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ABSTRACT

Mathematics learning and practice in school and out of school differ in some significant ways which are explained by the fact that: (1) problem in everyday situations are embedded in real contexts that are meaningful to the problem solver; and (2) the mathematics used outside school is a tool in the service of some broader goal. This paper discusses research which examined mathematics practice in everyday work situations by comparing in-school and out-of-school practice. It presents a framework for gaining insight into the interplay between socio-cultural and cognitive developmental processes through the analysis of practice. Discussion of the research illustrations includes goals of the activity, conceptual understanding, and flexibility in dealing with constraints. Suggestions for teachers in connecting in-school with out-of-school experiences are given. Contains 25 references. (MKR)

Mathematics Learning and Practice In and Out of School: A Framework for Making These Experiences Complementary

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Introduction

A variety of researchers in the last fifteen years have described how people use mathematics in out-of-school situations to solve problems and achieve goals (e.g., Lave, 1988; Masingila, 1992a; Millroy, 1992; Scribner, 1985). Furthermore, it is generally accepted that mathematics learning "is not limited to acquisition of the formal algorithmic procedures passed down by mathematicians to individuals via school. Mathematics learning occurs as well during participation in cultural practices as children and adults attempt to accomplish pragmatic goals" (Saxe, 1988, pp. 14-15).

However, there are differences between mathematics *practice* in and out of school, as well as mathematics *learning* in and out of school. Lave (1988) has found evidence that mathematics practice in everyday settings differs from school mathematics practice in a number of ways. In everyday settings: (a) people look efficacious as they deal with complex tasks, (b) mathematics practice is structured in relation to ongoing activity and setting, (c) people have more than sufficient mathematical knowledge to deal with problems, (d) mathematics practice is nearly always correct, (e) problems can be changed, transformed, abandoned and/or solved since the problem has been generated by the problem solver, and (f) procedures are invented on the spot as needed.

Researchers who have investigated how persons solve problems in school-like situations and solve mathematically-similar problems in everyday contexts found that in the former situation people "tended to produce, without question, algorithmic, place-holding, school-learned techniques for solving problems, even when they could not remember them well enough to solve problems successfully" (Lave, 1985, p. 173). When the same people solved problems in situations that appeared different from school, they used a variety of techniques and invented units with which to compute (Lave, 1985).

These differences in mathematics practice appear to be explained by the fact that: (a) problems in everyday situations are embedded in real contexts that are meaningful to the problem solver and this motivates and sustains problem-solving activity (Lester, 1989), and (b) "the mathematics used outside school is a tool in the service of some broader goal, and not an aim in itself as it is in school" (Nunes, 1993, p. 30)

Just as mathematics practice in and out of school differs, so does mathematics learning. Whereas school learning emphasizes individual cognition, pure mentation, symbol manipulation and generalized learning, everyday practice relies on shared cognition, tool manipulation, contextualized reasoning and situation-specific competencies (Resnick, 1987).

Knowledge constructed in out-of-school situations often develops out of activities which: (a) occur in a familiar setting, (b) are dilemma driven, (c) are goal directed, (d) use the learner's own natural language, and (e) often occur in an apprenticeship situation allowing for observation of the skill and thinking involved in expert performance (Lester, 1989). Knowledge gained in school all too often grows out of a transmission paradigm of instruction and is largely devoid of meaning (lack of context, relevance, specific goal). Furthermore, Resnick (1987) has argued that "[t]he process of schooling seems to encourage the idea that . . . there is not supposed to be much continuity between what one knows outside school and what one learns in school" (p. 15).

In some instances, the difference between mathematics practice in and out of school may be inherent. Sometimes a mathematical concept is understood and used differently in everyday situations than the way it is taught in school (e.g., de Abreu & Carraher, 1989). For example, percentage of change is a common concept in retailing and in school mathematics. In school, percentage of change is understood to be the amount of change from the original amount. A typical textbook exercise involving this concept might be the following:

Find the percent of change for a video game system that costs $29 in 1980 and $99 in 1990. (Davison, Landau, McCracken & Thompson, 1992, p. 262)

A student finding the answer to this exercise would subtract $29 from $99 to get a $70 increase, then divide $70 by $29 to get an increase of approximately 241%. Percentage of change in retailing, however, is understood to be the amount of change from the retail price. Thus, for the situation in the textbook exercise above, a retailer would divide $70 by $99 to get an increase of approximately 71%. Since the final result in retailing is sales, all percentages of change are based on retail prices. In this case, the solution process in the everyday context is different because of the different conceptual understanding of percentage of change (Masingila, 1993b).

We believe that while some differences may be inherent in mathematics learning and practice in and out of school, the differences can be narrowed so that instead of being disjoint activities that do not influence each other, mathematics learning and practice in and out of school can build on and complement each other. In this way, students can bring to bear their mathematical knowledge gained in out-of-school experiences on their school mathematics. Likewise, students can use their school mathematics in solving problems that occur in everyday situations. Acioly and Schliemann (1986), in their study of lottery game bookies in Brazil, found that the bookies with school mathematics experience were able to understand and solve novel problems while bookies who had not attended school were unable to do this. In this case, the schooled bookies seemed able to draw on their school mathematics to use in an out-of-school situation.

Individuals need both in-school and out-of-school mathematical experiences in our society. Without everyday mathematical experiences, in-school learning is solely for the sake of learning. Students need in-school mathematical experiences to build on and formalize their mathematical knowledge gained in out-of-school situations. This occurs in the classroom through the activity of constructing knowledge that is subject to "explanation and justification as students participate in the intellectual practices of the classroom community" (Cobb, Yackel & Wood, 1992, p. 7).

The question, then, is how can teachers use students' in-school and out-of-school experiences so that mathematics learning and practice in these contexts can be complementary? Before discussing and elaborating on a framework that we think sheds some light on this, we will present some research from several studies (Agwu, 1993; Davidenko, 1994; Masingila, 1992a, 1993b; Prus-Wisniowska, 1993) that illustrates some of the points made above and lays the groundwork for discussion of the framework.

Illustrations from Research

Structure of the Studies

In each of these studies, mathematics practice was examined in an everyday work situation. Workers in each context were observed and informally questioned as they worked. Data were collected from carpet laying estimators and installers, a dietitian, an interior designer, a retailer, a restaurant manager, and a textile designer. We analyzed the data through a process of inductive data analysis, looking for the concepts and processes that were involved in the mathematics practice in these contexts.

Selected problems that occurred in these contexts were then given to pairs of secondary students. We observed and informally questioned the pairs of students as they solved these problems. We analyzed the data by examining how each pair solved the problems, and then compared these with how the persons in the work context had solved the same problems.

Comparing In-School and Out-of-School Practice

The follow discussion focuses on three key differences that we found in our work comparing in-school and out-of-school mathematics practice. The differences involved the goals of the activity, the conceptual understanding of persons in each context, and flexibility in dealing with constraints.

Goals of the activity. In each context (carpet laying, dietetics, interior design, retailing, restaurant managing, school, textile design), the distinction between the goals of the individuals in the out-of-school contexts were in sharp contrast to the goals of the students in the school context as long as the students viewed the problems as school problems. However, when the students were able to place themselves in the everyday situation, they appeared to view the problems differently.

For the individuals in work situations, the goal was to make a decision. In the course of that decision-making process, problems needed to be solved and mathematics was a tool to be used to solve the problems. The students had as their goal to solve the problems we gave them; nothing beyond that was required of them. We offer two examples of this distinction.

In the restaurant management context, the restaurant manager was faced with the problem of changing a recipe obtained from a newspaper (Prus-Wisniowska, 1993). The recipe gave the ingredients for serving six persons (see Figure 1). However, the restaurant manager needed to use the recipe for a dinner party for 20 people. Her goal appeared to be to decide the amount of each ingredient needed and give instructions for the cooks while being efficient. To this end, she decided to make enough fruit salad for 24 portions and divide the remaining four portions among the 20 fruit cups.

When asked to change the recipe for exactly 20 portions, she computed the factor $20 \div 6 = 3.3$ on her calculator and used it to increase the amount of each ingredient. To make the recipe feasible for the cooks, she changed each decimal into a proper fraction. She noted that "we only work with halves, thirds, and fourths." For example, since the 2 cups of apples became 6.6 cups when multiplied by the factor of 3.3, she indicated that "Six and a half cups is fine for that." The restaurant manager used 1 1/2+ for the 1.65 cups of sugar in the increased recipe: "The cooks know to put just a little bit more in when they see the plus sign."

However, in the case of the cream, 1.65 cups of cream she noted as 1 1/2 cups and 1 Tbls. She indicated that it was important to be more specific for the cream. The restaurant manager figured that .8 cups of vinegar were needed for 20 portions and she reasoned as follows: "The .8 is a result of multiplying 3.3 by 1/4 so I won't be far off if I skip the .3 and multiply 3 by 1/4. So I'll just need 3/4 cup for that." She also decided that 7 eggs were needed.

When the restaurant manager was asked about buying the groceries needed for making the fruit and vegetable salad, she responded that she would first check to see if she already had all the ingredients and then order those that she needed. For instance, if she was low on sugar she would order a large quantity since it will be needed for other things as well: "Only in the case of very unusual meals, like scallop salad, do I order the exact amount."

A pair of secondary students was asked to change the same recipe so that it would make 10 portions. These students were second-year high school students in an advanced algebra course. They were on track to take calculus in their fourth year of high school. They immediately calculated $10 \div 6 = 1.7$ and began increasing each ingredient by this factor. When they multiplied the 1/2 cup of carrots by the factor 1.7 and obtained .833 cups, they were not able to interpret this as a proper fraction and abandoned this approach. Instead, the students set up a proportion for each ingredient. For example, for the apples they used the proportion $6/2 = 10/x$ and obtained 3 1/3 cups of apples. Using this method, the students found the increased amount for each ingredient, including 5/6 cup of carrots. When asked about how they would measure 5/6 of a cup, the pair decided to accept only halves, thirds, fourths, and eighths.

The students decided that 5/6 cup of carrots was equivalent to "2/3 cup + 1/2 of 1/3 cup." They said they could measure this by just filling up half of a 1/3 measuring cup. The students found that they needed 3 1/3 eggs and decided that they would mix together four eggs and then take out about 2/3 of an egg. We noticed that, unlike the restaurant manager, these students had no discussion about the level of accuracy that was needed for the ingredients involved. We also noted that the restaurant manager was able to use mathematics as a tool in order to modify the constraints and make decisions that made sense in her situation. Certainly to her, using fractions instead of decimals allowed her to make better use of her number sense and she was able to convert the decimals into fractions that suited her purpose.

The students had the mathematical knowledge to change their strategy when they ran into difficulties in dealing with .833 of a cup. However, their seemingly lack of number sense (e.g., to view .833 cup as a little more than 3/4 cup) prevented them from pursuing this strategy. But when they used proportions to obtain fractional representations, the students were able to make sense of the measurements. However, when asked to decide what groceries they would need to buy for the fruit and vegetable salad, the students' list contained items like 3 1/3 cups of apples and 1 2/3 tablespoon of butter. It appears that these students saw the goal of the problem was simply to obtain measurements for the ingredients without regard to their reasonableness in an everyday situation.

Another pair of students, however, after thinking about the problem noted that if they really had to make this salad they would make the salad for 12 portions by doubling everything and then divide the two extra portions among the 10 people. These students were second-year high school in a geometry course. While these students initially saw the problem as simply using proportions to find the increased amounts, they realized that they would do things less formally in an out-of-school context. Their list of groceries was more reasonable also, including items such as 10 apples, 1 small can of peanuts, 1/2 dozen eggs (the smallest amount they thought they could buy), the smallest possible bag of sugar and flour, a pint of cream. However, like the first pair, they did not consider whether they would have some of these items already in stock.

Another illustration of this difference in goals is from the interior design context. The interior designer was deciding on the amount of materials needed for a house that was being refurbished. One aspect of the redecorating involved wallpapering a number of walls of the house. The wallpaper that was chosen was 20" wide and had a repeat length of 9" (i.e., the pattern repeated every 9"). The interior designer told me that what she does in considering repeat lengths is to divide the desired length by the length of the repeat, in this case 96" ÷ 9". This gives the number of repeats in the desired length. She calculated that there were 10.67 repeats, "so there needs to be 11 repeats in each strip of wallpaper so 11 times 9" means 99" needs to be considered as the length of the wall. So, you figure the number of strips you need for the rooms and figure each strip to be 99" long. Of course, you also have to consider that you have to place the seams for the strips at least two inches past the corner so it stays down better. Also, wallpaper is sold in single, double, and triple rolls so you must figure the best deal for what you need." The interior designer's solution process is aimed toward the goal of making a decision the quantity of materials needed while considering cost efficiency.

We gave a pair of secondary students a problem concerning this same situation. These students were from the same advanced algebra course as the first pair of students who solved the recipe problem. In order to structure the situation a bit more and focus on how the students would deal with the repeat length constraint, we gave the following problem:

Suppose you decide to wallpaper your bedroom. If your room is 10' by 8' and the walls are 8' high, how much wallpaper do you need if the wallpaper has a repeat length of 9"? The wallpaper is 20" wide and a roll has 45 feet of wallpaper on it.

The concept of repeat length was explained and the students indicated that they understood. They began the solution process by calculating that the one wall was 120" long and the other was 96" long. The students decided that they would need 6 strips of wallpaper on each of the 10' walls and 5 strips of wallpaper on each of the 8' walls. They figured that each wall was 96" from ceiling to floor so that 576" of wallpaper were needed for each 10' wall. When asked to explain what the 576" inches meant, they said that there were 6 strips needed across the wall and each strip was 96" long "and 6 times 96 is 576." The students continued in this vein and determined that each 8' wall would need 480" of wallpaper. Totaling the amount needed for the walls, they indicated that 2112 inches of wallpaper were needed for the room.

The students appeared to act as if their solution was complete. Finally, as one student checked back over the statement of the problem, she noted that they had not considered the repeat length: "Well, how would we do this if we were really going to wallpaper a room? Maybe we should draw a picture to see what this looks like." At that point, the students drew a diagram of one 10' wall: "The first strip doesn't matter. The pattern doesn't have to match anything, so all we need is 96". But the next strip has to match that one."

The students decided to divide 96 by 9 to find out how many sections of the pattern are on each wall. They determined that there were approximately 10.7 sections of the pattern in the first strip and so three inches were going to have to be trimmed off the wallpaper before cutting the next strip. When asked to explain their reasoning, one student replied, "Since 99" would be a whole number of 9" sections, we have to cut off three more inches [after the 96"] so that the pattern will be starting again for the next strip." At this point, the students decided that they could treat all the remaining strips as 99" and come out with matching patterns. They totaled the strips and decided that 2175 inches of wallpaper were needed. The students made no attempt to determine the number of feet or number of rolls of wallpaper that would be needed but seemed satisfied to leave their solution in inches.

The approaches to the problem-solving activity were different in the in-school and out-of-school contexts because the persons involved had different goals. For both the restaurant manager and the interior designer, solving the problems was a necessary part of their job. They used mathematics as a tool to help them solve problems and not as the goal of the problem. The students, however, seemed to view the problems as mathematical exercises and immediately started using algorithms that they thought would be appropriate. Although two of the pairs gained some insight when they tried to put themselves in the everyday problem situation, they did not stick with this perspective totally and did not check the reasonableness of their solution with the everyday context (e.g., checking for groceries already on hand, converting the number of inches of wallpaper needed into the unit of rolls).

Conceptual understanding. We observed differences in conceptual understanding between individuals in the everyday work situations and the secondary students. While the students had the procedural knowledge to solve the problems, they were not able to understand the concept involved and apply the procedures. On the other hand, the workers understood the concept, at least in this context, and had the tools necessary to solve the problems. The following two examples illustrate the differences between the workers and the students.

In the carpet laying context, the concept of area is pervasive in the all work done by the estimators and installers. All the workers Masingila (1992a) observed converted square feet to square yards by dividing square feet by nine. This algorithm is essential in the carpet laying business since measurements are taken in feet but carpet must be ordered from a supplier in square yards. In a conversation, Dean, an estimator, explained why this algorithm worked.

Joanna:	If you just know the length and width of a room, how do you find how many square yards of carpet you need?
Dean:	Well, if the room is 12' by 8' then you take 12 x 8 ÷ 9.
Joanna:	What does the 9 mean?
Dean:	That's the way you convert square footage to square yardage.
Joanna:	Okay, but where does the 9 come from?
Dean:	I don't know. Maybe I don't understand the question. . . . Where does the 9 come from?
Joanna:	Yeah, why isn't it 8 or 6?
Dean:	Well, when you have square footage (draws diagram with 3 by 3 grid—see Figure 2) . . . each of these squares is a square foot and there are three feet in a yard (puts x's inside the three squares in the right column of the grid) and then three across (puts x's inside three squares in the top row)—so that makes 9. (pp. 114-115)

By using a diagram Dean was able to illustrate, although not fully articulate, that in one square yard there are nine square feet and to convert from square feet to square yards involves dividing by nine.

In contrast, several pairs of secondary general mathematics students were given a similar problem and did not understand that the concept of area was involved. The problem and conversation with two students, Jim and Matt, are given below.

Suppose you need a piece of carpet 12 feet by 9 feet. How many square yards should you order from the carpet supplier?

Matt:	I don't know nothin' about square yards.
Joanna:	Well, let's see. What does a piece of carpet 12' by 9' look like?
Jim:	(draws a rectangle and labels the dimensions 12' and 9')
Joanna:	Alright. Now how would you change that to yards?
Matt:	Divide by 3.
Joanna:	Why?
Matt:	'Cause it takes 3 feet to make a yard.

Jim:	(writes "4 yds" and "3 yds" and scribbles out 12' and 9')
Joanna:	Okay, now how many square yards is that?
Jim:	Square yards? Oh . . . well, there's two 4's and two 3's—one on each side. So that's 4 square yards and 3 square yards.
Joanna:	What does square yards mean?
Matt:	I don't know. (Jim shakes his head.)
Joanna:	What it means is area; finding the square yardage of this carpet is finding the area.
Jim:	So that's 4^2—8 and 3^2—6 and take 8 x 6.
Joanna:	Where did you get the 8?
Jim:	4^2—4 x 4—no, that's not 8. Area is length times width times height. (pause) I'm not sure.
Joanna:	Area of a rectangle is length times width. So what's the area of this carpet piece?
Jim:	You'd multiply 4 x 3—no, 8 x 6 because those are square yards.
Joanna:	So the area or square yards is what?
Jim:	8 x 6.
Joanna:	Matt, do you agree with that?
Matt:	Yeah. (Masingila, 1992a, pp. 235-237)

None of the six pair of students who worked this problem, including this pair, understood that finding the square yardage of the piece of carpet was the same as finding the area of the carpet piece. However, these students had studied area with square units for several years. In fact, in the textbook they had used the previous year there were exercises that were similar to this problem. The main difference may have been that the exercises were in a chapter on area and in a lesson on area of rectangles so the students knew what procedure to use.

Contrast this with Dean's explanation of how to convert from square feet to square yards. He knew the algorithm, dividing by nine—because he used it regularly in his job. However, Dean also understood that he was dealing with area, and that in one square yard there are nine square feet. It is our conjecture that if Dean had been asked to explain this conversion when he was a ninth grader, his explanation would not have been much different than that of these general mathematics students. However, through his day-to-day experience working with rectangular area, Dean had come to a fuller understanding of this conversion and was able to construct a reason for its mathematical validity.

In the dietetics context, the concepts of ratio, proportion, percentage, and conversion of units are used by dietitians in a variety of problem situations and in varying levels of difficulty. Most of the calculations and algorithms required to solve the problems are simple; however, we observed that a conceptual understanding of the concepts of ratio and proportion is necessary in order to properly interpret, model, and solve complex problems (Davidenko, 1994).

We gave a dietitian and several pairs of students the following problem:

You buy a 12 lb roast for which you pay $3.98 per lb. The waste, when removing fat and bones is about 18%. Then, when you cook it, the roast will shrink about 14%. What is the cost of a 3 oz portion of the cooked roast?

Judy, the dietitian, read the problem and looked a little puzzled. She read it again and said, "First, the goal is to find 32% of 12 pounds and find what is available." She worked a bit on some calculations and then realized that 32% was not right. Judy mentioned that she faces similar problems: "For example, we can buy two different brands of ham at different prices. One has 13% fat, the other has 15% fat, and the fat has to be removed. Then we have to compare prices per unit to see which is the best buy." During the ensuing discussion, it was clear that Judy understood the concept involved in these problems—finding the unit price of a product after discarding unusable parts—and was then able to solve the problem.

We gave this problem to several pairs of secondary students who were second-year students in a geometry course. We explained what was shrinkage and they all indicated that they understood; all the students approached the problem in the same way. The first step the students took was to multiply 12 x 3.98 to find the total price of the roast, $47.76. Next the students tackled the problem of accounting for the removal of fat and bones. One pair's dialogue is as follows:

Nina:	It is 18% waste, so let's take 18% of 47.75 and subtract it from the total price.
Todd:	No, it was 18% waste and 14% shrinkage, so that is 32% of $47.76.
Nina:	Let's take 18% of the $47.76 and then 14% of that.
Todd:	Wait! Shouldn't we take 18% of the 12 pounds?
Nina:	I don't know.

Nina and Todd discussed whether to take 18% of the cost or of the weight. Finally, with some guidance from one of the researchers, they convinced themselves of the procedure to use: "We have to take 18% of the weight because what we are doing is reducing the amount of roast, not reducing the cost. Then we take 14% of the usable part."

Whereas Judy understood the concept involved in the problem immediately, the students started by looking for an algebraic solution without understanding the concept. They first performed the calculations suggested by the first part of the problem. Then they continued with the next sentence, and in doing so they sought to reduce the price and not the weight. The students were able to deal procedurally with taking percentages of a number, but they did not understand (without guidance) the concept behind the procedure (i.e., removing fat and bones reduces the amount of usable meat but not the price paid).

We attribute the differences in conceptual understanding between the individuals in everyday work contexts and the students to a lack of experience on the part of students in dealing with these concepts in problematic situations where mathematics is used as a tool rather than an object. We observed with our respondents that the workers were able to understand the problems within the context and had the conceptual understanding to solve the problems within that context.

Flexibility in dealing with constraints. Problems that occurred in each of the everyday situations that we examined were filled with constraints. We observed noticeable differences in the ways that workers in these contexts and the students were able to deal with these constraints.

A problem that occurred in the restaurant management context is what we call the Order Problem (Prus-Wisniowska, 1993). One of the many responsibilities of the restaurant manager was to order necessary food and supplies while considering the constraints of delivery, storage space, and efficiency. Figure 3 shows the problem that the restaurant manager faced in ordering meat for each week.

In dealing with all the constraints, the restaurant manager chose to minimize the number of delivery days and maximize the amount of meat in the freezer. Another priority was to schedule all deliveries at the beginning of the week since the end of the week is often hectic with many functions at the restaurant scheduled on Thursday and Friday. Furthermore, Friday is the day she does inventory and plans deliveries for the next week; she preferred not be bothered with additional things like meat deliveries. Thus, the problem of ordering and scheduling deliveries was a real problem for the restaurant manager and she dealt with all the constraints through optimization and efficiency.

Two pairs of secondary students worked on this problem. One pair consisted of second-year students from an advanced algebra course and the other students were second-year students in a geometry course. Both pairs produced minimal order solutions: For each delivery day they decided to order only the amount of meat that would cover the needs until the next delivery day. For example, on Monday they decided to have 50 pounds delivered (25 pounds for both Tuesday and Wednesday), on Wednesday, 30 pounds to cover Thursday's needs.

When we questioned the students about if other solutions were possible, each pair reorganized their order schedule so that one less delivery day was needed; one pair eliminated delivery on Friday and the other eliminated delivery on Thursday. Neither pair seemed to consider that a delivery could be for more than 55 pounds (the capacity of the freezer) since each day some meat had to be taken out of the freezer to defrost for the next day. Both pairs struggled to keep track of all the constraints involved in the problem and appeared unable to consider all the constraints in formulating their solutions.

The carpet laying context contains a variety of constraints: (a) floor covering materials come in specified sizes (e.g., most carpet is 12' wide, most tile is 1' by 1'), (b) carpet pieces are rectangular, (c) carpet in a room (and usually throughout a building) must have the nap (the dense, fuzzy surface on carpet formed by fibers from the underlying material) running in the same direction, (d) consideration of seam placement is very important because of traffic patterns and the type of carpet being installed, (e) some carpets have patterns that must match at the seams, (f) tile and wood pieces must be laid to be lengthwise and widthwise symmetrical about the center of the room, and (g) fill pieces for both tile and base must be six inches wide or more to stay glued in place. Some particular situations have more constraints, such as a post in the middle of a room that is being carpeted (Masingila, 1992a).

The ninth-grade general mathematics students who worked problems from the carpet laying context often had difficulty dealing with the constraints involved in the problems. For example, in a problem involving the installation of tile, the students struggled to figure our a way to install the tile so that the constraints about lengthwise and widthwise symmetry and fill pieces being at least six inches wide were fulfilled (see Masingila, 1992a, for more discussion about the students' problem-solving work).

The students were also not as flexible as the experienced workers in seeing more than one way to solve a problem. In a pentagonal-shaped room that needed carpeting, the students were able to see only one way (without guidance) to install carpet. The estimator, on the other hand, was able to visualize how the carpet would be laid if it were installed with the nap running in the direction of the maximum length of the room and with the nap running in the direction of the maximum width of the room. By having more than one solution, he was able to weigh cost efficiency against seam placement and make a decision while considering these constraints (Masingila, 1992a, 1992b).

One of the interior design problems we gave to pairs of students was as follows:

You need to purchase some materials for upholstering some chairs. If you buy the whole bolt, which has 60 yards of material, it will cost $5.00 per yard. If you purchase less than a full bolt, it will cost an additional $1.50 per yard. At what point does it become more economical to purchase the whole bolt of material?

One pair of second-year geometry students approached the problem through trial and error. They first found that 30 yards of material would cost $195, 40 yards would cost $260, and 50 yards would cost $325. At this point, one of the students said, "Well, the whole bolt only costs $300 so it must be less than 50 yards." After some more calculations they decided that 46 yards was the cutoff point: "Since 46 yards costs $299, anything more than 46 yards would cost more than that—so it would be cheaper to buy the whole thing."

The interior designer, when faced with this problem, found that the whole bolt cost $300 and then divided by $6.50 to find that 46 yards and 5 inches is approximately the amount at which it becomes more economical to buy the whole bolt. However, she decided that if she needed an amount close to 46 yards, "like if I need 44 yards, I will buy the whole thing because I'm spending less than 5% over what I need and I can most likely use the material for something." For the students, this problem had only one answer; for the interior designer, the answer depended upon the situation.

Our interpretation of this difference in flexibility on the part of the students and the workers is that the students, for the most part, have not been exposed to problems with real-life constraints that must be considered and addressed in order to find solutions (Masingila, 1993b). Although there are many exercises in school textbooks that are set in these contexts, the exercises are typically devoid of real-life constraints and, as a result, do not require students to engage in the type of problem solving required in the everyday contexts (Masingila & Lester, 1992).

Interplay Between Mathematics In and Out of School

Saxe (1991) has delineated a "research framework for gaining insight into the interplay between sociocultural and cognitive developmental processes through the analysis of practice participation" (p. 13). The theoretical underpinnings of the framework are based on both Piaget and Vygotsky, but the framework moves beyond them in considering this interplay. Saxe explains:

> The framework shares the underlying constructivist assumptions of the Piagetian and Vygotskian formulations, and, with respect to core constructivist assumptions, the model . . . is consistent with both approaches. However, the framework . . . targets a level of analysis that is not addressed by either of these formulations. Unlike the Piagetian approach, my concern is to treat cognitive development on a level of analysis in which activity-in-sociocultural context is a critical focus and cognitive developmental processes are analyzed with reference to these contexted activities. Unlike the Vygotskian writings, which do not develop core developmental and sociocultural theoretical constructs with reference to systematic analysis of core domains of knowledge, the present approach is concerned with a systematic analysis of mathematical cognition that integrates cognitive developmental and sociohistorical perspectives. (pp. 13-14)

Although Saxe's framework is a method for studying the interplay between sociocultural and cognitive developmental processes, we find it helpful in thinking about working towards in-school and out-of-school mathematics learning and practice being complementary. Thus, we discuss his framework with illustrations from our own research, and then elaborate on ways to make this interplay between in-school and out-of-school contexts more deliberate.

Saxe's (1991) framework consists of three analytic components: (a) goals that emerge during activities, (b) cognitive forms and functions constructed to accomplish those goals, and (c) interplay among the various cognitive forms. Goals are "emergent phenomena, shifting and taking new form as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts" (p. 17). Forms are "historically elaborated constructions like number systems, currency systems, and social conventions." As these forms are "acquired and used by individuals to accomplish various cognitive functions" (e.g., counting, measuring), they become cognitive forms (p. 19). Interplay among the various cognitive forms occurs as individuals, "in order to accomplish goals in one setting, . . . appropriate and specialize cognitive forms linked" to another (p. 22).

Emerging Goals

Saxe outlines four parameters that influence the emergence of goals: (a) the goal structure of activities, (b) social interactions, (c) conventions and artifacts, and (d) an individual's prior understandings. Figure 4 illustrates this four-parameter model. We will use examples from our own research to illustrate these parameters.

The goal structure of an activity consists of the tasks that must be accomplished in the activity. For example, in order to run a store a retailer must buy and reprice items for sale. A principal concern for the retailer is to sell an item for as much money as possible while selling as many of the item as possible. Thus, mathematical goals that emerge in marking items up and down are guided by this economic concern.

Social interactions that occur during activities may also influence the emerging goals. In the carpet laying context, installers worked with helpers in a master-apprentice relationship. The discussion and interaction that occurred between installers and helpers often allowed helpers to engage in activities they would not have been able to unassisted (Masingila, 1992a).

Saxe (1991) writes of conventions and artifacts as "cultural forms that have emerged over the course of social history, such as . . . the Oksapmin indigenous body-part counting system and . . . a particular currency system" (p. 18). Sometimes individuals within a culture develop a set of conventions that may be unique to their particular situation. For example, the restaurant manager developed a notation system to keeping track of the restaurant's inventory. In counting items for inventory purposes she used different units for different items (e.g., pound, 5 pound, box, case, pack, each). These units were usually the same as the units used for delivery purposes.

However, she adopted a different unit for French fries. French fries were only delivered in full cases, where 1 case = 10 boxes and 1 box = 10 pounds of French fries. The restaurant manager found it difficult to operate with case as the unit for French fries since the restaurant was rather small and 10 pounds of French fries would often be more than was needed for a particular meal. So she decided to use zero to denote a case less than half full; a zero indicated to her not that there were no French fries, but rather that it would soon be time to order more.

After using this convention for some time, the restaurant manager found it, too, was inconvenient because sometimes five boxes of French fries sufficed for one week; other times it did not, and so the distinction between zero and one became critical. In the end, the restaurant manager decided to change her notation to using box as the unit. Even though the French fries continued to be delivered in cases, she reported each case as ten boxes and so from this time on her inventory indicated the amount of French fries with an accuracy of one box. Thus, the convention used by the restaurant manager influenced emerging mathematical goals of activities associated with the inventory.

The prior understandings that "individuals bring to bear on cultural practices both constrain and enable the goals they construct in practices" (Saxe, 1991, p. 18). In solving the carpet problem that involved converting from square feet to square yards, the students' prior understandings about area as a formula, dependent upon the geometric shape appeared to constrain their goals. However, for another problem that involved a pentagonal-shaped room to be carpeted, one student knew from personal experience that the room had to be treated as a rectangle and this enabled him to construct goals that were different from students who tried to determine how to lay carpet in a five-sided room (Masingila, 1992a).

Form-Function Shifts

The second analytic component of Saxe's (1991) research framework is the dynamic in the "shifting relations between cultural forms and cognitive functions as they are interwoven with the socially textured goals linked to practice participation" (p. 19). He describes how the cultural form of body counting shifted in function as individuals' levels of economic participation changed.

This phenomena also occurred in the carpet laying context as the helpers gained experience through participating in the practice of installing floor coverings. For example, one convention that was present in this context was an algorithm for laying tile. The algorithm was an agreed-upon procedure for laying tile so that the tile was lengthwise and widthwise symmetrical about the center of the room and that fill (partial) pieces were at least six inches wide (Masingila, 1992a). However, as the helper participated in the tiling process and, as was sometimes the case, became an installer, the procedure (form) became a cognitive tool (function) to be used for making decisions when complicating factors compounded the installation.

Interplay Among Various Cognitive Forms

In studying Oksapmin schoolchildren, Saxe (1985) found evidence that the children used out-of-school cognitive forms to bring to bear on in-school problems. Other researchers have determined that persons in out-of-school contexts may use knowledge gained in school to address problems they encounter (Acioly & Schliemann, 1986). Thus, there can be interplay between cognitive forms that may be appropriated and specialized in one setting and their use in another.

Saxe (1991) has specified a generalized portrayal (see Figure 5) of how cognitive developmental and sociocultural processes are "interwoven with one another in complex ways" (p. 186). Saxe notes:

> As the figure shows, in our daily lives, we are engaged with multiple practices. Within practices, goals emerge that must be accomplished, avoided, or reckoned within the achieving of larger objectives. Across practices, the understandings we generate in one may be appropriated and transformed to structure and restructure goals in another. (p. 186)

Linking the Framework to Classroom Practice

As mentioned previously, although Saxe's intent was to outline a framework for conducting research to better understand the interplay among various cognitive forms through practice, we find the framework useful in thinking about ways to bring about more and deliberate interplay between developmental processes in different settings.

We have discussed ways in which mathematics learning and practice often differ in school and everyday contexts. However, individuals do make use of knowledge in one context that was situated in another context when they view the problem situations as being similar (Stigler & Baranes, 1988). We suggest that if we, as teachers: (a) can create situations where students experience their mathematics learning and practice in school as similar to mathematics learning and practice out of school, and (b) encourage students to participate in activities out of school in which the mathematics learning and practice may be similar to their mathematics learning and practice in school, then these experiences can become complementary to each other.

Connecting in-school with out-of-school experiences. First, in order to create in-school experiences similar to out-of-school experiences, the goal structures of activities must be similar for in-school and out-of-school activities from which students may construct similar mathematical knowledge. This means that the curricula includes a wide variety of problem situations that engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations. Thus, problems are embedded in situations that are real and meaningful to students, and mathematics practice can be structured in relation to these problematic situations. It also means that mathematics is a tool to be used and that procedures and processes are learned as they are needed in the midst of accomplishing emerging goals.

We further suggest that in order to structure classroom experiences like this instruction should be via problem solving. In teaching via problem solving, "problems are valued not only as a purpose for learning mathematics, but also as a primary means of doing so. The teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems" (Schroeder & Lester, 1989, p. 33). Teaching via problem solving deviates from the traditional instructional approach of the teacher presenting information and then assigning exercises in which students practice and apply this information. Using a teaching via problem solving instructional approach means that mathematical understandings are constructed by students as they seek to accomplish emerging goals through problematic situations.

Second, social interactions are an essential part of this classroom mathematics practice. In working individually and collectively to accomplish emerging goals, mathematical knowledge is developed within a meaningful context and cognitive development occurs as students work together with peers and teacher to negotiate shared meanings. As Saxe (1991) noted, social interaction is a key influence on the emerging goals of an activity.

Third, in-school activities should make use of cultural artifacts and conventions that students can use to interpret problems and make sense of them. Students should also be encouraged to generate conventions that may be helpful to them in the course of accomplishing their emerging goals. For example, students may invent notation to indicate when objects are the same size and shape, in the course of working in a measurement context, before they have formalized the concept of congruence.

Finally, teachers can build on students' prior understandings. All students bring to school mathematical knowledge acquired in other contexts. This knowledge is often hidden and unused by students in school as they learn to use the mathematical procedures that teachers demonstrate and evaluate (Masingila, 1993a). If teachers engage students in conversation about their everyday experiences, listen to them, and encourage and observe their informal methods of mathematizing, they can learn much about students' prior understandings. Similarly, teachers can encourage students to bring to bear their prior understandings by having students: (a) create their own problem situations, (b) solve problems in more than one way and share their solution methods with each other (Lester, 1989), and (c) focus on semantics rather than syntax.

Connecting out-of-school with in-school experiences. Besides creating experiences in school that may complement out-of-school mathematics learning and practice, teachers can guide students in reflecting on how in-school learning and practice are used out of school. In a study examining middle school students' ideas about their out-of-school mathematics practice, Masingila (1994) observed that with encouraged reflection students were able to note a number of ways that they used mathematics outside of school. Sixth- and eighth-grade students were interviewed before and after keeping a log for a week in which they recorded their use of mathematics. Although students reported ways they used mathematics they classified as "non-school math," they also indicated many instances where they used knowledge they categorized as "school math."

We suggest that an important aspect of in-school and out-of-school mathematics experiences becoming more complementary is to encourage students to be aware of their mathematics learning and practice outside of school. This involves having students discuss their out-of-school experiences and what mathematics concepts and processes they used in those experiences. Additionally, teachers can have students reflect on how their in-school mathematical experiences influence this learning and practice. Teachers can also ask students to think of out-of-school experiences that are similar in some aspects to mathematical problem situations they have encountered in the classroom. Students and teachers can have a good discussion concerning similarities and differences between these situations that can help students to see the value of mathematics practice in both contexts.

In both in-school and out-of-school experiences. students participating in mathematics practice will become engaged with novel mathematics goals that require form-function shifts. Teachers who observe these gradual and complex shifts, gain valuable assessment information about students and can serve to facilitate the process of students acquiring mathematical knowledge to use as cognitive tools.

Concluding Remarks

Mathematics learning and practice in school and out of school differ in some significant ways. Some of these differences may be inherent because a concept is learned and used differently in school than out of school. However, we believe that many of the differences can be narrowed by creating experiences that engage students in doing mathematics in school in ways similar to mathematics learning and practice outside of school. The framework Saxe (1991) outlined for examining the interplay between sociocultural and cognitive developmental processes targets cultural practices as important contexts for study. Similarly, our discussion has used Saxe's framework to suggest how more and deliberate interplay can be encouraged between these developmental processes by focusing on mathematics learning and practice in everyday contexts as starting points. We believe that by making in-school and out-of-school mathematics experiences more complementary, student learning and practice in both of these situations can be enhanced.

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Apple and Vegetable Salad

Salad Ingredients	Dressing
2 cups diced apples	2 eggs
1/2 cup shredded carrots	1/2 cup sugar
1/2 cup chopped peanuts	1 Tbl flour
	1 Tbl butter
	1/4 cup vinegar
	1/2 cup cream

Mix apples, carrots, and peanuts together. Cook dressing until it thickens. Add cream and allow to cool. Pour dressing over salad mixture and serve. Serves 6.

Figure 1

Converting Square Feet to Square Yards

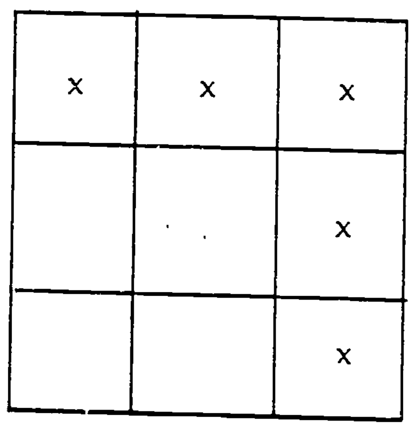

Figure 2

Ordering Problem

You are a manager in a small restaurant. Each Friday you have to decide what will be cooked during the period of the next week (from Tuesday to Saturday and Monday in the following week) and send a suitable order to the commissary shop. Today is Friday, January 14 and by looking at the catering book and taking into consideration possible trends, you found out that for each day you will need the following amount of meat (in pounds):

	Tue 1/18	Wed 1/19	Thu 1/20	Fri 1/21	Sat 1/22	Mon 1/24
Needed	25	25	30	20	10	20

The commissary shop offers you good quality and very cheap meat but they deliver their goods only four times a week: on Monday, Wednesday, Thursday, and Friday. You can purchase things in advance and keep them in storage but the meat freezer capacity is only 55 pounds. So on Friday, January 14, you have to plan very carefully how much meat has to be delivered each delivery day to cover your needs. Meat comes frozen so it needs one day to be put aside and defrosted.

Plan the delivery schedule for the coming week:

	Mon 1/17	Tue 1/18	Wed 1/19	Thu 1/20	Fri 1/21	Sat 1/22
Meat to be delivered in pounds		no delivery				no delivery

Figure 3

17

Four-Parameter Model*

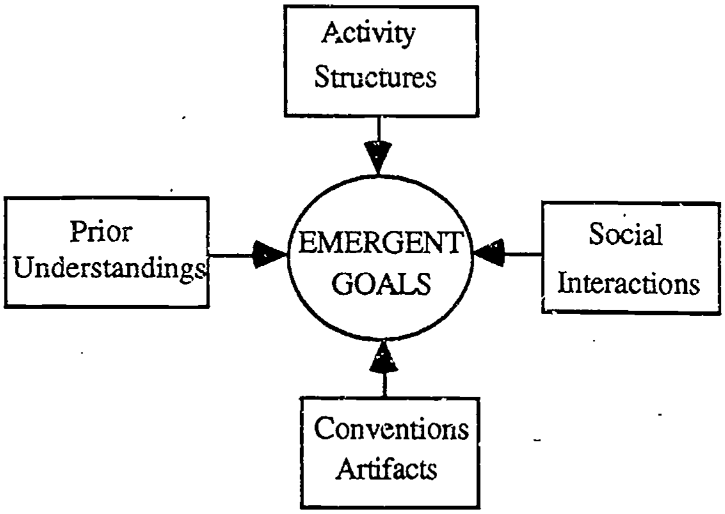

Figure 4

*(Saxe, 1991, p. 17)

Expansion of Four-Parameter Model*

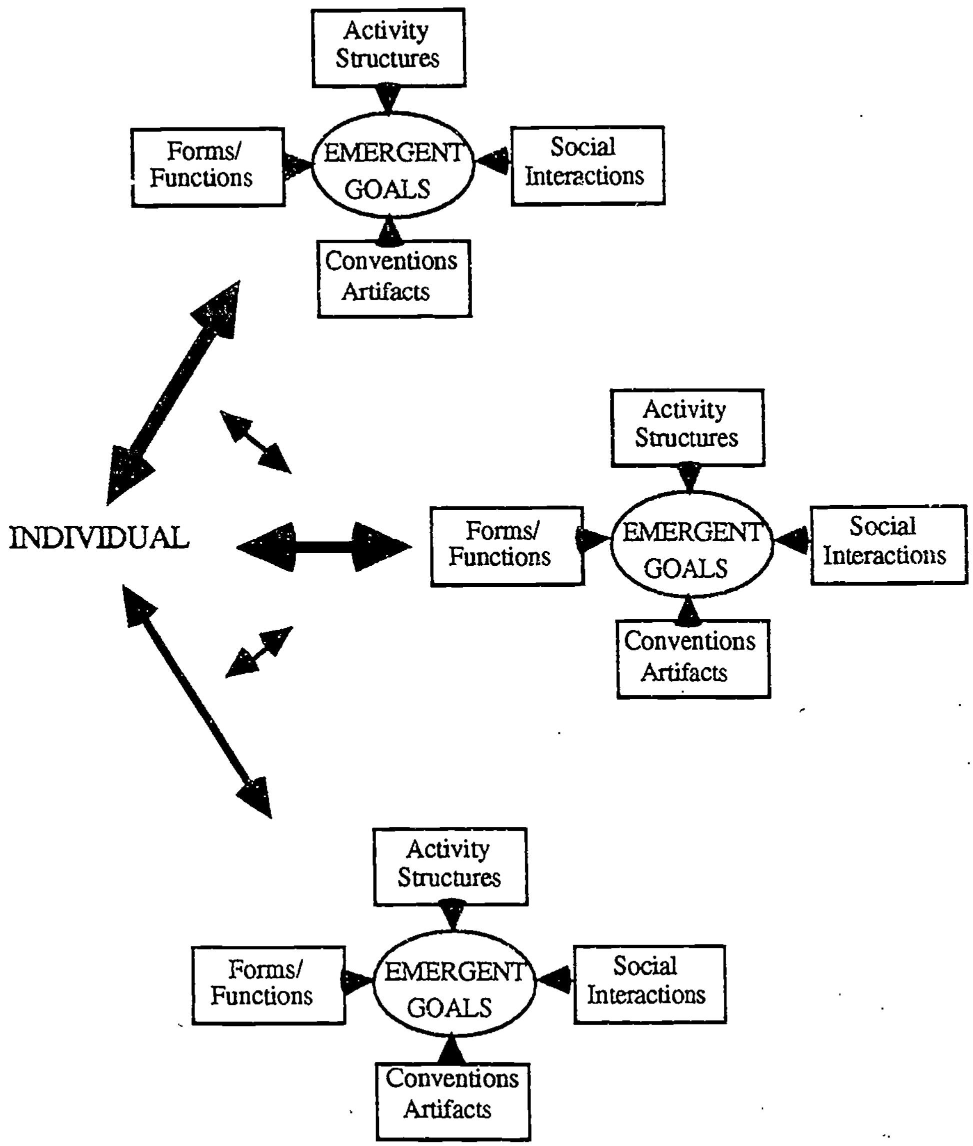

Figure 5

*(Saxe, 1991, p. 185)

19