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ABSTRACT

Algebra students typically pay little attention to choosing appropriate domains, ranges and scales of axes for graphing functions. The purpose of this study was to investigate the effect of introducing an assignment and employing the graphing calculator to examine and alleviate students' difficulties regarding the selection of appropriate domains, ranges and scales of axes for graphing functions and to examine difficulty students might have regarding the identification, construction and definition of function. The research sample consisted of 128 college algebra students enrolled in a north-central Florida community college. There were eight classes of students: six treatment classes and two control classes. All classes were intact, thus random assignment of students to the classes was not possible. Two classes were in each of the following groups: Graphing Calculator and Assignment Group, Graphing Calculator Only Group, Assignment Only Group, and the Control Group. The analysis of covariance was used to examine mean differences on two instruments between the four groups in the study. The Domain/Range/Scale Instrument and the Identification/Construction/Definition Instrument were both administered as pretests and posttests. The results of the study suggest that the treatments were in various ways interactively and independently effective regarding the students' understanding of the concept of function. Contains 25 references. (Author/MKR)

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EXECUTIVE SUMMARY

Algebra students typically pay little attention to choosing appropriate domains, ranges and scales of axes for graphing functions. Use of the graphing calculator in the teaching and learning of algebra, particularly regarding the concept of function, demands the need to consider reasonable domains, range, and scales of axes for graphing functions.

The purpose of this study was to investigate the effect of introducing an assignment and employing the graphing calculator to examine and alleviate students' difficulties regarding the selection of appropriate domains, ranges and scales of axes for graphing functions and to examine difficulty students might have regarding the identification, construction and definition of function.

The research sample consisted of 128 college algebra students enrolled in a north-central Florida community college. There were eight classes of students: six treatment classes and two control classes. All classes were intact, thus random assignment of students to the classes was not possible. Two classes were in each of the following groups: Graphing Calculator and Assignment Group, Graphing Calculator Only Group, Assignment Only Group, and the Control Group.

The analysis of covariance was used to examine mean differences on two instruments between the four groups in the study. The Domain/Range/Scale Instrument and the Identification/Construction/Definition Instrument were both administered as pretests and protests. The results of the study suggest that the treatments were in various ways interactively and independently effective regarding the students' understanding of the concept of function.

The following manuscript provides a detailed overview of the study. It is composed of an introduction, a section describing the methodology of the study, and implications following from the results. The information provided by the manuscript will assist practitioners, i.e., teachers of mathematics, in using appropriate assignments and graphing calculators to teach mathematics.

INTRODUCTION

This study was supported by the need in mathematics education to provide research data pertaining to the ability of public community college students enrolled in college algebra to select and recognize appropriate domains, ranges and scales of axes for graphing mathematical functions. The concept of function is a fundamental and unifying theme of mathematics (National Council of Teachers of Mathematics, 1989). The use of the graphing calculator in instruction requires that students consider how they select domains, ranges and scales of axes for graphing functions. One must be aware of the effects that these elements have on the visual representation of the concept. In addition to this concern, the conceptions that the students have prior to instruction with the graphing calculator may be in conflict with new knowledge presented to them, and if so this conflict should be capitalized to enhance students' understanding of the concept. This application of cognitive theory in mathematics education to improve conceptual understanding will provide a model for educators to adopt and/or adjust for instruction and, it will provide an initial reason for examining the resources of other disciplines for application in mathematics educations.

Moreover, this study also included an attempt to validate previous research findings regarding students' ability to identify, construct, and define function. Because very little research in mathematics education is conducted on the community college level, it is imperative that research results are not quickly applied to levels of instruction where the research rarely takes place. This portion of the study is not simply a replication, but it is an extension of previous research. This extension includes the use of a public community college sample, the use of a homogeneous sample (college algebra students), and the factors of graphing calculator use and concept assignment participation.

Theoretical Framework

Skemp (1987) described a concept as an "idea" and the name of a concept as a "sound." The concept of function can be identified by the written or spoken word "function." The word is just an identifier for the concept; the concept itself is an idea. Skemp suggested that in determining whether or not one had a concept, the main condition was whether or not the person behaved accordingly when presented with new examples of the concept. Gilbert and Watts (1983) found that these actions could be "...linguistic and nonlinguistic, verbal and nonverbal..." (p. 69). Historically, the mastery of a concept was described as "...the demonstrated ability to recognize or identify the definition, description, or illustration of a concept or the appropriate usage of the word or words naming the concept..." (Butler, 1932, p. 123).

The emergence of graphing calculators in education forces one to consider the advantages of emphasizing the graphical representation of function. Yerushalmy (1991) suggested that "only one topic in a traditional algebra course utilizes visual-graphic representation in addition to the symbolic one: investigating functions." (p. 42) Functions and graphs should be central topics in algebra, one reason is because they are at the heart of elementary calculus (Fey, 1984). Buck (1970) suggested that compared to other representations of the concept of function, the graphical form of functions was most useful for students. From information obtained by observing the graph, the learner can depict various characteristics of the relationship, such as the possible values of the independent variable (domain), that may not be as obvious through another representation and the corresponding dependent values (range). The learner can use the graphing calculator to examine the effects of changing the domain of a function or to examine the behavior of a function by restricting its domains. The graph has the potential of enhancing the concept of function (Clement, 1989), and the graphing calculator is the medium with which this can be done.

Learners using the graphing calculator will find it necessary to select appropriate domain, range and scale of axes in order to provide a useful graph of a function. Thus the students' concept of function is enhanced through realization that the domain dictates the resulting range and that the scale of the axes dictates the visual properties of the graph of the function. Researchers (e.g., Ayers, Davis, Dubinsky, & Lewin, 1988; Demana & Waits, 1988; Fey, 1990) are now beginning to emphasize the importance of scaling for providing appropriate graphs of functions. The consideration of the scale of the axes on which the graph appears reflects the importance one puts on viewing an appropriate graph of a function. If one is not careful to choose a reasonable scale for the axes, "critical features" of the graph can be overlooked (Goldenberg & Kliman, 1988).

Hewson and Hewson (1984) adopted a model for conceptual change, which emphasized that learning is an interaction between previous and existing knowledge with the outcome depending on the interaction. Nussbaum and Novick (1981, 1982) proposed a similar model, but their aim was to have the learner resolve conflict strictly by accommodation. The researchers suggested that accommodation is a process which can be prepared for, but not scheduled or guaranteed. However, they argued that all "...one can do is to try to characterize it and to look for instructional strategies that may facilitate its occurrence" (p. 186). The model they suggested for facilitating cognitive accommodation is summarized as follows:

1. Create a learning situation that encourages learners to examine his/her conceptions prior to instruction of a topic, express these conceptions orally and written. The teacher's role involves assisting the learner in expressing conceptions in a definitive manner and promoting an environment where learners can debate on the conceptions.
2. Create a situation where conflict is born between the students' (mis)conceptions and some academic truth or reality. ". . .the information must be presented in such a way as to challenge or stimulate the student, for it is through this process of conflict that he integrates the new material" (McMillan, 1973, p. 36).
3. Support the learners' accommodation of the concept.

METHODOLOGY

Research Population/Sample

The population for this study consists of students enrolled in college algebra at public community colleges. Community colleges, although rightfully deserving of a place in America's educational system, are often omitted from the network of educational research, at least more so than elementary, middle, and high schools, four-year colleges and universities. As of 1989, Wattenbarger reported that there were 1,300 community colleges in the United States and that 38% of all college and university students seeking bachelors degrees were graduates of a community college. Furthermore, 47% of all undergraduate, minority students are enrolled in community colleges (Koltai & Wilding, 1991). These statistics support the need to include the community college system in the mathematics education research base of undergraduate institutions. If this is not done, we as educators are taking the risk of omitting many learners from our research sample. These facts provide reason for an increased effort to include community colleges in more educational research.

The research sample consisted of 6 treatment classes and 2 control classes (128 students total) at a north-central Florida community college. All classes were administered a pretest and a posttest. Two treatment classes were allowed to use graphing calculators on in-class assignments and these students participated in a conceptual change assignment. Two treatment classes had access to the graphing calculator only, and two treatment classes participated in the conceptual change assignment. The two control classes were not provided with graphing calculators and did not participate in the conceptual change assignment.

All classes were intact, thus random assignment of students to the classes was not possible. The students in the classes represented characteristics of the population of students enrolled in this particular community college and community colleges in general. To aid with the demographics and other descriptors of the students in the sample, each student in the study completed an information sheet. The researcher obtained the following information regarding the subjects:

1. There were more females than males participating in the study, however, the arrangement of the students by gender in the four groups was nonsignificant.
2. The average age of a student participating in the study was 21 years.
3. A significant number of students who participated in the student were Caucasian/White for all four groups.
4. There were no significant differences between the groups regarding the number of students who attended college full-time and the number of students who attended college part-time.
5. The average grade point average was 2.98 (based on a 4.0 scale).
6. There were no significant differences between the four groups regarding the number of mathematics courses taken by the students.

Procedures

A nonequivalent control group design was used to collect data. This factorial (quasi-) experiment involved a 2x2 design: two levels of graphing calculator use, and two levels of in-class assignment. The objective was to determine the effect of the two independent variables, individually and interactively, on the dependent variable (posttest scores).

The first factor in the experiment was students' access and use of the graphing calculator during instruction. Interest in this factor developed from the need to determine if students could engage in concept development better with the use of the graphing calculator than without use of the graphing calculator. An implication is that if students perform better on function concept instruments with the use of the graphing calculator than without its use, then attention needs to be given to integrating this tool in mathematics education instruction.

The second factor in the experiment was participation in a concept development assignment focusing on students' (mis)conceptions. Interest in this factor resulted from the overwhelming existence of difficulties which students have with the concept of function. An implication is that if students perform better on the function concept instrument after participating in such an assignment, then more attention sold be given to focusing on students' (mis)conceptions before formally presenting concepts.

Analysis of the Data

The analysis of the data included the following:

1. Descriptive statistics for the treatment and control groups
2. Analysis of covariance to determine main and interactive effects
3. Categorization of the students' function definitions
4. Categorization of the students' function images
5. Frequency of students' mis-use of a correct function definition

Results

Descriptive statistics for the pretest and posttest results for the Domain/Range/Scale Instrument indicate that the groups were not successful with the instruments in regards to the percentage of items answered correctly. At the institution where the study was conducted, a score of 70% is the lowest score that is considered as a sufficient score for a student to "pass" an examination.

The analysis of covariance results revealed that when considering the posttest scores on the Domain/Range/Scale Instrument, there was a significant interaction effect between the factors of calculator and assignment. There were three significant group differences: The Calculator Only Group had a significantly higher mean score than the Calculator and Assignment Group. The Assignment Group had a significantly higher mean score than both the Calculator and Assignment Group and the Control Group.

Descriptive statistics for the pretest and posttest results for the Identification/Construction/Definition Instrument indicate that the groups were not successful with the instruments in regards to the percentage of items answered correctly. The analysis of covariance results indicated that when considering posttest scores on the Identification/Construction/Definition Instrument, there is a significant main effect for the factor of concept assignment. The Assignment Only Group had a significantly lower group mean than the students who did not participate in the assignment.

A categorization of the definition of function provided by the students revealed that 73% of all of the students who gave definitions gave a similar definition. In fact, these students all gave an ordered pair form for the definition. The second most common form was a graphical definition. Sixteen percent of students giving a definition gave a graphical definition.

The students' image of the concept of function was dominated by the vertical line representation. After examining the explanations given by the students for identifying and constructing functions, it became evident that 58% of all of the students who gave images gave the graphical representation in form of the vertical line representation.

In addition, the researcher denoted that more than 80% of the students who gave an acceptable definition for the concept of function neglected to properly apply this definition when responding to other questions on the instruments.

Conclusion

The interaction of the use of graphing calculators and participation in a conceptual change assignment was found to significantly affect the student's concept of function regarding application of the concepts of domain and range, and the selection of appropriate domain, range, and scale for the axes for graphing functions. Further analyses of the interaction revealed that students employing the graphing calculators only and the students participating in the conceptual change assignment only were more affected by the two separate treatments than the students who employed the graphing calculators and participated in the conceptual change assignment. Almost every student and five of the six instructors in the study were unfamiliar with graphing calculators before participation in the study. None of the instructors had formally applied conceptual change theory in their teaching of mathematics. Therefore, the graphing calculator and the conceptual change assignment were two new introductions into the calculator and assignment group. The students and instructor for this group were required to operate under circumstances involving two new issues into the classroom, while the calculator group and the assignment group only had to deal with one new introduction into the classroom.

In addition, the mean for the assignment group on the Domain/Range/Scale Instrument was significantly higher than the mean for the control group. The assignment was designed to focus attention on misconceptions that the students had about the concepts of domain and range and scales for graphing functions. Its design, based on findings from the literature, was purposely developed to assist the students to logically deal with their conceptions. The students were able to participate in the assignment using paper and pencil, which can be considered natural conditions for the community college mathematics classroom.

Overall, the students in the study were not successful with the Domain/Range/Scale Instrument. This applies to both the treatment groups and the control group. The instrument was particularly designed to examine the students' concepts of domain and range and the students' understanding of the scale of the axes for graphing functions. The results from the analyses of the Domain/Range/Scale Instrument indicate that the students had difficulty with the following:

1. Denoting the domain and range of functions given algebraically
2. Denoting the domain and range of functions given graphically
3. Identifying specific function values for functions given algebraically
4. Identifying specific function values for functions given graphically

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5. Graphing functions given algebraically with domain restrictions
6. Graphing functions given algebraically without domain restrictions
7. Identifying functions which satisfy specified domain and range restrictions
8. Distinguishing between the properties of the function and the properties of its graph
9. Choosing appropriate domain and range restrictions and reasonable scales to provide complete graphs of functions
10. Recognizing the effect that a domain restriction and scale of the axes may have on the graph of a function

Participation in the conceptual change assignment was found to significantly affect the student's concept of function regarding their identification, construction, and definition of function. The group mean for students who participated in the assignment was significantly lower than the group mean for students who did not participate in the assignment. This indicates that the assignment was not appropriate to assisting students with identification, construction, and definition of function. In fact, one can conclude that the assignment was a hindrance for students attempting to develop these abilities. Perhaps this is an indication that encouraging students to attend to their misconceptions and inconsistent knowledge is not useful for all activities in the mathematics classroom, and the length of time necessary for development of concepts should be greatly considered.

Further analyses of the Identification/Construction/Definition Instrument revealed that the students' definition of function was dominated by the ordered pair representation, and the students' image of function was dominated by the vertical line representation. The ordered pair representation and the vertical line representation for the concept of function both adhere to functions on a point-wise basis. The students did not seem to respond to the arbitrary nature of functions discussed by Even (1989, 1990, 1993). They viewed functions as collections of points of ordered pairs. The students may have exhibited such difficulties with the concept of function, because the students' concept of function was dominated by these two representations.

The difficulties with the concept of function that the students exhibited are consistent with the findings of several researchers (e.g., Dreyfus, 1990; Papakonstantinou, 1993). The students had difficulty mastering the definition of function and applying the concepts of domain and range. The students' concept image of function was dominated by a point-wise view of function. According to Hershkowitz, Arcavi, and Eisenberg (1987), students construct mental images of function according to the images that are emphasized in instruction.

Because of the results of the analyses, the researcher is led to propose that the students in the study had assimilated the concept of function, but they had not accommodated the concept. According to Strike and Posner (1985) and Posner, Strike, Hewson, and Gertzog (1982), this indicates that a major conceptual change was not required by the students. This is also in agreement with Nussbaum and Novick (1981, 1982) who suggested that accommodation is a process which can be prepared for, but not scheduled or guaranteed.

IMPLICATIONS

The results reported in this study have several implications for mathematics curricula. First and foremost is the need to truly embed concept of function and functional thinking in mathematics curricula. This includes emphasis on the concepts of domain and range and emphasis on the graphical representation of the concept. The results indicate that as an underlying concept of mathematics, the introduction or reintroduction of the concept of function and the concepts of domain and range at the postsecondary level does not provide a foundation for the concepts. The concepts of domain and range are two of the most important concepts surrounding the concept of function, and also must be thought of us as underlying concepts of mathematics. Attention also needs to be given to definitions of the concept of function. Care should be taken to determine the appropriate time to introduce formal definitions to students and to determine which definitions should be used in the curricula.

Formal definitions may not be appropriate for introducing or reinforcing the concept of function. Dominant definitions, such as the ordered pair representation, which do not assist students in understanding the concept of function should be complemented with other definitions which prove to be more useful for conceptual understanding. This indicates a need to include multiple representations of the concept of function in the algebra curriculum. In addition, a student's ability to provide a definition of the concept of function does not necessarily indicate that the student has ownership of the concept. Attention should be given to developing and/or collecting appropriate curricula materials and activities which will assist educators in assessing students' understanding of the concept of function.

These are implications for mathematics education resulting from the analyses in the study. Instruction focusing on functions without focusing on the concept of function does not truly address the concept or the idea of function. Students who do not own the concept of function can not be expected to be able to use the graphing calculator to its fullest benefit. They must have a basic understanding of the concept in order to understand the reasoning behind the operation of the graphing calculator. Otherwise, the student will see the graphing calculator as a machine for doing mathematics instead of a tool for learning. This is consistent with Yershulamy (1991) who suggested that stressing the visual alone without the use of a technological tool is not sufficient. The learner will still be inhibited if an understanding of the concept of function is not obtained.

The use of the conceptual change assignment indicates that a planned focus on students' misconceptions regarding the concepts of function, domain, and range can aid students in successfully dealing with their misconceptions. However, as indicated by the results of the study, caution should be taken regarding the selection of misconceptions or difficulties which are addressed.

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