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ABSTRACT

This article discusses a primary level mathematics research program, Cognitively Guided Instruction (CGI), which investigates the impact of providing primary teachers with access to a structured, coherent body of knowledge about children's thinking in mathematics on teachers' knowledge and beliefs, their instruction, and their students' learning. In CGI classrooms students spend most of their time solving problems. Little work is focused explicitly on the mastering of counting, basic facts, or computational algorithms. The article discusses knowledge about children's thinking, what research has shown about CGI, teachers, children, successful implementation, and the longitudinal study of CGI in primary schools, including: development of educational materials, urban settings, and preservice education. The study finds there is a pervasive view held by teachers, students, and the public that equates with telling students what to learn and how to learn it rather than with guiding students' learning process. Changes in instructional practices must accompany changes in materials. (MKR)

Cognitively Guided Instruction

> CGI is a philosophy versus a recipe ... You as a teacher have to take the knowledge that CGI is about problem types, about solution strategies, about how children develop cognitively and you have to apply that to your own teaching style. (Mazie Jenkins, First Grade Teacher)
>
> A CGI classroom is where you build on the math knowledge of your children according to what they know... You don't build objectives that say they should be doing this, this, this, and this. You sort of take what they know and build on from there. (Susan Gehn, First Grade Teacher)

Cognitively Guided Instruction (CGI) is captured in the above statements of experienced CGI teachers; it is about teachers making instructional decisions based on their knowledge of individual children's thinking. Currently in its seventh year of funding from the National Science Foundation, CGI was started by Elizabeth Fennema, Thomas Carpenter, Penelope Peterson, and Megan Franke as a research program to investigate the impact of research-based knowledge about children's thinking on teachers and their students. The project currently includes the investigation of childrens' and teachers' thinking in Kindergarten through 3rd grade, the study of CGI in urban schools, and the study of the impact of CGI in preservice teacher education. Teacher education materials are being developed for use with both inservice and preservice teachers.

What CGI Is

CGI IS NOT A TRADITIONAL PRIMARY school mathematics program. It does not prescribe the scope and sequence of the mathematics to be taught. Nor does it provide instructional materials or activities for children, or suggest that there is an optimal way to organize a class for instruction. Instead, knowledge about mathematics, in terms of how children think about the mathematics, is shared with teachers. The learning environments of CGI workshops are structured so that teachers learn how the knowledge about children's mathematical thinking can help them learn about their own children. With CGI support, teachers decide how to use that knowledge to make instructional decisions. As teachers implement and reformulate the plans they make, unique CGI classrooms emerge. Each teacher creates a teaching and learning environment that is structured to fit his or her teaching style, knowledge, beliefs, and children.

Even though CGI does not prescribe instruction, CGI classrooms do have similarities. Children in CGI classrooms spend most of their time solving problems, usually problems that are related to a book the teacher has read to them, a unit they may be studying outside of mathematics class, or something going on in their lives. Various physical materials are available to children to assist them in solving the problems. Each child decides how and when to use the materials, fingers, paper and pencil; or to solve the problem mentally. Children are not shown how to solve the problems. Instead each child solves them in any way that s/he can, sometimes in more than one way, and reports how the problem was solved to peers and teacher. The teacher and peers listen and question until they understand the problem solutions, and then the entire process is repeated. Using information from each child's reporting of problem solutions, teachers make decisions about what each child knows and how instruction should be structured to enable that child to learn.

Starting at the kindergarten level, CGI teachers ask children to solve a large variety of problems involving addition, subtraction, multiplication, or division. Children learn place value as they invent procedures to solve problems that require regrouping and counting by 10s. Little work is focussed explicitly on the mastering of counting, basic facts, or computational algorithms. Instead problems are selected carefully so that children count by 1s, 10s, or 100s depending on the child; discuss relationships between basic number facts; and invent procedures to solve problems involving two- and three-digit numbers.

The climate in a CGI classroom is one in which each person's thinking is important and respected by peers and teacher. Children approach problem solving willingly and recognize that their thinking is critical. Each child is perceived by the teacher to be in charge of his or her own learning as individual knowledge of mathematics is used to solve problems that are realistic to her or him. Mathematics is usually taught at least an hour a day; it is also integrated into the many other learning activities children do.

Author Notes

Work on children's thinking in geometry and measurement began with CGI teachers in January 1992; the results have not yet been implemented in classrooms.

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Knowledge about Children's Thinking

Recent research has provided a reasonably clear picture of how many basic mathematical ideas develop in children. This research has shown that children enter school with well developed informal or intuitive systems of mathematical knowledge that can be used as a basis for the further development of their understanding of mathematical concepts, symbols, and procedures. Even before children are introduced to formal notions of addition, subtraction, multiplication, and division—they can solve a variety of problems involving the actions of joining, separating, comparing, grouping, partitioning, and the like.

To understand children's intuitive problem solving processes, it is necessary to understand the different problem situations that characterize addition, subtraction, multiplication and division. Therefore we start by helping teachers develop a taxonomy of problem types. Some of the distinctions among subtraction problems and division problems are illustrated by the problems in Table 1.

The distinctions among the problems in Table 1 are critical because they reflect the ways that children think about and solve the problems. Initially children directly model the action or relations in the problem. They solve the first problem in Table 1 by making a set of 12 counters (to represent the stamps) and removing 8 of them. They solve the second by first making a set of 8 counters and then adding more until there are 12. The fourth problem is often solved by matching a set of 12 counters with a set of 8 counters. Many children cannot solve the third problem because it is difficult to model. They have no place to start because the initial set, i.e., how many stamps Sybil had to start with. is not known. The fifth problem is solved by putting 12 counters into groups of 4 and counting the number of groups. The last problem is solved by first dealing 12 counters into 4 groups, and then by counting the number of objects in each group. With the possible exception of the third problem in Table 1, all these problems can be solved by many children who are as young as Kindergartners or 1st graders, if they are given opportunity to model the problem situations.

These modeling or concrete strategies provide a foundation for the development of more abstract ways for solving problems and thinking about numbers that involve counting. For example, children come to recognize that it is not necessary to make a set of 8 objects to solve the second problem. They can find the answer just by counting from 8 to 12 and keeping track of the number of counts. Similarly, the first problem can be solved by counting back 4 from 12, and the fifth problem can be solved by counting by 4s (4, 8, 12). The sixth problem, on the other hand, is more difficult to solve by counting. Since the number of objects in each group, i.e., the number of stamps each child is to receive, is not known, children do not have a specific number to count by. Thus, this analysis of problems and children's strategies for solving them provides a principled basis for understanding differences in the difficulty of the problems and why children may have more difficulty in using particular strategies to solve certain problems.

In the process of solving problems, children learn number facts, not as isolated bits of information, but in a way that builds on the relationships between facts. Certain facts like doubles, e.g., 6 + 6, are learned earlier than other facts, and children use this knowledge to solve problems and to learn other facts. For example, consider how one child figured out that 8 + 9 is 17, "Well, 8 and 8 is 16 and 8 and 9 will be just one more. So it's 17."

Knowledge of place value and computational algorithms also can be developed through problem solving. The counting and modeling solutions that children use with smaller numbers are extended naturally to problems with larger numbers. Rather than using individual counters and counting by 1s,

Table 1

Addition, Subtraction, Multiplication and Division Problems

1 Sybil had 12 stamps. She gave 8 of them to George. How many stamps did Sybil have left?

2 Sybil had 8 stamps. George gave her some more and then she had 12. How many stamps did George give her?

3 Sybil had some stamps. George gave her 8 more and then she had 12 stamps. How many stamps did Sybil have before George gave her any?

4 Sybil had 12 stamps. George had 8 stamps. How many more did Sybil have than George?

5 Sybil had 12 stamps. She put 4 of the stamps on each page of a book. On how many pages will she put stamps?

6 Sybil had 12 stamps. She wants to divide them so that she and 3 friends have the same number of stamps. How many will each person get?

children use physical representations for 10s and 100s to model two- and three-digit numbers. They learn to use symbols by inventing procedures for solving two- and three-digit problems without counters as illustrated in Table 2.

Table 2

Children's Invented Procedures for Two-Digit Problems

Susan had 27 stickers. She bought 34 more stickers. How many stickers did she have then?

Megan: Mmmm 27, 37, 47, 57. Now I need 4 more. Well, 58, 59, 60. 61. (Megan mentally separated 34 into 30 and 4, counted 30 by 10s, and then counted 4 more.)

Todd: Well 20 and 30, that's 50; and the 4 and 7, that's 11. So it's 61. (Todd combined the 10s to get 50, combined the ones to get 11, and then combined the two sums.)

Roberto had 41 candies. He ate 23 of them. How many candies did Roberto have left?

Juan: Well 40 take away 20 is 20. But it was 41, so that's 21 take away 3, that's 20, 19, 18. He had 18. (Juan changed the numbers into 40 and 20 and then added the one he had taken away from the 40. He then took away the 3 he had taken away from the 23.)

Janice: 40 take away 20 is 20, take away 3 more is 17, but we have to put one back, so it's 18. (Janice had changed the numbers into 40 and 20. She then took away the 3 she had taken away from the 23 and added in the 1 she had taken away from the 41.)

Adam: 40 - 20 is 20, and 3 - 1 is 2; so I take 2 away from 20. That's 18. (Adam separated the 10s and then the 1s and then took the remaining 1s from the total.)

In summary, children start school with a conception of basic mathematics that is much richer and more integrated than that presented in most traditional mathematics programs. For example, in most textbooks subtraction is presented as only a separating or take-away action. However, subtraction can also be represented by the comparing, joining, and part-whole problems illustrated in Table 1, and young children can solve such problems. Symbols then, are learned not as abstractions, but as a way of representing situations that children already understand. Rather than expecting children to learn skills in isolation and then to learn how to apply those skills to solve problems, the learning of computational procedures is facilitated by problem-solving experiences that permit children to invent ways to calculate answers to problems.

What Research has Shown About CGI

Previous studies have investigated whether the CGI knowledge that we shared in the workshops had an impact on teachers and on students; the results have been reported in a variety of publications. The studies have used a variety of methodologies to study teachers including precise observations of teaching, paper and pencil assessments, individual interviews, and indepth case studies. To assess children's thinking, standardized tests, self-developed paper and pencil tests, and individual interviews have been used. The majority of the studies that have been reported to date have been concerned with the learning and attitudes of 1st grade children and with the thinking and instruction of their teachers. The findings from a number of studies that have been conducted over the last seven years are synthesized here.

Teachers and CGI

In general, teachers can learn the knowledge about the mathematical domains and children's thinking within those domains. The knowledge has proved useful to them. They are able to use it as they plan for and implement instruction and to assess what individual children know. Teachers can use this knowledge to make instructional decisions, both before and during instruction.

The instruction of teachers who have been through a CGI workshop is different than the instruction of teachers who have not been exposed to CGI, and children in CGI classrooms do different things when compared to those in non-CGI classrooms. When compared to non-CGI teachers, CGI teachers assess their children's knowledge more often and use a larger variety of procedures to gain knowledge about children. Much assessment is integrated into ongoing instruction, when the teachers gain knowledge of children by asking questions and listening to their children's responses. Some teachers supplement this informal assessment with individual interviews. Mathematics is integrated throughout the day and problems are situated in a variety of contexts which have meaning to children. Teachers find mathematics becomes more fun to teach when CGI principles are used.

Successful Implementers of CGI

While all teachers who have participated in a CGI workshop appear to change their instruction, some teachers are better able to implement CGI than others. A number of studies have identified teachers who appear to implement CGI better than others using some kind of inter-rater judgment or by measuring the learning of children in the classrooms. The two sets of teachers' characteristics have been compared and contrasted and the relationships between these characteristics and their children's learning examined.

One characteristic critical to any implementation of CGI is the knowledge that teachers have: knowledge of content analyses and children's thinking in general, as well as knowledge of the thinking of specific children in their classrooms. Before any exposure to CGI, many teachers have an intuitive knowledge of content analyses and how children solve problems. However, that knowledge does not appear to be particularly well integrated and organized. It is not particularly useful to them as they make instructional decisions. After participating in CGI workshops and using the knowledge as they teach, the knowledge becomes integrated into a more coherent network and used as a basis for making instructional decisions. The knowledge of the better implementers of CGI is more highly integrated than the knowledge of those who implement it less well. The degree of knowledge that teachers have about CGI and their children's learning is correlated with what their children learn in mathematics.

Teachers' beliefs about mathematics instruction, i.e., their role and students' role in learning mathematics, is another important characteristic. Those teachers who hold beliefs more closely aligned with the philosophy of CGI are better able to implement CGI. The degree to which these beliefs are held is positively correlated with their children's learning.

More successful CGI teachers believe more strongly than less successful CGI teachers that: 1) children's learning should be considered as they make instructional decisions; 2) children have informal knowledge that enables them to solve problems without instruction; 3) the teacher's role is to build a learning environment where children can construct their own knowledge rather than where the teacher is a transmitter of knowledge; and 4) the learning of procedural skills does not have to come before children can solve problems.

Becoming a CGI teacher is not done overnight, nor is it accomplished by the end of a workshop. It takes time and interaction with children to learn CGI knowledge, and to incorporate it into a classroom. The more the knowledge is used to gain an understanding of individual children's thinking and ability, the more important it becomes to teachers. They increasingly ask questions that elicit children's thinking, listen to what children report, and build their instruction on what is heard. Teachers increasingly come to believe in the importance of children's thinking as they see what children are able to do and what they are able to learn when given the opportunity to engage in problem solving appropriate to their ability.

Children and CGI

The learning and beliefs of children who spent one year in a classroom taught by teachers who had attended a CGI workshop have been compared with those of teachers who had no CGI education. Children in the CGI teachers' classrooms spent more time solving problems and talking about mathematics with their peers and teacher and less time working on computational procedures than did children in non-CGI teachers' classrooms. They reported more confidence in their ability to do mathematics and a higher level of understanding than did non-CGI students. When compared to non-CGI students, children in CGI classrooms were better problem solvers; in spite of the fact that they spent only about half as much time explicitly practicing number fact skills, they actually recalled number facts at a higher level than did non-CGI students.

CGI children became more flexible in their choice of solution strategies and increased their fluency in reporting their mathematical thinking. Children in CGI classrooms learned much more than has been expected of children in traditional classrooms. They learned to solve a larger variety of addition/subtraction and multiplication/division problems; their understanding of place value increased; and they learned to be flexible in their use of invented strategies to solve multidigit problems.

CGI In 1992

The Longitudinal Study of CGI in the Primary School

The purpose of current CGI research is to study the impact of providing primary teachers with access to a structured, coherent body of knowledge about children's thinking in mathematics on teachers' knowledge and beliefs, their instruction, and their students' learning over a three-year period. Research based knowledge about children's thinking in addition/subtraction, multiplication/division, place value, early ideas of fractions, geometry, and measurement has been identified. Workshops have been developed and taught to most of the Kindergarten through 3rd grade teachers in participating schools. Currently how teachers come to understand their students' thinking, how teachers use children's thinking to develop and provide instruction, the impact of the knowledge of children's thinking on teachers' knowledge and beliefs, and the cumulative effect of being in CGI classrooms for three years on students' mathematics learning are being studied. Additional studies are looking at the scope, sequence, and pedagogical presentation of mathematical ideas by two different, expert CGI teachers per year to obtain rich descriptions of CGI in Grades 1-3.

The Development of CGI Educational Materials

The materials that were written to enable CGI to be implemented are being revised into a coordinated program which can be used with either preservice or inservice teachers. These materials will include chapters detailing CGI philosophy; the content analyses of addition/subtraction, multiplication/division, place value and multidigit algorithms, functions, and geometry; children's thinking; and video tapes that illustrate children's thinking and prototypic classrooms. If feasible, hypermedia will be made available to help teachers interact with CGI ideas. These CGI educational materials currently are being tested using a variety of procedures to organize workshops. Descriptions of the procedures will be made available to assist future workshop developers.

CGI in Urban Settings

Under the direction of Deborah A. Carey, a further study of CGI's impact on 1st grade children and their teachers is underway in six magnet schools with 60 percent or more racial/ethnic minority populations in Prince George's County, Maryland. Of particular interest is the change in teachers' expectations as they learn to assess their children's knowledge. The change in instructional behavior which happens as teachers learn to use children's knowledge is being documented. Both quantitative and qualitative research methodologies are being used. Any modifications of workshop materials which are necessary when working in multicultural settings will be noted and incorporated into the teacher education materials that are being produced.

CGI and Preservice Education

A further investigation is studying the conditions under which CGI can be incorporated into preservice teacher education programs. Of interest is whether and under what circumstances successful inservice teacher education programs can have similar effects on preservice teachers. This study, directed by Donald Chambers, is a collaboration with teacher education faculty at Queens College, City University of New York; San Diego State University; and the University of North Carolina at Greensboro, and the primary-grade teachers in schools used by those institutions for field-based preservice teaching experiences.

The first phase of this project, beginning in February 1991 and lasting about 18 months, is designed to help the teacher education faculty and primary-grade teachers at each of the three sites develop sufficient expertise in CGI knowledge and its application in primary-grade classrooms so that they can serve as experts in the education of preservice teachers. Activities during this period will include workshops at each of the three sites, a two-week summer conference in Madison, Wisconsin, and visits to the classrooms of the site team teachers. Not only are the project members at each site becoming familiar with the application of the principles of CGI, but they are also planning for implementing CGI into their preservice teacher education programs through university course modifications and field assignments. Some piloting of instructional activities is also taking place at this time. Workshops are being conducted at two sites to expand the cadre of CGI mentor teachers available to supervise the field experiences of preservice teachers.

The actual intervention will begin in the fall of 1992. Team members at each site are developing the site's intervention plan which will be finalized during the second summer conference in July 1992. Plans for the evaluation of the impact of CGI in the preservice teacher education program are also being developed for each site. The evaluation will look at changes in the beliefs and knowledge of preservice teachers and their ability to use CGI principles in their interactions with students. Interventions will be documented and modified over a two-year period in an attempt to achieve the maximum possible impact.

The results of the study will be published in a monograph that will include an overview of CGI, descriptions of the activities and results at each of the three project sites, an evaluation of each site, a cross-site synthesis, and suggestions for the incorporation of CGI into teacher education programs at other institutions. This monograph should become available in the spring of 1995.

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> The students were excited about ideas— they were thinking and interpreting problems that were real and not contrived. No one said, 'When will I ever need this?' They were learning to listen to each other as some raised valid points others had not considered . . . The creativity and ingenuity of many of the students was exciting. All of the students had the opportunity to succeed and, in doing so exceeded our expectations. Consistently there was evidence of higher-order thinking and analysis in all of the classes, not just in the honors class. (p. 194-195)

The authors of the study conclude that, based on their experiences while carrying out the study, there is a danger that schools, teachers, educators, and publishers will make only "nominal" changes in their practices and then consider themselves in line with the NCTM *Standards* (1989). There is a pervasive view—held by teachers, their students, and the public—that equates teaching with telling students what to learn and how to learn it rather than with guiding students' learning process. Changes in instructional practices, according to this study, must accompany changes in materials.

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The bibliography contains only a partial list of CGI publications. A complete list of CGI publications can be obtained by writing NCRMSE.

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