DOCUMENT RESUME

ED 375 008 SE 055 324

AUTHOR	Dockweiler, Clarence J.
TITLE	Children's Attainment of Mathematical Concepts: A Model under Development.
PUB DATE	Aug 94
NOTE	18p.
PUB TYPE	Reports - General (140) -- Viewpoints (Opinion/Position Papers, Essays, etc.) (120)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Concept Formation; Elementary Secondary Education; *Mathematical Concepts; Mathematics Education; *Mathematics Instruction; Models
IDENTIFIERS	*Mathematics Activities

ABSTRACT

This paper describes a model of student attainment of mathematical concepts and its development. In this model three types of activities (developmental, connecting, and abstract) are considered as an overlay o: the three ways of representing mathematical concepts (physical/visual, oral, and symbolic). Each activity type involves some means of representing mathematical concepts, and the activities are sequentially presented as the learner attempts to master a mathematical idea. Initial exposure comes from Developmental Activities, followed by Connecting Activities, and finally, Abstract Activities are utilized. (Author/MKR)

Children's Attainment of Mathematical Concepts: A Model Under Development

by

Clarence J. Dockweiler
Texas A&M University

The preservice and in-service education of mathematics teachers is indeed a rewarding experience. There is a frustration or two along the way, but, generally, efforts to help teachers help children learn mathematics is fulfilling. Observations of the teaching of mathematics and teachers education in a few other countries, indicates the difficulties are not unique to the United States.

Often the difficulties arise because the presentation of mathematical ideas occurs in abstract, symbolic ways. To emphasize the necessity for concrete representations for young learners, I have frequently used the following expression on preservice pedagogy classes to emphasize the importance of providing children with physical models for mathematical concepts: 'If you can't model it, don't teach it.' I have come to the realization that the physical and/or visual modeling of mathematical concepts is of the utmost importance for children to internalize and master these concepts. I continue to challenge my undergraduate and graduate students to come up with a mathematical concept or idea, to be taught at elementary school levels, for which this modeling is not necessary. Nothing yet! There is, obviously, more to teaching mathematical concepts than just being creative in modeling them in some physical way. My purpose is, therefore, to present a way of thinking about the student acquisition of mathematical concepts which incorporates the modeling of those concepts. This simple model is not an entirely new idea and has been influenced by many factors over the years. Not the least of which is the observation and analysis of children's efforts to understand mathematical mysteries.

Contributions to the Structure of a Theoretical Model

The development of my approach has occurred over a considerable time and some reference to the origins may provide some credibility and permit the reader to participate in the developmental process. A number of years ago, I had the occasion to re-read a portion of Vincent Glennon's contribution to the 17th Yearbook of the National Council of Teachers of Mathematics (Glennon 1963). His premise was that a balance was necessary in elementary school mathematics and that balance needed to occur between the three theories of curriculum organization. He identified these three theories as: the mathematical, the social, and the psychological. Glennon translated these three theories into 'needs-of-the-subjects,' 'needs-of-society,' and 'needs-of-the-child,' respectively. These became corners of a triangle so that when all three needs were exerting a similar pressure, a curricular balance was achieved. At the time, it was an intriguing idea and caused considerable thought on my part and was useful in the development and future application of this triangular model. (See Figure 1)

Later this balance was used in a development by Trafton (1975) to present the necessary interactions which occur in the student development, mastery, and application of mathematical concepts. His model also used a triangular figure to illustrate his idea. The corners of his figure represent 'Concepts,' 'Skills,' and 'Applications' as shown in Figure 2. A simplistic interpretation of this model suggests that

- concepts may give rise to skills and vice versa,

- applications may require certain skills and vice versa, and
- concepts may lead to applications and vice versa.

The interactions are described as important to the proper development of mathematical concepts in students.

The formation of my model has its beginning. The triangular figure seems to fit. The concern, however, is to construct a representation which will adequately reflect the total internalization and mastery of a mathematical concept. The corners of a the triangle, therefore, assume roles which reflect ways of expressing concepts. That is, how can mathematical ideas be presented to children in a meaningful way?

One way to communicate an aspect of a concept is in a physical (or visual, if appropriate) manner. That assumes that a mathematical abstraction is derived from a physical material. We refer to these physical materials as manipulatives. For the young learner, the concept of number is attached in some way to a set of blocks or a group of family members. Ideas about geometric shapes may be found in the shape of a window or a ball.

A second way (or corner of the triangular model) to represent a mathematical idea is through oral communication. Early efforts with children are full of attempts to help them know the number names and attach the proper mathematical descriptors to ideas, such as, straight and long. This oral communication will eventually be extended to terms appearing in written form, in printed text.

The third corner of the model is the symbolic form used to represent mathematical ideas (i.e. the symbol 23 represents the number twenty three). This is, obviously, the most abstract representation and is the most difficult for children to fully understand. These three representations are to include all possible ways children are exposed to mathematical concepts.

This transitional stage of the model's development was referred to as "The Six-Way Key to Understanding." The implication is that mastery of the six possible inter-relationships between these factors is critical to complete understanding of a mathematical concept and its use. To be assured of a child's understanding of a mathematical idea, for example, the number 23, the child should be able to communicate the idea in all six ways as follows:

1. Given a physical representation of 23 with place value blocks, say the number name (physical to oral).

2. Say the number name, twenty three, and construct the physical representation from place value blocks (oral to physical).

3. Construct twenty three from place value blocks and have the child write or identify the proper symbolic form (23) for the number (physical to symbolic).

4. Present the symbolic form (23) and construct the number with place value blocks (symbolic to physical).

5. Say the number name, 'twenty three,' and write or identify the proper symbolic form (23 for the number (oral to symbolic).

6. Show the symbolic form (23) and have the student say the proper number name.

It seems reasonable to apply this model to other mathematical concepts of the elementary curriculum and to become convinced, at least in this writer's view, that the physical representation of mathematical ideas and concepts are an important part of the process. Even if the readers are convinced of the value of the model and its applicability to many concepts, it still does not provide means or methods of teaching children.

My more recent insights led me to include methodological considerations. In part, these extensions were triggered by Marilyn Burns' writings and her work with teachers (Burns, 1982). She describes mathematics instruction of young children as occurring at three levels (Concept Level, Connecting Level, and the Symbolic Level). My adaptation of these three levels converts them from levels to activities, since I choose to describe the presentations to children in terms of activity types.

Of particular importance to the development of the model was an initial opportunity to become aware of the work of Professor Richard Skemp of England. Over the years, he has painstakingly developed his theory of intelligence and, in particular, his ideas of how children learn mathematics. With regard to my model for children's attainment of mathematical concepts, Skemp contributes the following (Skemp, 1987, p. 15):

> Only by being detachable from the sensory experiences from which they originated can concepts be collected together as examples from which new concepts of greater abstraction can be formed.

His comment supports the emphasis on connecting activities and the necessity for the learner to eventually detach the connection of the abstract from the physical to assure complete understanding of the concept.

The Model

Attempts to explain the attainment of mathematical concepts have led me to consider the three types of activities (Developmental, Connecting, and Abstract) as an overlay on the three ways of representing mathematical concepts (Physical/Visual, Oral, and Symbolic). Each activity type involves some means of representing mathematical concepts and the activities are sequentially presented as the learner attempts to master a mathematical idea. Initial exposure comes from Developmental Activities, followed by Connecting Activities, and, finally, Abstract Activities are utilized.

Developmental Activities

Developmental activities are those activities which permit children(or any learner) to experience a mathematical concept and to become familiar with the proper terms to describe that concept. In terms of the model described previously as 'A Six Way Key to Understanding,' activities which relate physical representations to oral representations or oral to physical are Developmental Activities. These activities are beginning experiences which permit children to do hands-on manipulation with an emphasis on oral communication and the introduction of terms which describe the particular concept in question. Developmental activities are designed to provide basic understanding of the concept. This basic understanding begins the mental process of abstracting the concept from its physical representation. The symbolic form, however, is not involved in these activities (See Figure 4).

This type of activity is illustrated by young learners counting blocks and making the correct association of terms and counting order to the blocks as they are counted. Similarly, a block in the shape of a triangle is named and characteristics are described (number of corners, straight sides, etc.).

Connecting Activities

After the learner has had adequate experience with Developmental Activities, they should be followed by Connecting Activities. These activities are designed to connect the early mathematical conceptual understandings as represented by physical modeling and the oral representation to their mathematical symbols. With respect to the model, Connecting Activities bring together physical modeling, the oral expression for the concept, and the symbolic form; all three corners of The Model. A natural sequence of activities is suggested. Concepts or ideas are introduced by means of Developmental Activities and as the acquisition of concepts progresses, Connecting Activities are used to give meaning to the appropriate mathematical symbols. The learner is still encouraged to 'see' the mathematical idea in the physical representation, but is gradually introduced to the symbolic form to permit a lasting association to be made (See Figure 5). The example of counting blocks, previously used, now includes a written record of the symbols for the numbers as the blocks are counted, the number names are said and the symbols are referred to.

Abstract Activities

Activities of the third type are referred to as 'abstract' to denote the absence of physical models. Oral and symbolic means of representation are incorporated into Abstract Activities. This is the ultimate level in communicating mathematical ideas and the most difficult level of concept attainment to reach. Appropriate use of Abstract Activities can only occur after the physical and oral aspects of a mathematical idea have been extensively explored in a meaningful way through the use of Developmental and Connecting Activities. (See Figure 6) The learner might be expected to retain the mental image of place value blocks, but, for all practical purposes, the number twenty three (23) is used in conversation and in symbolic form without physical representations. The abstraction has been successfully "abstracted" from the physical representation and is now comfortably represented by the symbolic and oral representations.

Physical - Symbolic Activities?

The thoughtful reader should be wondering about activities which relate physical and symbolic representations without the oral representations, since the full model seems incomplete. Such a combination may productively occur. For example, teachers may provide learners with activity sheets which present pictures of place value blocks and ask for the symbolic form which represents the number shown. This model, however, has been developed with concept attainment in mind and, to the writer, that means the communication of the developing concept to an interested teacher. In ascertaining the level of conceptual understanding it seems necessary to include an oral component to provide further evidence of clear understanding. It has been concluded, therefore, that the physical-symbolic activities may be of some use, but the importance of such activities do not compare with the other activities delineated.

Summary

The model can be a useful tool to teachers who are interested in reaching their students in an appropriate way. There is, obviously, no simple recipe for the application of the various types of activities. Introduction of new concepts to young learners suggests the use of Developmental Activities. However, the appropriate time to change to Connecting Activities is left to the discerning eyes and ears of the experienced teacher. As students gain confidence and understanding of mathematical ideas, the teacher will ascertain the right moment to provide symbolic representations to the physical and oral aspects of the Developmental Activity. For the normal classroom, the Connecting Activity may be the type of activity most often utilized. It is the student's connection of the mental construct which has been established through visual and oral means to the symbolic representation which is a key ingredient to successfully utilizing that understanding in it's abstract form. Moving to Abstract Activities should occur only after the learner has the concept well-established. Traditional classroom activities have relied to heavily on the abstract/symbolic forms prior to conceptual understanding. Consequently, the ability of the student to successfully utilize the concepts in future developments is hindered. (See Figure 7)

An appropriate closing comment is taken from the K-4 section of the *Curriculum and Evaluation Standards for School Mathematics.* One of several assumptions for this section includes the following statement:

> A conceptual approach enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstraction to emerge from empirical evidence. (NCTM, 1989, p. 17).

References

Burns, Marilyn. Meeting the Math Challenge. Palo Alto, California: Learning Institute, Pitman Learning,Inc. 1982.

Glennon, Vincent J. "Some Perspectives in Education." Enrichment Mathematics for the Grades. Washington, D.C.: 27th Yearbook, National Council of Teachers of Mathematics, 1963.

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. Curriculum and Evaluation Standards for School Mathematics. Reston, Virginia: National Council of Teachers of Mathematics. 1989.

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. Professional Standards for Teaching Mathematics. Reston, Virginia: National Council of Teachers of Mathematics. 1991.

Skemp, Richard. Psychology of Learning Mathematics. Hillsdale, New Jersey: Lawrence Erlbaum Associates. 1987.

Trafton, Paul R. "The Curriculum." Mathematics Learning in Early Childhood. Washington, D.C.: 37th Yearbook, National Council of Teachers of Mathematics. 1975.

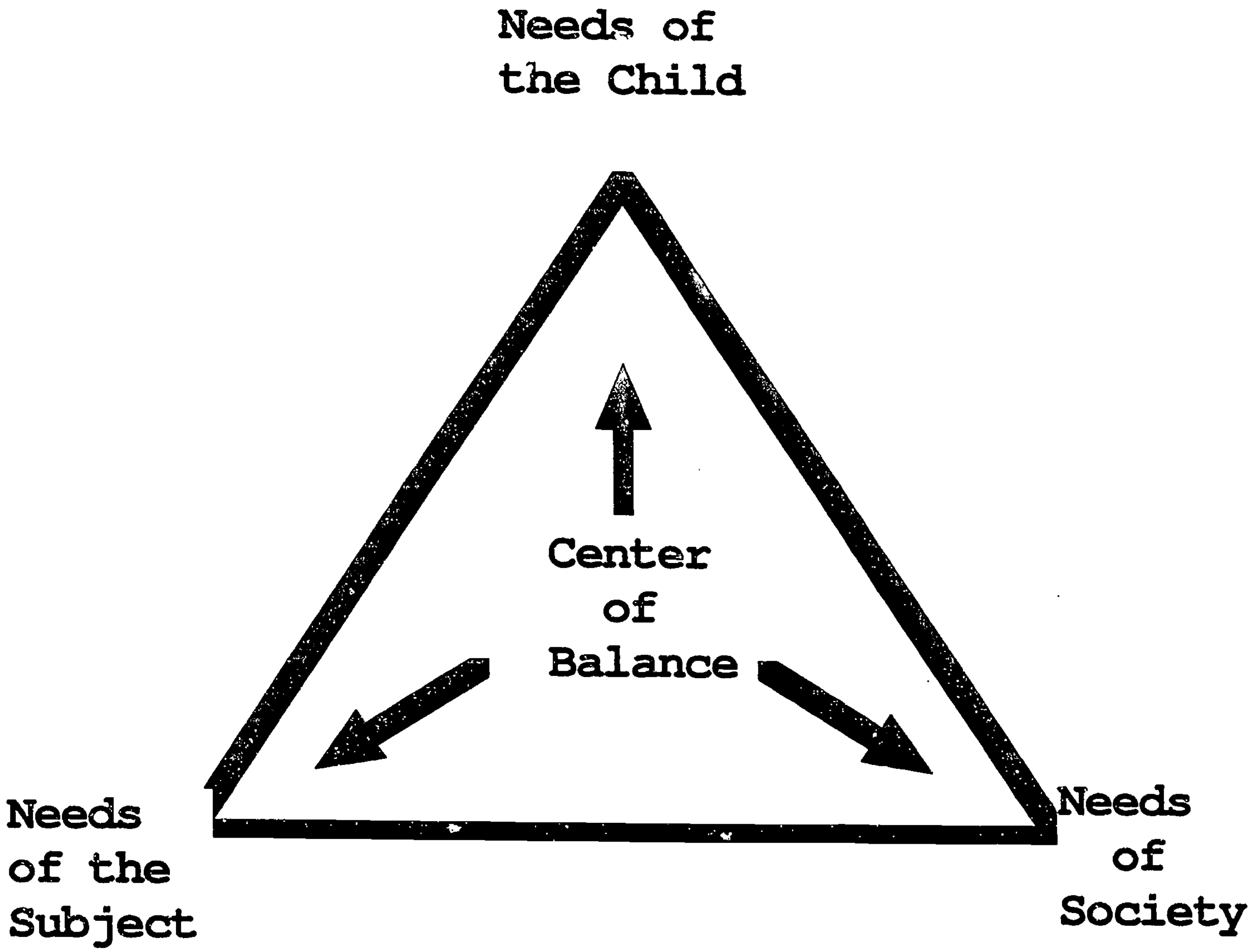

Figure 1
Glennon's Triangle

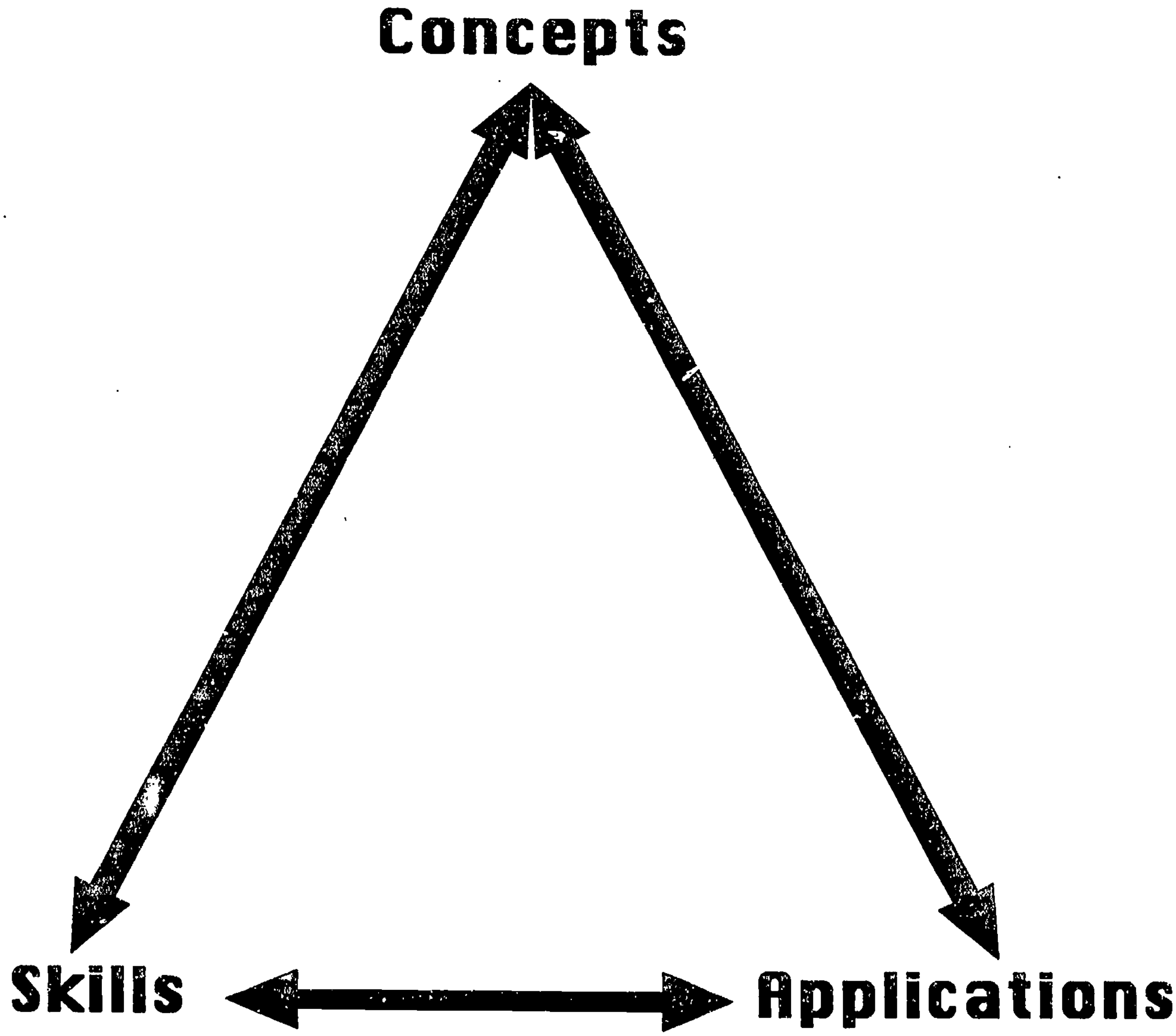

Figure 2
Trafton's Triangle

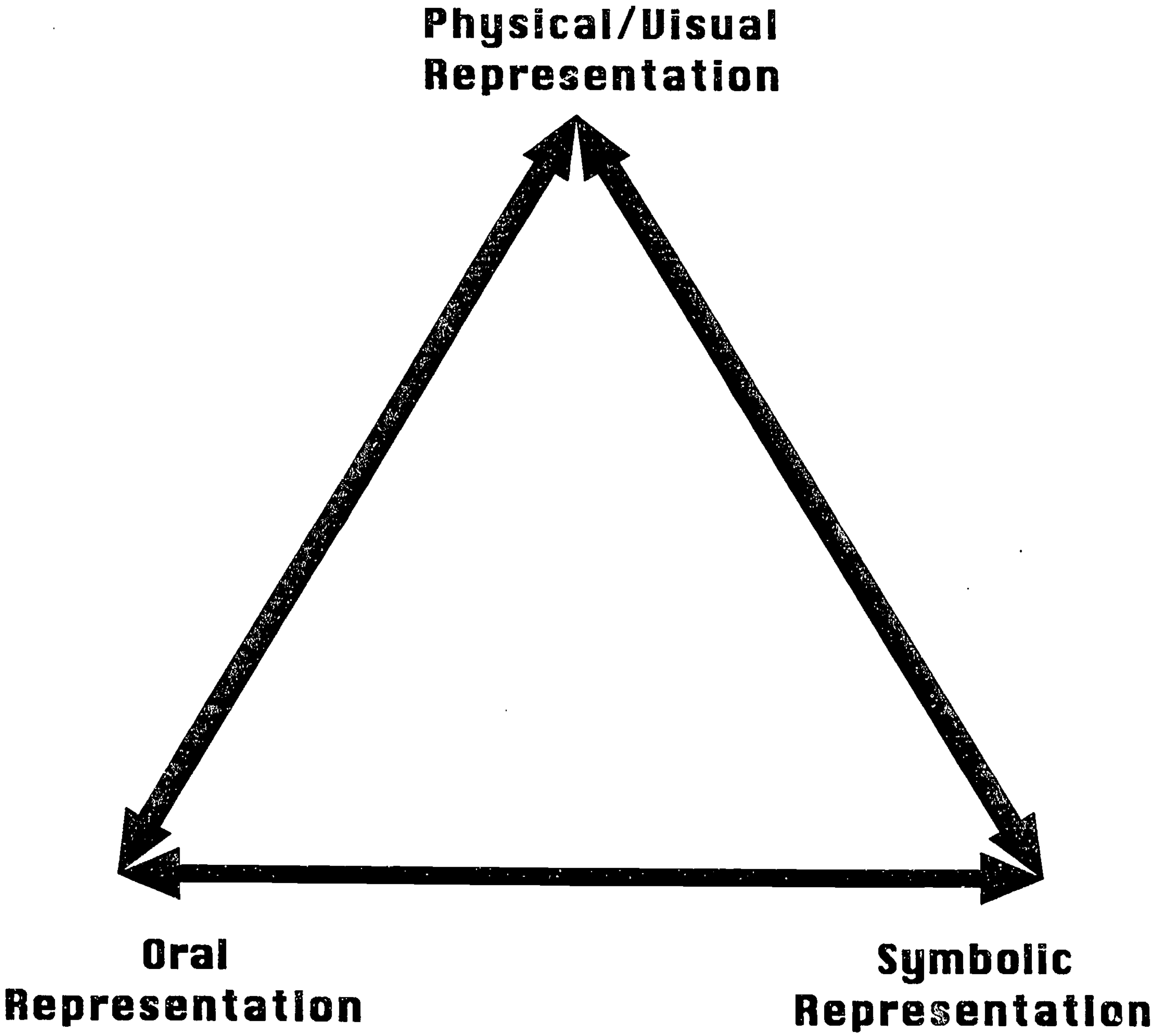

Figure 3
Representing Ideas

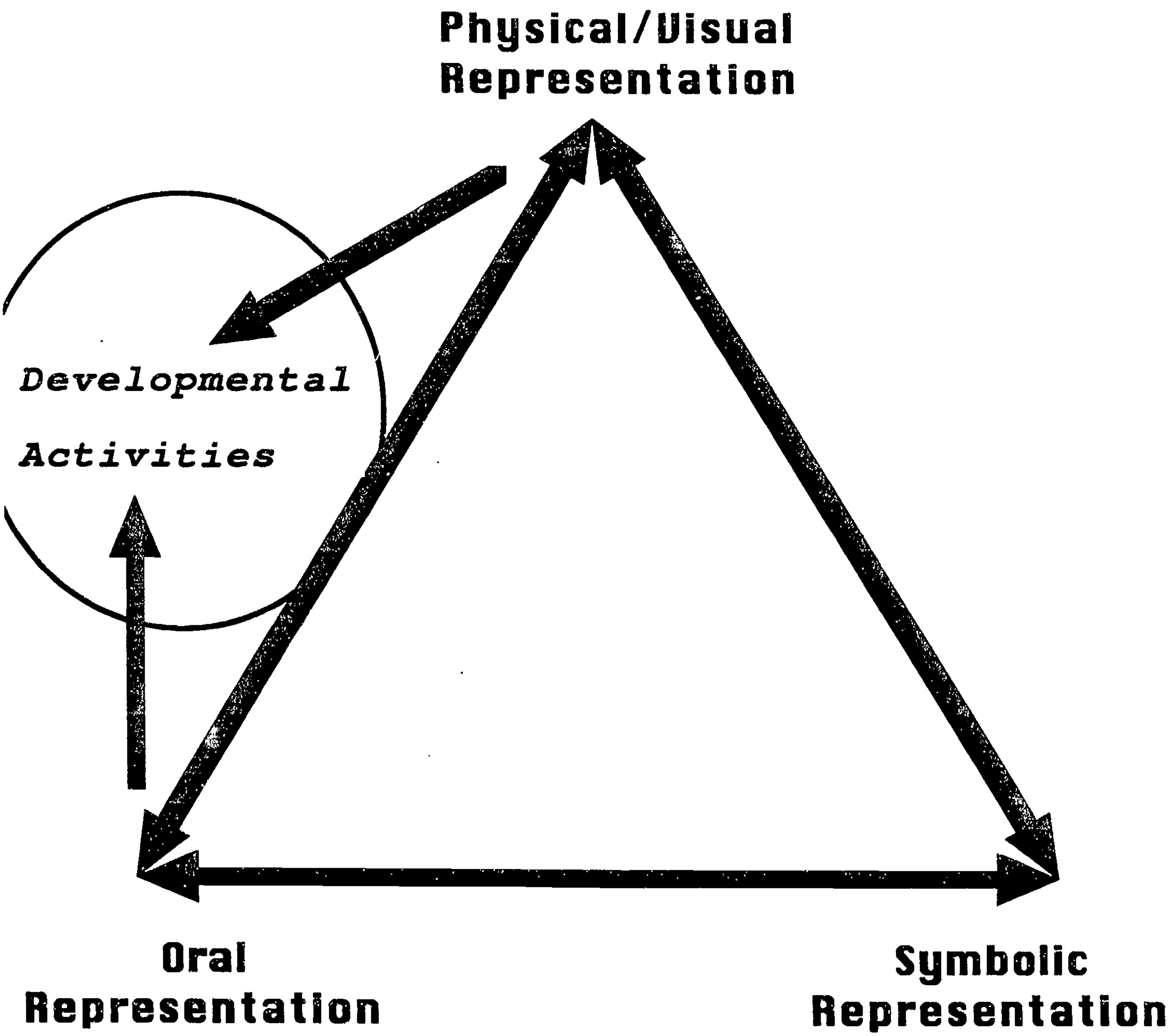

Figure 4
Developmental Activities

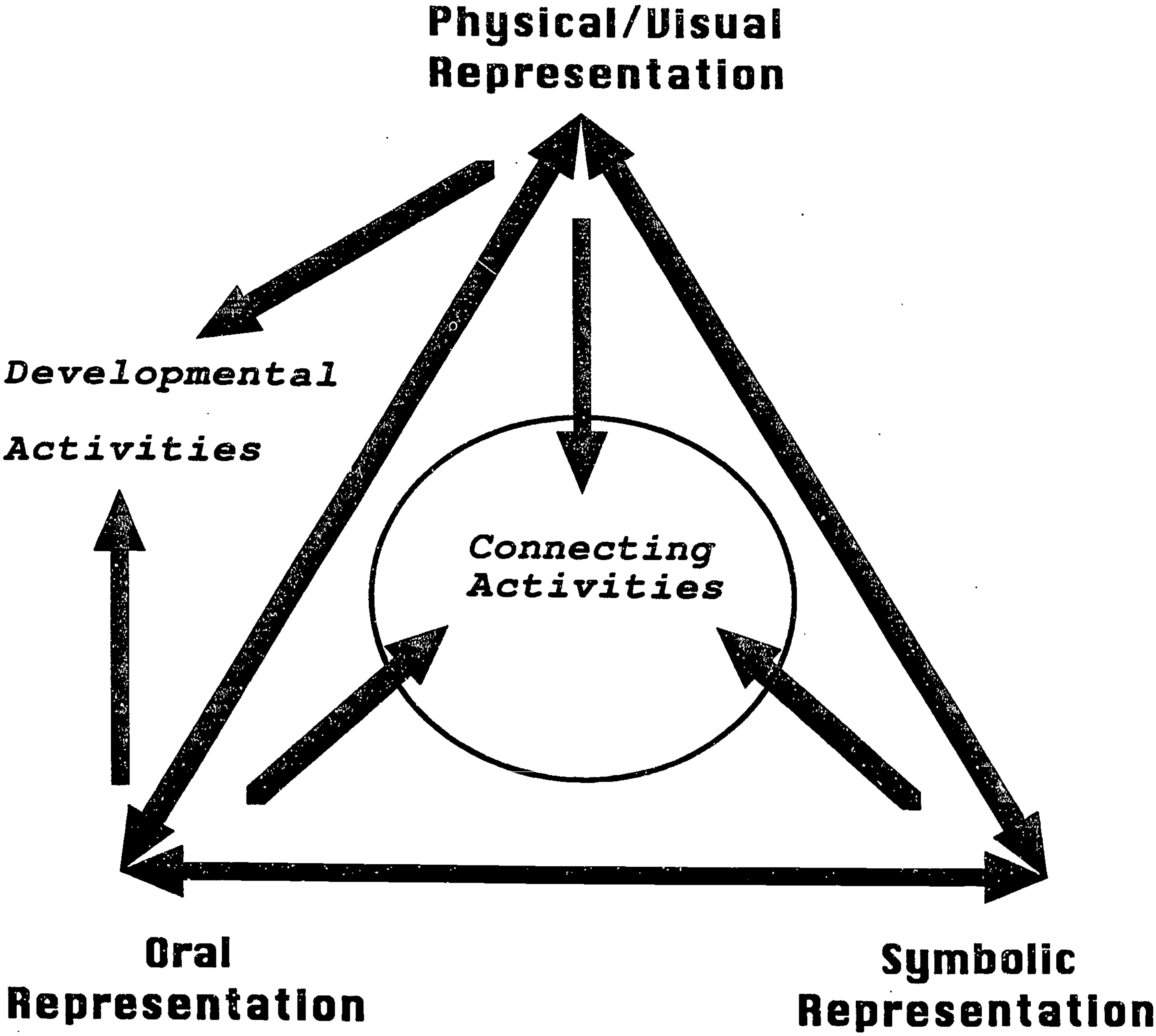

Figure 5
Connecting Concepts
to Symbols

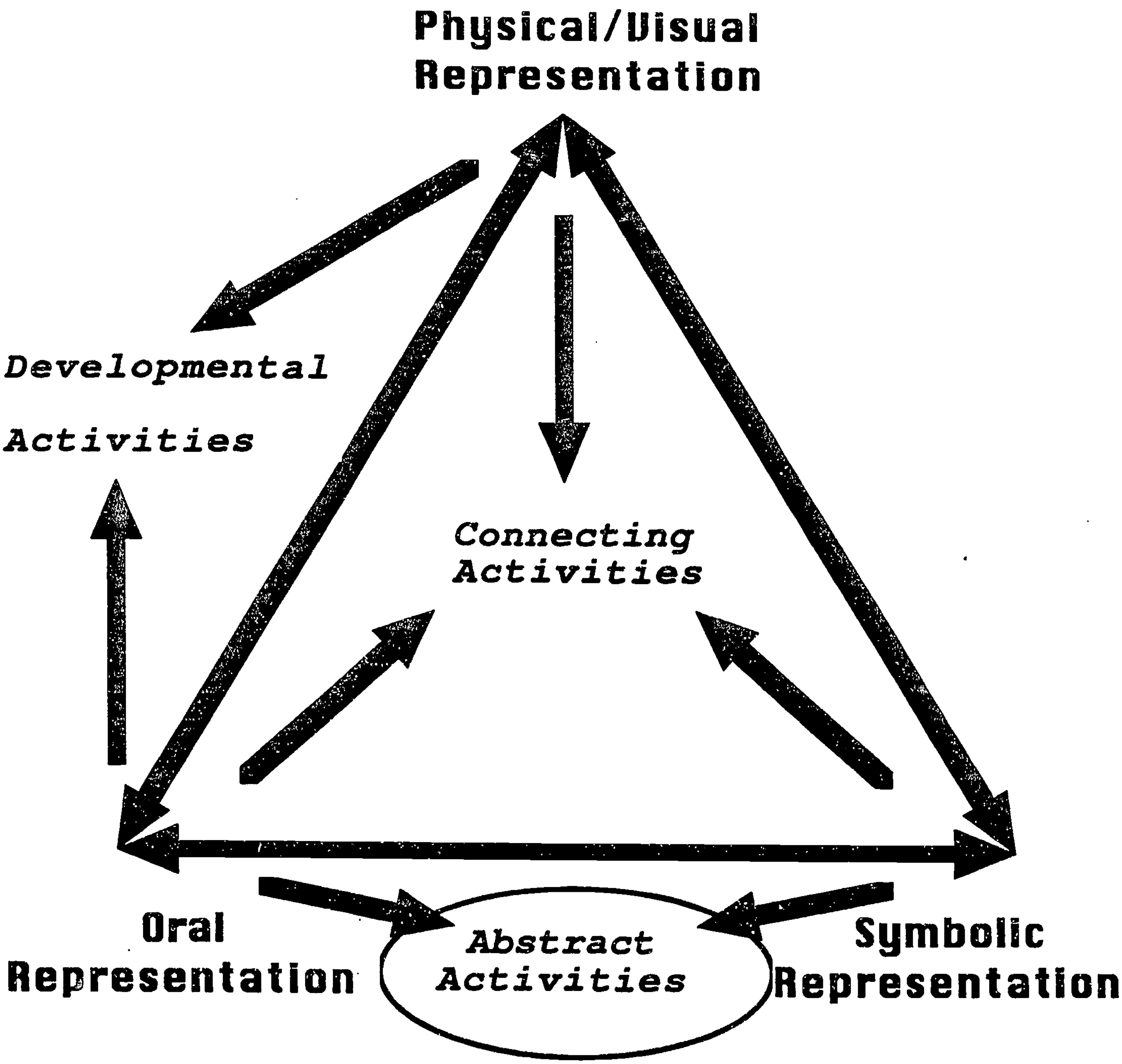

Figure 6 - Abstract Meanings

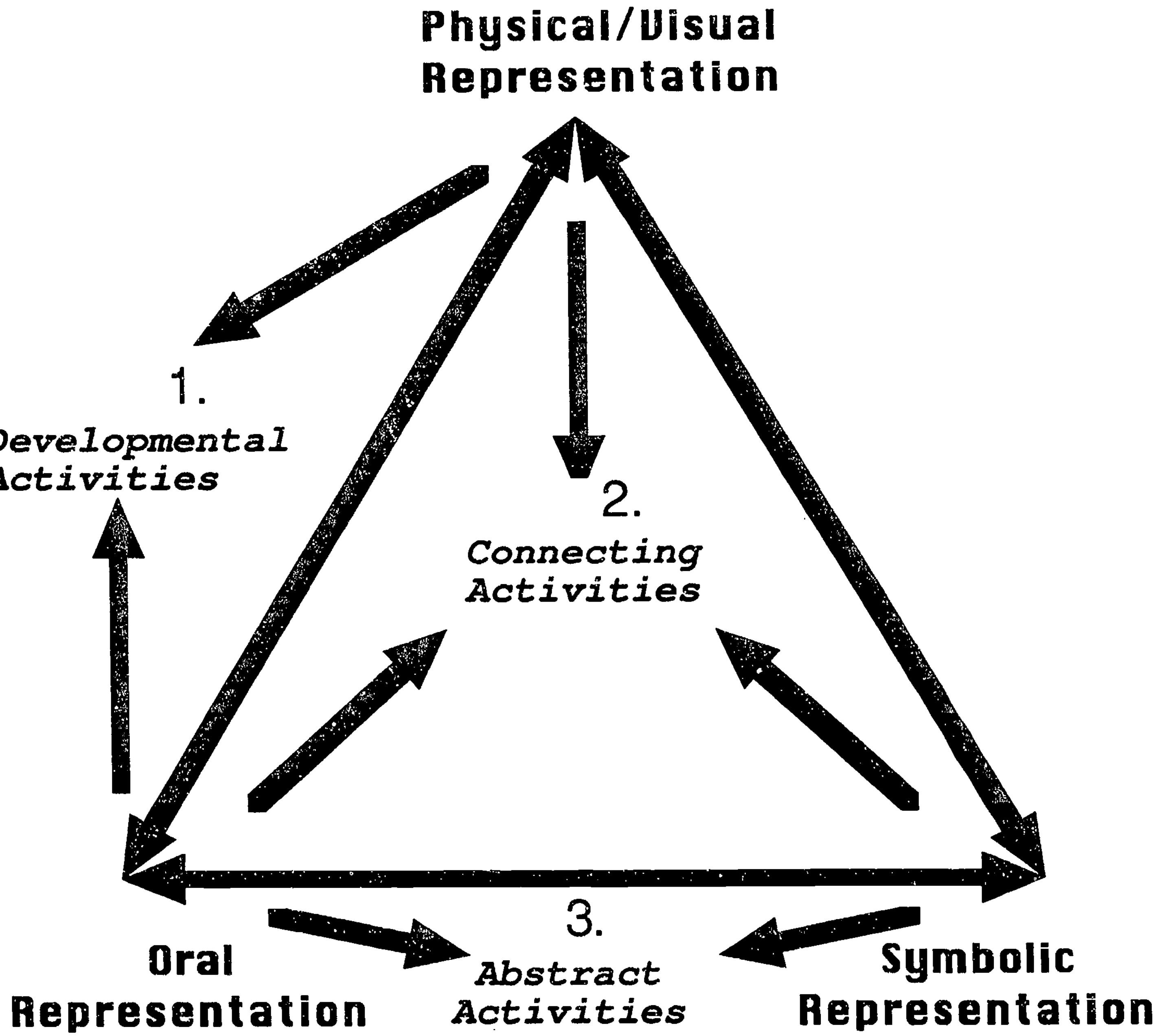

Figure 7
Full Model