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ABSTRACT

To test the theory of mental calibration for mental addition it was hypothesized that the mean reaction time for problems in homogeneous sets will be faster than for problems in heterogeneous sets and that the reaction time was inversely related to ability and development (grade level) and independent of gender. Three sets of 40 addition problems were selected from the basic facts, and from combinations of addends in the range of 10 through 29. These combinations were presented by a computer program in a true-false format with the correct answers, and with answers differing by plus or minus one and plus or minue two. The problems were selected so that the digits 0 through 9 appear equally often as the first and last digit, and that the incorrect sum of one half of the problems was an odd number. Three groups of 40 problems (1-digit, 2-digit, and mixed) were presented in various orders. Results for grade, ability, and task were significant as predicted. One-digit problems in the heterogeneous set took significantly longer than 1-digit problems in the homogeneous set, but there was no significant difference in the context-related processing of 2-digit problems. It appears that mental calibration accounts for some of the processing resources in mental addition, but only in tasks that are in the declarative knowledge of the subject. (Author)

Context and Ability Effects on Children's Development of Mental Addition

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Abstract

To test the theory of mental calibration for mental addition it was hypothesized that the mean reaction time for problems in homogeneous sets will be faster than for problems in heterogeneous sets and that the reaction time was inversely related to ability and development (grade level) and independent of gender. Subjects were 72 students in grades 4 through 6, matched by grade level, ability, and gender.

Three sets of 40 addition problems were selected from the basic facts, and from combinations of addends in the range of 10 through 29. These combinations were presented by a computer program in a true false format with the correct answers, and with answers differing by ± 1 and ±2. The problems were selected so that the digits 0 through 9 appear equally often as the first and last digit, and that the incorrect sum of one half of the problems was an odd number. Three groups of 40 problems (1-digit, 2-digit, and mixed) were presented in various orders.

Results for Grade, Ability, and Task were significant ($p < .05$), as predicted. One-digit problems in the heterogeneous set took significantly longer than 1-digit problems in the homogeneous set, but there was no significant difference in the context-related processing of 2-digit problems. It appears that mental calibration accounts for some of the processing resources in mental addition, but only in tasks that are in the declarative knowledge of the subject.

Context and Ability Effects on Children's Development of Mental Addition

Mental addition has been investigated extensively with adults and children. Over the past quarter century, several types of models of mental addition have been proposed--for example, models hypothesizing that analog (Restle, 1970), counting (Groen & Parkman, 1972), or memory network retrieval (Ashcraft & Battaglia, 1978) processes are used to arrive at the solution for a given addition problem. Mental addition may not depend solely on one source of processing but on multiple sources depending upon the context of the addition problems. The present study proposes a model of mental addition that extends the theory of processing addition problems and provides evidence in support of this model.

Although a great deal is already known about the manner in which persons process addition tasks, a comprehensive model that identifies several elementary processes underlying problem solution has not been developed (Widaman et al., 1989). Widaman et al. (1989) proposed and evaluated a general processing model specifying the processes required to solve mental addition tasks of any magnitude. The 800 problems used in their study comprised 200 problems of each of four types: two single-digit addends, one single-digit and one double-digit addend, two double-digit addends, and three single-digit addends. These problems were randomly assigned to 200 quartets of four problems so that each quartet contained, in random order, one of each of the four types of problem. Two sets of 100 quartets were created. Subjects were administered the two sets in two sessions on different days. Their results verified the power of the componential model to represent reaction time data for each problem type and the combined data set.

Componential models are robust in explaining the variance in reaction times for the mental addition and ability problems. We do not dispute the validity of the componential models in accounting for the reaction time data for addition problems and in providing stable estimates of structural variables incorporated in such models. We argue that subjects process addition problems differently depending upon the composition of a specific set of problems. The difference in processing is in the initial allocation of mental calibrator to predict the type of problem to be encountered and set the ready state of the system accordingly. After the number of problems threshold is reached in a set of problems, the subject sets the ready state in a particular memory network retrieval module and stays in that module until uncertainty is encountered to the previous ready state. According to the prediction of our theory, reaction time for solving addition problems of one genre in a set will be faster than solving the same problems in a set of problems of mixed genres. We predict consistent developmental and ability differences, but no gender differences, in mental calibration for mental addition problems.

Method

Sample

From grades 4, 5, and 6 each in a local elementary school 12 boys and 12 girls were selected. From each grade, 6 boys and 6 girls were of high mathematical ability, and 6 boys and 6 girls were of low ability. For the participating children, teachers' ranking of children's mathematical ability were obtained in terms of the decile a given child was judged to belong. Participating boys and girls were matched on teacher's ranking in each grade.

Tasks

Three sets of 40 addition problems were prepared for individual computer administration. Type I problems were one-digit addition problems selected from the basic 100 simple problems used in many previous studies (e.g., Widaman et al., 1989). The basic 100 problems result from the Cartesian product of using the digits 0 to 9 as the first addend and the same set of digits as the second addend. A random sample of 20 problems was selected with the constraint that digits 0 through 9 appear equally often as the first and the last addend. Presenting each problem once with the true sum and once with an incorrect sum, differing from the true sum by ± 1 or ± 2, resulted in 40 problems. Ten problems differed from the true sum by ± 1 and 10 problems differed from the true sum by ± 2. Also, the incorrect sum of one half of the problems was an odd number and the incorrect sum of one half of the problems was an even number.

Type II problems were two two-digit addends. A constrained random sample of 20 of the 400 addition problems defined by the Cartesian product of the numbers 10 through 29 as the first addend and the same set of numbers as the second addend was chosen. The constraint resulted in the use of each of the 20 numbers 10-29 appearing once as the first addend and once as the second addend with at least one of the problems being a tie problem, i.e., the same number as the first and the second addend. Ten problems were such that their true sum was an odd number and 10 problems had an odd number as the true sum. Presenting the Type II problems once with the true sum and once with the incorrect sum resulted in 40 problems. Of the 20 incorrect-sum problems, 10 had their sum differing from the true sum by ± 1 or ± 2, while the remaining 10 problems had their sums differing by ± 10.

Type III problems were a constrained random sample of 40 problems selected from Type I and Type II problems. Ten true-sum problems were randomly selected from each Type I and Type II problems. Of the remaining 10 true-sum problems in each Type I and Type II problems, their incorrect-sum versions were selected.

Procedure

Children were required to indicate by pushing the designated keys of an IBM microcomputer whether the presented problem was true or false. The response of the child, T (True) or F (False), and the reaction time with an accuracy of ±1 ms for each problem were recorded by the computer.

Problems of the three types were presented as three different sets, with about two minutes of rest after a set, in six possible orders. A child from each ability, gender, and grade group was administered one of the predetermined order of the problem types such that 6 children in each group took six different orders of the problem sets.

Instructions to each child were presented on the computer screen and ten practice problems were presented before the experimental problems were administered. It was emphasized that the appropriate key, indicating the accuracy and speed, should be depressed as quickly as possible. If there was any misunderstanding by the subject in terms of the keys to depress and the speed and accuracy, those were rectified. Each subject used the index finger of the dominant hand for indicating that the sum was correct. The index finger of the non-dominant hand was used to indicate that the sum was incorrect. After the three sets had been presented, the child was given a five minute structured interview. In this interview, each child was asked to indicate the difficulty of each set of tasks, to state the strategy use in solving the problems, and to provide the reasons for their perceived difficulty of a given set of problems.

Analysis

A 3 (Grade) X 2 (Ability) X 2 (Gender) X 3 (Task Set) repeated measures analysis of variance (anova) with repeated measures on Task Set (T) was applied to analyze the reaction time data. The $\log_{10}$ transformed sum of the reaction times for the problems in each set was used as the dependent variable. Subjects' post-interview responses were summarized. Two other anovas using the same design except that the repeated measures factor in each case consisted of two levels: in the first the $\log_{10}$ transformed average reaction time on the one-digit problems set was used with the corresponding transformed average reaction time on the one-digit problems in the mixed set and similarly in the second the transformed average reaction times on the two-digit problems from the two-digit set and the mixed set were used.

Results

Observed means for the three between-subject main effects on the three types of tasks are given in Table 1. Significant ability main effect (F (1, 60) = 11.85, $p < .05$) was observed such that the average reaction time for the high ability group, as expected, was faster than the low ability group. Multivariate analysis of variance (manova) also confirmed this conclusion, and it further indicated that on each set of tasks the high ability group was faster than the low ability group.

Insert Table 1 about here

Grade main effect was also significant (F (2, 60) = 8.67, $p < .05$). Grade 6 children were significantly faster than each of grade 5 and grade 4 children. But, the average speeds of processing of Grade 4 and Grade 5 children were not significantly different. Manova analysis of the data also produced significant Grade effect with Grade 6 group significantly faster in mental addition than Grade 5 and Grade 4 children, respectively. Univariate anova analysis on each set of tasks also confirmed the above results.

The gender main effect, as hypothesized, was not significant. The within-subject factor, Type of Task (T), was significant (F (2, 120) = 155.54, $p < .05$). Problems in the one-digit set took significantly less time than either of the other two sets. Problems in the two-digit set took significantly longer time to respond than the time required for the problems in the mixed set. But, this result is confounded because the problems in the mixed set consisted of 20 one-digit and 20 two-digit problems. To address this issue, two other anovas were performed.

For these other two analyses, the between-subject effects were consistent with those reported above. The within-subject effect for one-digit problems (F (1, 60) = 11.10, $p < .05$), but not for two-digit problems was significant. Figures 1 and 2 portray the average reaction time for the 20 items in the one-and two-digit problem sets by ability and context, homogeneous and heterogeneous set of items. One-digit problems in the heterogeneous set took significantly longer to solve than in the homogeneous set. In none of these analyses significant interactions were observed suggesting that processing of addition problems in heterogeneous context takes longer across development, ability, and gender of respondents.

Insert Figures 1 and 2 about here

The post-test interview data showed that subjects found the problems in the heterogeneous set relatively more difficult to solve than in the homogeneous sets. It was also almost consistently reported that subjects used either memory of facts or computational approaches to solve the problems. No use was made of any metacognitive strategies. In fact, children could not name any strategy or rule, when prompted in the interview, that could have been used effectively in some situations.

Discussion and Conclusions

Our results partially support the hypothesis that mental calibration takes up some of the processing resources in mental addition. Reaction time for 1-digit addition items in homogeneous context required significantly less time than for the corresponding items in the heterogeneous context. Context of the addition problems determines the ready state of the respondent. The mental calibrator allocates the procedural and/or strategic resources before the problem is presented for solution. But, if the context is unknown, some mental resources are wasted in order to engage the appropriate computational and strategic retrieval network. The recognition of this source of variation in parameter estimation, using componential models, is expected to enhance their predictive power. Our further analysis of the data would determine whether significant improvement in prediction is obtained to what was reported by Widaman et al. (1989).

Since significant differences for the context effect were obtained only for the 1-digit item set, not for the 2-digit, we speculate that mental calibrator is operative only in tasks that are in the declarative knowledge domain of the child. Our subjects were school children from grades 4, 5, and 6, not numerate adults as in many studies in the area (e.g., Ashcraft & Battaglia, 1978; Ashcraft, Fierman, & Bartolotta, 1984; Ashcraft & Stazyk, 1981; Widaman et al., 1989). Further indication of the role of the mental calibration phenomenon is the evidence that Grade 6 children were significantly faster than Grade 4 or Grade 5 children in processing the mental addition tasks in all three sets, but there were no significant differences in the average reaction times of Grade 4 and Grade 5 children. Because Grade 6 children as compared with the younger children in grades 4 and 5 to some extent had automatized the addition of the presented addends, the mental calibrator was effective in the differential performance of subjects in our study.

As there were no significant interactions observed in our study, the context effect on mental addition is systemic. This implies that ability or development of children does affect reaction time for mental addition systematically, not differentially. Further, it is indicated by these results that the knowledge of the context adds a constant amount of reaction time in solving mental addition problems despite the ability, development, or gender differences of the subjects. In the memory network retrieval models involving tabular or nontabular processes, components representing context of the task, competence of the subject, and developmental level should add to the prediction of the criterion variable, reaction time in mental addition. Our further analyses of the data are directed toward this end.

The use of declarative knowledge in mental addition is limited to the access of the presented item from memory if available. If access is not available immediately then the subject resorts to the procedural knowledge and makes the decision. In the post-test interview, it was evident that subjects did not use the strategic knowledge at all in mental addition. It may well be that in speeded processing strategic knowledge is not optimal. We are attempting to address this question in our ongoing research.

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Table 1

Observed Means for the Main Effects on Three Type of Tasks

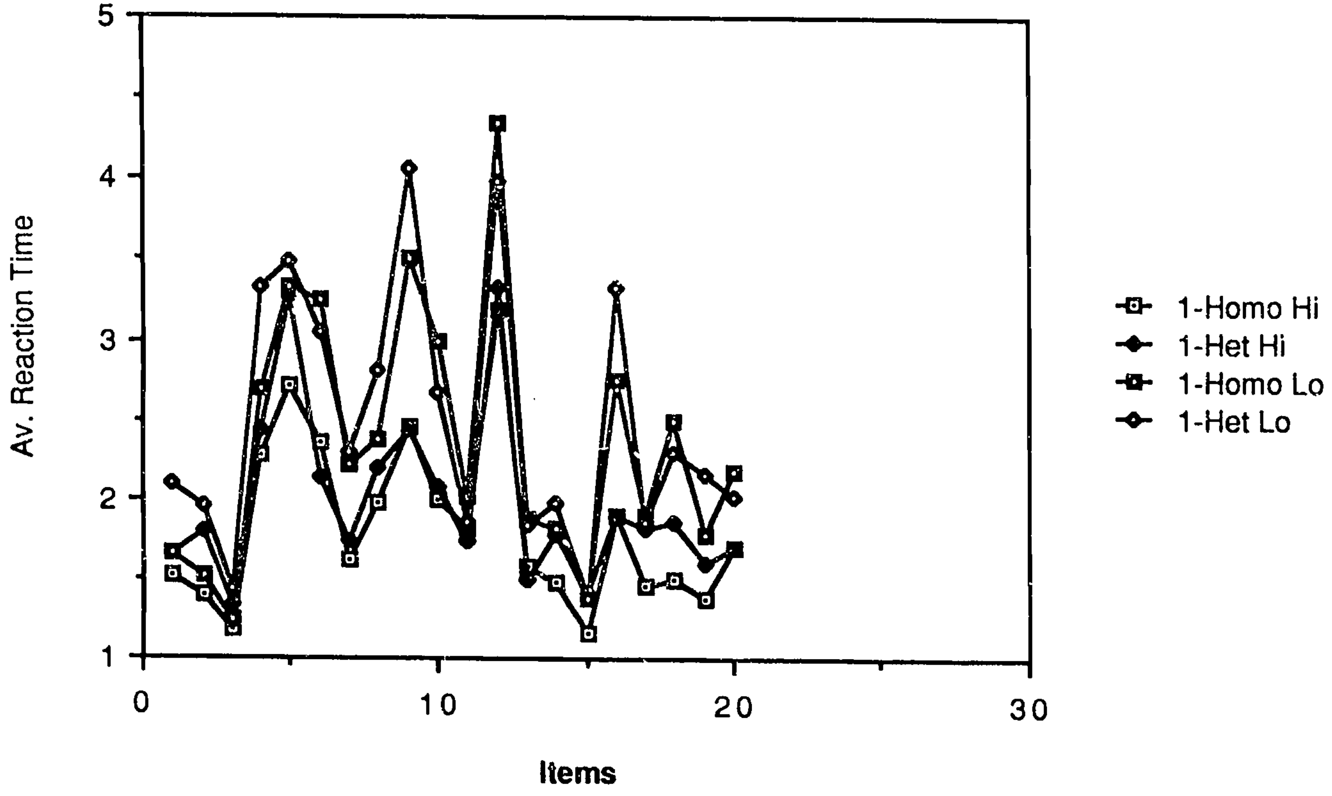
Main Effect	Level	Task		
		1-Digit	2-Digit	1- & 2-digit
Ability*	High	73.04	125.11	102.39
	Low	94.33	152.51	123.66
Grade*	4	95.26	147.30	120.79
	5	89.67	152.75	123.78
	6	66.12	116.39	94.50
Gender	Male	78.89	141.60	115.04
	Female	88.48	136.02	111.00

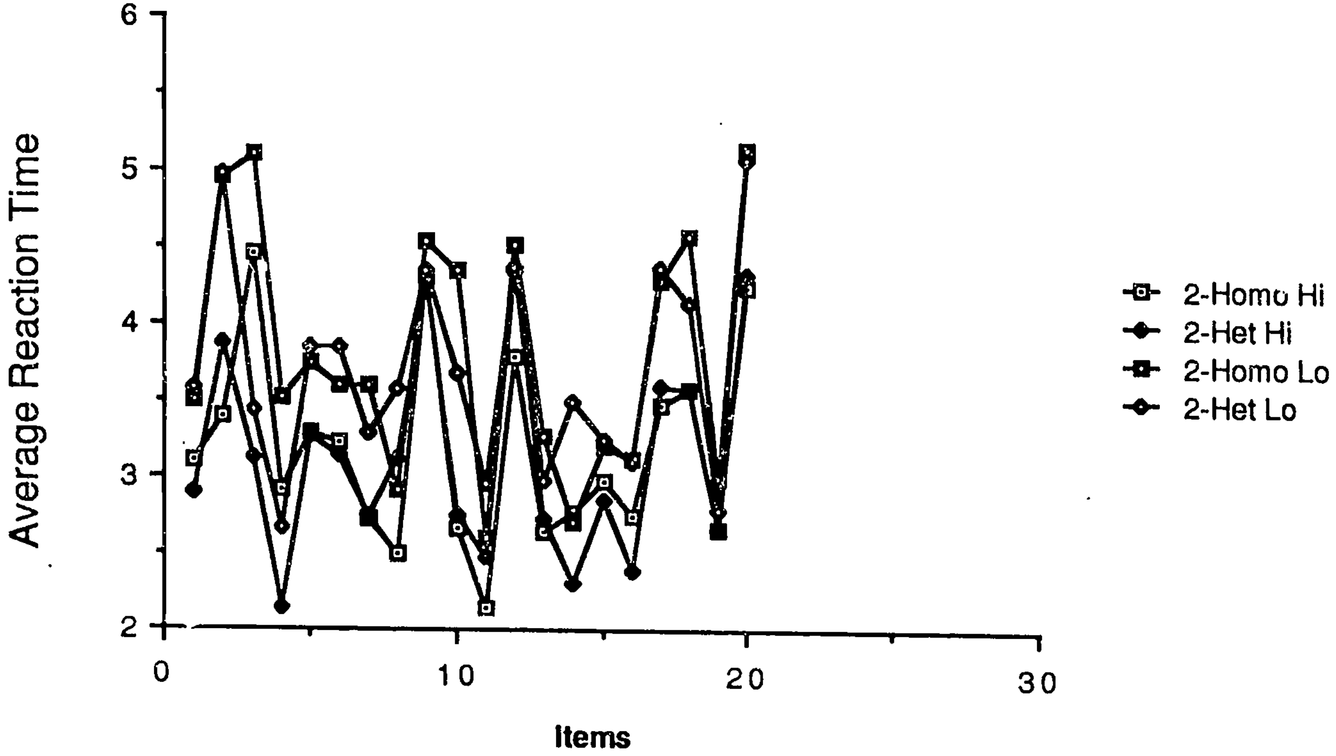
$^{*}p < .05$ for the indicated main effect.

Figure Captions

Figure 1. Reaction times for the 20 1-digit common mental addition items in homogeneous and heterogeneous sets by ability.

Figure 2. Reaction times for the 20 2-digit common mental addition items in homogeneous and heterogeneous sets by ability.


5
4
3
2
1
Av. Reaction Time
0
10
20
30
Items
1-Homo Hi
1-Het Hi
1-Homo Lo
1-Het Lo


6
5
4
3
2
Average Reaction Time
0
10
20
30
Items
2-Homo Hi
2-Het Hi
2-Homo Lo
2-Het Lo