As the current reform effort in school mathematics education continues to gain momentum, an understanding of the process by which teachers make changes in their instructional practices becomes increasingly important. The first part of this paper describes a model for teacher change using an extension of the constructivist model for learning. According to the model, six factors drive the teacher change process: (1) experiencing a perturbation, (2) having a commitment to change, (3) constructing a vision of what specific changes might look like within a teacher's own classroom, (4) projecting the teacher's self into that vision, (5) deciding to make changes within a given context, and (6) being a reflective practitioner. The remainder of the paper describes a case study designed to test the model. A middle school teacher was chosen as a subject for the study, which found that each of the six factors was demonstrated by the subject. A vignette illustrates how each of the six factors reveals itself. An appendix contains the activities used in the study. Contains 18 references. (MKR)
USING A MODEL TO UNDERSTAND THE PROCESS OF CHANGE IN A MIDDLE SCHOOL MATHEMATICS TEACHER

Thomas G. Edwards
The Ohio State University

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A critical problem facing mathematics education today is the translation of the vision of mathematics teaching and learning contained in two National Council of Teachers of Mathematics' (NCTM) standards documents (NCTM, 1989, 1991) into actual practice in our schools. This vision suggests learning environments that are quite different from the lecture-dominated norm that exists today (Goldsmith & Schifter, 1993; Hart, 1991; NCTM, 1991). Clearly some of what Richardson (1990) terms "significant and worthwhile changes" in mathematics teachers will be a necessary condition for the realization of that vision (NCTM, 1991). Moreover, "as teachers implement important, timely, and exciting changes, they will require continuing programs of professional support" (Mathematical Sciences Education Board [MSEB], 1990, p. 48).

During the 1992-93 school year a middle school mathematics teaching improvement project was developed and implemented. Most of our project teachers worked in a large urban district in the midwest, but some worked in a suburban school and others in a parochial school. To assess the impact of the project on our project teachers, a theoretical frame for studying change in teaching practice was constructed.

A Framework for Change

A current theoretical viewpoint that is ubiquitous in mathematics education is constructivism: the theory that learners actively construct their own knowledge through interaction with their environment. If teachers are viewed as reflective thinkers who use a problem-solving approach to instructional practice, then such a cognitive theory can be extended to provide a theoretical framework for the study of teacher change.

A Constructivist View of Teaching and Learning

Confrey (1991) and Underhill (1991) both describe construction of knowledge as a cyclic process. Confrey (1991) describes the process of reflective abstraction in these terms:
We act through sensory-motor and cognitive operations. We use tools and previously familiar systems of representation. Then we monitor the results of our actions to see if the problematic has been resolved and equilibration restored. This may end the sequence, lead to a reconsideration and perhaps alteration of the problematic, and subsequently a new cycle of action and reflection. (p. 118)

It is significant that Confrey, in describing such an important Piagetian construct as reflective abstraction, ascribes a prominent role to the "tools and symbols" which so interested Vygotsky (Kozulin, 1990, pp. 110-150). This suggests a sociocultural, as well as a cognitive grounding for constructivist theory in mathematics education.

Underhill (1991) also sees the construction of knowledge as a cyclic process. His description posits cognitive conflict and curiosity as the primary motivational devices in the learning process. He sees peer interaction as a catalyst in the production of cognitive conflict, cognitive conflict as a catalyst to individual reflection, and reflective activity as inducing a cognitive restructuring. Finally, since the process must occur within the individual's experiential field, any

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**Figure 1: A cyclic model of the construction of knowledge**

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cognitive restructuring must eventually be followed by further peer interactions, whence a new cycle begins. Figure 1 provides a simple model of one possible interpretation of Underhill's description of the cycle.

Perhaps the most powerful aspect of Underhill's thinking is that it yields a form of empowerment. The learning cycle just described empowers learners, because they are now perceived as being "... in control of their own learning" (p. 230).

An Extension of the Constructivist Model

It is clear that successful educational reform demands an understanding of the processes by which teachers change (Hart, 1993; Richardson, 1990; Schifter, 1993; Shaw & Jakubowski, 1991; Wood, Cobb, & Yackel, 1991). Often, an understanding of complex processes can be enhanced through the development of a model. Indeed, with respect to the construction of a model for studying teacher change, Goldsmith and Schifter (1993) note that "... the means by which teachers develop their practice are as yet little understood. It is critical that we develop some models for the growth of teaching practice if we are to succeed in stimulating such change on a wide scale" (p. 124). Thus, it is to the construction of a model that attention is now directed.

Hart (1991) believes that learning is a process of providing structure and organization to one's world to make sense of experience. Moreover, she believes that learning occurs as knowledge is modified in dealing with problematic situations. Thus, it is an easy extension for her to suggest that teachers, in attempting to change, will modify their knowledge and beliefs about teaching and learning when their attempts are made problematic.

Cooney (1993) acknowledges that the teaching of mathematics is, by its very nature, a problematic activity. Furthermore, he suggests a fundamental role for reflectivity in the process of teacher change. In his view, "... the notion of reflection is rooted in the constructivist notion of adaptation. The relevance of
reflection and adaptation ... is that neither can meaningfully take place from a closed, dualistic perspective" (p. 45).

Shaw, Davis, and McCarty (1991) likewise ground their theoretical framework of how teachers change in constructivism, but carry the theory a step further with their notion of teacher change precipitated by perturbation, or mental dissonance. They argue that those who are interested in effecting change in teachers' practice should expose teachers to alternatives in theories of learning and teaching as well as classroom activities. Such exposure is believed to cause perturbation, followed by frustration and discomfort, followed by the reflection that leads to change. Shaw and Jakubowski (1991) also note the importance of peer support and collaboration in the process of teacher change. Thus, the same, or similar features from the model of the construction of knowledge developed in Figure 1 can be seen as descriptors of the teacher change process.

Figure 2 suggests a cyclic model for teacher change. The "peer interactions" could take several forms (discussions with colleagues, discussions with administrators or researchers, exposure to "new" ideas), but the key steps would seem to be the manufacture of some form of perturbation which, in turn, elicits reflective activity in the teacher. Thus, a framework has been

![Figure 2: A cyclic model of teacher change](image)

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A Model For Teacher Change

built for studying teacher change by extending some of the major tenets of a constructivist view of learning to a perspective which views teachers as learners and teaching as a problem solving activity.

**A Constructivist Model of Teacher Change**

Shaw and Jakubowski (1991) believe that their research supports the existence of six factors that drive the teacher change process. They believe that in order for substantive changes to occur, teachers must:

- experience a perturbation,
- have a commitment to change,
- construct a vision of what specific changes might look like within their own classroom,
- project themselves into that vision,
- decide to make changes within a given context, and
- be reflective practitioners.

Shaw and Jakubowski see *perturbation* as a necessary condition to change in practice, because in its absence, individuals are likely to be satisfied with their current practice of teaching. A perturbation is needed to upset the equilibrium. However, perturbed or not, change is not likely without *commitment*. This is the force that sets the change process in motion.

Nevertheless, Shaw and Jakubowski believe that perturbation and commitment alone are not sufficient to elicit change, and that teachers must *construct a vision* of what their changed practice might look like. This requires viable alternatives. Moreover, peer collaboration and support often aid in the construction of such visions.

Having constructed a vision of a changed practice, it is then necessary to *project oneself into that vision*. Shaw and Jakubowski believe that those unable to do so are unlikely to change. Those who are capable of such a projection and attain a certain level of comfort in that projection then often *decide to change* within a given context. Finally, a *reflective practice* is necessary to compare one's
actual practice with one's vision. In fact, in this model, it is teacher's reflection on their practice that is seen as the force that drives the entire process of teacher change.

Outline of the Study

Some studies have examined the process of teacher change, particularly in the context of developing theories and defining factors which are associated with the change process. However, few, if any, previous studies have set out to test a model for teacher change as a constructivist learning experience. To do this, a study within a study was designed. The research project within which this smaller study was conducted was partially funded through the Dwight D. Eisenhower Mathematics and Science Program sponsored by the Ohio Board of Regents. The larger study consisted of 16 middle school teachers who voluteered to participate in a project designed to study teacher change while implementing the NCTM Standards, particularly through introduction to inquiry-based modes of instruction. This article will report only the findings from the smaller study.

Method

Subject and Procedure

Rich was the subject for study and worked in the one suburban setting in the larger project, teaching five classes of seventh grade mathematics. The research associate visited Rich on nearly a weekly basis for an entire school year—and thus was defined as the measuring instrument for this study. Data were collected through observations, field notes, project journals, interviews, and videotape. The reliability and validity of these data were confirmed through triangulation of data sources and perspectives as well as through a member check.

The research questions which organized the smaller study were (a) Are there identifiable episodes in which the factors of the teacher change model are embedded in the process of change?; (b) What changes occurred?; and (c) Does this study provide evidence to confirm or deny the tested model?
Data Analysis and Interpretation

To test the model, data were studied to find episodes that reflected the components of the model. To do this, at the close of the larger study, the qualitative data that had been collected were examined for indications of change in teaching practice. Once such episodes had been identified, the research associate wrote a short narrative report detailing both the indicators of change that had been observed and the context within which they occurred. Finally, the resulting vignette was analyzed in terms of the proposed model of teacher change. This report will detail an interpretation of one such series of episodes which emerged from this process.

Results

The vignette developed in this section will illustrate how each of the six factors suggested by Shaw and Jakubowski (1991) reveals itself relative to a series of incidents involving the project teacher, Rich, and one of the research associates in the project. This vignette occurred approximately four months into the project and spanned approximately three weeks. It is related from the perspective of the research associate. The Geometric preSupposer is an interactive software package that allows students to explore geometric properties in a plane.

The Change Process in One Teacher: A Vignette

Rich asked me to present an activity in his classes that uses The Geometric preSupposer. He acquired the software a few years ago, and has it stored in his classroom. He had never used the software, but told me, "I know I ought to be doing something with it, but I'm not sure what to do nor how to do it." I sensed in Rich a certain reluctance about asking me to do this. A possible reason for such reluctance surfaced during the planning of the activity. Rich indicated that he had encountered some difficulties when using the preSupposer with students in one-to-one situations. He was not specific about those difficulties, but seemed very dubious about the efficacy of using the software with the whole class.
We jointly planned a set of activities that would utilize the preSupposer. We met twice to do so. During these sessions, we examined an activity book that accompanies the software (Chazan, 1989). Rich indicated that he would like his students to work on activities related to properties of angles. I suggested activities aimed at the development of three related concepts: the measure of vertical angles, the measure of the angles formed when parallel lines are cut by a transversal, and the sum of the measures of the angles of a plane triangle. Rich reviewed each of these activities and decided that they would be appropriate for his students. By then, it had occurred to me that it might be desirable to use the property of angles that form a linear pair as a lead-in to the vertical angles activity. As there was nothing comparable in the sourcebook that we were using, I designed and wrote such an activity (see Figure 3). As with the others, Rich reviewed and approved this activity for use in his classes. We thus decided to use these four activities. (See the Appendix for complete descriptions of each of these activities.)

Rich indicated that he could arrange to have all of the instructional computers in the school moved into his room for about a week, and that this would allow for groups of two or three students working at one computer. He also felt that he should share the availability of the computers with his fellow seventh grade mathematics teachers, so we settled on two days for the completion of the four activities, plus one day for a follow-up to help his students consolidate their learning.

We decided that I would introduce the software and some of its functions to his students and then let them explore and interact with the software as they completed tasks aimed at investigating properties of angles. Since Rich indicated that he was not yet totally comfortable with the preSupposer software, he decided that it would be appropriate if he observed his students and worked along with some of them during this introduction. During this planning, I suggested to Rich that these activities would lend themselves to a team approach. For example, the
Task: To examine the angles in a linear pair.

Steps:
- Start with segment AB that is 6 units long.
- Label point C on segment AB by subdividing the segment into 2 parts.
- Label a movable point D anywhere not on segment AB.
- Draw segment DC.
- Your drawing should look something like this:

![Drawing of a linear pair]

- Sketch your drawing below.
- Measure angles ACD and DCB and record on your drawing.
- Starting with a new segment, repeat all of the steps above.

Observations:
- What do you notice about the measures of the angles?

- Pairs of angles like the ones you have drawn are called linear pairs. Can you make a conjecture about linear pairs?

Figure 3: The First Activity
Task: To examine angles formed by the intersection of two line segments.

Steps:
- Start with a segment AB that is 6 units long.
- Place moveable points C and D on either side of segment AB and draw segment CD to connect them.
- Label the intersection of segments AB and CD with point E.
- Your drawing should look something like this:

```
    C
   /|
  /  |
 /    |
 A---------E---------D
    |
   \
    B
```

- Sketch your drawing below.
- Measure angles AEC, CEB, BED, DEA and record on your drawing.
- Starting with a new segment, repeat the steps above.

Observations:
- What do you notice about the measures of the angles?

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- Do you think these patterns will always be true? Why or why not?

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Figure 4: The Second Activity
second activity, "Angles Formed by Intersecting Lines" (see Figure 4) follows quite logically from the first activity, "Linear Pairs."

In "Linear Pairs", the students were asked to use the preSupposer software to construct the diagram shown in Figure 3 and then to use the "Measure Angle" feature of the software to measure angles ACD and DCB. Similarly, in "Angles Formed by Intersecting Lines", the students were expected to use the software to draw the diagram in Figure 4, measure the four angles (AEC, CEB, BED, and DEA). Both of these activities asked the students to look for any relationships and to make any conjectures that seemed appropriate.

I felt that Rich could easily carry out the second activity after I had completed the first with his students, so I invited him to share the teaching with me. He declined, however, saying that he preferred to just watch this time. I was anxious to get him directly involved in some way, so I suggested that he help me monitor the students' explorations. Rich liked that idea, and said that he would probably interact with just one or two groups of students.

As it turned out, Rich did everything during these sessions that he said he would - and more. He worked through each of the activities at least once, interacting with one of his student groups in the process. He and I both monitored all of his students. It was difficult for me to know exactly what form Rich's monitoring of students took, but from what I could see, he made a conscious effort to assess each groups' interactions with the preSupposer software. He did so by asking students what they had done to construct a figure, or which angles it would be appropriate to measure, or what patterns they could see in their data, or what the source of their conjectures had been.

Rich was impressed with the way that the activities held his students' interest. Near the end of the second day of the activities, he told me, "I will feel very comfortable using this software in the future."
A caveat is perhaps in order here. Rich is a secondary-certified mathematics teacher. The strength of his mathematical background enabled him to easily interact with the preSupposer himself. He also had past experience using computers in instructional settings. Therefore, he was able to focus on the pedagogical aspects of using the software. It seems likely that a middle school mathematics teacher having a weaker mathematical background or little experience with computers would require much greater support in such an endeavor than Rich did.

Coincidentally, I had occasion to visit Rich's school on another matter nearly a year later. Of course, I could not resist the urge to stop in to see how he was doing. He made it a point to tell me that he planned to use the preSupposer activities for three or four days this year, and invited me to visit when he did. Although other commitments prevented me from directly observing Rich's use of the preSupposer, I was able to visit at the end of his week-long use of the computers.

At that time, he described the previous days work with the preSupposer as a fairly free-wheeling set of student explorations. If his use of another piece of software, a computer game called Taxman, on that last day is any indication, it must have been an exciting week of inquiry for his students.

In using Taxman, Rich purposely neglected to provide his students with the rules of the game. "Oh, it's a pretty easy game," he told them. "You'll probably get clobbered by the Taxman the first few times, but once you figure out his rules, you should be beating him just about every time." And they were! It was gratifying to see Rich's decision to use The Geometric preSupposer and other software truly result in student explorations that were in the spirit of inquiry.
Discussion

This vignette can be matched for its degree of fit with the Shaw and Jakubowski model for teacher change. It will be shown that Rich demonstrated all six of the factors in that model.

That he had been *perturbed* can be seen in his initial statement that he knew he ought to be doing something with *The Geometric preSupposer*. His *commitment* to change revealed itself through his action in asking for assistance, in spite of his reluctance to do so. Moreover, the way he used *Taxman* demonstrated some comfort as risk-taker, something that had not previously been observed in his practice. This also evidences his commitment to change. The decision not to share in the teaching, but to "just watch this time", was necessary for the *construction of his vision* of what students working with *The Geometric preSupposer* would look like in his classroom. Rich was able to *project himself into that vision* in a literal, as well as a figurative sense. His active role in monitoring and assessing his students during their activities was a more proactive role than he originally envisioned and is a literal embodiment of such a projection. His *decision to change* within a given context was apparently made when he spoke of being very "comfortable" using *The Geometric preSupposer* in the future. As he subsequently actually brought that decision to life, this is a second literal embodiment of the model.

Finally, it is argued that Rich's *reflective activity* was at work throughout this incident. His original perturbation seems clearly the product of a reflective practice, as was his commitment to seek the assistance of the research associate. Likewise, Rich's decision to remain an observer during these lessons was a reflective action designed to better position himself for further reflection. It seems clear that reflection was at work when he literally projected himself into the monitoring aspect of the teacher's role, especially given the evaluatory direction that his activity eventually took. Finally, his decision to actually use *The Geometric preSupposer* in the future, implied by his statements voicing comfort with that prospect, was clearly the act of a reflective practitioner.
The overarching role that has been ascribed to Rich's reflective activity in the interpretation of the foregoing vignette suggests that the model for teacher change that was proposed in Figure 2 does not adequately consider the role of reflection in this process. Rather than one point in a cyclic process, it seems more likely that the teacher's reflective activity is the context within which the cycle occurs. Figure 5 illustrates a way to think of the process of teacher change that better accounts for the role of reflection.

Figure 5: Teacher Change as a Reflective Cycle

Finally, one segment of the cycle of teacher change described in Figure 5 can be better understood by incorporating Shaw and Jakubowski's (1991) six cognitive requisites into the reflective turn between perturbation and change. If one visualizes portions of this process in linear terms, Figure 6 illustrates a means of conceptualizing the transformation of a perturbation into change.
Figure 6: From Perturbation to Change - A Reflective Turn
Conclusion

As the current reform effort in school mathematics education continues to gain momentum, an understanding of the process by which teachers make changes in their instructional practices will become increasingly important for those interested in effecting significant and lasting change. It has been shown that the nature of this process can be made more transparent through the construction of a model and the application of that model to a particular case. Shulman (1986) argues persuasively for the development of case knowledge to assist in the preparation of teachers. His argument should be extended to the development of a case literature to assist in an understanding of the teacher change process.

References


APPENDIX: The Activities (Adapted from Chazan, 1989)

Activity 1

Linear Pairs

Task: To examine the angles in a linear pair.

Steps:
- Start with segment AB that is 6 units long.
- Label point C on segment AB by subdividing the segment into 2 parts.
- Label a moveable point D anywhere not on segment AB.
- Draw segment DC.
- Your drawing should look something like this:

```
  D
 /|
/  |
A---C---B
```

- Sketch your drawing below.
- Measure angles ACD and DCB and record on your drawing.
- Starting with a new segment, repeat all of the steps above.

Drawings & Measurements

Observations

What do you notice about the measures of the angles?

Pairs of angles like the ones you have drawn are called linear pairs. Can you make a conjecture about linear pairs?
Activity 2
Intersecting Lines

Task: To examine angles formed by the intersection of two segments.

Steps:
- Start with segment AB that is 6 units long.
- Label moveable points C and D on either side of segment AB and draw segment CD.
- Label the intersection of segments AB and CD with point E.
- Your drawing should look something like this:

```
  C
   |
   |
   E
   |
  A

  D
   |
   |
   B
```
- Sketch your drawing below.
- Measure angles AEC, CEB, BED, and DEA and record on your drawing.
- Starting with a new segment, repeat the steps above.

Drawings & Measurements

Observations
What do you notice about the measures of the angles?

Do you think these patterns will always be true? Why or why not?
Activity 3

Corresponding and Alternate Interior Angles

**Task:** To investigate measure of angles formed when a segment crosses two parallel segments.

**Steps:**
- Start with a segment AB that is 6 units long.
- Using "Moveable Point" label a point C about 2 or 3 units from segment AB.
- Draw a parallel through C to segment AB. Define the parallel by moveable points D and E.
- Using "Moveable Point" label points F and G, one on either side of the two parallel segments.
- Draw segment FG; erase label C.
- Label the intersections of segment FG with segments DE and AB. Your drawing should look something like this:

```
   F
   \______
   |     |
   |     |
   |     |
   H     I
   |     |
   |     |
   G
   \______
       A
       D
```
- Measure the four angles that share point H and the four angles that share point I.
- Record your drawing and mark the angle measures on the drawing.
- Repeat these steps.

**Drawings & Measurements**
Observations

- Based on your observations, shade in the angles you think will be equal in measure on the drawing below. Use different shadings for different measures.
Activity 4
Triangle

Task: To investigate the sum of the interior angles in a triangle.

Steps:
- Start with any triangle ABC.
- Draw a line segment through A parallel to side BC.
- Measure angles ABC, ACB, and BCA.
- Record your drawing, measurements, and any conjectures.
- Repeat these steps on another triangle.

Drawings & Measurements

Conjectures