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ABSTRACT

This paper reviews literature documenting gender differences in beliefs related to mathematics achievement. The areas included in the discussion are: (1) gender differences in mathematics achievement and self-referenced cognitions, (2) the effect of confidence and anxiety on mathematics performance and expenditure of effort, and (3) confidence and doubt while solving mathematics problems. Contains 3 figures and 21 references. (MKR)

CONFIDENCE AND DOUBT IN RELATION TO MATHEMATICS

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CONFIDENCE AND DOUBT IN RELATION TO MATHEMATICS

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Gender Differences in Mathematics Achievement and Self-referenced Cognitions

Within mathematical problem solving a distinction can be made between cognitive and affective aspects. Examples of cognitive aspects are operations on the knowledge base, which include committing facts, definitions, rules, and principles to memory and activating and applying this type of knowledge on a later occasion. Doing mathematics requires also an understanding of when and how to use this knowledge (meta-cognition). Furthermore, decisions that have to be taken during mathematical problem solving can be influenced by all sorts of affective factors, such as confidence in one's mathematical ability, the capacity to motivate oneself to start on a problem, and mental processes that link personal goals to self-regulatory activities. It is important to note that both cognitive and affective processes involved in problem solving are subject to monitoring and control and that such self-regulatory activities are crucial elements of problem solving (cf., Boekaerts, 1994a).

Several authors drew attention to the influence of affective states on the problem solving process. For example, Mandler (1989) clarified that affective responses may occur prior to any attempt to solve the problem. The beliefs that students have about mathematics could be seen as a factor affecting their confidence and anxiety. When students believe that all mathematical problems have to be solved by applying specific rules activated from memory, they may feel stuck after having tried in vain to activate a rule. These students may easily doubt that they can solve the problem, which may in turn lead them to experience anxiety and be hesitant to start or continue with a problem. Another factor that may cause anxiety or doubt are beliefs and attitudes in relation to one's ability in math.

There is a vast body of knowledge illustrating that differences in achievement-related beliefs favor males (Sherman, 1980; Oakes, 1990; Leder, 1992). Gender related differences in mathematics performance seem to be present from an early age. By the time students reach high school, boys score higher on achievement test that require complex mathematical problem solving (Martin & Hoover, 1987) and they take more advanced math coursed than girls (Eccles et al., 1985). These differences in achievement and participation have been paralleled by differences in achievement related beliefs and attitudes. More specifically, boys have higher performance expectations than girls even if girls have equal or better results (Eccles et al., 1985). They also view math as more personally relevant than girls who successfully completed math courses (Van de Werf, 1988). Boys also show a more positive attitude towards mathematics than girls (Sherman, 1980) and link success more to capacity and failure to luck (Fennema, 1985). There is general consensus in the literature regarding the negative effect of these trait characteristics on math performance and participation. But, the exact mechanism through which these self-referenced cognitions exert their negative effect is unknown.

In order to gain insight into the joint effects of cognitive and affective variables on effort expenditure and on actual math performance, I measured affective variables in actual mathematics learning situations with the On-Line Motivation Questionnaire (Boekaerts, 1985, 1988). In later research, an extension of this questionnaire was used, namely the confidence and Doubt Scale. Results so far warrant the conclusion that, at the end of primary education, Dutch boys and girls do neither differ in mathematical problem solving ability nor in solution time. However, they differ in the way they make sense of mathematics learning situations, which in turn influences their math performance. More specifically, it was found that when math assignments were given to students, aged between ten and twelve, in a setting that resembled test taking situations (i.e., individual seat work and handing in the completed assignments when the task is finished) boys outperformed girls. The differences between the sexes were more pronounced when difficulty of the items increased (see, Seegers & Boekaerts, 1993). However, no differences were found in mathematical problem solving capacity when students were asked to do math tasks under individual testing conditions, i.e., with the experimenter informing the students whether an assignment was correctly completed or not and giving them a second chance to solve the problem if they wished to do so (Boekaerts, Seegers & Vermeer, in press). Nonetheless, marked gender differences were noted on the affective variables: Before starting with the math tasks, boys expressed more confidence and judged the math tasks as more fun than did girls. They also expressed a higher learning intention and displayed a more positive emotional state than did girls. But, contrary to the results reported by other authors, no differences were noted in the students' judgement of the relevance of mathematics tasks. There were also no differences in self-assessment, reported effort expenditure and emotions after completing the math assignments. Furthermore, boys and girls attributed their result to the same causes, except that boys attributed their perceived positive outcome more to task attraction than did girls.

Seegers and Boekaerts (in press) used the LISREL 7 program to investigate whether gender differences in the scores on the affective variables reflect a different pattern of relations between the variables. They concluded that the same underlying mechanism induces learning intention for math tasks in boys and girls and affects their math performance. However, inspection of the parameter values and the variance explained warrants the conclusion that boys display more confidence in their capacity to do math than do girls. These positive beliefs about their capacity to do math turn math situations into interesting activities that deserve effort and into successes for which they take full credit. Girls, on the other hand, reveal a lower degree of control over math situations and these unfavorable beliefs about their capacity to do math surface in actual problem situations and turn math experiences into threatening, non-interesting experiences with low performance scores.

The Effect of Confidence and Anxiety on Math Performance and on Effort Expenditure

Gender differences in affective variables were also found in a sample of secondary school students, aged between twelve and fourteen. In this longitudinal study (Boekaerts & Otten, in preparation), the on-line motivation questionnaire was used, as well as a trait measure of fear of failure, and Kuhl's Action Control Scale (see, Kuhl, 1985, 1994; and Boekaerts, 1994b). The action control scale measures the psychological processes that lead to the initiating, maintaining and disengaging of desired behavior. The first subscale, initiative, measures the students' capacity to couple a behavioral intention to an action plan in the orientation stage of every-day activities. The second subscale, persistence, measures their capacity to keep these two aspects coupled during the execution stage. The third subscale, disengagement, assesses their capacity to uncouple intentions and actions when the objectives cannot be reached. It was found that girls scored higher than boys on persistence. No significant differences were found on initiative or disengagement. Girls scored also higher on fear of failure. With respect to task-specific variables, boys scored higher than girls on confidence and lower on state anxiety (bodily feelings) measured just before starting with the task. There were no differences with respect to perceived relevance or task attraction. After completing the math task the girls assessed their performance lower than the boys did. There were however no gender differences in capacity to do math, as judged by the students' score on grade-point-average (GPA) at the end of the semester following each of the two periods of data collection. The same gender differences were found in the two data waves investigated so far. But, interestingly, the girls' performance on the curriculum based math tasks used in this study was significantly lower than the boys' in the first but not in the second testing session. This finding could signify that the girls gave a different interpretation to the second testing session than to the first. It is easy to envisage that when the girls have become more familiar with the tasks and activities they appraise them as less threatening.

In order to investigate the effect of negative cognition and affect on upcoming and ongoing math performance, Boekaerts (1994c) constructed a structural model and applied it to the data from the second wave of the longitudinal study. In the model fear of failure (trait), disengagement (trait), and GPA in mathematics were selected as exogenous variables, task-specific confidence, anxiety and perceived personal relevance as the endogenous variables, and math performance and reported effort as the outcome variables. It was hypothesized that fear of failure and the students' capacity to cope with failure by uncoupling behavioral intentions and action plans (Kuhl's disengagement construct) would affect math performance and effort expenditure through their effect on task-specific confidence and anxiety. Two mechanism were postulated: viz. a cognitive mechanism and a motivational mechanism. More specifically, it was hypothesized that students who are doubtful that they can find adequate strategies to solve math problems will be hindered more by intrusive thoughts before starting the task and during the task. This pre-occupation calls for divided attention which impairs math performance (cognitive mechanism). In a dissimilar vein, it is hypothesized that when these intrusive thoughts raise the level of anxiety, students will interpret the experienced bodily symptoms as a threat signal. Such emotional turbulence, rather than doubt as such, will induce behavioral tendencies to disengage from the task (either mentally or behaviorally) reflected in lower reported effort at task-offset (motivational mechanism).

Further, it was theorized that students who have access to self-regulatory control, especially disengagement, will suffer less. More concretely, students who score high on disengagement (i.e., those who are not pre-occupied with failure, but, can uncouple behavioral intentions from action plans in a context-sensitive way) will experience less doubt before and during math tasks resulting in higher performance. In other words, disengagement can counteract the cognitive mechanism. These students will also experience less anxiety resulting in higher reported effort after the task (disengagement also counteracts the motivational mechanism). These hypotheses were tested with the aid of the LISREL 7 program (Jöreskog & Sörbom, 1989) on the second data wave.

Figure 1 about here

Close analysis of the model revealed that the postulated links between confidence and math performance (cognitive mechanism) and between anxiety and effort expenditure (motivational mechanism) were confirmed. Confidence directly affected math performance, but not reported effort; whereas state anxiety directly affected reported effort, but, not performance. The students' perception of task relevance was not linked to either confidence or state anxiety, but had a direct positive effect on reported effort. Fear of failure did neither affect the cognitive mechanism nor the motivational mechanisms when disengagement was in the model. The students' ability to regulate their behavior when intrusive thoughts or emotional turbulence occurs (disengagement) affected confidence directly and state anxiety both directly, and indirectly via confidence. These results allow for the inference that self-regulatory control, in the sense of being able to uncouple behavioral intentions and action plans in a context-sensitive way, facilitates positive thinking about math tasks (confidence) and decreases anxiety before the task. Boys and girls did not differ with respect to disengagement, but girls scored lower on task-specific confidence and higher on task-specific anxiety.

Both doubt about one's ability to do math task and emotional turbulence imply a disturbed person-task relationship which is reflected in an imbalance in the performance-oriented and in the effort-oriented pathways. This imbalance can be restored by the capacity to use self-regulatory strategies, especially disengagement, in a flexible and context-sensitive way. A puzzling question is why the person-task relationship is more disturbed in girls than in boys?

Confidence and Doubt While Solving Mathematics Problems

The Confidence and Doubt Scale makes it possible to explore the mechanism by which gender differences in achievement-related beliefs and attitudes might influence the processes that occur during mathematical problem solving. As can be seen in Figure 2, this scale can be used to measure the students' use of problem solving rules as well as the degree of confidence and doubt displayed at each of the steps in the problem-solving process. The students are asked to write down their solution process and calculations in as detailed a manner as possible and to put a mark under one of the faces arranged along a five point scale, ranging from a very sad face (low confidence in finding an adequate solution) to a very happy face (high confidence). This scoring system provides at least three scores of confidence and doubt, namely an initial indication after first reading the problem statement (orientation stage), a second indication during the actual problem solving process (aggregation of the degree of confidence and doubt expressed at each separate step of the execution stage), and finally an indication of confidence and doubt in relation to the solution (verification stage). It is assumed that the students' estimation of the extent to which they can (still) succeed on a task is determined by (1) their beliefs and attitudes about mathematics learning, (2) their expectancies concerning successful goal attainment as the problem solving process unfolds, and (3) their reactions to success and failure when it actually occurs.

Figure 2 about here

Several pilot studies have been conducted with the confidence and doubt scale. Boekaerts et al. (in press) reported that confidence and doubt expressed in the different stages of the problem solving process is not necessarily consistent with the use of a correct problem solving strategy. For some problems girls expressed less confidence than did boys, even though they equally succeeded in doing the problems. These findings are in line with the results obtained with the on-line motivation questionnaire and they parallel the results obtained with trait-measures of motivation. In a further study a distinction was made between algorithmic problems and word problems. The former types of problems are easier than the latter because the sequence of steps which students have to take in order to solve these problems is less ambiguous and more systematic than that required to do a word problem. Vermeer, Seegers and Boekaerts (1994) displayed that, in 11- to 12-year olds, there were no gender differences in confidence ratings after first reading an algorithmic problem statement (divide 357,75 by 28,80), but that girls expressed considerably more doubt than boys after first reading the word problem printed in Figure 2. On the basis of a content analysis of the answers generated by the students, efficient and inefficient problem solving strategies were identified. This procedure revealed that 92% of the students used an efficient strategy for the algorithmic problem (96% girls and 88% boys) whereas only 55% used an efficient strategy (61% boys and 48% girls) for the word problem.

Figure 3 (a,b,c) about here

In Figure 3 (a and b) gender differences in confidence ratings with respect to the three stages of the problem solving process are displayed, separately for the algorithmic and the word problems. Girls expressed equal or higher confidence than did boys in relation to the algorithmic problem, but they scored significantly lower than the boys with respect to the word problem in all stages of the problem solving process. Students who used an efficient strategy for the word problem differed from students who used an inefficient strategy in the degree of confidence expressed in the orientation and execution stages. There were no significant differences in the verification stage. It is interesting to note, that these two groups had similar confidence ratings *before* starting with the math assignments. Neither were differences noted in task attraction, learning intention and emotional state before starting on the tasks. But, students who used an efficient strategy reported more positive emotions than the students who did not *after* they had completed the series of math tasks. No differences were noted in result assessment and reported effort. In Figure 3c gender differences are displayed. As can be seen, girls who used an efficient strategy expressed more doubt in the orientation stage of word problems, they gave a more conservative indication of their confidence in the execution stage, and scored equally high in the verification stage.

Putting It All Together

I have referred to a vast body of literature that documents gender differences in achievement-related beliefs. These differences in beliefs, measured at the general or middle level, are held responsible for girls' lower math performance and lower degree of participation. In our own studies with the on-line motivation questionnaire situation-specific measures of affective variables are used. Our findings showed that differences in achievement-related beliefs are associated with favorable or unfavorable appraisals of actual mathematics tasks. In threatening settings boys outperformed girls on complex math tasks. They also had a different scoring pattern on the affective variables. Nevertheless, it was demonstrated that the same mechanism underlies the relation between the appraisals, on the one hand, and the intention to expend effort for math tasks and math performance, on the other. When students were tested individually, there were neither gender differences in math performance nor in solution time. The gender differences in confidence expressed before starting with the math assignments did not show up in the students' confidence ratings *while* involved in algorithmic problems. But, these differences were evident in the orientation stage of solving the word problem. Looking at the scoring pattern of boys and girls who used an efficient strategy revealed that the former showed more confidence in relation to the word problem than with respect to the algorithm. This was most evident in the execution stage. For the girls the picture was reversed. Girls who used an efficient strategy displayed lower confidence ratings in the orientation stage of word problems than did boys.

Why should girls who can generate an efficient strategy for solving mathematics problems doubt their capacity to solve word problems and not their capacity to solve algorithmic problems? Doubt implies a disturbed person-task relationship that flexible strategy use is meant to change. When girls realize after first reading a problem statement that they have access to a systematic plan for solving it, e.g., because an explicit or implicit cue is available, they may classify the problem tentatively as a 'practicable problem'. Such labelling may signify that they have and also anticipate neutral or positive feelings. Actual interruptions due to false starts or slips of strategy may still occur in the execution stage. But, I would suggest, that when such interruptions occur they only elicit negative feeling states to the extent that the student does not have access to an alternative plan of action. Boekaerts, Seegers and Vermeer (in press) reported that some students switch blindly from one algorithm to another (first divide, then multiply, then subtract, then add) without showing a dip in their confidence ratings. This blind switching from one routine calculation to another suggests, firstly, that they do this without any understanding of the math principles involved, and secondly, that negative feelings and avoidance behavior are postponed till they actually got stuck. At the point when they cannot perform any more routine calculations, students may realize that they incorrectly classified the problem as 'practicable'.

It may be theorized that some students, and girls in particular, have the tendency to give the label 'impracticable' to those problems that require modeling, i.e., those problems that must be transformed into a new problem statement by exploration, experimentation and reflection. When this is the case they perceive the problem as 'ambiguous' or 'high on decision making' (a problem solving situation which requires considerable understanding of the elements involved). It is hypothesized that mathematics problems perceived as 'ambiguous' by the students are likely candidates for creating a disturbed task-person relationship. For algorithmic problems such imbalance may be situated later in the problem solving process (i.e., when students get stuck in the execution stage or when the problem did not work out) than for word problems. For the latter type of problem the imbalance will already be apparent in the orientation stage.

It is reasonable to assume that individual differences in mathematical disposition (cf., Perkings, Jay & Tishman, 1993) and the ways in which mathematics teaching compliments these differences determines to a large extent the degree to which students express confidence and doubt about mathematical problem solving. These authors emphasize that the three components of mathematical disposition, viz., ability, inclination, and sensitivity jointly affect mathematics learning. Some students may have the ability, but, lack either the inclination (motivation) or the sensitivity (feeling for and alertness to opportunities for using the actual skills). Traditional textbooks of mathematics do not set the scene for developing inclination and sensitivity. A heavy emphasis is still put on computational skills and the activity of modeling mathematics problems is still largely neglected. This implies that students who can mentally represent the problem and who possess the skill to generate the necessary cognitive strategies on the basis of their meta-cognitive knowledge develop favorable beliefs about mathematics. Students who need explicit instructions and plenty of practice to model new types of math problems are at a double disadvantage: they do not acquire the meta-cognitive knowledge necessary for generating cognitive strategies in a context-sensitive way, and neither do they develop a positive attitude toward exploration and reflection in a mathematical context.

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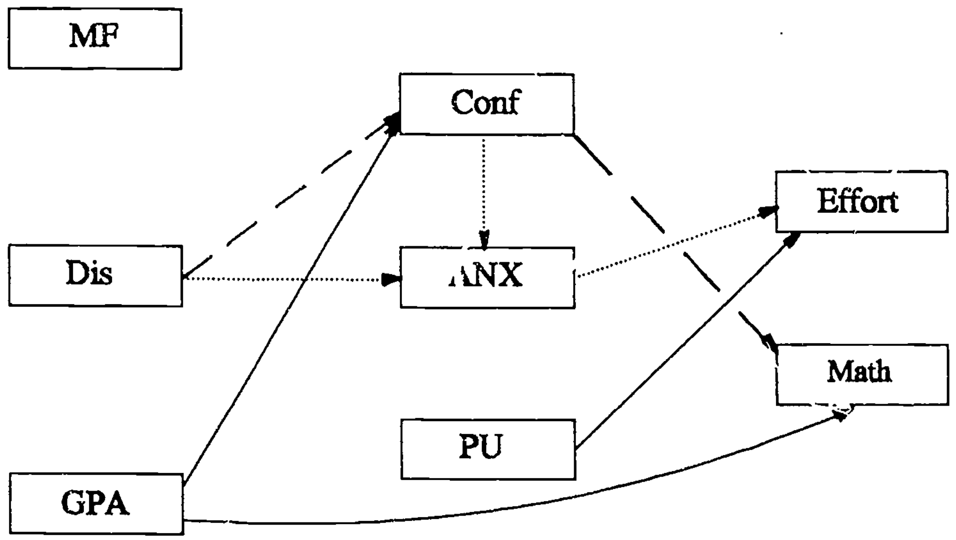
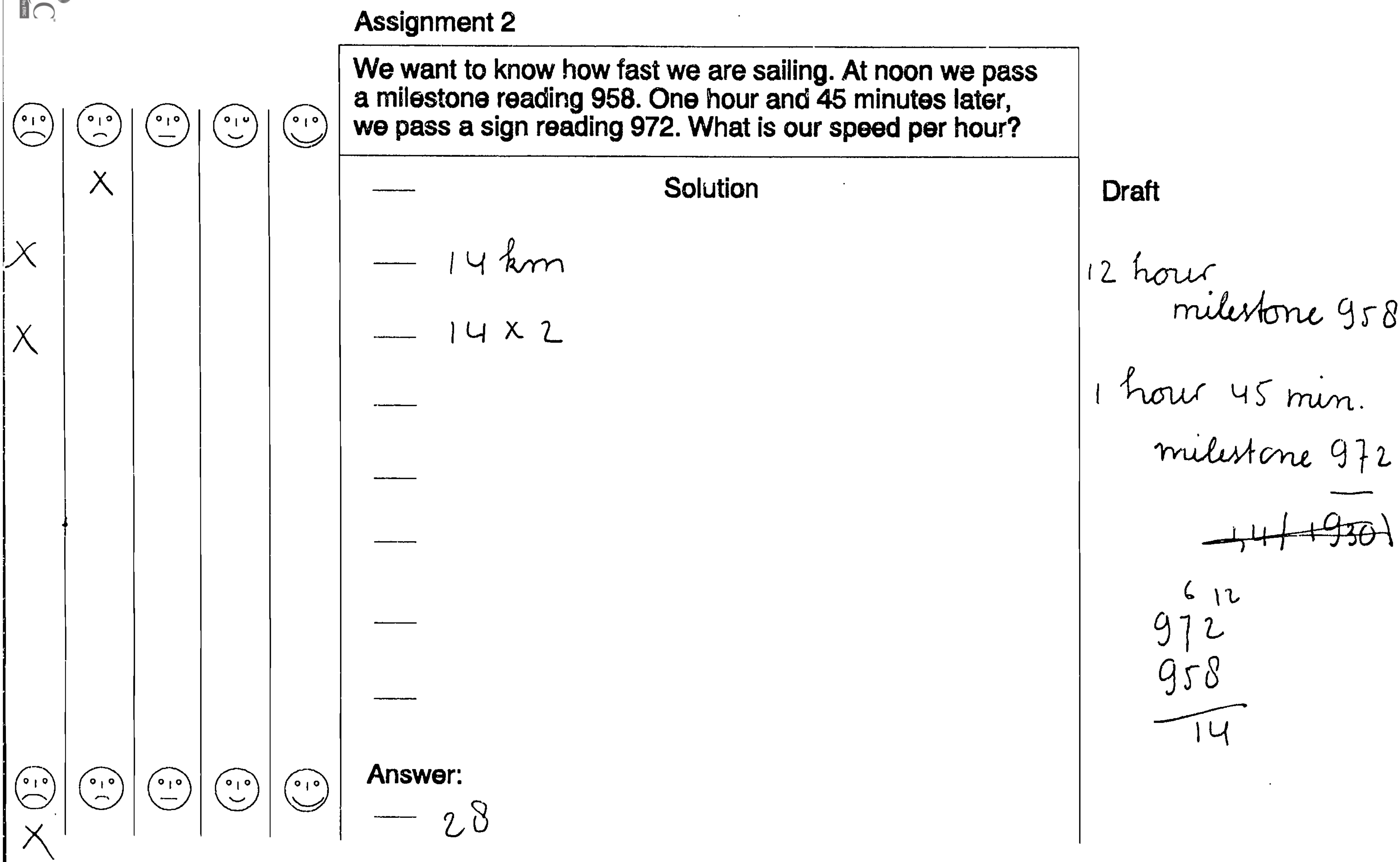


Figure 1.
Evidence for a cognitive mechanism affecting math performance (symbolized by - - -) and a motivational mechanism affecting effort expenditure (symbolized by .....)



Assignment 2

We want to know how fast we are sailing. At noon we pass a milestone reading 958. One hour and 45 minutes later, we pass a sign reading 972. What is our speed per hour?

☹	🙁	😐	🙂	😊	Solution
	X				—
X					— 14 km
X					— 14 x 2
					—
					—
					—
					—
					—
☹	🙁	😐	🙂	😊	Answer:
X					— 28

Draft

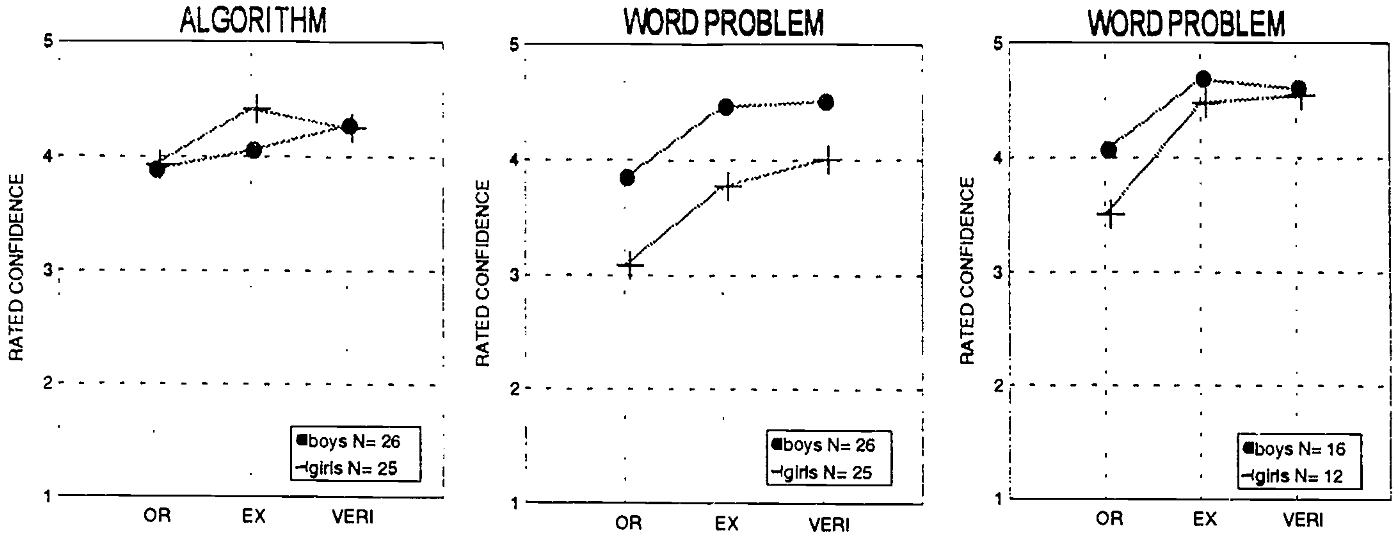
12 hour
milestone 958

1 hour 45 min.
milestone 972

~~14/1930)~~

$$\begin{array}{r} \overset{6}{9}\overset{12}{7}2 \\ 958 \\ \hline 14 \end{array}$$

Figure 2.
Confidence and Doubt Scale as applied to a word problem



a

b

c

Figure 3.
Rated confidence in the 3 stages of the problem solving process. a for algorithms. b and c for word problems (in c only the students who used an efficient strategy were included)