math - unlike every natural language - as an unambiguous language. If that were true, we might fault them only for their lack of tact. But it is not entirely true. Some symbols have multiple meanings, and the minus sign is an outstanding example of this. It denotes: 1) the operation of subtraction, 2) part of the name of a negative number, and 3) the "additive inverse of." Good students eventually realize that though the expression "5 - 3" is ambiguous from one point of view, from another point of view that doesn't matter because all three interpretations (5 subtract 3, 5 plus negative 3, 5 plus the additive inverse

of 3) yield the same result. Still, we assert as a matter of principle that the symbolism ought to be improved. A stab was made in this direction during the 1950's and 1960's, when in some books there were symbols such as $^{-}3$ or 3^{-} for the number negative three. This attempted reform did not catch on. One may hope that eventually the symbolism will be improved, but in the meanwhile we have to live with it. If you are teaching General Math, emphasize subtraction (5 subtract 3). If you are teaching pre-algebra, put more emphasis on the interrelationships between adding and subtracting signed numbers.

40. MIXED NUMBERS

The use of calculators leads to a more frequent use of decimal numbers. That's fine, but we should not lose sight of the utility of MIXED NUMBERS. In carpentry, in recipes, in interior decoration, and in many other applications, MIXED NUMBERS are the rule and not the exception. The board is 21 3/8 inches long, not 21.375 inches; the amount of sugar is 1 1/4 cups, not 1.25 cups; the curtains are 32 1/4 inches wide, not 32.25 inches; etc. Of course, when the US finally adopts the metric system (as it will eventually), then we shall be in a different situation.

Note that certain words are used as names of important sets of numbers: the PRIMES, the INTEGERS, the RATIONALS, etc. Other words (such as MIXED NUMBER, FRACTION, DECIMAL NUMBER) focus on the format in which a given number is presented.

41. MULTIPLE OF

The phrase MULTIPLE OF has traditionally been used in the context of the integers: B is a multiple of A if A divides B with zero remainder. Some textbooks use the phrase INTEGRAL MULTIPLE OF. In defense of this longer phrase we note that it makes explicit that the context is the set of integers. On the other hand, in the broader context of real numbers, every non-zero real number is a multiple of every other non-zero real number; and this makes the concept useless in that context. On balance, we believe that after a suitable explanation of the concept and its context, the shorter phrase (MULTIPLE OF) is preferable.

42. MULTIPLICATIVE INVERSE

MULTIPLICATIVE INVERSE is a mouthful. Even adults have trouble pronouncing it (with the accent on PLI). To avoid this difficulty you have two options: 1) use RECIPROCAL, or 2) use INVERSE FOR MULTIPLICATION. RECIPROCAL has the advantage of being shorter. INVERSE FOR MULTIPLICATION has the advantage of planting the seed of an idea: the word INVERSE always has reference to some operation.

43. OF

OF is one of those trickly little words that cause trouble both for grammarians and for math students. Grammarians distinguish between the possessive OF (the wheel OF the car) and the objective genitive (the very thought OF you). Math students have to make additional distinctions:

a) When OF is preceded by a fraction or a percent and followed by a number, OF is a cue that you should multiply ("One-half of 12 is 6" and "50% of 12 is 6").

b) When OF is both preceded and followed by positive integers, OF is a cue to divide the first number by the second if you want to compute a rate.("He won 6 of 12 matches for a success rate of 50%.")

44. ORDER OF OPERATIONS

ORDER has many meanings in mathematics. Perhaps students would learn about ORDER OF OPERATIONS more rapidly and use their knowledge more consistently if teachers talked a bit more about the concept of order. In general, to order objects is to arrange them according to some criterion (or set of criteria). Thus your collection of books can be ordered (placed on shelves) according to height, according to color, alphabetically according to author's last name, etc. Numbers are ordered on the number line according to their magnitudes. Operations are ordered according to priority of performance: which is to be done first, which second, etc. For example, if you wish to add 7 and 8 first, then multiply by 6, you write $6(7 + 8)$ or $(7 + 8)6$, where the parentheses indicate the first priority.

Be sure your students realize that the only foolproof system for showing such priorities is to use a pair of parentheses (or another grouping symbol) for each operation after the first. Beyond that, there is a general agreement on how to interpret expressions when grouping symbols have been omitted. One such arrangement is summarized in the slogan "My Dear Aunt Sally" (Multiplication, Division, Addition, Subtraction), another by "Please Excuse My Dear Aunt Sally" (Parentheses, Exponentiation, Multiplication, etc.). The only difficulty is that manufacturers of calculators and computers decide for themselves what agreement they expect the user to accept. Thus you must check each machine to see what the manufacturer had in mind. Sometimes the answer you get depends on how fast you press the keys!

45. PERCENT, PERCENTAGE

A poll of US citizens would probably reveal that very few can define PERCENT vs. PERCENTAGE. A student in pre-algebra clearly needs to be taught this distinction--it can never be assumed. It may help to paint a scenario: four students collaborate in mowing someone's lawn, for which $40 is paid. Have students recite: "My share is 25%: my percentage is $10." The problem here is not with PERCENT; everyone recognizes percent, because there is a special symbol (%) after the number (or the word PERCENT). The problem is with the word PERCENTAGE, which denotes the number of dollars (or bushel or acres or whatever) that results when you multiply the total number of dollars (or bushels or whatever) by the decimal version of the percent. Thus 25% is equivalent to .25; and $40 multiplied by .25 is $10. PERCENT always includes the symbol "%" or the word PERCENT. PERCENTAGE always includes or implies a unit such as dollars or acres.

One could argue that the word PERCENTAGE is more trouble than it is worth. Instead of asking "What's Jean's percentage?," one could ask, "What's Jean's share?" Admittedly, PERCENTAGE has a very precise technical meaning (unlike SHARE); but if students never learn or quickly forget that meaning, how valuable is the word?

46. PROPER FRACTION, IMPROPER FRACTION

These terms are less used than formerly. At the pre-algebra level they are not very important. Their utility comes later, when students study statements that are true for proper fractions (that is,

rational numbers between zero and one) and not true for improper fractions. For example, x squared is less than x, if x is a proper fraction.

47. QUADRILATERAL

This word--from the Latin QUATOR (meaning "four")--provides the opportunity to enlarge your students' vocabularies. Ask them to find words that start with QUAD or QUAT. (Note that the Latin T sometimes changes to a D.) They may come up with examples such as QUADRICEPS, QUADRIPLEGIC, QUADRANGLE, QUADRUPLETS, QUATRAIN, etc. All of these involve the number 4 in some way. By the way, it would be more consistent of us to use QUADRANGLE than QUADRILATERAL, since all our other terms for polygons focus on angles (TRIANGLES, PENTAGONS, HEXAGONS, etc.).

48. ROUND

Now that many students have calculators, the concept of rounding a decimal number to a certain number of places (called ROUNDING OFF in some texts) needs to be taught as early as possible. For when students do calculations (especially division and finding roots), the answer on the screen may contain lots of digits. For example 34 + 7 may show up as 4.857142857. As you know, there are two issues that must be dealt with. The first issue, which should be dealt with early, is the question: In a decimal number, how many decimal places should you use? The answer is twofold. With pure numbers (as opposed to measurements) the teacher should give instructions ("Round your answers to two places."). In applications, however, the situation determines what would be a reasonable answer. For example, .01 inches is rarely important in rough carpentry, but it may be very important in the manufacture of auto engine components. The second issue concerns the accuracy of data obtained by measurement and what happens to that accuracy when you perform an operation on two or more such numbers. This second issue, which may come much later in the course, is usually dealt with at length in pre-algebra texts and requires no further comment here.

49. SAME, DIFFERENT

In arithmetic, SAME and DIFFERENT are easy to deal with because the idea being expressed is that two numbers are equal or unequal. Thus: "6/8 is the same as 3/4" and "In 3 • 4 + 5 , if you

add first you get a different answer." However, in algebra and geometry (and in real life!) SAME and DIFFERENT are more subtle. Consider the phrase "two different rectangles". Different in what respect? Area? Location? Proportion? Perimeter? Or consider the sentence, "You kids are all the same." In our saner moments we know that this sentence is at worst ridiculous and at best ambiguous. Students probably won't have much trouble with SAME and DIFFERENT in abstract arithmetic, but they do need to struggle with the subtleties of these words. One appropriate place to discuss these words is in geometric applications. For example, draw three rectangles on the board and label them A, B, and C. Label the sides of A as 3 and 16, the sides of B as 4 and 12, and the sides of C as 6 and 10. Tell the students that these are rectangles and ask if A is the same as B, if A is the same as C, etc. The questions inevitably lead to the idea: Same in what respect? Eventually, students should be able to rephrase your initial questions in a more precise form (*e.g.*, "Is A the same as B in terms of its area?").

50. SOLUTION, SOLVE, ROOT, SATISFY, SOLUTION SET

At least in pre-college math, the word ROOT is used in a very limited way (with reference to equations). Given an equation in only one variable, a number which, when substituted for the variable, turns the equation into a true statement is called a ROOT of the equation. Thus, 5 is a root of the equation $2x + 3 = 13$; for when you replace x by 5, you get $10 + 3 = 13$. That number is also called a SOLUTION; the process of finding that number is called SOLVING the equation; the collection of all solutions is called the SOLUTION SET; and each solution is said to SATISFY the equation. However, for equations containing two or more variables, the word ROOT is never used. Thus for the equation $y = x + 3$, the ordered pair (4,7) is called a SOLUTION (but not a ROOT) of the equation.

A case can be made that ROOT (in the sense of SOLUTION) is unnecessary and should be dropped. On the other hand, it can be argued that historically there has been a close link between ROOT as an operation (SQUARE ROOT, CUBE ROOT, etc.) and ROOT as a solution. Thus a number is called the cube root of 8 if it is a ROOT of the equation $x^3 = 8$. On balance, we think that ROOT as SOLUTION can be dispensed with. SOLUTION will take care of all cases.

A case can also be made for dropping the word SATISFY from mathematical vocabulary. First, its mathematical use has little or nothing in common with its everyday use, where its reference is primarily to the appetites, thoughts, and wishes of humans. Secondly, a good substitute is available. Instead of saying "Does 3 satisfy the equation $2x = 10$?", we can say "Is 3 a solution of

the equation 2x = 10?" or "Does 3 solve the equation 2x = 10?" We encourage you to use one of the latter options.

It is true that in logic and in law, SATISFY is used to indicate that some conditions or requirements have been fulfilled. However, students in pre-algebra and first year algebra are not usually expected to know very much logic or law, so we still suggest dropping SATISFY from the language of pre-algebra.

51. SUM

We have identified 18 pairs of homophones (words that sound the same but have different spellings, origins, and meanings) in the language of pre-algebra. Probably the most important of these are: ADD/AD, CENT/SENT/SCENT, EIGHT/ATE, FOUR/FOR, ONE/WON, SUM/SOME, TWO/TOO/TO and WEIGHT/WAIT. Students should be told about these, because a misunderstanding could impede mathematical activity. For example, the spoken sentence "Find the sum of these numbers" could be misheard as "Find some of these numbers." Now that we have piqued your interest, we shall tell you the 9 other pairs of homophones: BASE/BASS, CARRIES/CARIES, MEAN/MEIN, PER/PURR, PI/PIE, RAISING/RAZING, SIGN/SINE, TIME/THYME, TON/TUN, and WHOLE/HOLE. (Some people would say QUART/COURT are homophones, too.) If you find any others please let us know!

52. TAKE AWAY

TAKE AWAY is a good example of a scaffolding term. Some teachers will swear that TAKE AWAY is absolutely essential for teaching children the concept of subtraction. That may well be. But once it has done its job, it should be discarded, because it duplicates other terms that are more powerful.

53. THAN

Many ESL students will have little difficulty with THAN when it follows a comparative adjective (GREATER THAN, etc.) for the same type of structure is found in many other languages (*e.g.*, French, Spanish, German, Russian, Arabic, Vietnamese). However, some students may need

some special coaching because their native language uses a different type of structure for comparisons (*e.g.*, Japanese, Chinese, Urdu). (See also the annotation on COMPARATIVES).

54. TIMES

For ESL students you need to point out that TIMES has two meanings:

1) "m times" means "on m occasions"
2) "m times n" means "m multiplied by n."

The distinction, obvious to native English speakers, may not be to others.

55. TWICE

TWICE is one of those words that native English speakers learn at an early age and then take for granted for the rest of their lives. ESL students, on the other hand, need some help. TWICE has two senses: 1) as a substitute for a counting number ("I have been to Chicago twice" instead of "I have been to Chicago on two--count them--occasions."); and 2) to denote multiplication by two ("Twice three is six.") A simple matter--unless you are learning the language and nobody bothers to explain.

56. VARIABLE

To a teacher who has taken at least high school math the term VARIABLE has become familiar and obvious. In fact, it conceals some rather subtle differences in meaning, and these differences can be stumbling blocks for students. If you are interested in a good discussion about VARIABLE, see Schoenfeld and Arcavi, "On the meaning of variable" (*Mathematics Teacher*, September 1988, pp. 420-427).

57. VERTICAL, HORIZONTAL

In arithmetic, possibly the only time that students will see these terms is in a description of how numbers to be added can be arranged in a column or a row. Please be sure that students learn VERTICAL and HORIZONTAL because these words will be used much more frequently in algebra and analytical geometry.

58. VOLUME, AREA, PERIMETER

Some students have trouble getting a clear picture of the concepts expressed by these three words. The most common mistake is confusing AREA and PERIMETER, probably because in both area problems and in perimeter problems, the student is shown a two-dimensional figure such as a rectangle. The key to understanding is the concept of dimension:

3 dimensions	....	volume	....	cubic unit of measure
2 dimensions	....	area	....	square unit of measure
1 dimension	....	perimeter	....	linear unit of measure

What requires most emphasis is that perimeter is a one-dimensional measurement--you are measuring the lengths of line segments and curves, not the size of the region bounded by them. If your students need it, make up some simple exercises on rectangles that represent a part of a lawn marked off with chalk lines. Let the rectangles be 2' by 18', 4' by 9', and 6' by 6'. In each case tell the student to measure the total length of the chalk lines and to measure the number of square feet of grass. Then draw some rectangles that have the same perimeter but different areas. You may also use a geoboard or some other manipulatives. (For some interesting teaching experiences, see Virginia Simpson, "What do we mean by area and perimeter?, *Mathematics Teacher*, May 1989, pp. 342-344)

59. WHOLE NUMBER, NATURAL NUMBER, COUNTING NUMBER

The use of these terms is another good example of how we sometimes have more than one name for an object in English. Is there any distinction here? Yes and no. You may have seen a textbook in which the author says that zero belongs to one of these sets but not to another. That is an author's prerogative, but you should remember that there is no universally accepted definition that

creates such a distinction. To us, there is nothing unreasonable in considering zero to be a WHOLE NUMBER (after all, it is not a fraction or irrational) or a NATURAL NUMBER (what is so artificial about zero?) or a COUNTING NUMBER ("I have one, you have two, he has none."). In short, if the question arises in the classroom, tell your students that some authors include zero in one or more of these sets and some do not.

60. WIDTH, LENGTH

In the best of all possible worlds, words such as WIDTH would have a clear definition. In fact, we are not consistent in our usage of WIDTH and LENGTH--neither individually nor as a society.

Imagine that we are in a backyard, looking at a flower bed that is 3 ft. by 15 ft.; and that we are standing next to a 3 ft. side and looking down the longer side. Most people, if they want to know the longer dimension, would ask: "HOW LONG is this bed?" However, if we are standing near the middle of the longer side, some would say: "HOW WIDE is the bed?"(because WIDE means left to right), but others would say: "HOW LONG in this bed?" (because LONG is always for the longer of two sides). In brief, usage is inconsistent and certainly not dictated by any logic. Mathematically, the important thing is noting that the variables L, W, H, and D (cf. annotation on DEPTH) appear in formulas as linear measures of line segments and that the choice of which letter for which dimension is not important and not rigidly prescribed.

A good classroom exercise is to draw a rectangular box (three-dimensional) on the blackboard. Then select three edges that meet at a common point and label the edges A, B, and C. Ask your students to write down what word they would use for edge A, what word for edge B, and what word for edge C. Have them share with the class their choices, and they will discover a lack on uniformity. You can then point out that in English there is no universally accepted system for naming the edges of a box.

If you are not convinced of the arbitrariness of our labels, imagine that you are an astronaut free-floating inside a space capsule, looking at a rectangular box that is also free-floating. Which dimension is the height of the box?

PART III

LANGUAGE/MATHEMATICS ASSESSMENT TECHNIQUES

Introduction

Increased attention is being given to the instructional integration of language and content skills. Recent proposals for the revision of the elementary and secondary school mathematics curriculum have called for a process approach to the development of conceptual and communicative skills. The National Council of Teachers of Mathematics (NCTM) suggests increasing the opportunities given to students to communicate so that they can:

- model situations using oral, written, concrete, pictorial, graphical and algebraic methods;
- reflect on and clarify their own thinking about mathematical ideas and situations;
- develop common understandings of mathematical ideas including the role of definitions;
- use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
- discuss mathematical ideas and make conjectures and convincing arguments;
- appreciate the value of mathematical notation and its role in the development of mathematical ideas.

It can be seen from the above recommendations that the emphasis throughout is on using language to communicate ideas. The general approach suggested is one in which oral skills are emphasized at the primary grades, followed by the development of reading and writing skills at the upper elementary level. The process culminates with a focus on the use of formal mathematical language and the development in the students of the ability to devise strategies for solving mathematical problems.

The purpose of the Language/Mathematics Assessment Techniques is to provide teachers with strategies through which the development of language skills can be assessed within the context of daily instruction in mathematics. These activities are designed for grades 7-9, and they reflect the content which is typically covered in a "pre-algebra" course. These assessment techniques are part of an effort to assist ESL teachers incorporate content into language instruction, and to help mathematics educators become aware of ways in which the language needs of limited English proficient students may be addressed.

Assumptions

The assessment techniques presented here are based on five assumptions. They are presented below with some comments about their relevance to present trends in language and mathematics instruction.

Assumption One

The primary purpose of student assessment is to improve learning.

The assessment techniques presented are designed to be used as tools to assist teachers determine the extent to which students are learning the language (and content) skills presented in class during the process of instruction. This information can then be used to adapt or create instructional activities to improve student progress. The techniques do not lend themselves for use as summative measures of student achievement.

Assumption Two

Assessment must be closely related to instruction.

The techniques given are samples or illustrations of assessment strategies which can be used in class. It is important that they be adapted to reflect the specific learning objectives intended for a particular lesson. Again, these techniques are *not* designed to serve as assessments which can be used in the placement of students in ESL or math levels.

Assumption Three

If assessment is to improve learning, it must emphasize the measurement of skills in the process of development.

Given Assumption Two, it is important to reinforce the idea of assessment that takes place as an integral part of the process of instruction. Figure 1 (p. 96) illustrates this idea.

Assumption Four

Assessment must be related to instruction which emphasizes active involvement by the students in the learning process.

Effective learning of mathematics involves active student participation. The same is true of language learning. Assessment of language skills must reflect this approach. This assessment

must take place in contexts which closely match the processes used in the teaching of content and/or language. The emphasis on the use of cooperative learning and manipulative materials in mathematics teaching lends itself to the assessment of language skills in such a setting.

Assumption Five

Language and mathematical skills cannot be totally separated in assessment.

A question concerning the techniques--whether they assess language or content skills, or both--may come up after an initial examination. In many instances the clear separation of language skills from mathematical skills may be apparent. In other cases, *e.g.*, problem solving, it may appear that only content skills are being measured while language is secondary. These perceptions are accurate; they reflect the nature of language used in the context of learning mathematics.

Even though language and mathematics skills cannot be totally separated in assessment, efforts need to be made to identify students' weaknesses as language or content-based. The determination of the source of problems is essential in the selection of appropriate instructional strategies. Teacher intuition is often used in deciding whether a student error is based on a language difficulty or a content misconception. The assessment techniques are meant to provide an objective basis for the intuitive decisions made in instruction.

Organization of Assessment Techniques

The assessment techniques presented in Part III have been organized according to a framework which focuses on four areas of the mathematics curriculum and two areas of language skills.

Mathematics Content Areas - The mathematics curriculum for grades 7-9 may be viewed as addressing (a) concepts, (b) operations, (c) word problems, and (d) problem solving skills. A brief description of each follows.

Concepts. A review of commonly used "pre-algebra' texts was conducted to identify the concepts most frequently addressed. The Pre-Algebra Lexicon contains a list of these concepts together with other terms and language structures.

Operations. Terms associated with addition, subtraction, multiplication, and division are included in the inventory and addressed in the assessment techniques.

Word Problems. A distinction is made in the framework between "word problems" and "problem solving skills". Word problems are those problem settings in which an available algorithm can be used to obtain a solution. These problems have been referred to in the literature as "routine" problems. The solution of new problems can be achieved by applying algorithms found appropriate for like "classes" of problems.

Problem Solving. As opposed to word problems, problem solving settings require students to develop strategies to deal with new and unique problem situations; a solution algorithm is not readily available. These problems have been named in the literature "non-routine". The teaching/learning of problem solving skills is being emphasized as one of the primary goals of the new trends in mathematics education.

Language Skill Areas - The assessment techniques have been also classified as addressing receptive or productive language skills. The receptive category encompasses listening comprehension and reading skills, while the productive category addresses writing and speaking skills. At this point a note must be made concerning the language skills related to problem solving. Given the higher order cognitive skills involved in problem solving and the emphasis on the communication of solution processes, the distinction between receptive and productive language skills has been eliminated. At this level it is extremely difficult (and the research literature supports this) to make a discrete distinction between these language skills. Language assessment will focus on the integration of both of these skill areas in the context of a problem solving setting.

Cooperative Learning Tasks - Cooperative learning tasks have been emphasized in the new math curriculum proposed by NCTM, so a number of illustrative math/language assessment activities have been included in Part III. These tasks reflect communicative language skills which do not lend themselves to discrete classification within the framework described here. As assessment techniques, they are presented as alternatives to techniques already included in the framework. They are numbered 1A, 5A, 8A, 11A, 13A, and 15A.

Figure 2 (p. 97) shows graphically the content/language framework used in the classification of the assessment techniques.

General Description of the Assessment Techniques

The assessment techniques presented have certain common characteristics:

1. The techniques represent examples of how assessment of language skills for a particular content area may be made. The ones given illustrate procedures teachers may use to assess students.

2. Suggestions are given for the development of assessment items. These suggestions should facilitate the further writing of items to be used with students. It is recommended that an odd number of items be used to assess a particular area. This strategy will facilitate deciding whether a student has achieved minimum mastery of a skill. An ambiguous situation is created if an even number of items is given, *e.g.*, four, and the student responds correctly to only two of them. What evaluation can be made concerning mastery of skills if the student shows a 50-50% correction rate? This can be avoided by using an odd number of assessment items.

3. Recommendations for the analysis of language skills are also given. These suggestions are general in nature. Depending on the instructional goals in the ESL class, the instructors may wish to do a more detailed analysis of the language performance of the students. Follow-up instructional suggestions have been kept general and to a minimum. Please remember that the primary purpose of the techniques is to provide you, the teacher (ESL or math), with a basis on which to structure further instruction. Suggestions for specific class activities are beyond of the scope of this guide.

Suggestions For Using the Assessment Techniques

The following suggestions have been derived from the comments made by teachers as they pilot-tested the techniques. They are designed to make the implementation of the assessment activities easy and effective.

1. *Incorporate the techniques as part of your classroom activities.* On-going diagnostic assessment (feedback) should be an integral part of instruction.

2. *Implement techniques which you feel will give you needed information about the students' communication skills.* Mathematics teachers may wish to use a

strategy to find out what language skills the students have learned after a particular math lesson. ESL teachers may use the assessment to determine the extent to which language skills are being developed within the context of mathematics. In either case, you will be responsible for making the decision of whether a skill deficiency is language or content based.

3. ***Try the technique on yourself, then use it with students.*** In order for the techniques to be successfully implemented, you need to be thoroughly familiar with them.

4. ***The techniques can be used with the whole class to obtain feedback from each student, or they may be used singly or in small groups.*** The cooperative learning tasks, of course, require grouping of the students.

5. ***A number of 'alternate' assessment activities have been given to obtain feedback on students' communicative proficiency in small group settings.*** The focus of the assessment is on how well students can use language skills to communicate as part of the process of a problem or task the activity requires them to solve. The language skills assessed in these techniques have been classified as General due to the integrated manner in which language skills are used in such settings.

6. ***Do not use the techniques as a means to give grades in the ESL and/or math class.*** These assessment techniques are designed to be part of the instructional process as feedback tools to improve learning.

7. ***The given sequence of techniques does not imply any specific order in which the assessment techniques must be used.*** The techniques have been numbered only for reference purposes. The guide is to be used as a 'catalogue' of techniques to be implemented according to the objectives and instructional activities which take place in class.

Pre-Algebra Lexicon
Language/Math Assessment Technique #1

A. Inventory Category: Receptive/Reading
Concepts

B. Purpose: To assess the student's knowledge of standard units of measurement; and reading skills for expressions of measurement.

C. Sample Assessment Item(s):

Directions: Please complete the following table. In Column B circle the word which represents the kind of measurement given in Column A - standard or non-standard. In Column C circle the word which represents what the unit measures - weight, volume, distance, or time.

Column A	Column B	Column C
1. 21 steps	standard nonstandard	weight volume distance time
2. 200 pounds	standard nonstandard	weight volume time distance
3. 75 truckloads	standard nonstandard	weight volume distance time
4. 35 minutes	standard nonstandard	weight volume distance time
5. 1 stone's throw	standard nonstandard	weight volume distance time

D. Suggestions for the analysis of student performance: You have two areas to assess - first, whether the students can tell if the measure is standard or nonstandard; second if the correct classification of the measure is identified. Therefore,

- Analyze responses to Column B first, determine if there are any error patterns.
- Analyze Column C, look for error patterns.

E. Notes on item construction:

1. These items are fairly simple to construct.

2. Be sure to have a balance between standard/nonstandard and weight/volume/time/distance items.

Pre-Algebra Lexicon
Language/Math Assessment Technique #1A

A. Inventory Category: Cooperative Learning Tasks
Concepts

B. Purpose:
- To assess the student's knowledge of standard units of measurement; and reading skills for expressions of measurement.
- To assess the students' communicative skills in a small group setting.

C. Sample Assessment Item(s):

Directions: Divide the class into small groups of three students each. Give each group a set of color coded cards (use any colors you may wish). Set A contains a measurement; Set B, the type of unit (standard or nonstandard), and Set C, the classification of the measure (weight, volume, distance, time). Have the students match the cards appropriately. The students should display the results on their work table by arranging the cards in the correct order, as follows:

[Set A] » [Set B] » [Set C]

Set A	Set B	Set C
1. 21 steps	standard nonstandard	weight volume distance time
2. 200 pounds	standard nonstandard	weight volume time distance
3. 75 truckloads	standard nonstandard	weight volume distance time
4. 35 minutes	standard nonstandard	weight volume distance time
5. 1 stone's throw	standard nonstandard	weight volume distance time

D. Suggestions for the analysis of the student's performance:

You have two areas to assess - first, whether the students can tell if the measure is standard or nonstandard; second if the correct classification of the measure is identified.

- Check the arrangement of the cards.
- If the students have appropriate language skills, you may want to ask them to justify their arrangment and classification.

The analysis may be accomplished as you move around the groups. You may also have the groups report in a whole class setting.

E. <u>Notes on item construction</u>:

1. These items are fairly simple to construct.

2. Be sure to have a balance between standard/nonstandard and weight/volume/time/distance items.

3. The sets of cards should be color coded.

Pre-Algebra Lexicon
Language/Math Assessment Technique #2

A. Inventory Category: Receptive/Reading
Concepts

B. Purpose: To assess knowledge of definitions of types of numbers.

C. Sample Assessment Item(s): Matching

Directions: Column A contains a list of numbers. On the line at the left of each statement, write the number of the term in Column B that best fits the statement. Each response in Column B may be used once, more than once, or not at all.

Column A	Column B
____A. 10	1. composite number
____B. 7.62	2. prime number
____C. -234	3. terminating decimal
____D. 7.43	4. fraction
____E. .333...	5. mixed number
____F. 7 3/8	6. repeating decimal
____G.11	7. negative number

D. Suggestions for the analysis of student performance:

- Get a frequency count of the items missed.
- Determine which of the definitions assessed were missed the most.
- Follow-up with appropriate instructional activities.

E. Notes on item construction: For matching format items the following principles should be followed:

1. Keep responses (Column B) homogeneous - all from the same content area.
2. Use "explanations" of the terms, rather than "formal" definitions" .
3. Make Column B longer than Column A so the final items do not just "match up," but require some student reflection.
4. Keep the list short.

Pre-Algebra Lexicon
Language/Math Assessment Technique #3

A. Inventory Category: Receptive/Reading
Concepts

B. Purpose: To assess knowledge of definitions of number properties.

C. Sample Assesment Item(s): Matching

Directions: Column A contains a list of descriptions of mathematical properties for numbers/operations. On the line at the left of each statement, write the letter of the term in Column B that best fits the statement. Each response in Column B may be used once, more than once, or not at all.

Column A

____1. The order in which numbers are added does not change the answer.

____2. It tells you how many times a number is used as a factor.

____3. $(3 \cdot 5)4 = 3(5 \cdot 4)$

____4. 45^3

____5. Two numbers whose product is 1.

Column B

A. Commutative property of multiplication

B. Reciprocals

C. Commutative property of addition

D. Distributive property

E. Associative property of multiplication

F. Associative property of addition

G. Exponent

D. Suggestions for the analysis of student performance:

- Get a frequency count of the items missed.
- Determine which of the definitions assessed were missed the most.
- Follow-up with appropriate instructional activities.

E. Notes on item construction: For matching format items the following principles should be followed:

1. There should be an odd number of items under Column A.
2. Keep responses (Column B) homogeneous - all from the same content area.

3. Use "explanations" of the terms, rather than "formal" definitions".
4. Make Column B longer than Column A.
5. Keep the list short.
6. Balance numerical examples with verbal "explanations" of the properties.

Pre-Algebra Lexicon
Language/Math Assessment Technique #4

A. Inventory Category: Production/Writing, Speaking
Concepts

B. Purpose: The objective is to assess the degree to which students can:
-write examples, OR
-give oral descriptions, OR
-construct models using manipulatives, OR
-draw representations
of given mathematical concepts.

C. Sample assessment item(s):

Directions: For each of the terms given below have the students
-write an example, OR
-give an oral descriptions, OR
-construct a model using manipulatives, OR
-draw a representation.

[Note: You may wish to divide the class into small groups. Have each group decide the mode they wish to use to describe the term -example, oral description, model or drawing. The students may use a different mode for each of the terms given.]

TERM

1. Average

2. computer program

3. less than

4. area

5. triangle

D. Suggestions for the analysis of student performance:

- First, analyze the responses for mathematical (conceptual) accuracy - were the students able to communicate a characteristic or characteristics of the concept given? Here we are looking for communicative competence.
- Second, determine the appropriateness of the communication mode used - example, model, drawing, or oral description. In more advanced ESL classes you may wish to have the students explain
  (a) their choice of communication mode, and
  (b) the representation of the term.

E. Notes on item construction:

1. Be sure the concepts you are asking the students to describe have been thoroughly explained in class and the students are familiar with them.

2. Select two or three concepts from the same content area. To illustrate the assessment procedure the sample items have been drawn from a variety of areas.

Pre-Algebra Lexicon
Language/Math Assessment Technique #5

A. Inventory Category: Production/Writing Concepts

B. Purpose: To assess the students' skills in applying definitions of geometric concepts.

C. Sample Assessment Item(s):

For each one the following, draw the geometric figure asked. Then write a sentence or phrase that describes the essential features of the figure.

Directions	Figure	Description
1. Draw an isosceles right triangle.		______
2. Draw a hexagon		______
3. Draw a rectangle that has a perimeter of 10 units.		______

D. Suggestions for the analysis of student performance:

- First, check the appropriateness of the figures drawn.
- Second, determine if the student has included the essential features of the figure in the description. For example, in describing the isosceles right triangle, the student should have included the fact that two sides are congruent (or equal) and there is one right angle.
- The descriptive part of the assessment will be difficult for many students. Praise students for the correct features described. As necessary, impress upon the students the need for completeness. A concept correlates with a well-defined set, but an incomplete description can lead to a different set.

E. Notes on item construction:

1. Select geometric concepts that include some classification scheme, as in the case of polygons, triangles, etc.

2. Give conditions for the students to draw a particular type of figure. Have them support their drawing.

Pre-Algebra Lexicon
Language/Math Assessment Technique #5A

A. Inventory Category: Cooperative Learning Tasks
Concepts

B. Purpose: To assess the students' skills in applying definitions of geometric concepts.

C. Sample Assessment Item(s):

Directions: Divide the class into small groups (3-5 students each). Give each group two geoboards. [Note: If geoboards are not available, provide for students appropriate materials (toothpicks, straws, etc.) for them to construct the figures.] For each one the following, construct the geometric figure asked. Then have each group write a sentence or phrase that describes the essential features of the figure.

Directions	Figure	Description
1. Draw an isosceles right triangle.		______
2. Draw a hexagon		______
3. Draw a rectangle that has a perimeter of 10 units.		______

D. Suggestions for the analysis of student performance:

- First, check the appropriateness of the figures constructed.
- Second, determine if the group has included the essential features of the figure in the description. For example, in describing the isosceles right triangle, the student should have included the fact that two sides are congruent (or equal) and there is one right angle.
- The written descriptive part of the assessment will be difficult for many students. Students may do this orally (have the group appoint a reporter). Praise students for the correct features described. As necessary, impress upon the students the need for completeness. A concept correlates with a well-defined set, but an incomplete description can lead to a different set.

E. Notes on item construction:

1. Select geometric concepts that include some classification scheme, as in the case of polygons, triangles, etc.

2. Give conditions for the students to construct a particular type of figure. Have them support their construction.

Pre-Algebra Lexicon
Language/Math Assessment Technique #6

A. Inventory Category: Production/Writing
Concepts

B. Purpose: To assess the degree to which students can use appropriate units of measurement in the context of a given situation.

C. Sample Assessment Item(s):

Directions: Fill-in the missing word in each statement. Use any word that makes sense.

1. Our bus traveled a ______________ of 250 miles in five hours.
2. Patty bought a one-half ___________container of milk.
3. The World Trade Center in New York City is 1350 _________ high.
4. Mr. Jones is supposed to pick up 150 bushels of wheat to take to the local market. He drives at 50 _________ per ________. It takes him 3 _________ to go to the market.
5. _____________ in many European countries are given in kilometers. For example, Munich and Vienna are 450 _____________ ____________.

(Note: These items were adapted from instructional activities presented in Crandall, et. al., English Skills for Algebra, 1989).

D. Suggestions for the analysis of student performance:

•Answer key:

1. distance
2. any liquid measure unit
3. any measure of distance
4. any measure of distance; time; miles or hours
5. Distances; kilometers; apart

• Analyze responses on the basis of the appropriateness to the context of the sentence; determine error patterns.

•Also give credit to students who complete the sentence with correct expressions rather than a single word. For example: "Our bus traveled on a lousy road for a total of 250 miles."

E. Notes on item construction:

1. Write a sentence or short paragraph which deals with measurement ideas. You may choose selections from the mathematics text used in class.

2. Delete references made to units of measure. Present these as blanks in the test item.

3. Give students a short list of units of measure. Have the students write a short sentence using each of the terms.

Pre-Algebra Lexicon
Language/Math Assessment Technique #7

A. Inventory Category: Receptive/Reading
Operations

B. Purpose: The objective of this assessment is for students to match the numerical expression given in words with the corresponding symbolic representation.

C. Sample assessment item(s):

Directions: Match each numerical expression written in words with the same expression written in numerals and symbols. Have the students draw a line from the term in Column A to the appropriate numerical expression in Column B.

Column A	Column B
The sum of twenty-eight and thirty	$8(72 + 12)$
The product of six and the square of ten	$28 + 30$
Eight times the quantity, seventy-two plus twelve	$(6)(-10) - 7$
The sum of ten squared and twenty-two cubed	$6(10^2)$
Seven subtracted from the product of six and negative ten	$(10 + 22)^5$
	$10^2 + 22^3$

D. Suggestions for the analysis of student performance:

- Obtain a frequency count of the items missed.
- Make a list of the expressions which were not matched correctly.
- Follow-up with appropriate instructional activities.

E. Notes on item construction:

1. Make Column B longer than Column A.

2. The textbook used in your class should be an excellent source of numerical and algebraic expressions.

Pre-Algebra Lexicon
Language/Math Assessment Technique #8

A. Inventory Category: Receptive/Reading
Operations

B. Purpose: To assess the student's knowledge of the correct usage of terms dealing with mathematical operations.

C. Sample Assessment Item(s):

Directions: The following items will ask you to choose the meaning of the underlined word or phrase as used in a sentence. Write the letter of the choice that best answers the questions in the space given.

1. What is the meaning of average in the following sentence? Julio's grade average is 75.
   a. Julio received a 75 in math.
   b. Adding all of Julio's grades and dividing by the number of grades, Julio received a 75.
   c. Julio's highest grade is 75.
   d. Julio's lowest grade is 75.

2. Your teacher has asked you to **arrange the numbers 75 and 15 horizontally and find their sum.** You are supposed to:
   a. Multiply the numbers.
   b. Write the numbers in a column, then add them.
   c. Write the numbers in a row, then add them.
   d. Write the numbers in a column, then subtract them.

3. What is the meaning of the underlined word in the following sentence?
   What is the <u>estimate</u> for fixing the car?
   a. The exact cost of the repair.
   b. The approximate cost of the repair.
   c. The cost of the repair is difficult to figure out.
   d. The car cannot be fixed.

4. Which of the following shows 10 raised to the third power?
   a. 10_3
   b. 10-3
   c. 10^3.
   d. 10+10+10

5. If the text asks you to **reduce all of the answers to lowest terms,** you must be working with ________.
   a. Decimals
   b. Prime numbers
   c. Fractions
   d. Whole numbers

D. Suggestions for the analysis of student performance:

- The object of the assessment is to determine the extent to which students can understand the meaning of mathematical expressions used in a communicative context.
- Determine any error patterns in the responses. Is there an expression which causes misunderstanding? What is the choice most frequently selected?
- Use this information to plan and implement appropriate instructional activities.
- Discuss the misconceptions used as item distractors. Be sure to review with students why these are wrong responses.

E. Notes on item construction:

1. Select phrases which use mathematical terms in the context of every day use. The textbook, newspapers and the way in which you present content in class are good sources.
2. Follow the format given in the sample items.
3. Use misconceptions of the term or phrase as distractors.

Pre-Algebra Lexicon
Language/Math Assessment Technique #8A

A. Inventory Category: Cooperative Learning Tasks
Operations

B. Purpose: To assess the student's knowledge of the correct usage of terms dealing with mathematical operations.

C. Sample Assessment Item(s):

Directions: Give each student in a small group a card with one of the following assessment items written on it. Give them time to think about what the bold faced term or expression means. Ask them to say or write the meaning of the terms. As each student reports, have the other members of the group listen.

1. What is the meaning of **average** in the following sentence? Julio's grade **average** is 75.
2. Your teacher has asked you to **arrange the numbers 75 and 15 horizontally and find their sum.** What are you are supposed to do?
3. What is the meaning of the underlined word in the following sentence?
   What is the estimate for fixing the car?
4. What does the expression 10^3 show?
5. What are you supposed to do if the text asks you to **reduce all of the answers to lowest terms**?

D. Suggestions for the analysis of student performance:

- The object of the assessment is to determine the extent to which students can understand the meaning of mathematical expressions used in a communicative context.
- Determine any error patterns in the responses. Is there a expression which causes misunderstanding? What is the choice most frequently selected?
- Use this information to plan and implement appropriate instructional activities.

E. Notes on item construction:

1. Select phrases which use mathematical terms in the context of every day use. The textbook, newspapers and the way in which you present content in class are good sources.
2. Follow the format given in the illustrations above.

Pre-Algebra Lexicon
Language/Math Assessment Technique #9

A. Inventory Category: Receptive/Listening comprehension
Operations

B. Purpose: The objective of this activity is for students to identify the correct operation symbol from a given numerical expression.

C. Sample assessment item(s):

Read each expression. Have the students circle the operation they hear (on a response sheet, like the one below).

	Expression	Response sheet Operation			
1.	fifteen times fifteen	+	-	x	/
2.	sixty-two divided by four	+	-	x	/
3.	the quotient of 20 and 5	+	-	x	/
4.	one less than twenty-seven	+	-	x	/
5.	eight increased by two	+	-	x	/
6.	four-ninths of twenty-seven	+	-	x	/
7.	fifteen-sixths divided by one-half	+	-	x	/
8.	seventy-seven decreased by seventy	+	-	x	/
9.	forty-four minus twelve	+	-	x	/
10.	the sum of three and twenty-four	+	-	x	/

D. Suggestions for the analysis of student performance:

- First, determine which operations were identified correctly.
- Second, list those in which mistakes were made. You may wish to do a classroom performance profile to determine which operations were missed most frequently by students. Use this profile for follow-up instructional activities.

E. Notes on item construction:

1. This is a very simple type of item to write and score. Do not attempt to use this assessment approach with power and root and "other" terms listed under Operations in the Inventory. There are terms such as, all together, partial product, multiplication table, which can be better assessed through different procedures.

2. The technique may be also carried out in a small group setting. Give each student in the group a set of cards containing the symbols for each of the four operations. Read the phrase, and at a signal have the students in the group show the card with the symbol for the operation given.

Pre-Algebra Lexicon
Language/Math Assessment Technique #10

A. Inventory Category: Production/Writing
Operations (Algebra)

B. Purpose: To assess students' skills in completing phrases which represent symbolic algebraic expressions.

C. Sample assessment item(s):

Directions: Look at the algebraic expression on the left. Use the correct words to complete the phrases on the right which are verbal "translations" of the symbolic expression. An example is done for you.

Example: $x + 7$	a. the *sum* of x *and* seven b. x *plus* seven
1. x^2	a. x ________ b. the ________ of x
2. $x - 12$	a. x ____________ twelve b. twelve __________ ______ x
3. $x^2 + y^2$	a. x _________ plus y _________ b. the _______ of x _______ and y _____
4. x/y	a. x _________ by y b. the ______________ of x ____ y *c. x times the _______ _______ y
5. (x - 1)3	a. the _______ of x minus one ______ three b. the ___________ of x and 1 _______ three

(*Note - This item assesses the students' ability to express division as the inverse of multiplication. This might be very difficult for students to realize. A correct response is "x times the reciprocal of y".)

D. Suggestions for the analysis of the student's performance:

- Make a list of all the terms used correctly, and those missed. (You may allow minor misspellings.)
- Develop appropriate instructional activities to reinforce the correction of mistakes.
- Discuss other correct answers that students have submitted (even if they require a different number of blank spaces.

E. Notes on item construction:

1. Very easy items to construct. Take an appropriate sample of algebraic expressions from the text used which your students may have encountered.

2. This activity may also be carried out orally with the class.

Pre-Algebra Lexicon
Language/Math Assessment Technique #11

A. Inventory Category: Production/Writing
Operations

B. Purpose: Students will write the verbal expression for a number expression given in a symbolic form.

C. Sample assessment item(s):

Directions: Write the corresponding verbal expression for the numerical expression given in symbolic form.

a. $0.1 + 0.9 - 1.7$ ____________________

b. $2(15 + 8)$ ____________________

c. $3(25) + 4$ ____________________

d. $(15/32)(-6/7)$ ____________________

e. $31^2 + 64^2$ ____________________

f. $2/3 - (1/10)^2$ ____________________

g. $(70 - 13)^2$ ____________________

h. $7(8 + 5)$ ____________________

i. $-150/20$ ____________________

j. $4[15/(5)^2]$ ____________________

D. Suggestions for the analysis of the student's performance:

Analyze students' responses in the following areas -

- Use of correct terms for numerals and operations.
- Use of correct sequence of terms for the operations.
  For example,
  $4[15/(5)^2]$ - is not four times the square of the quotient of fifteen and five.
- Allow for variations in the way in which a numerical expression may be presented verbally; *e.g.*, the above expression can be stated -
  (a) "four times fifteen divided by five square"
  (b) "four multiplied by ...."
- Use your judgment concerning the spelling of the terms.

E. Notes on item construction:

1. These are very simple items to construct.
2. Be sure to have a variety of operations represented in the items.
3. This activity may also be done orally.
4. Keep the items fairly simple. It is difficult to write complicated expressions in words without introducing ambiguity.

Pre-Algebra Lexicon
Language/Math Assessment Technique #11A

A. Inventory Category: Cooperative Learning Tasks
Operations

B. Purpose:
a. Students will write the numerical expression for a verbal expression dictated.
b. Students will state orally the verbal expression for a numerical expression given.

C. Sample assessment item(s):

Directions: There are a number of alternate strategies that can be used to implement this technique.

- •Dictate each of the expressions given in the list, OR
- •Have the students orally state the given expression.

a.	$0.1 + 0.9 - 1.7$	f.	$2/3 - (1/10)^2$
b.	$2(15 + 8)$	g.	$(70 - 13)^2$
c.	$3(25) + 4$	h.	$7(8 \div 5)$
d.	$(15/32)(-6/7)$	i.	$-150/20$
e.	$31^2 + 64^2$	j.	$4[15/(5)^2]$

D. Suggestions for the analysis of the student's performance:

Analyze students' responses in the following areas -

- Use of correct terms for numerals and operations.
- Use of correct sequence of terms for the operations.
  For example,
  $4[15/(5)^2]$ - is not four times the square of the quotient of fifteen and five .
- Allow for variations in the way in which a numerical expression may be presented verbally; *e.g.*, the above expression can be stated -
  (a) "four times fifteen divided by five square"
  (b) "four multiplied by ...."
- Use your judgment concerning the spelling of the terms.

E. Notes on item construction:

1. These are very simple items to construct.
2. Be sure to have a variety of operations represented in the items.
3. Keep the items fairly simple. It is difficult to write complicated expressions in words without introducing ambiguity. In a dictation, items should be kept short.

Pre-Algebra Lexicon
Language/Math Assessment Technique #12

A. Inventory Category: Production/Writing
Operations

B. Purpose: To determine the student's ability to "translate" expressions from numerical to verbal format.

C. Sample Assessment Item(s):

Directions: Look at each numerical expression below. Then look at the group of words next to it. Put the words in the correct order to say the expression. **Caution!** - some words in the list may not be needed, they may be extra. Use commas where needed. Write the expression in the blank.

1. 16/8

divided
eight
sixteen
by
times

____
____

2. 55-14

fifty-five
than
fourteen
subtracted
less

____
____

3. 1/6 + 19/2

plus
nineteen halves
one-sixth
minus
one-six

____
____

4. $17^2 + 12$

square
seventeen
twelve
plus
squared
twenty-one

____
____

5. $\frac{10-15}{4-1}$

the, of, all, and
fifteen
over
one
plus
ten
sum
difference
four
minus

____
____
____

D. Suggestions for the analysis of the student's performance:

- Correct for accuracy.
- Analyze incorrect responses by item.
- Determine if there are any patterns to the incorrect responses.
- If incorrect patterns exist, identify the errors the students are making.
- Use this information to plan appropriate instructional activities.

E. Notes on item construction:

1. Follow the format presented in the sample items.
2. If this is the first time for this type of technique, modify the task and do not include the additional words (distractors) in the list. Add them later, when students are more familiar with the task.
3. Be sure to include additional words in the "word column". These distractors should look feasible. For example, if the students have difficulty differentiating between difference and sum, include both in the list.
4. This item type is sometimes difficult to write, *e.g.*, the expression 3(4+5) can be read "3 times the sum of 4 and 5" or "3 times the quantity 4 plus 5. A student who is locked into one way of reading this expression may get frustrated trying to make sense of your list of words.

Pre-Algebra Lexicon
Language/Math Assessment Technique #13

A. Inventory Category: Receptive/Reading
Word Problems

B. Purpose: The students will identify information presented in a word problem.

C. Sample assessment item(s):

Directions: Read each word problem. On your answer sheet, check those items which are needed to solve the problem.

1. Problem: In a shallow part ci Greenslime swamp there were seventeen turtles resting on a log. A hungry alligator ate some of them. Nine turtles were left. How many turtles did the alligator eat?

____ There were seven turtles resting.
____ There were seventeen turtles on a log.
____ Nine turtles were eaten.
____ Eight turtles ran away.
____ In this problem you have to find how many turtles were eaten.

2. Problem: Susan is planning a party. She has twenty-seven chairs. There are 35 people coming to the party. Each is bringing two presents. How many more chairs will Susan need if she wants everyone to be able to sit?

____ Susan has 35 chairs.
____ Susan has 27 chairs.
____ Susan is expecting 35 guests.
____ Each guest is bringing two presents.
____ The problem asks you to find the total number of presents the guests are bringing.
____ The problem asks you to find the number of chairs Susan will need to seat everybody.

3. Problem: Jaws, the friendly shark, caused $750 worth of destruction to the beach at Sunnyrock. The citizens could only come up with $634 to repair the damage. In order to get the rest of the money, the citizens of Sunnyrock are going to sell T-shirts to the tourists for $15. If it costs $5 to make each shirt, how many shirts will they have to sell in order to come up with the rest of the money?

____ The story takes place in the town of Sunnyland.
____ Each T-shirt will sell for $5.
____ It costs $5 to make each T-shirt.
____ The tourists will buy the T-shirts for $15.
____ The problem asks you to find out how many T-shirts the citizens will have to sell.
____ The citizens could only come up with part of the money to repair the damages caused by Jaws.

D. Suggestions for the analysis of the student's performance:

- Analyze incorrect responses. Make a list to determine if there are any recurring patterns (*i.e.*, some aspect of the problem which was not understood).
- Follow-up with some listening comprehension exercises.

E. Notes on item construction:

1. Select appropriate word problems from the text used in class.

2. Write sentences which agree with the information given in the problem. Use paraphrases as much as possible. If the sentence: "The problem asks you to find 'x'", is given, be sure never to use 'x' for a quantity given in the problem. The sentence should clearly refer to the problem question.

3. Write appropriate distractors.

Pre-Algebra Lexicon
Language/Math Assessment Technique #13A

A. Inventory Category: Cooperative Learning Tasks
Word Problems

B. Purpose:
- The students will identify information presented in a word problem.
- The students will use diagrams and/or manipulatives to represent the word problem.

C. Sample assessment item(s):

Directions: Divide the class into small groups. Have each group do the following:

First, make a list of the facts given in the problem. This is information needed to solve the problem.

Second, draw a diagram or use objects to represent the problem.

Third, write or be ready to tell how the problem can be solved.

1. Problem: In a shallow part of Greenslime swamp there were seventeen turtles resting on a log. A hungry alligator ate some of them. Nine turtles were left. How many turtles did the alligator eat?

2. Problem: Susan is planning a party. She has twenty-seven chairs. There are 35 people coming to the party. Each is bringing two presents. How many more chairs will Susan need if she wants everyone to be able to sit?

3. Problem: Jaws, the friendly shark, caused $750 worth of destruction to the beach at Sunnyrock. The citizens could only come up with $634 to repair the damage. In order to get the rest of the money, the citizens of Sunnyrock are going to sell T-shirts to the tourists for $15. If it costs $5 to make each shirt, how many shirts will they have to sell in order to come up with the rest of the money?

4. Problem: Steve rode his bicycle for 2 hours. He averaged 10 mph. How many miles did he ride in all?

5. Problem: Jeanne ran 10 km in 1.5 hours. Maggy ran 7 km in one hour. At these rates whose average speed was faster?

6. Problem: One gallon of water weighs about 8.3 pounds. About how much does 1 quart weigh?

D. Suggestions for the analysis of the student's performance:

- Analyze incorrect responses. Make a list to determine if there are any recurring patterns (*i.e.*, some aspect of the problem which was not understood).
- Analyze the way in which the problems were represented. Determine if there are any differences between the identified elements of the problem and its representation.
- Have the groups exchange papers. Have each group check each other's description of the solution .

[Note: This assessment activity is recommended for ESL students who have the required language skills to accomplish the tasks outlined.]

E. Notes on item construction:

1. Select appropriate word problems from the text used in class. Be sure to select word problems that lend themselves to graphic/object representation.

2. Be sure to try out the problems before assigning them to the students.

Pre-Algebra Lexicon
Language/Math Assessment Technique #14

A. Inventory Category: Receptive/Reading
Word Problems

B. Purpose: To determine the extent to which students can identify key features of a problem.

C. Sample Assessment Item(s):

Directions: Read each problem carefully. Then answer the questions. Select (circle) the letter of your choice.

Problem A. Steve rode his bicycle for 2 hours. He averaged 10 mph. How many miles did he ride in all?

1. What does this problem ask you to find?
   a. The time it takes Steve to go one mile.
   b. The total distance Steve went on his bike.
   c. The average speed that Steve went.
   d. The type of vehicle Steve was riding.

2. To answer this problem, you need to know Steve's speed and:
   a. how far he was going.
   b. how long he rode.
   c. how fast he was going.
   d. where he was going.

Problem B. Jeanne ran 10 km in 1.5 hours. Maggy ran 7 km in one hour. At these rates whose average speed was faster?

1. What does this problem ask you to find?
   a. Who ran faster, Jeanne or Maggy.
   b. How fast Maggy ran.
   c. How fast Jeanne ran.
   d. How far Jeanne ran.

2. To answer this problem, you need to know how fast Maggy ran in an hour and:
   a. how far Maggy and Jeanne ran together in an hour.
   b. how fast Jeanne ran in an hour.
   c. how long it would take Jeanne and Maggy to run 17 km.
   d. how far Jeanne and Maggy ran.

Problem C. One gallon of water weighs about 8.3 pounds. About how much does 1 quart weigh?

1. What does this problem ask you to find?
   a. How much a gallon of water weighs.
   b. What the weight of a quart of water is.
   c. What the weight of water is.
   d. What the weight of water in your body is.

2. In order to solve this problem, you must know
   a. How many gallons are in a quart.
   b. How many pounds are in a quart.
   c. How many quarts are in a gallon.
   d. How many gallons of water are in your body.

(Note: Problems A and B were adapted from J. Crandall, et. al. English Skills for Algebra, 1989.)

D. Suggestions for the analysis of the student's performance:

- Analyze incorrect responses in terms of -
  - can the students identify the question posed? AND
  - are the students able to identify what is needed to solve the problem?
- You may wish to construct a matrix to develop a profile of the problem areas. For example -

Student	Question	Data
Chou, J	X	X
Lopez, M	-	X
......	.....	.....

...and so on...
(Note: X indicates non-mastery)

E. Notes on item construction:
1. Follow the format of the above illustrations.
2. You may wish to select problem statements from the text used in class.
3. Write two types of items -
   - One should deal with what is being asked in the problem. Be sure to paraphrase the question as given in the problem.
   - The other should address the process needed to solve the problem.
   - Write feasible distractors for the multiple choice format.

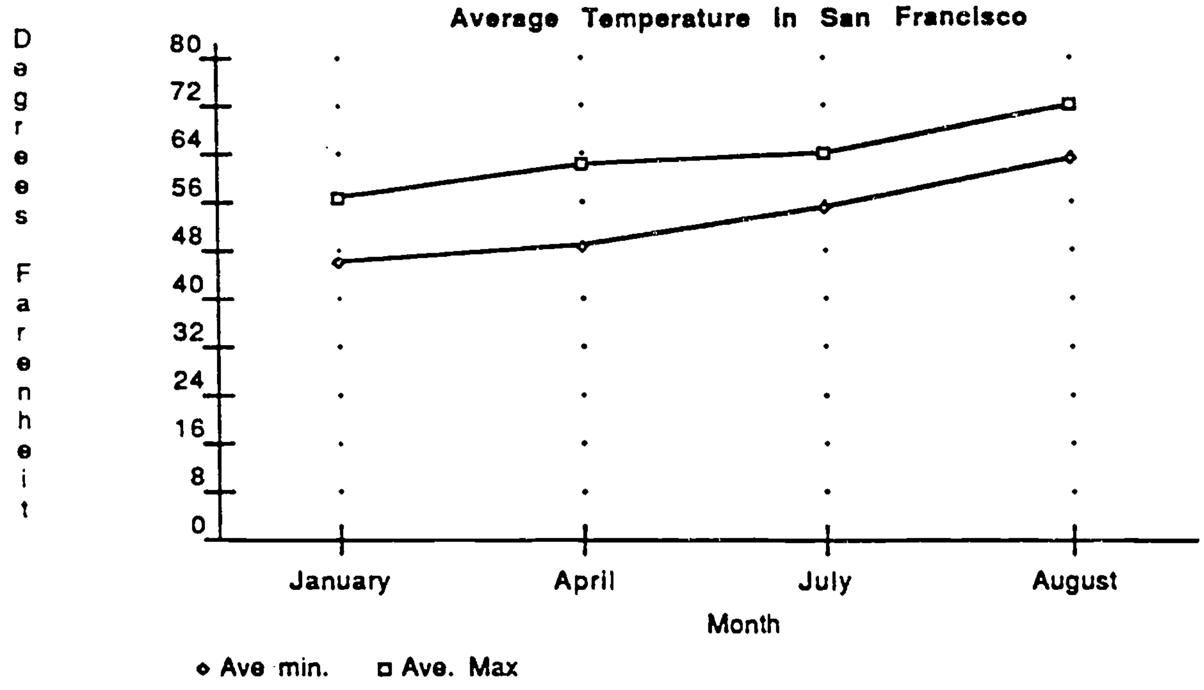
Pre-Algebra Lexicon
Language/Math Assessment Technique #15

A. Inventory Category: Receptive/Reading
Word Problems

B. Purpose: To assess the student's ability to interpret information presented in a graph.

C. Sample Assessment Item(s):

A. Directions: Use the graph below to answer the questions. Select the letter of the choice which best answers the question.



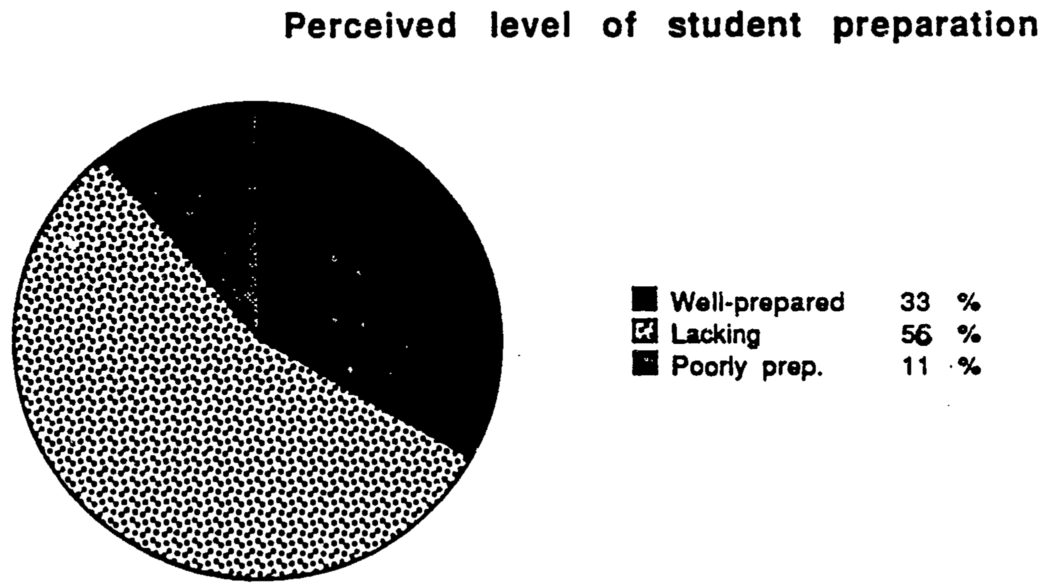
1. In which month was the average maximum temperature the highest?
   a. January
   b. April
   c. July
   d. August

2. In which month was the average maximum temperature the lowest?
   a. January
   b. April
   c. July
   d. August

3. In which month(s) was the average minimum temperature higher than 50 degrees?
   a. January
   b. January and July
   c. July and August
   d. April and August

B. Use the graph below to answer the questions. Select the letter of the choice which best answers the question.

Mrs. Jones just received the results of the test the students took to be able to graduate from high school. Some of the students showed they were well-prepared, others lacked some skills or were poorly prepared. Only those who are well-prepared will be able to graduate.

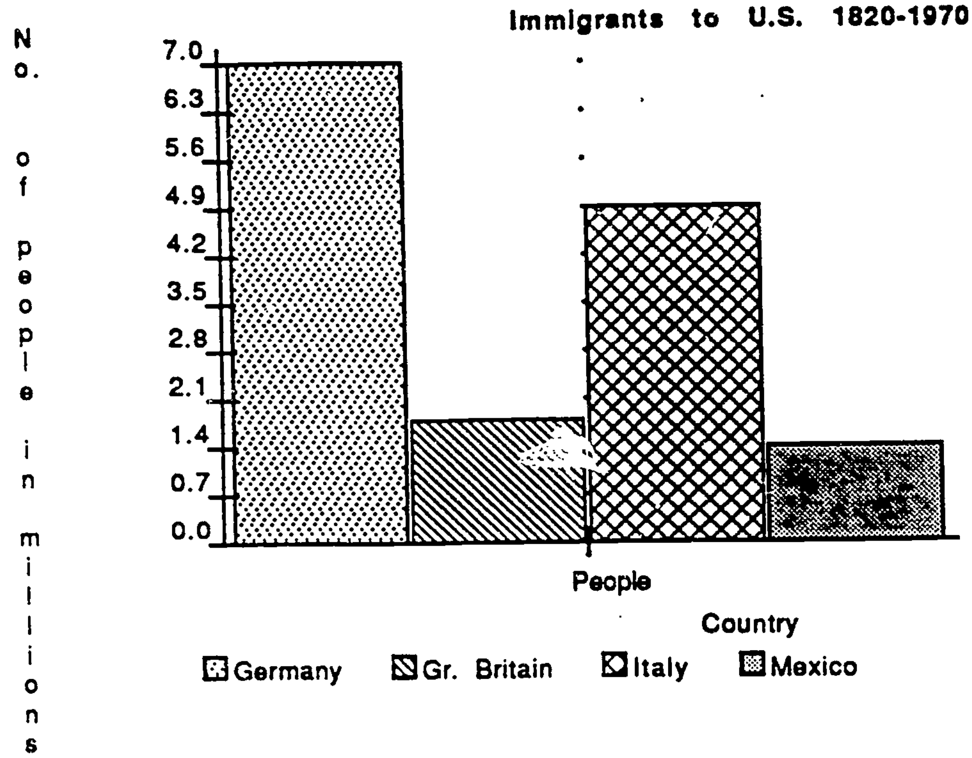


1. What percentage of Mrs. Jones' students did well on the test?
   a. 33%
   b. 56%
   c. 11%
   d. 67%

2. How many students were well-prepared to graduate from high school?
   a. 33
   b. 56
   c. 11
   d. can't tell from the information given

3. What percentage of students did the poorest on the test?
   a. 33%
   b. 56%
   c. 11%
   d. 67%

C. Use the graph below to answer the questions.



1. From which country have the most immigrants come?
   a. France
   b. Germany
   c. Great Britain
   d. Italy

2. From which country have the fewest immigrants come?
   a. Italy
   b. Germany
   c. Great Britain
   d. Mexico

3. From what country(ies) have more than 3 million immigrants come?
   a. Italy and Mexico
   b. Germany
   c. Italy and Germany
   d. France and Mexico

D. <u>Suggestions for the analysis of the student's performance</u>:

- Correct responses for accuracy.
- Analyze incorrect responses for error patterns.

E. Notes on item construction:

1. There are a number of places where information presented in graph form may be obtained. First, look in the support materials which accompany the mathematics text used. Second, there are newspapers, such as USA Today, which use graphs extensively.

2. Once a graph has been selected, construct multiple choice questions about the information presented. Be sure the student responses do not require them to do computation (unless this is one of your objectives). For example, in graph #1 above, it would be better not to ask a question such as - What is the difference in the average maximum temperature between July and August?

3. Avoid data whose description is highly technical or verbose -- *e.g.*, "Number of items per person per square foot per second."

4. For students who have intermediate or more advanced language proficiecy levels, you may wish to ask the questions without the choices. Have these students write out the answers.

Pre-Algebra Lexicon
Language/Math Assessment Technique #15A

A. Inventory Category: Cooperative Learning Tasks
Word Problems

B. Purpose: To assess the student's ability to organize and represent information using a graph.

C. Sample Assessment Item(s):

Directions: Give students one of the following sets of data. Have them construct a graph to represent the information given. When the task is completed, have the students describe the graph orally.

Data Set #1: Number* of hamburger restaurants built in the past four years.

Restaurant	1986	1987	1988	1989
McPartland's	9,000	10,000	11,000	12,500
Burger Queen	5,000	5,500	5,900	6,200
Laurel & Hardy's	3,800	3,900	4,000	4,200
Windy's	3,000	3,300	3,800	4,100

*Note: Numbers are approximations.

Data Set #2: The following are prices paid by two students, Sam and Stan for popular brand-name products.*

What Sam paid	Item	What Stan paid
$10	Bon Jovi tape	$7
$35	Hombre Cologne	$30
$40	Fancy watch	$35
$130	Soleil Portable Cassette	$100
$160	Intendo Power Set	$150
$310	Zoom - Zoom camera	$260

*Note: Adapted from Penny Power, October, 1989, p.21.

D. Suggestions for the analysis of the student's performance:

- Check the graphs/charts for accuracy.
- Assess the graphs in terms of representation mode used - bar, line or pie charts.
- Assess the oral presentation of the students. Were they able to communicate the information on the graphs accurately? If you asked them questions about the graph, were they able to respond appropriately?

E. Notes on item construction:

1. Be sure to avoid complex tables of information during early stages of skill development.

2. There are a number of sources where information presented in tabular form may be obtained. First, look in the support materials which accompany the mathematics text used. Second, there are newspapers, such as USA Today, and youth magazines, such as Penny Power, which use numerical information extensively.

Pre-Algebra Lexicon
Language/Math Assessment Technique #16

A. Inventory Category: Production/Writing, Speaking
Word Problems

B. Purpose: The assessment items are designed to determine the degree to which students can correctly write a word problem which matches a pictorial representation of its components.

C. Sample assessment item(s):

Directions: Write a word problem for each of the following pictures or diagrams. After you write the problem, solve it.

1. Picture of two pizzas. Each pizza is cut into eighths. Three-eighths are shaded.

PROBLEM STATEMENT:

Possible problem: *Patty bought two pizzas. Each pizza was cut into eight pieces. How many pieces are left for her friends if she eats three pieces?*

SOLUTION

2. Picture of 20 plants; fifteen of them healthy, five wilted.

PROBLEM STATEMENT

Possible problem: *Tom has twenty plants. One-fourth of them wilted. How many wilted?*

SOLUTION

3. Picture of a rectangle; label sides 55 ft and 379 ft.

PROBLEM STATEMENT

Possible problem: *Farmer Jones has a plot of land 55 ft. wide and 379 ft. long. What is the area of the plot?*

SOLUTION

D. Suggestions for the analysis of the student's performance:

- First, be sure that the problem written does match the pictorial representation and information given.
- Second, make a list of incorrect usages of terms, phrases, etc.
- Determine how much the responses deviate from a model problem with respect to the terms used to communicate the process of solution. (Note: it is obvious that it would be difficult to expect students to reproduce verbatim a problem that has been developed beforehand.)
- Analyze solutions.

E. Notes on item construction:

1. Write a word problem mathematically appropriate for the level of the students.

2. Draw a picture of diagram representing the problem. This will become the assessment item.

3. Write all of the solution steps.

Pre-Algebra Lexicon
Language/Math Assessment Technique #17

A. Inventory Category: Production/Writing
Word Problems

B. Purpose: To assess the student's ability to construct a meaningful word problem, given its components.

C. Sample Assessment Item(s):

Directions: Read each group of phrases and/or sentences. Then write them in order to form a word problem. Use the blank lines on the right to write the problem.

1. for 6 hours on Saturday.
   How much did she earn?
   Helen worked
   at $4.75 per hour.

2. Julia was 17 years old
   ago.
   Find Julia's present age.
   five years

3. does he have?
   Francisco has some dimes
   How many coins of each kind
   and quarters
   totaling $4.05.
   He has five more quarters
   than dimes.

D. Suggestions for the analysis of the student's performance:

- Answer key -
  1. Helen worked for 6 hours on Saturday at $4.75 per hour. How much did she earn?
  2. Julia was 17 years old five years ago. Find Julia's present age.
  3. Francisco has some dimes and quarters totaling $4.05. He has five more quarters than dimes. How many coins of each kind does he have?
- Analyze the responses in terms of the parts misplaced in the statement of the problem. See if there are any patterns.

E. Notes on item construction:

1. Obtain problem statements from the mathematics textbook used in class. If the students are at a beginning language level, you may want to use a text two or three grades below grade level.

2. Break the problem statement down into "cohesive components". Each component should have some meaning by themselves. (If desired, increase the level of difficulty by scrambling all of the words in the problem.)

3. Scramble the phrases and/or sentences.

Pre-Algebra Lexicon
Language/Math Assessment Technique #18

A. Inventory Category: Production/Writing, speaking
Word Problems

B. Purpose: To assess the student's ability to write/tell a word problem using given information.

C. Sample assessment item(s):

Directions - Write a word problem that fits the solution/information given. Be sure to use complete sentences.

Information	Problem
1. \$120 - (3 X \$30) = \$30	
2. 4 X 17 = ? rose bushes in the garden	
3. 3 X 25 = 75	
4. 45 students	
5. Write a word problem that requires division to get the answer.	

D. Suggestions for the analysis of the student's performance:

- First, be sure that the problem written does match the solution and information presented.
- Second, make a list of incorrect usages of terms, phrases, etc.
- Determine how much the responses deviate from a model problem with respect to the terms used to communicate the process of solution. (Note: it is obvious that it would be difficult to expect students to reproduce verbatim a problem that has been developed beforehand.)

E. Notes on item construction:

1. Write a word problem mathematically appropriate for the level of the students.

2. Write all of the solution steps. This will become the assessment item.

Pre-Algebra Lexicon
Language/Math Assessment Technique #19

A. Inventory Category: Production/Writing, Speaking
Problem Solving

B. Purpose: To assess the student's skills in solving a problem and in communicating the steps necessary for the solution.

C. Sample Assessment Item(s):

Directions: Assume your favorite calculator only has the following working keys -

2 6 + - X =

Using, at most, ten key presses, find at least one way to get the following numbers on your calculator display screen.

Number	Keystrokes									
12	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
13*	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

(Note: Activity adapted from Family Math, Stenmark, Thompson, & Cossey, 1986)

D. Suggestions for the analysis of the student's performance:

Have each group post on newsprint how many ways they discovered for calculating each of the numbers (one sheet per number). Have groups compare each of their approaches. Also have students explain why a certain group's sequence of keystrokes may have given the wrong answer. Record which groups were able to complete the task and explain their work.

* Teachers have the option to include a number like "13" which cannot be solved with the available working keys. This type of number will challenge more advanced students and, in this case, determine whether or not students recognize a prime number.

E. Notes on item construction:

The emphasis here is on the student's ability to communicate the strategies used and to provide a rationale for them. Questions and/or assessment activities should be structured to elicit such skills.

Pre-Algebra Lexicon
Language/Math Assessment Technique #20

A. Inventory Category: Receptive and Production
Problem Solving

B. Purpose: To determine the student's ability to (a) explain what a problem asks and (b) draw a picture or diagram of the problem situation/solution.

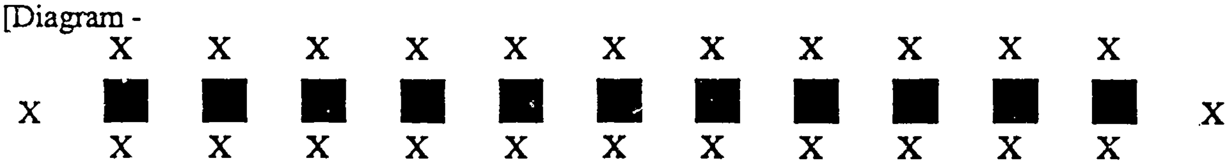
C. Sample assessment item(s):

Directions: Read the problem below. Your teacher will ask you to explain what the problem is all about. Then, draw a diagram of what you think the problem is asking.

Twelve couples have been invited to a party. The couples will be seated at a series of small square card tables, placed end to end so as to form one large long table. Only one person sits on a side. How many of these small tables are needed to seat all 24 people? (Krulik and Rudnick, 1980, p.29)

[Answer: 11 tables]

[Diagram -



D. Suggestions for the analysis of the student's performance:

- Assess the degree to which the students can communicate orally what the problem is all about. The key factor which needs to be mentioned in the explanation is how the tables will be set up.
- Determine to what degree the students were able to generate the above picture.

E. Notes on item construction:

The reference given below has a variety of problem solving examples which can be used in assessment activities such as this one. The emphasis here is on individual performance rather than group work.

Krulik, S, & Rudnick, J. (1980). *Problem solving: A handbook for teachers.* Boston: Allyn and Bacon.

Pre-Algebra Lexicon
Language/Math Assessment Technique #21

A. Inventory Category: Receptive and Production
Problem Solving

B. Purpose: To assess the student's ability to explain orally how they attempted to solve a problem.

C. Sample assessment item(s):

Directions: Give the following problem to the class, individually or in small groups. (You may wish to write it on the board.)

> You are at the bank of a river with two pails. The first holds exactly three gallons of water, the second, five gallons; and the pails are not marked for measurement in any other way. By filling and emptying pails, or by transferring water from pail to pail, find a way to carry exactly four gallons of water away from the river. (Luger, 1984)

After the students have had sufficient time to work on the problem, ask them the following questions* -

1. What are the important facts, conditions in the problem?
2. Do you need any information not given in the problem?
3. What question is asked in the problem?
4. Describe how you solved the problem.
5. Do you think you have the right answer? Why? Why not?
6. How did you feel while you were solving this problem?
7. How do you feel after having worked on the problem?

[Note: Questions adapted from Charles, R.; Lester, F., & O'Daffer, P. (1987) *How to evaluate progress in problem solving*. Reston, VA: National Council of Teachers of Mathematics.]

D. Suggestions for the analysis of the student's performance:

- The above questions can be asked individually or in small groups in an interview setting.
- The primary purpose of the assessment is to determine how well the students can communicate orally the processes they used to solve the problem.
- You may wish to make notes as you interview the students concerning points that were not expressed clearly. The emphasis is on communicative competence.

E. Note on item construction:

You may select any problem solving item which is appropriate for the grade level of the students.

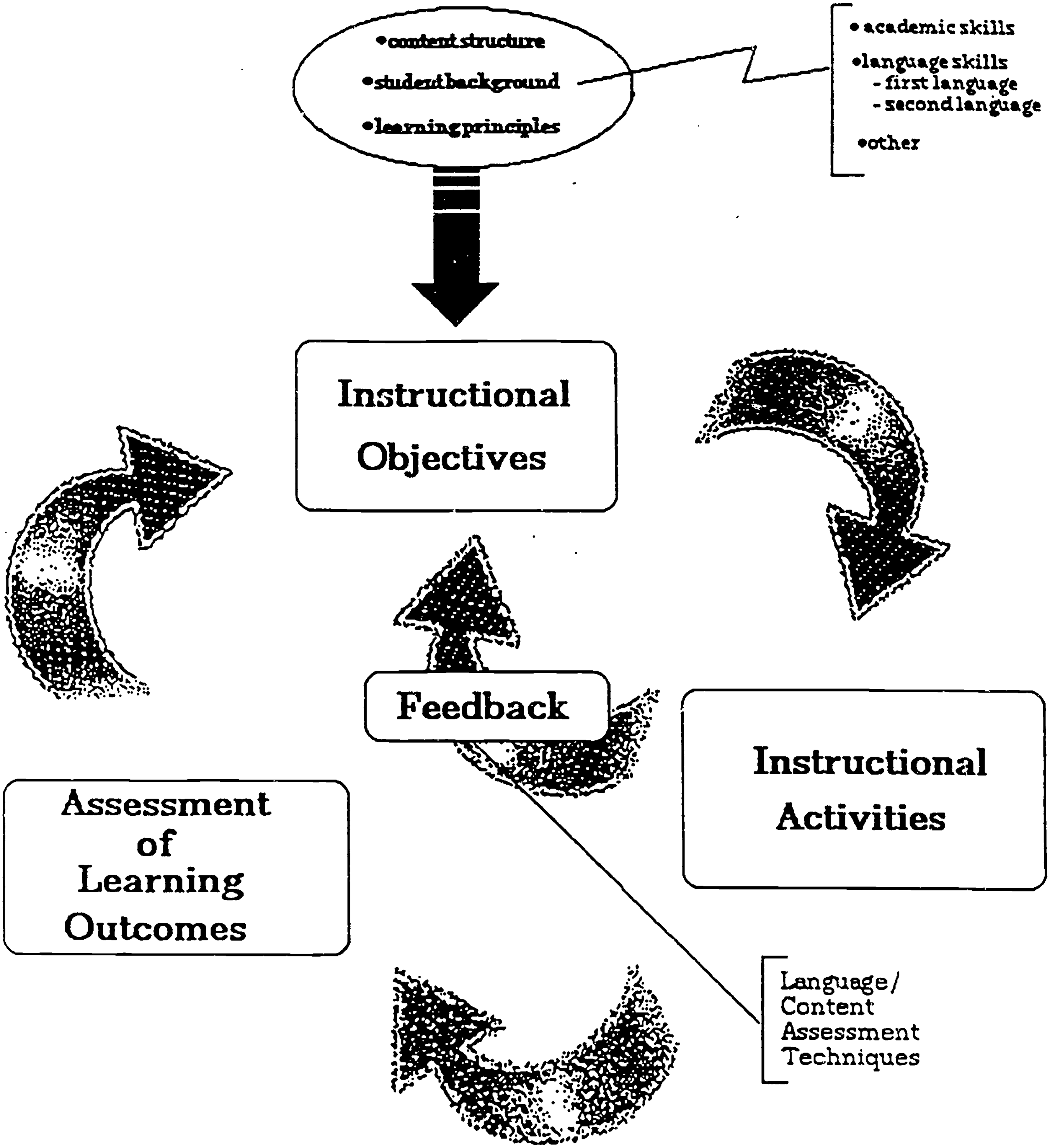

•content structure
•student background
•learning principles
• academic skills
•language skills
- first language
- second language
•other
Instructional Objectives
Instructional Activities
Feedback
Assessment of Learning Outcomes
Language/ Content Assessment Techniques

Figure 1

MATHEMATICS CONTENT AREAS

MATHEMATICS CONTENT AREAS	RECEPTIVE	PRODUCTION
PROBLEM SOLVING	Techniques 19-21	
WORD PROBLEMS	Techniques 13-15	Techniques 16-18
OPERATIONS	Techniques 7-9	Techniques 10-12
CONCEPTS	Techniques 1-3	Techniques 4-6

LANGUAGE SKILL AREAS

Figure 2

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