DOCUMENT RESUME

ED 372 947 SE 054 400

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TITLE Pupil-Generated Multiplicative Word Problems as a Test of the Theory of the Primitive Intuitive Models.
PUB DATE Apr 94
NOTE 9p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 4-8, 1994).
PUB TYPE Speeches/Conference Papers (150) -- Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Age Differences; Division; Elementary School Students; Elementary Secondary Education; Foreign Countries; High School Students; Mathematics Instruction; Models; *Multiplication; *Schemata (Cognition); *Student Developed Materials; Student Teachers; *Word Problems (Mathematics)

ABSTRACT

There is substantial empirical evidence showing the impact of the type of numbers in the solution of problems with a multiplicative structure. The theory of the intuitive models of arithmetic operations is today the most plausible theoretical account for these findings. In an attempt to contribute to a better understanding of the effects of number type, a study was carried out in which student-generated word problems were used as data to test a series of predictions derived from the theory of the intuitive models. Subjects were 107 12-year-olds, 107 15-year-olds, and 99 elementary student teachers in Belgium. Results concerning the influence of the type of number sentence on performance are reviewed and the congruence of the generated problems with the primitive models are discussed. Several findings of this study support the theory of Fischbein et al. (1985) that the selection of an operation for solving multiplication problems is mediated by primitive intuitive models of the arithmetic operation. (Author/MKR)

Pupil-generated multiplicative word problems as a test of the theory of the primitive intuitive models

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April 1994

Paper presented in a Structured poster session on "Children's understanding of mathematical knowledge", organized at the Annual Meeting of the American Educational Research Association, New Orleans, LA, April 4-8, 1994.

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PUPIL-GENERATED MULTIPLICATIVE WORD PROBLEMS AS A TEST OF THE THEORY OF THE PRIMITIVE INTUITIVE MODELS

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Abstract
There is substantial empirical evidence showing the impact of number type on the solution of problems with a multiplicative structure. The theory of the intuitive models of arithmetic operations is today the most plausible theoretical account for these findings. In an attempt to contribute to a better understanding of the number type effects, a study was carried out in which student-generated word problems were used as data to test systematically a series of predictions derived from the theory of the intuitive models.

Introduction

Research has shown that the type of numbers in multiplicative word problems is an important determinant of their difficulty level. More specifically, students of different ages have difficulties with decimal multipliers and divisors, especially when the decimal is smaller than 1, and with divisors that are larger than the dividend (see Greer, 1992). For example, De Corte, Verschaffel, and Van Coillie (1988) have shown that with respect to multiplication problems 12-year-old students are better at choosing the correct operation from a series of six alternatives when the multiplier is an integer than when it is a decimal larger than 1; but problems with a decimal multiplier smaller than 1 are still more difficult. The most frequently occurring error is dividing instead of multiplying. Similar number type effects have been reported for division problems. In a study with large groups of 10-, 12-, and 14- year-olds, Fischbein, Deri, Nello, and Marino (1985) found that only a small percentage of pupils succeeded in selecting the correct operation for problems with a decimal divisor smaller than 1; the most common error consisted of multiplying both numbers. Problems with a divisor larger than the dividend caused also many errors; most of the mistakes originated from dividing the larger number by the smaller one.

The most plausible theoretical account today for these findings is provided by the concept of <u>intervening primitive intuitive models</u> of arithmetic operations introduced by Fischbein et al. (1985). Research relating to this concept is mostly based on multiple-choice tests in which the subjects were asked to choose the correct operation. This work provides only partial support for the theory of Fischbein (Greer, 1992). This article reports a study in which an alternative technique was used, namely asking students to generate multiplicative verbal problems as a window to their difficulties with this kind of tasks, and, consequently, as a contribution to the further validation of this theory.

The theory of primitive intuitive models of operations

According to Fischbein et al. (1985) pupils acquire a primitive intuitive model for every arithmetic operation. The choice of an operation to solve a word problem is mediated by these intuitive models, especially by their implied constraints. When the numbers in a word problem are incongruent with one or more of the constraints of the underlying model, the probability that a wrong arithmetic operation is chosen increases.

The primitive model associated with multiplication is repeated addition: a number of sets of the same size are put together. Under the repeated addition interpretation, one number (i.e. the number of equivalent sets) is taken as the multiplier, the other (i.e. the size of each set) as the multiplicand. Two numerical constraints derive from this model: 1. the multiplier must be an integer; 2. multiplication always produces a result that is bigger than the multiplicand.

In a similar way Fischbein et al. hypothesize the existence of two primitive models for division: the partitive and the quotitive models. In the partitive model a given quantity is divided into a specified number of equal subsets, and one has to determine the size of each subset. This model implies three constraints: 1. the dividend must be larger than the divisor; 2. the divisor must be an integer; 3. the quotient must be smaller than the dividend. In the quotitive model one has to find how many times a certain quantity is contained in a larger quantity. This model involves only one numerical constraint: the dividend must be larger than the divisor.

This theory of the primitive intuitive models accounts for a number of the empirical findings reported in the literature. For instance, the robust multiplier effect mentioned above involves that problems with a decimal multiplier larger than 1 are more difficult than those with an integer as multiplier, and that problems with a decimal multiplier smaller than 1 are even harder. This is in line with the theory: in the first case one numerical constraint of the repeated addition model is violated, namely that the multiplier must be an integer; in the second case the additional constraint - the outcome must be larger than the multiplicand - is also violated.

But, some other observations with respect to multiplication problems are less consistent with the theory. For example, the multiplier effect is much weaker when students are asked to solve problems as compared to the choice of operation task (De Corte et al., 1988). And, with respect to division the explanatory power of the constraints of the partitive and quotitive models has so far remained rather limited (Greer, 1992). For instance, remarkably good results have been reported on partitive problems with a decimal divisor larger than 1 when the dividend is also a decimal larger than 1 (e.g., 11.44 : 4.51) (Bell, Greer, Grimison, & Mangan, 1989), and problems with an integer dividend smaller than the divisor (e.g., 5 : 15) have been found to be more difficult than those with a decimal dividend smaller than a whole divisor (e.g., 4.5 : 6) (Fischbein et al., 1985). The theory of the intuitive models offers no sound explanation for both results.

Referring to the latter finding as well as to their own work, Harel, Behr, Post, and Lesh (in press) have put forward the idea that the "dividend larger than divisor" (D>d) constraint is not as robust as the other two constraints of the partitive model. Fischbein et al. found that in the case mentioned above of a decimal dividend smaller than a whole divisor, very few students reversed the numbers in solving such problems, as they frequently did when both were integers; an explanation for this observation is that they avoided the decimal divisor resulting from such a reversal. In their own study Harel et al. observed that the majority of the pre-service and in-service teachers who failed on a problem that had to be solved by the operation 11 : 2.53 chose the inverse operation 2.53 : 11. This supports the differential robustness idea: while the correct operation violates the constraint that the divisor must be an integer, the applied inverse expression breaks the larger-dividend constraint.

In an attempt to further unravel the number type effects on the solution processes of multiplicative problems, we carried out a study in which we used student-generated verbal problems to test systematically a series of hypotheses and predictions derived from the theory of intuitive models.

Method

Subjects were 107 12-year-olds, 107 15-year-olds, and 99 elementary student teachers. They were given a collective test consisting of 12 multiplicative number sentences, 6 multiplications and 6 divisions (see Table 1). Two versions were developed differing with respect to the sequence of the number sentences, and, for the multiplications, with regard to the order of the numbers. The students were asked to write for each sentence a story problem that could be solved with the given operation; however, they were warned that "pseudo word problems" like "Mother had to divide 0.6 by 0.8. Can you help her?", were not allowed. To restrict the possible influence of computational skills, the outcome of each number sentence was mentioned in brackets.

Table 1. Multiplicative number sentences used in the present study

Multiplication		Division	
a) 9 x 3	= 27	g) 24 : 3	= 8
b) 6 x 2.8	= 16.8	h) 6.3 : 9	= 0.7
c) 8 x 0.9	= 7.2	i) 5 : 25	= 0.2
d) 7.4 x 3.8	= 28.12	j) 6 : 4.8	= 1.25
e) 5.3 x 0.6	= 3.18	k) 4 : 0.8	= 5
f) 0.7 x 0.2	= 0.14	l) 0.6 : 0.8	= 0.75

On the basis of a pilot study a schema in the format of a flow chart consisting of a series of seven questions, was designed to separate appropriate story problems from statements that cannot be accepted as appropriate problems, and to classify the latter into categories. Out of the 3756 collected statements, 2089 were considered as appropriate; only 61 inappropriate statements did not fit in one of the designed categories. An interrater reliability of 95% obtained on a random sample of 118 problem statements, showed the utility of the schema.

Hypotheses, predictions, and results

Because of space restrictions, the findings of the study can only be reported partially in this article. First, the results concerning the influence of the type of number sentence on performance are reviewed. Next, the congruence of the generated problems with the primitive models is discussed.

Influence of the type of number sentence on performance

The general hypothesis was that the nature of the multiplicative number sentence would strongly influence students' performance on the problem-generating task. More specifically, we expected an increase in the level of difficulty of the task depending on the amount of constraints of the underlying intuitive model that needed to be violated. From this hypothesis a series of predictions were derived. To test these predictions chi-square values were calculated for the subgroups of subjects separately as well as for the total group; to assess significance alpha was set at the usual level of 0.5.

Multiplication

1. A significantly higher number of appropriate problems was predicted for the sentences a, b, and c (see Table 1) which are congruent with the repeated addition model, than for d, e, and f. This prediction was confirmed in all groups.

2. The presence of an integer in b and c allowed to state a problem that conforms to the repeated addition model. Therefore, no difference was expected between those sentences and a. The results support this prediction in the groups of 15-year-olds and student teachers. Apparently, the presence of a decimal in b and c raised problems for the 12-year-olds.

3. In line with the intuitive model and with the empirically robust multiplier effect, it was also expected that the multiplications d and c would yield more appropriate story problems than f. Indeed, whereas stating a good problem for d and e requires the violation of the first constraint only (the multiplier must be an integer), with respect to f the second constraint has to be violated also (multiplication makes always bigger). In none of the groups a significant difference between e and f was observed. A significant better performance was found for d than for f in the 12- and 15-year-olds only.

4. The presence of a decimal larger than 1 in sentence e makes it possible to formulate a word problem that violates only the first constraint of the intuitive model (multiplier must be an integer). Therefore, no difference between d and e was anticipated. In contrast to this prediction e yielded significantly less good problems in all groups. Apparently, the presence of a decimal smaller than 1 in e complicated the task seriously.

Division

1. Sentence g is the only one that satisfies the constraints of the quotitive and especially the partitive division models. Therefore, a significantly higher number of appropriate problems was predicted for g than for all other sentences. This prediction was confirmed in all groups, however, with two exceptions.

Indeed, in the 15-year-olds and in the student teachers groups no difference occurred between the congruent division g and sentence h which contains a decimal dividend larger than 1 and a whole divisor larger than this dividend. These observations are in line with the idea of Harel et al. (in press) that the D > d constraint of the partitive model is less robust than the other two.

2. On the basis of this differential robustness hypothesis it was anticipated that h and i would both yield significantly more good problems than j, k, and l. While h and i violate the D > d constraint, j, k, and l break one or two of the other intuitive rules. The data provide support for this prediction in all cases but one; there was no difference between i and k in the group of student teachers.

3. Again in line with the idea that the D > d constraint is less robust than the other intuitive rules, a significantly higher number of appropriate problems was expected for h than for i. The basis of this prediction is that the presence of a decimal dividend in h will reduce the number of reversals of dividend and divisor as compared to i. The data confirm also this prediction, and, thus, provide additional support for the differential robustness hypothesis.

4. Finally, it was anticipated that l which is most incongruent with the constraints of the intuitive model, would yield significantly more inappropriate statements than j and k. The results are in line with this prediction too.

Congruence of generated problems with the intuitive models

We hypothesized that students would try to generate as much as possible word problems that are congruent with the intuitive models. This hypothesis was tested in two ways.

1. It was predicted that, when confronted with a multiplication sentence involving an integer and a decimal number (the sentences b and c in Table 1), students would strongly avoid to use the decimal number as multiplier of their word problem. The data strongly support this prediction. For both number sentences there was an overwhelming and, thus, statistically significant preference for the integer as multiplier. For instance, for sentence b (6 x 2.8) in 253 out of the 260 appropriate problems the integer was used as the multiplier. The results for sentence c were analogous.

2. A second expectation was that a qualitative analysis of the inappropriate statements would reveal a substantial amount of unallowed modifications of the given numbers and calculations to conform the numerical data to the constraints of the intuitive models. The following findings confirm this expectation.

To avoid violation of the constraint that the multiplier or divisor must be an integer, students rather often changed a decimal into an integer. This error type was very typical for sentence f (0.7 x 0.2): 10% of the inappropriate statements belong to this category (e.g., "One candy weighs 0.2 gr. What is the weight of 7 candies?").

Another incorrect modification consists of reversing the role of the given dividend and divisor. Of all inappropriate statements for sentences j (6 : 4.8) and e (4 : 0.8), 22% and 10% respectively were of this type (e.g., "Four grandchildren are visiting grandma. They all like milk. But grandma has only 0.8 litre of milk. How much does each child get?"). In both cases such a reversal avoids violation of the constraint that the divisor must be an integer; and, for sentence e it allows to bypass the intuitive rule that the quotient should be smaller than the dividend. This finding also supports the differential robustness hypothesis of Harel et al. (in press): apparently, students prefer to state a problem with a divisor larger than the dividend rather than one with a decimal divisor.

Reversing dividend and divisor was also observed in 10% of the inappropriate statements for sentence i (5 : 25) (e.g., "Five friends have together 25 sandwiches. How many does each of them have?"). This reversal avoids violation of the D>d constraint. Interestingly, not even one such a reversal was found for sentence h (6.3 : 9) which would result in having a decimal divisor. This finding involves additional evidence for the differential robustness hypothesis concerning the constraints of the intuitive division model.

Finally, another representative error consisted of using decimals to refer to objects that can only be denoted accurately by integers. By doing so, students used decimal numbers in a way that is incongruent with the intuitive models. For instance, this error type accounted for 25% of the inappropriate statements for sentence d (e.g., "A chicken lays 3.8 eggs a week. How many eggs do 7.4 chickens lay?"), and for 28% of the incorrect problems for sentence j (e.g., "4.8 people measure together 6 m. How much for each of them?").

Conclusions

Several findings of this study support the basic hypothesis of the theory of Fischbein et al. (1985), that the selection of an operation for solving multiplicative problems is mediated by primitive intuitive models of the arithmetic operations. In the three age groups students generated significantly more appropriate story problems for number sentences that are congruent with the intuitive models than for the incongruent ones. The impact of the models was also manifested in students' tendency to generate problems that conform to their constraints.

But, the study also shows that the theory of the intuitive models cannot appropriately account for all the available empirical data. Indeed, several findings with respect to multiplication were not in line with the predictions derived from the theory. And, for division, the present results largely support the idea put forward by Harel et al. (in press) that the D>d constraint is less robust than the other intuitive rules of the partitive model. These findings even raise the question whether this constraint is an essential feature of the primitive division model (see also Greer, 1987).

In summary, while the results of this study are fairly consistent with the the theory of Fischbein et al. (1985), they also suggest that in order to fully account for the available empirical data, it is necessary to aim at the design of a more comprehensive theory which takes into account that solving problems with a multiplicative structure is influenced by a large variety of factors interacting in multiple and complex ways. In view of the development of such a theory, an attempt towards combining the different approaches in research on multiplicative structures described by Nesher (1992), could possibly open new and productive research perspectives.

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