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ABSTRACT

Mathematical problem solving has been the focus of much concern. This study investigated the relationship of various cognitive factors, attributions, and gender to the solution of mathematics problems by 100 high school seniors. The independent variables examined in this study included: (1) mathematics knowledge as measured by a score on the mathematics section of the Preliminary Scholastic Aptitude Test (PSAT); (2) metacognitive regulation as measured by a score on the Assessment of Individual Mathematical Metacognition (AIMM); (3) beliefs, including attributions and generalized beliefs. as measured by scores on selected questions from the Inventory of Students' Mathematical Beliefs and Behavior (ISMBB); (4) metacognitive awareness as measured by a score on the Metacognitive Awareness Assessment (MAA); and (5) gender. No gender differences were found in any of the variables except in the high knowledge group: (1) high knowledge boys outperformed high knowledge girls on one problem, and (2) high knowledge girls were less likely to attribute their success in mathematics to effort than were high knowledge boys. Contains 53 references. (MKR)

Adolescent Mathematical Problem Solving:
The Role of Metacognition, Strategies and Beliefs

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Mathematical problem solving has been the focus of much concern in recent years. Toward this end, researchers have investigated several factors that appear to influence competence in mathematical problem solving. These factors include cognitive skills such as knowledge of mathematics (Sternberg, Guyote, & Turner, 1980), strategies and metacognition (Garofalo & Lester, 1985; Kilpatrick, 1967; Kraus, 1982; Schoenfeld, 1979, 1980, 1983a), as well as non-cognitive factors such as attributions (Lester & Garofalo, 1987; Schoenfeld, 1987), and gender (Fennema & Peterson, 1985).

Cognitive Factors. Individual differences in mathematics knowledge for solving problems have been suggested as important in studies of mathematics (McKee, 1979; Threadgill-Sowder, 1985). Mathematical knowledge has been viewed as the ability *inferred* from a person's performance on a mathematics task (Kilpatrick, 1987), such as the Preliminary Scholastic Aptitude Test (PSAT), the Differential Aptitude Test (DAT), and the National Assessment of Educational Progress (NAEP) (Benbow, 1992; Benbow & Stanley, 1980; Feingold, 1988, 1993; Wilder & Powell, 1989). Although researchers have focused on mathematics-knowledge differences on standardized tests, this approach has been criticized as inadequate, because such tests do not provide information about how a student thinks (Goldin, 1984; Krutetskii, 1976; Lester & Schroeder, 1983).

Although research on individual differences in problem solving generally has focused on the differences between experts and novices (Chi, Feltovich & Glaser, 1981; Larkin, 1981a), more detailed results in processing and thinking differences in mathematics knowledge has come from research on characteristics of mathematically talented individuals. The most well-known work in this area is the work of Soviet psychologist Krutetskii (1976), who attempted to characterize the cognitive processes used by mathematically talented students during problem solving. However, there is little research on differences between high knowledge and low knowledge problem solvers who are neither experts, novices, nor gifted, even though these groups comprise the majority of high school students represented in statistics often cited in relation to competency of high school students in mathematical problem solving.

Research in strategies and metacognition, including metacognitive regulation and knowledge, suggests that both are tied to mathematical ability and successful performance on novel mathematics problems (Artz & Armour-Thomas, 1992, Hecht & Tittle, 1990; Lester, Garofalo & Kroll, 1989; Schoenfeld, 1987). Some metacognitive research has focused on knowledge of cognition. Wang (1991) found no differences in knowledge of cognition between gifted and non-gifted groups. Swanson (1990) found that, regardless of aptitude, high metacognitive students performed better than low metacognitive students. He also hypothesized that development might be a factor in level of metacognitive knowledge and suggested that research explore differences in older groups.

The research examining metacognitive regulation has also been sparse. Artz and Armour-Thomas (1990) identified students' processes and determined the relative contribution of each process to problem solving. Their study included seventh graders working in groups. Results indicated that all groups used the processes including understanding, analysis, exploration, planning, implementation, and verification. Fortunato, Hetch, Tittle and Alvarez (1991) found that all students used the processes investigated. The study also examined the relative contribution of processes to problem solving. They found that students most often used rereading, followed by planning and understanding. Both studies examined younger children and focused on whether a process was evident. Schoenfeld (1985) examined expert-novice differences and found that novices did not plan and engaged only in reading and exploration. In contrast, experts monitored problem solving on an ongoing basis and engaged in analysis, planning, implementation and evaluation of their work.

Research has been hampered by problems related to task selection, age group, and setting. Specifically, finding appropriate levels of novelty and difficulty in tasks to elicit metacognitive behaviors has been problematic (Lester, 1985, Lester et al., 1989b, Peverly, 1991). More importantly, explorations into metacognitive regulation have been limited to identification of processes as opposed to examining the quality of those processes.

In relation to strategy use, several researchers (Briars, 1982; Lesh, 1981; Lesh & Akerstrom, 1982) have reviewed the results of expert-novice problem-solving performance and determined that good problem solvers use domain-specific or problem-specific strategies while poor problem solvers use general strategies.

Several studies have suggested a link between the use of specific strategies and achievement (Peterson & Swing, 1982; Peterson, Swing, Stark, & Waas, 1984). Researchers (Lester & Garofalo, 1989b; Schoenfeld, 1985, 1987) have emphasized the importance of choosing more complicated problems in order to explore differences in strategy use, paticularly among older adolescents. Kelly-Benjamin (in preparation) identified cognitive strategies used by high school students solving math SAT problems.

One strategy that has been the focus of renewed interest among researchers in problem solving is reasoning (Sternberg, 1986; Swanson, 1990); little work has been done on the strategy of reasoning. Researchers, primarily those who have focused on the gifted, have repeatedly mentioned non-verbal reasoning ability as important to successful mathematics problem solving (Benbow & Brody, 1990; Benbow & Stanley, 1980). In a meta-analysis on studies in problem solving, Hembree (1992) found that the best problem solvers were those who excelled at reasoning. He concluded that students need to be exposed to problem solving that included abstract thinking and reasoning skills since problem solving is a complex mental activity, especially in middle and high school.

While the Hembree work has documented that mathematics knowledge and ability to reason stand out as important skills, no systematic research has attempted to explore which adolescents use reasoning and what other strategies students ultimately use to solve problems. Additionally, no one has examined differences in the quality of solution strategy use.

Non-cognitive factors. Research on gender differences in mathematics has suggested a pattern of emerging gender differences in favor of males in adolescent problem-solving performance (Armstrong, 1982; Fennema, 1974; Fennema & Sherman, 1977). Additionally, the NAEP (1986) indicated that the gap between males and females increases in more difficult problems (Dossey et al., 1988). More recent research has suggested that the differences in ability and performance are diminishing except at the highest levels (Feingold, 1988,1993; Hyde, Fennema, & Lamon, 1990). Ramist and Arbeiter (1986), in a meta-analysis of MSAT performance, found a considerably large difference in favor of males. They pointed out that differences may exist in problem solving at the higher levels of competence and with older adolescents. Additionally, they recommend examining other factors such as beliefs and attributions.

Researchers have examined generalized beliefs (Schoenfeld, 1985), and attributions (Fennema, 1974, 1977). Fennema (1974, 1977) explored the relationship between attributions and mathematics achievement among younger children. For example, she (1977) found that fifth and sixth grade males were more likely than females to attribute their *successes* in mathematics to ability, while females were more likely to attribute their *failures* to lack of ability. In addition, females tended to attribute their successes to extra effort more so than males, while males tended to attribute their failures to lack of effort.

Wolleat, Pedro, Becker, and Fennema (1980) explored attributions with adolescents. They found significant differences on four of eight subscales in the direction predicted by attribution theory. Females used effort (unstable) more often than males to explain their successes. Males pointed to ability (stable) to explain their successes more than females. When students failed, females were more likely than males to draw on the attributions of ability and task difficulty (both stable) to performance. In contrast to the results of Wolleat et al., a meta-analysis conducted by Whitley, McHugh, and Frieze (1986) found that gender differences in causal attributions of achievement were not significant among adolescents and adults. The authors emphasized that gender differences in *causal* attributions were not supported by research.

Schoenfeld (1989) also did not find gender-related differences in beliefs. Both studies (Schoenfeld and Wolleat) used ninth and tenth graders as well as causally worded questions. Schoenfeld's results confirmed those of the Whitley et al. meta-analysis regarding attribution theory. Neither study attached the survey to a measure of math achievement. Thus, students were not answering relative to a specific situation, a valid criticism of the analyses by Whitley. Since few studies of attribution have examined how perceived causes of success and failure are related to academic achievement in an actual problem-solving situation (Stipek & Weiz, 1981), such study might clarify the different findings of the Wolleat et al. and Schoenfeld research.

PURPOSE

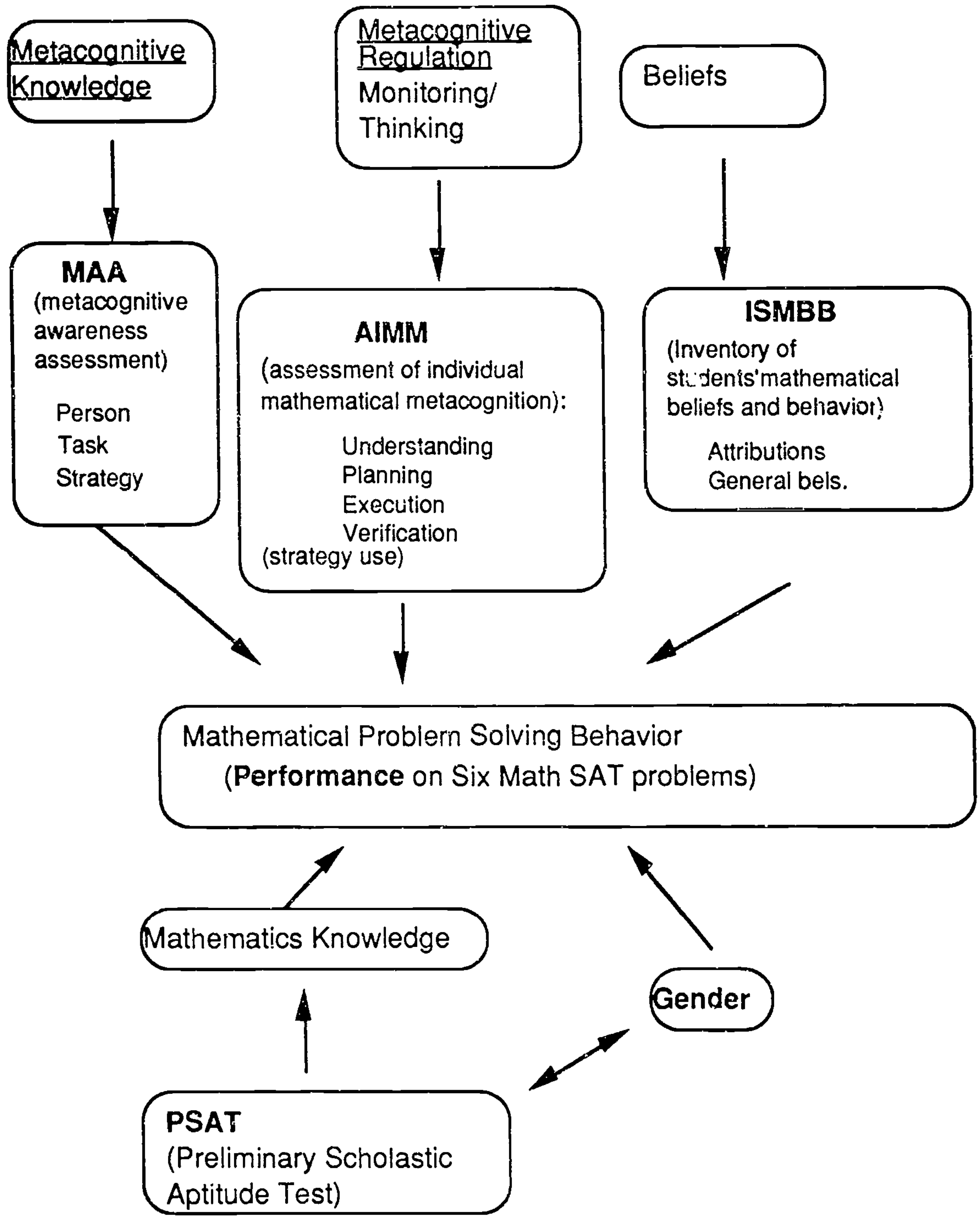
This study investigated the relationship of various cognitive factors (mathematics knowledge, metacognition - including knowledge and regulation of cognition-, and strategy use), attributions, and gender to the solution of mathematics problems by adolescents.

With regard to the influence of gender and mathematics knowledge on the variables under study: a) On performance, it was predicted that there would be differences in favor of males and better students, b) On metacognitive regulation, it was predicted that there would be differences in favor of high knowledge students and no differences as a function of gender, c) On metacognitive awareness, no predictions were made, and d) On attributions, it was predicted that there would be no differences in gender or mathematics knowledge.

With regard to predictors of mathematics performance, it was hypothesized that mathematics knowledge and gender would predict performance, and that metacognitive regulation, metacognitive awareness, attributions or generalized beliefs would not predict performance.

The figure below presents a graphic illustration of the six variables under investigation in the study, with the actual measures used to measure each variable highlighted in bold script:

Variables in Mathematical Problem Solving



METHOD

Design

The present study employed a multivariate correlational design to examine relationships among independent and dependent variables. The independent variables examined in this study included: (1) mathematics knowledge as measured by score on the mathematics section of the Preliminary Scholastic Aptitude Test (MPSAT); (2) metacognitive regulation as measured by score on the Assessment of Individual Mathematical Metacognition (AIMM); (3) beliefs, including attributions and generalized beliefs as measured by scores on selected questions from the Inventory of Students' Mathematical Beliefs and Behavior (ISMBB), (4) metacognitive awareness as measured by score on the Metacognitive Awareness Assessment (MAA); and (5) gender. The dependent variables include: (1) metacognitive regulation (AIMM); (2) math performance as determined by score on task problems; (3) beliefs (ISMBB), and (4) frequency of solution strategy use.

Subjects.

To participate in this study, a total of 100 high school seniors (50 females and 50 males) were selected from approximately 400 students on the basis of PSAT scores. The students were blocked into two groups: high knowledge and low knowledge, on the basis of scores on the mathematics section of the PSAT (MPSAT). There were 25 females and 25 males in each group. Scores for the high group were 520 or above (≥ 60th percentile) and scores for the low group were 420 or below (≤ 40th percentile). The mean MPSAT score for the high males and females was 59.1 (s.d. = 10.2); for the low males and females it was 39.6 (s.d. = 10.5). The decision to include mathematics knowledge as a factor was partially based on a pilot study that explored the relationship of gender to performance that held mathematics knowledge constant; the results of that study showed no difference in performance by gender for average students. In that study, the average MPSAT was 51.4 for the females and 51.55 for the males.

The students were selected from schools with similar mathematics curricula for ninth through eleventh grades. The schools consisted of two private-parochial schools, one private-independent school, and two public high schools. More than 70% of the students from these schools go on to college. They were ethnically and socio-economically diverse, representative of the New York metropolitan area, and were chosen partially to represent a variety of students and types of schools.

Instruments.

Inventory of Students' Mathematical Beliefs and Behavior (ISMBB). This inventory was used to assess attributions and generalized beliefs. It contains 70 questions in a Likert format which yields scores from 1 to 4, as measuring feelings or thoughts about the questions as "very true" to "not at all true."Selected questions were utilized in the study. The first ten questions (1-10) were used to explore students' attributions of success or failure and five other questions (63, 64, 65, 66, 68) were used to explore students' generalized beliefs about academic performance, mathematics performance, the role of effort, ability and motivation.

Mathematics Problems. Six mathematics problems from the 1987 SAT math test, form 7H, were administered during the individual session. Five of the six questions were selected from those items that had at least a 10 % difference between females and males, favoring males. Multiple-choice problems were chosen because they elicited the greatest differences in favor of boys. Two arithmetic, two geometry, and two algebra problems were included. Each content area contained one *difficult* and one ***very difficult*** question. Difficulty of questions is defined as follows: *difficult* questions were taken from those questions on the 1987 SAT Math examination that were correctly answered by 41-69% of the test takers; *very difficult* questions were taken from those questions that were correctly answered by 39% or fewer of the students (Rossner, 1989). Three of the six problems were word problems, including two arithmetic and one algebra.

The AIMM. This measure, the Assessment of Individual Mathematical Metacognition (AIMM), was devised for this study by this researcher and involves a multi-method assessment including observation during the problem-solving phase and during the interview following the problem-solving phase. The inventory consisted of a form with space to indicate question number, followed by the categories of understanding, planning, execution, and verification, which were the basis for assessing metacognitive competency. Points were allotted for level of performance in each of these areas, with understanding and planning receiving 0-4 points each, and execution and verification each receiving 0-2 points. Metacognitive strategies were not coded independently of cognitive strategies, but rather a total score was given based on how well the subject performed in each category. For example, for understanding, competency was determined by level of understanding and ranged from "no understanding" to "complete understanding with no irrelevant information used." Competency in planning ranged from "no planning" to "planning that would lead to a correct solution based on complete interpretation" (complete understanding). Scoring in the execution category ranged from "no execution" (guessing) to "execution with no computational errors." Verification ranged from "no verification" to "verfication that included an evaluation of planning and understanding." The highest possible score for each problem across all 4 categories was 12 (72 across all 6 problems). A guideline for assessing performance, including weighing of cognitive strategies, was formulated. Interrater reliability was .93; this reliability was based on the assessment of 8 subjects, 2 from each group.

Metacognitive Awareness Assessment (MAA) This measure, constructed by this researcher, consisted of three questions, each related to the three elements of metacognitive awareness: task, person, and strategy variables (Flavell, 1985, Lester & Garofalo, 1987; Schoenfeld, 1989). Students' responses on the first element, task difficulty, were scored according to how closely they estimated difficulty level compared to difficulty level, as assessed on the 1987 exam and as discussed in the section under mathematics problems. For example, if they completely agreed with the predetermined level of difficulty, they received 4. If they completely disagreed they received 0, with other scores in between these two. Their second response was scored according to their confidence in obtaining the correct answer. If they were very confident and also obtained the corrrect answer, they received 4. If they were very confident but obtained the wrong answer, they received 0. Other scores fell in between. Their third response was graded according to whether they used a strategy and the quality of that strategy, with the score ranging from 0 (no strategy) to 2 (very good strategy). Total score on each problem ranged from 0 to 10, with a range of 0 to 60 for all the problems.

Solution Strategy. Based on a pilot study exploring cognitive strategy use, a list of strategies used for the *final solution* was formulated. This form recorded the strategy used by each student. This list is a modified list based on one formulated by Kelly-Benjamin (in-preparation).

Procedures

Group Sessions. Students were initially seen in a large group setting to inform them about the experiment. Later, a smaller group, drawn on the basis of MPSAT scores, were given two math problems from the 1987 ETS test, and then asked to complete the ISMBB. The materials were collected and an individual follow-up session was arranged.

Individual Session At the individual sessions, a brief explanation of the experiment was again given. The students were then told to solve the problems as they would normally solve them under test conditions, showing all work on the page. They were told they had 20 minutes to solve the six problems (approximately 3.25 minutes per problem). This decision was based on data from a pilot study in which problems were given in an untimed condition, but total time was recorded. Boys averaged 2.32 minutes and girls averaged 2.56 minutes; on the MSAT, the average time per problem is 1.02 minutes. Total amount of time for solving each problem was recorded.

While the students solved the problems, the examiner observed and recorded any information needed to obtain a written sequence of activity during problem solving in order to facilitate the retrospective clinical interview. When the students finished solving the problems, the examiner interviewed them about their work, asking them to explain how they solved the problems, one by one, what they were thinking, and why they used a particular method. The sheet with the observer's notes as well as the student's worksheet allowed the examiner to develop a transcript of the student's performance and was used to guide the interview process. All interviews were recorded in order to analyze scoring of cognitive and metacognitive behaviors and to provide opportunity for validity and reliability checks.

RESULTS

Gender, Mathematics Knowledge, Performance and Metacognition

Since the correlations between performance and the AIMM and between the AIMM and MAA were significant, a MANOVA was used to evaluate the relationship of mathematics knowledge and gender to performance, AIMM and MAA. Means and standard deviations may be found in Table 1. The MANOVA was significant (Wilks $F(18, 257) = 6.12$, $p < .001$). The univariate tests indicated that the differences were in performance and the AIMM, but not the MAA. Special contrasts, shown Table 2, found that group-related differences were due to mathematics knowledge and not gender. The interaction was not significant, using the Bonferroni correction method ($p < .003$).

The prediction of mathematics knowledge followed by gender as the best predictors of performance was not confirmed. Examination of the results of regression indicated that the variables accounted for 76% of the variance in performance ($R^2 = .76$). The univariate analysis on individual variables indicated that the AIMM was the only significant predictor of performance ($F = 89.9$, $p < .01$) (See Table 3).

The finding that the AIMM predicted performance better than mathematics knowledge suggests that studies of knowledge-related differences in mathematics performance must be supplemented by analyses of the process of regulating the application of knowledge in problem solving, i.e. metacognition.

Metacognitive Regulation (AIMM)

Correlations (See Table 4) between *each* of the four categories and performance for each of the four groups indicated that understanding, planning, and execution were significantly correlated with performance for all four groups and verification for the high group only.

Since the four categories of the AIMM were correlated, a MANOVA was used to examine differences in this variable by gender and mathematics knowledge. The multivariate test was significant (Wilks $F(12, 285) = 8.23$, $p < .001$). Contrast testing using the Bonferroni correction method, indicated a main effect for mathematics knowledge on all four variables (See Table 5).

Finding significant differences by mathematics knowledge in each of the four metacognitive categories indicates that some students are better able to monitor and regulate their problem-solving activity and are better able to develop a meaningful sense of problem elements than low knowledge students. The low level students are less strategic in developing a plan and carrying it through to the execution and verification stages. To some extent, poor planning results from poor understanding. Low knowledge students are also less likely to verify; when they did verify they more likely checked computations than verified their understanding of the problem and plans for solution.

Solution Strategies

Although students often invoked several different strategies during the four phases of problem solving, interaction with metacognitive activity finally led students to solve problems with specific, codable strategies, or in the absence of strategies, to guess. Log linear analyses indicated significant differences between high and low groups in strategy use on 4 out of 6 problems and in use of specific strategies vs. guessing: high knowledge students were more likely to use a specific strategy and less likely to guess (See Table 6).

Relationship of Gender and Mathematics Knowledge to Attributions

Means and standard deviations for the attribution questions are reported in

Table 7. Since some attributions were significantly correlated, a MANOVA was used to test the hypothesis that beliefs would not be systematically related to mathematics knowledge or gender. The MANOVA was significant (Wilks $F(30, 256) = 1.92$, $p < .004$). Contrast testing using the Bonferroni correction method ($p < .002$) indicated no main effects due to mathematics knowledge or gender, but a significant interaction on Beliefs (S1, $t = 3.79$, $p < .0003$). High knowledge males are more likely than high knowledge females to think that a good grade in math is due to hard work. No other gender differences were found.

DISCUSSION

Mathematics Knowledge and Performance

It was predicted that mathematics knowledge, but not the AIMM, would predict performance in a regression. It was found, however, that although mathematics knowledge significantly *affected* performance, the AIMM was the only variable to significantly *predict* performance.

A major focus of this study was to explore the process and thinking of performing, not the final product of performance. For example, data obtained from the 1987 MSAT indicated a 10% difference in favor of boys among the better students on Problem 5. Those results indicated gender differences on an arithmetic problem, but nothing more. By contrast, the AIMM provided a more detailed analysis of some differences in problem-solving strategies, such as boys' use of a key metacognitive strategy (elimination of irrelevant information) and the somewhat surprising possibility that the high knowledge boys demonstrated better computational skills. More importantly, the AIMM demonstrated that the differences were based on the strategic application of that knowledge (elimination of irrelevant information) or the strategic application of basic skills (computation). Clearly, our impression of some of the differences is much richer because of our on-line assessment.

Indeed, the AIMM is a measure of the active application of the knowledge assessed by the PSAT; in that respect it incorporates the PSAT. This is confirmed by the fact that both the PSAT (the measure used to evaluate mathematics knowledge) and the AIMM were significantly correlated to performance and, more importantly, to each other. However, when regression analysis was performed to evaluate the relative size of the effect of the different variables including mathematics knowledge, AIMM, the MAA, and beliefs, mathematics knowledge (PSAT) was not as predictive as the AIMM. Thus, the AIMM is an informative instrument for assessing problem-solving behavior and a better predictor of performance on a small set of problems.

Metacognition

Regulation. The AIMM examined metacognitive regulation and found that it was significantly affected by mathematics knowledge but not by gender differences; these results were both predicted and they confirm previous findings which document the importance of metacognition in successful problem-solving (cf. Brown, 1978, 1979; Brown et al., 1983; Chi & Glaser, 1988; Flavell, 1979, 1981, 1984; Garofalo & Lester, 1985, Garofalo, Lester, & Kroll,1989; Kantowski, 1977; Schoenfeld, 1985, 1992; Silver, 1985). Data indicated that subjects used the processes described in the framework, including understanding, planning, execution and verification. In that regard, the results for the most part confirm other research that suggests the inclusion of these categories in exploring problem-solving (e.g. Artz & Armour-Thomas, 1992; Lester, Garofalo, & Kroll, 1989b; Polya, 1957; Schoenfeld, 1985, 1987). Equally as important, however, this measure expands the research on this construct in its exploration of the quality of metacognitive activity.

Specifically, prior research has examined *whether* subjects engage in aspects of metacognitive activity, but not *how well*. Previous research indicates that problem solvers engage in behavior to *understand* a problem; the current research indicates that not only do problem solvers engage in behavior to understand, but there are individual qualitative differences in how they engage. For example, high knowledge problem solvers integrate more information during understanding than low knowledge problem solvers, and some of the best problem solvers eliminate irrelevant information in understanding certain problems. The present study, therefore, expands the research in this area and provides a new approach to the individual assessment of competency, as well as enabling the more traditional comparisons of competency between groups in a more microscopic manner.

Besides the previous distinction. the AIMM may be distinguished from other frameworks in that it does not identify separately the cognitive and metacognitive processes used. Problem solving involves an interplay between metacognitive and cognitive processes (Artz & Armour-Thomas, 1992; Flavell, 1985; Lester, Garofalo, & Kroll, 1989b; Schoenfeld, 1985, 1987. 1992). Although one can distinguish the dual nature of cognitive processing as involving cognitive and metacognitive activities, the distinction is often difficult in practice. Some researchers suggest that some episodes of problem-solving activity cannot be purely cognitive or metacognitive, but rather depend on the predominant process used (Arzt & Armour-Thomas, 1992). In their framework, Arzt and Armour-Thomas considered understanding and planning as primarily metacognitive, whereas they considered implementing and verifying as cognitive and metacognitive, depending on the predominance of one type of behavior. They also considered reading a separate category and saw it as cognitive initially and metacognitive when it occurred a second time.

By contrast, in the current study, the emphasis was not on distinguishing between the two types of activity, but rather on the quality of the overall behavior, including making inferences about understanding and planning from cognitive and metacognitive behaviors. For example, the first reading as well as any rereading was part of understanding, with no emphasis on which reading was cognitive or metacognitive, but rather with an emphasis on whether it resulted in a deeper, more integrated understanding of the problem.

Metacognitive Awareness. No group differences were predicted or found on this measure of the subjects' knowledge of themselves as problem solvers, of task difficulty, and of strategy use. Researchers have explored metacognitive knowledge generally (Baker & Brown, 1984; Chi & Glaser, 1985; Flavell, 1979), but not in the area of mathematics or with older adolescents (Brophy, 1986; Lester & Garofalo, 1985). In this study, all subjects demonstrated a similar level of awareness representing 65% of the total score, considered good metacognitive awareness (Swanson, 1990). Perhaps, as was the case with all subjects in this study, those subjects who have had substantial exposure to high level mathematics coursework have reached a threshold where metamemory differences are negligible (as in knowledge of task difficulty and problem-solving awareness), and that what differentiates performance shifts to other factors such as metacognitive regulation and strategy use.

Solution Strategies

Significant differences in strategy use between high and low mathematics knowledge groups were evident in four of the six problems. (The two problems which showed no significant differences, 1 and 6, were the easiest and the hardest respectively.) Recall that there were several strategies that were more acceptable ways than answer elimination or guessing to solve different problems. These different strategies (with the exception of reasoning) were collapsed into one category to examine differences by group in strategy use, reasoning, answer elimination and guessing. Significant differences between groups were evident in use of these specific strategies vs. guessing, a somewhat predictable result, even though the "low" knowledge students are not poor students.

The more important finding became clear when the two groups who had used specific strategies or guessing were removed and the problem solvers who remained were examined. These students, who could not engage the problem through thorough understanding, yet did not want to give up, ultimately chose to use reasoning skills. There were no significant differences between these groups since both high and low students were willing to attempt the use of reasoning. This in itself reveals that low knowledge students do not choose to guess all that quickly, but rather try to think of an alternate way sometimes. However, it was evident that low knowledge students were less successful when they used reasoning. This discrepancy indicates a need to emphasize the teaching of reasoning skills. It also suggests the interrelationship between reasoning and metacognitive regulation.

Gender

This study addressed the relationship of gender to performance, metacognition, strategy use, and beliefs, in particular, attributions. Gender differences were predicted in only one of the variables in this study, performance. Specifically, it was predicted that there would be significant differences in favor of boys in performance.

No gender differences were found in any of the variables, with two small exceptions, in the high knowledge group. Those exceptions were: (a) high knowledge boys outperformed high knowledge girls (as well as both other groups) on Problem 5, and (b) high knowledge girls were less likely to attribute their success in mathematics to effort than were high knowledge boys.

The finding of no gender differences in performance was not expected given the findings of gender differences in the literature and the items selected for this study. Studies of mathematics ability and/or achievement have consistently found sex differences favoring males among students (Fennema, 1974; Fennema & Sherman, 1977; Benbow, 1988, 1992). Additionally, five of the six items selected for this study were chosen because there was a 12% difference favoring males with mathematics grade point avererages (MGPA) ranging from A+ to B+ on the 1987 MSAT. Other item characteristics that favored males were also considered. Since word problems are considered harder for girls, four of the six problems chosen were word problems (Chipman, 1988; Graf, 1972; Rosser, 1989) and the content of four of the six problems was algebra or geometry, content that has favored boys (Becker, 1990; Donlon, 1973; Engelhard, 1990). The cognitive complexity of the items was varied systematically since research has suggested that as the cognitive complexity of items tends to increase, differences in performance in favor of boys tend to increase (Armstrong, 1981; Bock & Moore, 1986; Dossey et al., 1988; Fennema, 1974; Marshall, 1984). The most difficult items, numbers five and six, were placed last since research suggests that placing the harder problems last favors boys (Becker, 1990). Finally, research has also indicated that boys have been more successful than girls on multiple-choice questions of the type chosen for this study (Becker, 1990; Benbow, 1992; Strassberg-Rosenberg & Donlon, 1975). Thus, the items were chosen and placed to facilitate the exploration of gender differences. The finding of no gender differences may be due to two controls used in this study: exposure to coursework and the quality of schooling.

All of the participants in this study had taken at least three and a half years of mathematics, including trigonometry and/or precalculus (a more in-depth exposure to functions) and geometry. Although the discrepancy between male and female participation in high school mathematics has diminished over recent years (Armstrong, 1985; Ekstrom, Goertz, & Rock, 1988), males still appear to take more math courses than females at the higher levels (Dossey et al., 1988; Wilder & Powell, 1989). Additionally, males are more likely to be placed in advanced classes earlier (Battista, 1990) and to receive higher levels of instruction than females (Armstrong, 1985;

Eccles et al., 1983; Fennema & Sherman, 1977; Wilder & Powell, 1989).

This study also controlled for type of schooling experience. The schools chosen all had more advanced course offerings, placed their low knowledge students in appropriately difficult mathematics courses in the eleventh grade, and used textbooks, not review books, as the primary teaching resource; thus the schooling experience was similar and strong for all subjects.

Perhaps some of the studies that assumed their procedures for controlling the course differential factor and schooling experience were adequate, in reality, did not control for these important factors appropriately. Indeed, Doolittle and Cleary (1987), in attempting to design a study of gender-based differential item performance for high school seniors, tried to control course selection similar to the current study and were unable to completely reduce differences in course taking. Thus, it may be that when girls are exposed to the same courses, particularly those at higher levels (as was the case in this study), a significant amount, though not all, of the variance in performance disappears. The two controls described here may have contributed to the finding of no gender differences, suggesting that the factors that lead to gender differences in performance may indeed lie in other areas than those previously cited areas of difference.

The finding of no gender differences may also suggest that gender differences in cognitive abilities, particularly quantitative, are decreasing. Recent research examining cognitive abilities indicates that gender differences in the verbal and quantitative areas have been decreasing in recent years (Hyde et al., 1990; Feingold, 1988, 1993; Wilder & Powell, 1989). Feingold (1993) has demonstrated that cognitive gender differences are developmentally related. He found that although cognitive gender differences have remained constant for children, they have decreased for older adolescents. Benbow (1992) showed that there remain significant gender differences among highest ability students at seventh and eighth grades; she claims that there are probably no differences in coursework or schooling at that level. Despite her view regarding no differences in coursetaking at that level, tracking for advanced mathematics classes generally begins earlier than seventh grade pre-algebra, often in fifth grade. Additionally, the best students often obtain the more experienced teachers at this very critical period. Conclusions drawn on supposedly equal coursework and schooling experiences should be drawn cautiously. Likewise, studies need to specify the age group included in any generalizations. The results of this study support the most recent research suggesting that differences are diminishing for adolescents.

Problem 5 Gender x Ability Interaction

There was a significant difference related to gender on Problem 5: high knowledge boys performed significantly better than high knowledge girls. Although several recent studies found diminishing differences related to gender, they have also noted the importance of exploring areas where small differences may still exist, especially in late adolescence on difficult problems (Hyde et al., 1990), and where the potential effects of item characteristics may be examined (Becker, 1990; Doolittle & Cleary, 1987; Engelhard, 1990). Significant gender differences were evident in three factors related to Problem 5: differences in verification, strategy use, and the attribution of effort in the high knowledge group. Discussion of these differences follows.

Differences in Verification. Analysis of the AIMM found one significant difference related to gender. For boys in the high group, understanding was significantly correlated with planning, execution, and verification. For girls in the high group, understanding was significantly correlated with planning and execution, but not verification. Gender-related differences were confirmed by the log linear analysis. A gender by mathematics knowleoge interaction indicated that high knowledge girls were less likely to verify than high knowledge bc..

There are a few possible hypotheses for these differences. It may be that for harder problems, such as number 5, verification becomes more important; girls may not understand its importance and just check their calculations. These girls may have a more superficial understanding of the role of verification in problem solving and at the highest levels, such an understanding affects their performance. Another possible hypothesis may be that high knowledge girls found the problem so difficult they did not attempt to verify. Schoenfeld (1985) found that college freshmen rarely verified, and if they did, it was at the local (calculations), not global level (solution process). Recognizing the importance of verification is important for all students. More research is needed to examine differences in the verification process.

Strategy Use Differences. Analysis of the AIMM for Problem 5 also revealed a significant gender-related difference in the use of a key strategy, eliminating irrelevant information. The most strategic way to solve this problem was to understand that depth was irrelevant. Researchers have cited eliminating irrelevant information as an important metacognitive activity since it involves recognizing something extraneous, reorganizing a problem and having the confidence to remain with that decision (Goldin & McClintock, 1984; Lester, Garofalo, & Kroll,1989; Schoenfeld, 1985). Eight boys and only one girl chose this strategy. An equal number of girls and boys chose volume, and significantly more girls solved the problem visually with depth. However, both methods are less strategic and more prone to error. Also, fewer boys than girls resorted to eliminating answers or guessing, the strategies of last resort. Twenty-four boys compared to 16 girls in this group of 50 obtained the correct answer, the difference being the eight boys who chose the elimination of depth strategy. The implication here is that in certain problems, particularly a hard one with computations, high knowledge boys are more likely to choose the "best" or "most efficient" strategy.

Problem 5 prompted the use of different strategies by high knowledge boys and girls but in Problem 6, the most complex problem, there were no significant differences in Performance or strategy use by gender *or* mathematics knowledge. Problem six was different than problem five in that there was no irrelevant information but it did require an exceptionally good conceptual grasp of the problem in order to distinguish between two of the possible answers. It is a characteristic of the last few questions on the SAT's that answer choices have subtle yet real differences. Even the best students struggled here and it bodes well for females that high knowledge girls performed equally as well as boys, considering that the conditions which supposedly favor boys (word problem, multiple choice, most difficult problem, serial effect, algebra problem) were more evident in this problem than any other.

One possible hypothesis might be it is probably "riskier" to eliminate information, and the best boys may have been more confident in doing that than the girls. In contrast, however, distinguishing between the two subtlely different answers required a *complete* understanding of the relationship of distance, time and rate, with no apparent strategy to simplify the problem.

Differences in Effort Attribution. One other hypothesis may be that high boys used more effort than high girls on this problem. If a student did not eliminate depth or solve visually, then Problem 5 required more mathematical computations, a skill which favors girls. Although an equal number of boys and girls (10) used the volume, all boys solved it correctly whereas only three girls solved it correctly. Additionally, it is important to recall the significant differences in attributing effort to performance, with high knowledge boys significantly more likely to attribute success to effort than high knowledge girls. Perhaps the boys worked harder to get a final answer, suggesting that at some point in the problem solving process, especially in more complex problems, boys are willing to exert more effort.

It could be that the nature of the differences seen between high knowledge boys and girls on Problem 5 is indeed multifaceted and includes choice of metacognitive strategy (thinking to eliminate irrelevant information) as well as affective decisions (maintaining effort to get to the solution), because of the type of problem. On Problem 6, there were fewer, if any, calculations, which reduced the need for effort related to that kind of activity. However, more complexity in thinking and monitoring of understanding was required. High knowledge boys were no more successful in integrating the information in Problem 6 than were the girls, suggesting that the multifaceted factors influencing complex problem solving are problem dependent.

In summary, using stringent controls, this study found no gender differences for the most part. This study supports recent research that shows that gender differences are diminishing. Similarly, these results suggest that if gender differences do emerge in early adolescence, they may diminish by late adolescence; therefore studies must be designed to be more precise regarding the focus of the age group for purposes of generalizability. Further, this study emphasizes the need to explore gender differences in mathematical problem solving more microscopically, at the item level (Becker, 1990; Doolittle & Cleary, 1987; Engelhard, 1990; Kimball, 1989; Feingold, 1993; Hyde et al., 1990; Lester, Garofalo, & Kroll, 1989; Schoenfeld, 1987, 1989). The structure of the AIMM, based on the metacognitive frameworks described above, is an initial attempt in the direction of microscopic research. Some findings regarding the relationship of metacognitive regulation and strategy use, as measured by the AIMM, to other variables is presented in the next section.

Attributions

No significant differences by gender or mathematics knowledge were found in subjects' attributions. These results confirm other work suggesting that there are no differences in causal attributions by gender or ability (Frieze et. al, 1982; Hyde, 1990; Sohn, 1982; Schoenfeld, 1987). One possible hypothesis may be that older male and female adolescents who may have had successful mathematics experiences (staying in high school mathematics for at least 3-1/2 years) have modified their attributions with relation to their schooling experience.

These findings do not support some early research on attributions and mathematics achievement, for several reasons (Fennema, 1974, 1977, 1980; Parsons et al., 1982; Ryckman & Peckham, 1987; Stipek, 1984; Wolleat et al., 1980). Much of that research was done on middle school children, or ninth and tenth graders. Further, at least one study mentioned that there was a considerable amount of variability in terms of schooling experiences such as number of students enrolled in college preparatory mathematics classes, and number and extent of mathematics offerings (Wolleat et al., 1980).

Findings with regard to ability and effort were especially noteworthy. The current study found a significant interaction for effort but not ability, and the direction of the interaction was different than that found in previous studies. In particular, high knowledge girls were significantly less likely to attribute their success to effort than were the high knowledge boys. These findings may reflect some of the differences in methodology described above; causally-related statements were used and these questions were given immediately after a problem solving experience. Additionally, the nature of this particular sample may also have been a factor in these findings. The implication of the results of this study is that girls who remained in the most advanced classes might have well believed themselves to be hard workers; yet, interestingly, they were significantly less convinced than boys that success was due to effort and at least as convinced as the boys that it was due to their ability. These results contrast with those of other researchers (Deboer, 1986; Fennema, 1985; Licht & Dweck, 1983; Stipek, 1984; Wolleat et al., 1980).

In addition, research on children's reasoning about the relation between effort and ability (Nicholls, Jagacinski, & Miller, 1986; Nicholls & Miller, 1984) indicates that up until adolescence, children do not differentiate ability from effort and do not have a fully developed idea of ability as a capacity limiting the effect of effort on performance. Some researchers suggest that the concept of a *stable* ability, referred to as an "entity" concept of ability, is developmental (Dweck & Bempechat, 1983) and limits the effect of effort on performance. Clearly, high knowledge girls in this study perceived the role of effort in higher order problem solving differentially than boys, even though they were as confident as the boys about their ability to perform competently. This awareness may be a partial explanation of the gender difference in the high group on Problem 5. If girls' perception of the role of effort in success is different from boys' perception, they may be less willing to work when faced with the hardest problems, such as Problem 5. Becker (1990) suggested this as a possible explanation for why girls seem to do less well on the harder, later SAT problems. Possibly, by late adolescence, the notion of ability as limiting the effect of effort on performance is more defined, albeit differentially by gender, for high achieving mathematics students; this is, of course, a rather provocative interpretation. Is it possible that high achieving late adolescent girls, who generally receive better grades in high school math courses than boys because of hard work (Kimball, 1989), may use less effort than boys when faced with the most difficult problems, even when they think they are as capable as boys? If so, then it is important that girls in the most advanced math classes understand what kind of effortful behavior is necessary for extremely high level achievement. Future exploration of this conjecture is needed.

While they readily grasp ability and effort as impacts on their performance, the best male students ascribe more influence to effort than the best female students. This study expands our understanding of the role of attributions regarding mathematics in older adolescents. Lack of gender differences suggests that girls and boys change their perceptions during the four years that begins with adolescence until they leave high school as young adults; particularly affected are those students who prepare to continue in college mathematics.

From all of this, one questions remains: Where are the high knowledge girls? If girls with comparable mathematics knowledge essentially use similar problem-solving skills as boys, as this study suggests, then we may rule out processing differences. However, there are still twice as many boys as girls with MSAT scores above 600. Recently, The New York Times revealed that 18,000 boys were Merit Scholarship Semi-Finalists this year, as compared to 8,000 girls. Other factors that are keeping girls out of the most advanced classes must be investigated. This research strongly indicates that if girls are in those classes, they perform equally as well as boys. However, the problem remains: there are not enough girls in the most advanced classes. Unfortunately, the only question that seems possible to ask of that conclusion is "why?" And what must schools do to encourage girls?

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Tables

Table 1
Means and Standard Deviations for Performance, AIMM, MAA by Group

	Performance		AIMM		MAA	
Group	M	SD	M	SD	M	SD
Low Boys	2.72	1.69	32.08	14.01	38.60	4.56
Low Girls	2.52	1.01	33.80	12.12	38.24	4.41
High Boys	4.88	1.17	57.48	8.09	40.52	5.17
High Girls	4.60	1.63	53.60	7.59	40.08	3.08
Total Sample	3.68	1.63	44.24	15.65	39.36	4.41
Total score	Perf. = 6.00		AIMM = 72		MAA = 60	

Table 2
Contrast Testing for Performance and the AIMM

Group	Performance t-value	AIMM t-value
Gender	.96	.49
Math. Know'edge	8.53*	10.43*
Interaction	-.16	-1.29

* Significant at .05 with Bonnferoni correction, p < .003.

Table 3

Regression: Performance on PSAT, Gender, AIMM, THE MAA, Luck, Skill, Effort, Gen.Belief in Ability, Gen. Belief in Effort, and Gen. Belief in Motivation.

	Variance Explained:	
R Square	.76	
Analysis of Variance	MS	DF
Regression	19.86	10
Residual	.71	89
F	28.01	Sign F <.0001

Variable	Standard. Coef.	F for Coef.
PSAT	.07	.77
Gender	-.04	.46
AIMM	.82	89.98*
THE MAA	-.03	.35
Luck Attrib	.03	.22
Skill Attrib.	.02	.13
Effort Attrib.	.10	3.33
Belief in ability	-.01	.05
Belief in effort	-.02	.02
Belief in Motivation	-.01	.01

* p < .01

Table 4
Correlations Among Variables and Standard Discriminant Function

Variable	Performance	AIMM	MAA	Stan.Disc.Func
Performance	1.00	-	-	.13
AIMM	.86**	1.00	-	.93
MAA	.28	.37**	1.00	.11
PSAT	.66**	.73**	.21*	-
Gender	-.07	-.04	.05	

*p < .05 **p < .01

Estimates for Performance, AIMM and MAA.

Group	t -value	Sig. t
Performance		
Gender	.96	.336
Math Know.	8.53	<.001*
Interaction	-.16	.872
AIMM		
Gender	.49	.619
Math Know.	10.43	<.001*
Interaction	-1.29	.199
MAA		
Gender	.46	.648
Math Know.	2.15	.034
Interaction	-.05	.963

* Significant at .05 with Bonnferoni correction, p < .003

Table 5
Means and standard deviations for categories of CMSA by group (1 = low boys, 2 = low girls, 3 = high boys, 4 = high girls).

Group	Understanding		Planning		Execution		Verification	
	M	S.D.	M.	S.D.	M.	S.D.	M.	S.D.
1	12.64	5.56	13.60	5.77	5.00	3.12	.76	1.13
2	13.08	4.01	14.08	5.21	5.56	2.34	1.44	2.45
3	20.68	2.29	22.32	1.80	9.96	2.17	4.32	3.24
4	19.92	2.60	21.16	2.41	10.04	1.81	2.48	3.17
Sampl.	16.58	5.33	17.79	5.73	7.64	3.37	2.25	2.93

Contrast testing for Understanding, Planning, Execution and Verification by gender, mathematics knowledge and interaction

Group	t-value for Understanding	t-value for Planning	t-value for execution	t-value for verification
Gender	.21	.41	-.66	1.10
Math Know.	9.69*	9.48*	9.74*	4.36*
Interaction	.78	.98	.50	2.39

* Significant at .01 with Bonnferoni correction , p < .004

Table 6
Hierarchical Log Linear Analysis: Strategy Use by Gender and PSAT

Prob#	Best Gen. Class	Likelihood ratio Chi Square	DF	Prob.
1	Strategy only	14.84	15	.46
2	PSAT x Strategy	6.60	10	.76*
3	PSAT x Strategy	8.32	10	.60*
4	PSAT x Strategy	5.32	10	.87*
5	PSAT x Strategy	4.90	10	.90*
6	Strategy only	23.73	15	.07

* Significant differences in strategy use between high and low groups.

Contrast Testing for Strategy Use, Reasoning, Elimination, Guessing

Group	t -value for Strategy Use	t-value for Reasoning	t-value for Elimination	t-value for Guessing
PSAT	-8.40**	1.63	2.18	5.57**
Gender	.08	-.10	.84	-.45
Interaction	-1.95	2.59*	-1.17	1.11

** Significant at the .05 level with Bonnferoni correction, $p < .004$.

* Not significant with Bonnferoni correction, $p < .004$

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Table 7
Means and Standard Deviations for Attributions (1-10) regarding success or failure in mathematics (1= low boys, 2= low girls, 3 = high boys, 4 = high girls) (N= 100).

	Group 1		Group 2		Group 3		Group 4		Entire Sample	
Attribution	M	SD	M	SD	M	SD	M	SD	M	SD
When I get a good grade in math...										
1. it's because I work hard	1.7	.8	1.4	.6	1.2	.4	1.8	.6	1.5	.6
2. it's because the teacher likes me	3.4	.7	3.6	.6	3.5	.7	3.6	.7	3.6	.7
3. it's just a matter of luck	3.6	.6	3.1	.7	3.6	.7	3.2	.8	3.4	.7
4. it's because I'm always good at math	2.2	1.0	2.5	1.1	2.4	.9	2.2	1.0	2.3	1.0
5. I never know how it happens	3.8	.5	3.6	.5	3.8	.6	3.4	.9	3.7	.6
When I get a bad grade in math										
6. It's because I don't study hard enough	1.6	.9	1.6	.9	1.6	.6	1.7	.7	1.6	.8
7. It's because the teacher doesn't like me.	3.4	.7	3.9	.3	3.6	.7	3.7	.7	3.7	.6
8. It's just bad luck	3.7	.7	3.5	.7	3.5	.8	3.4	.8	3.5	.7
9. It's because I'm just not good at math	3.2	.9	2.7	1.1	3.2	1.0	3.3	.9	3.1	1.0
10. It's because of careless mistakes.	2.0	.9	1.7	.61	1.8	.6	1.8	.9	1.8	.8

1 = very true; 2= sort of true; 3= not very true; 4= not true at all.