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ABSTRACT

This publication describes how writing in mathematics classes in Australia can be beneficial to students and their teachers, including: journal writing, expository writing, and transactional writing. In addition, it reports the results of a study that examined benefits derived by five secondary school mathematics teachers from reading their students' responses to timed, in-class, impromptu writing assignments. The bulletin presents suggestions for getting started for teachers who decide to implement writing activities in their classrooms. Contains 15 references. (MKR)

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WRITING IN MATHEMATICS CLASSES

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STUDENTS OFTEN do more writing in mathematics classes than in many other subjects, but typically the writing involves mathematical, not verbal, symbols. Mathematics is considered by many to be a language in its own right, with its own symbol system through which practitioners can think and express themselves. For this reason, the use of everyday spoken and written language in the learning and teaching of mathematics often is not appreciated.

The use of students' natural written language in the process of learning mathematics has been the focus of many research studies over the past decade. Mathematics educators in both the northern and southern hemispheres have been investigating ways of using writing in mathematics classes to enhance learning, diagnose misconceptions, and assess understanding and attitudes. Advocates of writing in mathematics believe that writing activities develop students' abilities to read, define and hypothesise; inculcate methods of problem solving; assist with the construction of knowledge; recognise attitudes; and promote individual teacher-student interaction.

The use of writing in mathematics classes can benefit both students and teachers. Some of these benefits are described in this publication.

BENEFITS TO STUDENTS

VARIOUS TYPES of writing have educational potential for mathematics teaching. These include: *journal writing*, which focuses on getting students to express their feelings about mathematics and the problems which they encounter in the learning process; *expository writing*, which has proven to be an effective and practical tool for teaching problem solving; and *transactional writing*, which can be used to motivate students towards developing an interest in mathematics and how it is used outside the classroom. Some benefits found for each of these types of writing are described below.

Journal Writing

Burton (1985) describes journal writing as 'brainstorming with oneself'. Everything is written down without evaluation. Thus, a journal becomes a student's 'think book' in which thinking about learning mathematics is done in writing. An example of one student's think .g is given by Borasi and Rose (1989):

What is a linear programming problem? I still don't know! It has something to do with solving a system of equations to find the maximum or minimum value. It's used for things such as maximizing profits and minimizing costs. The object is to find the highest and lowest solutions. I forgot about the constraints! (p. 355)

Waywood (1991), an Australian researcher investigating the use of journal writing in the learning of mathematics, believes that journal writing can benefit students cognitively by leading them to summarise and reflect on the mathematics that they are learning. In a study involving approximately 500 students in Years 7-11, he saw evidence in journal entries of how different students organise their learning differently. For example, three students might record the same fact from a lesson, but one will merely state the fact, one will give an example of how the fact might be used, and the third might enter into a discussion of how the fact is related to other facts and how its status is questioned. The first student is simply recounting what happened in class; the second student is labelling the content in order to gain mastery; and the third has entered into a dialogue of interaction between a number of ideas.

The power of journal writing to influence attitudes is documented in the work of Nahrgang and Peterson (1986), Borasi and Rose (1989) and Clarke, Stephens and Waywood (1989). The work of these researchers illustrates how journal writing benefits students by providing them an opportunity to express their anxieties about and attitudes towards mathematics and the problems that they encounter in the learning process. The writing seems to have a therapeutic effect on the emotional components of learning mathematics, allowing students to vent their frustrations and reflect on their past and present feelings about school mathematics.

Expository Writing

Asking students to explain, in writing, their thinking about a nonroutine problem or mathematical investigation can be an effective and practical tool for teaching problem solving (Bell & Bell, 1985). More than reading and speaking, writing provides a context that encourages learning and thinking. In a study using two Year 9 mathematics classes in the United States, Bell and Bell (1985) demonstrated that writing positively affected students' progress in mathematical problem solving. One class was taught using a method which combined traditional teacher-centred, chalk-and-talk techniques with structured expository writing. For students who judged a specific problem to be easy, their writing described the process used in solving the problem and explained why it was easy.

Writing about problem solving can improve students' problem solving skills.

For students who had difficulty in solving a problem or could not complete it, the writing became an opportunity to record the procedures used and the point at which confusion began. In other words, students put into words what was understood and what was not understood. The second class was taught using only the traditional teacher-centred, chalk-and-talk method. Both classes were given the same assignments, examples and assessments during the four-week period of the study. Only the writing component in the experimental class was different for the two teaching methods.

The results of the study support the use of expository writing in teaching problem solving. Pretests given to both groups before the study began showed no significant difference in the problem-solving skills of the two groups of students. However, four weeks after the study had been completed, the results of the posttest showed that the students doing the expository writing had better problem-solving skills than students following traditional methods.

Transactional Writing

Transactional writing is public writing meant to be read by an audience. In mathematics classes, the audience consists of other students

or a teacher. It attempts to inform or persuade others towards a particular line of thinking. Essays about famous mathematicians and student-composed word problems are examples of transactional writings that can be used in mathematics classes. In general, Australian researchers, Reuille-Irons and Irons (1989), suggest that transactional writing activities 'encourage children to be creators of their mathematical knowledge. When they create their own knowledge, they gradually build a picture of concepts and ideas that will be useful in problem-solving situations' (p. 98).

Ellerton (1986) investigated children's ability to make up problems and, through interviews, noted the need for more opportunities for children to share their often unspoken problems and beliefs about mathematics. Del Campo and Clements (1990) reinforced Ellerton's observations in their own research, which concluded that having students write creative stories helped them to construct more elegant mathematics, and reduced the passive nature of the mathematics classroom.

BENEFITS TO TEACHERS

MUCH OF THE RESEARCH on writing in content areas has focused on student benefits. But what about benefits that teachers can derive from reading students' writings? Borasi and Rose (1989) suggest that teachers, too, could be equally affected in their teaching by reading their students' writings. Birken (1989) admits to learning a great deal about her students' mathematical thinking, particularly their misconceptions and where their thinking went wrong. My own research (Miller 1991) documents how reading students' writings in first-year algebra classes revealed how students comprehended or misconstrued specific concepts and algorithms.

In general, however, there is a dearth of literature addressing the effect which the use of writing in mathematics has on teachers. Thus, the purpose of a study conducted in Perth, Western Australia, was to examine the benefits which secondary teachers derived from reading their students' writings in mathematics. The remainder of this publication focuses on the study and its findings.

DESCRIPTION OF THE STUDY

THE STUDY was undertaken by a research team consisting of three secondary mathematics teachers, each having 15 or more years of teaching experience, and two mathematics educators from the Key Centre for Teaching and Research in School Science and Mathematics at Curtin University. The teachers were from three different schools, one private and two government. Each teacher agreed to implement writing activities in one of his/her classes. One chose Year 11 *Introductory Calculus* students, one elected to implement writing in a *Mathematics for Living* class for Year 9 students, and one asked a class of Year 10 students to write in their mathematics class. The Year 10 and 11 students were considered to be above average academically by their teachers and the Year 9 *Mathematics for Living* students were described as below average.

Writing is a way to find out what students are thinking.

In-class, impromptu writing prompts — simply-worded statements or questions directing students' thoughts to the explanation of a single concept, skill or generalisation — were utilised in this study. For example, one prompt asked Year 9 students to explain to a primary-aged student how the year is divided into units of time. (For a more detailed description of writing prompts, see Miller, 1990.) The writing period was timed. Two teachers chose to allow five minutes for the activity; one teacher allowed students to write for 10 minutes. All three teachers asked students to write at the beginning of the class and agreed to use writing prompts approximately three out of every five teaching days. The prompts were

either written on the chalkboard or given to each student on a sheet of paper. The teachers collected writings during the second and third terms of the 1991 school year. In addition to reading the students' writings, the teachers agreed to produce a written response to their dominant impressions of the students' writings. The whole research team met four times during the course of the study to discuss the students' writings and, specifically, the benefits being derived by the teachers as a result of reading their students' writings.

WHAT TEACHERS LEARNED FROM THE STUDENTS' WRITINGS

Students' Understanding Is Not Always What Teachers Assume It To Be

ALL THREE TEACHERS believed that their assessment of individual students' understanding of mathematical concepts, skills and generalisations was enhanced by reading the students' responses to the in-class, impromptu writing prompts. One instance described by the teacher of the Year 9 students related to how she used a prompt to diagnose students' prior knowledge about averages. She asked them to respond to the question 'What is an average?' Of the 11 students in the class, four briefly outlined how the numbers were added and then divided by the number of numbers added. That is, they described how to find a mean, but did not use the word 'mean'. Four students used phrases like 'a number in between other numbers,' and one student said that an average was the number used most often, interpreted by the teacher as meaning the mode, but the student did not use the word 'mode' in his/her writing. Two students wrote meaningless, unacceptable responses.

One week later, after much teacher and student activity involving the three different types of averages (mean, median and mode) the teacher repeated the prompt 'What is an average?' to assess the students' understanding after instruction. Twelve students were present that day but, to her disappointment, only one student from the original 11 significantly revised his response. Six students referred to average as being 'the number in the middle'. Four of these six students used the word 'mean' in reference to 'the number in the middle'. One student described how to find a mean using the word 'mean' in her writing. Two students accurately described and used the word 'mode' and one student described mode but did not use the word 'mode'. One student described all three types of averages using the words 'median', 'mode' and 'average' for mean. One student chose not to respond. The teacher's writing after reading this collection of papers is given below:

> *We had discussed the fact that there are other types of 'averages' not just the mean. However only one student has mentioned this. Back to the drawing board again! Do they have preconceived ideas which they hang on to?* (Teacher's log, 21 May 1991)

Teachers' assumptions about students' prior knowledge often surfaced in comments and writings by the teachers. During a team meeting, another teacher said:

> *I found by reading the students' writings that, in some instances, the points I have emphasised in teaching a concept did not always coincide with what the students needed to hear to actually gain an understanding of the concept. After reading what they wrote, I knew what I should have emphasised to more fully explain the concept. I had made wrong assumptions about their knowledge and background. That sort of thing has happened to me in different situations.* (Transcribed tape, 3 October 1991)

All three teachers described instances, both verbally during team meetings and in their individual writings, when they had retaught a topic because the students' writings did not reflect a comprehension of the topic covered in a previous lesson. All three talked about how they had assumed that the students understood what was 'going on', but that their writings often indicated that they did not. Comments referring to students' understanding as reflected by their ability to manipulate symbols and numbers were frequent. However, when an acknowledgment of their understanding was requested using written language, the

students often failed to submit acceptable responses.

Students' Understanding and Ability to Use Mathematics Vocabulary is Limited

Mathematics teaching and learning is an interactive process which depends on the understanding of carefully defined terms and symbols. Understanding, in turn, is dependent upon a student's knowledge of mathematics vocabulary. Teachers use mathematics vocabulary routinely in the teaching-learning process, assuming that students have previously acquired meaningful definitions for words which might have been introduced several years previously. Without a command of the language used in mathematics teaching, the task of comprehending a teacher's comments, reading a mathematics textbook or solving a word problem becomes extremely difficult for students.

After only two writing prompts, one of the teachers in the study wrote: 'At this stage, I'm already concerned that a number of students do not have a good understanding of key mathematical words.' (Teacher's log, 3 May 1991). He continued to voice his concern at the next team meeting by saying:

> *I've been worried about their lack of understanding of key words. I find, when I look at the explanations, that they're not very good explanations. For example, in their writings they all say 'the number in front of x' or whatever the variable is. They do not use the word 'coefficient' and I know that they know what it means.* (Transcribed tape, 27 May 1991)

Throughout the study, the teachers reiterated their dismay at students' lack of use and inappropriate use of mathematics vocabulary. The following three responses are representative of the 16 students in one Year 11, *Introductory Calculus* class. The prompt asked students to 'Explain what occurs when h is a divisor and $h \rightarrow 0$':

Student A: *When* h *is a divisor and* h → 0, ~~*the quotient being divided*~~ *the answer arrived at would become increasingly larger as* h *decreases. At zero,* h *is* ~~*infinite*~~ *undefined as nothing can be divided by zero.*

Student B: *As* h *approaches zero the dividend of the equation becomes very large.*

Student C: *If the number that the divisor is going to be divided into is small and the closer* h *gets towards zero, then the larger the answer is 'going to be'.*

Student A uses the word 'quotient' but marks it out to signal some doubt as to the use of the word. He is more comfortable saying 'the answer arrived at'. Student B has confused 'dividend' with 'quotient', while Student C describes but does not use the word 'dividend'. Rather than saying 'quotient', Student C also chose to say 'the answer'. In this study, the lack of use and the inappropriate use of mathematics vocabulary by students ranging in academic ability from below average to above average was disturbing to the three teacher researchers.

Two of the three teachers indicated that they changed their teaching practices to accommodate what they were learning about students' knowledge and use of mathematics vocabulary. All three confirmed that one value of writing in mathematics classes was giving students an opportunity to use mathematics vocabulary and for teachers to assess their use and understanding of specialised terms. In an interview at the end of the study, one of the teachers said:

> *You know, teachers have got to change their language to become better role models for the students. We have got to use the correct vocabulary. I tried to emphasise key words throughout the course after the first five or six prompts. After I started using the right words, I think that the students started becoming more comfortable and the words automatically came to their mind a lot more quickly with the writing.* (Transcribed tape, 3 October 1991)

STUDENTS' WRITINGS INFLUENCE TEACHERS' PRACTICES

IN ADDITION to enhancing teachers' assessment of students' understanding and informing teachers of students' knowledge and use of mathematics vocabulary, the writing activities influenced the teachers' practices. Various comments made at team meetings and in teachers' logs confirmed that reading students' writings helped

teachers to know when they had successfully achieved a lesson's objectives. The writings sometimes suggested when a lesson should be retaught and what should be restated, perhaps in a different way, in order to assist students in constructing their knowledge about a specific concept or topic.

The comments and writings of the teachers in this study lend support to the results of a previous study which suggested that writing in mathematics classes can influence teaching practices in at least five ways (Miller, in press). After reading their students' writings, teachers can decide to (a) immediately reteach a lesson or concept, (b) delay an assessment because a lack of understanding is reflected in the students' writings, (c) schedule a revision based upon what was learned from the students' writings, (d) initiate private discussions with individual students who have mathematical misconceptions, and (e) use writing prompts on assessments where students are given a mark for their ability to communicate their knowledge and understanding of mathematics in writing.

Preparing the prompts and reading the students' responses do take time. At the end of the study, all three teachers were asked if they would continue to use impromptu writing prompts during the fourth term, even though the study was officially over. Everyone said yes, although they felt that they might not ask the students to write as often. One teacher's comment represents the sentiments of all three:

> *Yes, I was telling [teacher's name] that I would continue, especially with the group that is doing it now. I will be teaching newer areas and I'm sure that I'll try to think of something to get them to write. It probably won't be as frequent, because it does take a fair bit of time to read through and do these sorts of things. But, when there is a degree of uncertainty coming through in my mind as to whether they are understanding something, or if we have something new, then I'll be looking at it [writing] as a way to find out what they are thinking.* (Transcribed tape, 3 October 1991)

GETTING STARTED

The following five suggestions could prove helpful to teachers who decide to implement writing activities in their mathematics classes:

Suggestions for Getting Started

1. Decide on a definite period of time for in-class writing and when you want the students to write. Teachers who have used impromptu writing prompts recommend having students write at the beginning of class for a five-minute interval.
2. When preparing a lesson, write prompts that relate to that lesson; however, be flexible, because sometimes ideas for prompts surface during a class. If this happens, use that idea the next day rather than a preplanned prompt.
3. As past research suggests that students write more if they address their comments to someone (Miller & England, 1989), student responses could be addressed to their best friend.
4. Do not always offer students extrinsic incentives such as points towards a final grade. Talk with students about the purpose of the experience and solicit their cooperation on the merits of it being a meaningful and beneficial experience for you and them. Likewise, do not penalise students when they do not write. In past studies, even the most difficult student, with time, has started writing.
5. Be patient. The benefits of writing activities in mathematics classes do not surface immediately. Students first must get accustomed to the idea of writing in mathematics class. Initial writings can be very brief and meaningless to the teacher. Give students time to learn how to write in mathematics and the rewards should follow.

CONCLUSION

THIS PUBLICATION describes how writing in mathematics classes can be beneficial to students and their teachers. More specifically, it has reported the results of an Australian study which examined benefits derived by teachers from reading their students' responses to timed, in-class, impromptu writing prompts.

Writing in mathematics classes is not a panacea for every teacher. It works and is very rewarding for some teachers, but not necessarily for others. Questions raised by some critics include: 'What's so unique about writing in mathematics classes?' 'Can't teachers learn the same things by asking students questions in class?' While teachers can ask open-ended, thought-provoking

questions in class or pose them in a context of small-group activities, only a few students become involved in these discussions. Allowing sufficient time for students to formulate and respond to open-ended questions orally can create a lull during which other students can become off-task mentally, if not physically, too. Within the time allotted for teacher-student interaction in class, a teacher usually cannot interact with every student in a class of 20-30 students. A unique feature about impromptu writing prompts is that each student has the opportunity to express her/himself, in writing, to the teacher during every class in which a prompt is used. Writing also allows students an opportunity to examine their thoughts and make changes in their statements prior to submitting their response. Generally students answering orally do not have the chance to say something, reflect upon what they have said and then make changes to those statements. The teachers in this study found that five minutes was sufficient time for students to read and respond to a prompt, but it did not disrupt their other teaching practices.

Another benefit of writing in mathematics classes is that it gives students an opportunity to communicate their understanding of mathematics and to use mathematics vocabulary in writing. *A National Statement on Mathematics for Australian Schools* (1990) suggests that all students should learn to communicate mathematically and that a command of mathematical terminology is essential in learning mathematics. The purpose of this publication has been to stimulate teachers to think about the use of writing in mathematics, first, as a means to assess students' understanding of mathematics and use of mathematics vocabulary and, second, as a way to diagnose students' needs in the process of learning.

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