ED 368 779 TM 021 285

AUTHOR Romberg, Thomas A.
TITLE World Class Standards: The Mathematical Sciences Education Perspective.
PUB DATE Apr 94
NOTE 23p.; Paper presented at the Annual Meeting of the National Council on Measurement in Education (New Orleans, LA, April 5-7, 1994).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Criteria; Curriculum Design; *Curriculum Development; Definitions; Educational Assessment; *Educational Change; Educational Objectives; Elementary Secondary Education; Expectation; Foreign Countries; Instructional Improvement; *Mathematics Education; Outcomes of Education; *Science Education; Technical Education
IDENTIFIERS National Education Goals 1990; *NCTM Curriculum and Evaluation Standards; Reform Efforts; School Culture; Standard Setting; *World Class Standards

ABSTRACT

The use of the term "world class standards" grows out of the rhetoric surrounding the National Education Goals of 1990. The perspective of the mathematical sciences education community on world class standards is offered through a description of their efforts on behalf of reform of mathematics education and a discussion of what such standards should be. In pursuit of reform of mathematics education, the National Council of Teachers of Mathematics decided to produce a series of standards documents that curriculum specialists and teachers could use to create a demand for new texts and staff development. The resulting "Curriculum Standards" have become the operational definition of world class standards. No international norm exists against which these standards can be measured, but examination of the mathematics frameworks of eight other countries supports their label as world class and their alignment with the mathematics goals of other nations. To assess them in reality it would be necessary to compare the variations in student outcomes, expectations for students, programs, and school cultures across the countries. Some ongoing efforts in these areas are described, but it is finally argued that agreement on criteria that could be called world class may not be possible. Six tables and one figure illustrate the discussion. (Contains 15 references.) (SLD)

WORLD CLASS STANDARDS:

THE MATHEMATICAL SCIENCES EDUCATION PERSPECTIVE

by

Thomas A. Romberg

University of Wisconsin-Madison

Paper for the Annual Meeting of the

National Council on Measurement in Education

April 5-7, 1994

The use of the term "World Class Standards" in the current political debates grows out of the rhetoric surrounding the National Education Goals (see Table 1). Goals 3 and 4 explicitly mention mathematics, and mathematics is implicit in Goal 5. Furthermore, in the debates, the *Curriculum and Evaluation Standards for School Mathematics* produced by the National Council of Teachers of Mathematics (1989) are often cited as the exemplar for establishing "world class content standards" for the core disciplines. This claim is both flattering and surprising to the mathematical sciences education community: Flattering because we never thought politicians would take recommendations from mathematics teachers seriously, and surprising because comparing our efforts to those of other countries was not our intent when the *Curriculum Standards* were produced. Nevertheless, since we have been drawn into the debate, let me offer the mathematical sciences education community's perspective on "world-class standards" by: (1) describing the intent of our reform efforts with respect to school mathematics, (2) examining the problem of judging what is "world class," and (3) explaining the mathematical sciences education views about "world-class standards."

Insert Table 1 about here.

REFORM IN SCHOOL MATHEMATICS

During the past decade, the mathematical sciences education community has been attempting to change mathematics as it is taught and learned in this country. The need for new and different course materials, instructional procedures, assessments, and evaluation procedures was based on the fact that:

> In the past quarter century, mathematics and mathematical techniques have become an integral, pervasive, and essential component of science, technology, and business. In our technically-oriented society, "innumeracy" has replaced illiteracy

> as our principal educational gap. One could compare the contribution of mathematics to our society with needing air and food for life. In fact, we could say that we live in an age of mathematics--the culture has been "mathematized." (Jaffe, 1984, p. 117)

By "mathematizing," we mean the ability to represent quantitative and spatial relationships in a broad range of situations, express those relations using the language of mathematics, use technology to carry out numerical procedures and make predictions, and interpret the results. In particular, school mathematics programs need to be designed so that students learn to quantify variables from phenomena in physical, biological, and social contexts; represent relationships between variables arithmetically, algebraically, geometrically, graphically, and/or statistically; use techniques and properties of number systems to calculate, find solutions, and make predictions (the techniques need to be appropriate to the problem context and include estimating, approximating, and using calculation algorithms with technological assistance); use algebraic, geometric, graphical, and/or statistical techniques to find solutions and/or make predictions about patterns, models, functions, and relationships; and interpret mathematical characteristics of pattern, shape, dimension, data, chance, and change in numerous contexts in order to understand their significance in the world.

Based on these apparent needs, the National Council of Teachers of Mathematics (NCTM) decided to produce a series of standards documents. Our intent was to develop documents that curriculum specialists and lead teachers could use to create a demand for new texts, tests, staff development programs, and so forth. The strategy was informed by publishers, and other educational suppliers, who argued that they could not produce new materials without a demand, since they operate in a "supply and demand" economy (Romberg, 1993).

Our belief was that if school mathematics was approached from this perspective, then students would (1) learn to value mathematics, (2) become confident in their ability to do mathematics, (3) become mathematical problem solvers, (4) learn to communicate mathematically,

and (5) learn to reason mathematically (NCTM, 1989, pp. 5-6). These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity. Furthermore, society would benefit because such programs would result in (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate (NCTM, 1989, pp. 3-5). Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives.

This vision for school mathematics has been widely endorsed by the mathematical sciences community and is now hailed by policy makers as the model other disciplines should follow to establish "world-class" standards.

Are NCTM's *Curriculum Standards* World Class?

Since NCTM's *Curriculum Standards* have become our operational definition of a "world-class" mathematics program, this question deserves to be answered. To do this, my staff and I at the National Center for Research in Mathematical Sciences Education (Romberg et al., 1991) examined the mathematics frameworks of eight countries (Australia, France, Germany, Japan, The Netherlands, Spain, Norway, and The United Kingdom) and compared those frameworks with NCTM's *Curriculum Standards*. We found

> that there is considerable variation in what is taught, in when ideas are introduced, and in what is emphasized. Thus, there is no international norm against which one can compare American views of what it is important to teach and learn in school mathematics. . . . Nevertheless, we believe that the following statements about the

vision of school mathematics presented in NCTM's *Curriculum and Evaluation Standards for School Mathematics* qualify the *Standards* as a world-class mathematics reform document. (1991, p. 40)

1. The NCTM *Standards*, when compared with the national curricula of other countries, do not represent a "radical" or "romantic" vision of school mathematics (p. 40).

2. The manner in which each country builds a detailed rationale for these reforms and, specifically, which changes are emphasized, depends on their past practices (p. 41). For example, "number sense" and "estimation" are specifically mentioned in NCTM's document for Grades K-4, but not in most others. This does not mean either topic is unimportant in these countries. They have been central to their curricula for decades (but not in ours); thus, no emphasis is needed.

3. The four standards for mathematics teaching and learning in NCTM's *Standards*, problem solving, communication, reasoning, and connections, are reflected in all eight national curricula, not just in the reform curricula. But while the terms used may be different, the underlying themes are consistent (p. 41).

4. We are convinced that the variation in emphasis with respect to particular mathematical topics also is related to past cultural practices in different countries (p. 41).

5. In Grades K-4, the *Standards*, while including topics new to the American curriculum, still put more emphasis on whole number arithmetic than other countries. This same finding applies to all work with numbers up to Grade 8. However, there is no standard on "number" for Grades 9-12. Other countries appear to be more balanced in their approach to this important aspect of mathematics (p. 41).

6. While geometry is now included at all levels in the *Standards*, which, of course, is not the case in most American classrooms, it still receives less emphasis than in the majority of other countries (p. 41).

7. The *Standards* include more emphasis on statistics, probability, and discrete mathematics than in other countries (p. 42).

8. Although the beginning principles of calculus are now being suggested for all students in the *Standards*, most other countries have long assumed this to be important and, in fact, expect much more than is advocated by NCTM (p. 42).

9. Although a different program for students is common in other countries, their students are expected to study mathematics every year they are in school and are often offered several options. In the United States, the radical recommendation in the *Standards* that all American students study "real" mathematics for at least three years of high school falls short of the expectations of most other countries (p. 42).

10. The most striking difference between the NCTM *Standards* and the curriculum documents from other countries lies in their emphasis on the social, attitudinal aspects of schooling. Schools need to be "a secure environment and place of trust," "social behaviors" need to be taught, "students should realize that mathematics is relevant," "students should gain pleasure from mathematics," and "personal qualities should be nurtured" are common statements in these documents. Such statements put an emphasis on what happens in classrooms that is different from the focus on either cognitive learning or economic imperatives in the *Standards* (p. 42).

In conclusion, one cannot study the curricular documents from these countries without realizing that the current mathematics curriculum in the United States is far from being "world class." On the other hand, NCTM's *Curriculum Standards* (1989) presents a vision of content that is significantly in line with what other countries are now doing and with what they are planning to do. The expectations expressed in this vision, if realized in the schools, would bring all American students more in line with the expectations for students in the rest of the world.

Judging Whether Something is "World Class"

Given the difficulty of our attempt to judge whether NCTM's *Standards* were "world class," it is apparent that the political rhetoric in the National Goals implies we know how to judge whether students' achievements in any country are indeed "world class." In retrospect, to judge

"something" implies either certifying that certain criteria have been met, or rank ordering a "set of somethings" on the basis of specific criteria. For National Education Goal 3, the "somethings" are students whose work is to be compared against the certification criterion "competency in challenging subject matter" at Grades 4, 8, and 12. Also competency is to be defined so that all students learn to use their minds, are prepared for responsible citizenship, further learning, and productive employment (a difficult set of inferences). For Goal 5, similar certification criteria are implied for all adults. One would need to establish the connections between specific "knowledge and skills" in mathematics to be both employable and a responsible citizen. For National Education Goal 4, the "something" is a composite profile of U.S. students on achievement to rank order the American profile with those of other countries. Note that the judgments for Goals 3 and 4 are quite different judgments based on different criteria. Then, to judge whether something is "world class" involves either determining that both the certification criteria are comparable among nations and that the percentage of somethings that meet those criteria are comparable, or that the method of rank ordering on an attribute is reasonable across nations.

Thus, for one to argue that judgments related to Goals 3 and 5 are "world class," one would have to build the case that the American certification criteria for all students (and adults) and goals are comparable with those of other nations, and also the percentage of students (adults) who meet those criteria is comparable. Evidence to build such a case for Goal 3 could be drawn by comparing the expectations in different countries for all students at these grade levels, along with comparisons of instructional programs and of school cultures. Evidence for Goal 5 for mathematics would involve a similar argument. For Goal 4, one could build a case for appropriate rank ordering of profiles if one could agree on how and when to assess mathematical achievement. In summary, to build such arguments about what is "world class," one would need at least to examine the variations in student outcomes, expectations for students, programs, and even school cultures across countries. The mathematical sciences education community assumes that any such comparisons should be based on our vision of school mathematics and not on current practice.

Compare Outcomes

A straightforward way of making comparisons between the mathematical achievement of students in several countries is to administer a common test to a sample of students at the same level of schooling in each country. This has been done several times in the recent past and another similar study is now in the final stages of planning. The central question that needs to be addressed is:

How valid is the test as a measure of student achievement across nations?

To study the "world class" validity of such tests, during the past few years my staff and I were asked by the National Center for Educational Statistics to examine the items administered in the two past comparative studies with respect to NCTM's *Curriculum Standards*: The test battery for the Second International Mathematics Study (SIMS) administered by the International Association for the Evaluation of Educational Achievement (1985) and the battery for the International Assessment of Educational Progress (IAEP) administered by Educational Testing Service (1990). The question being addressed was: Does the content of these tests reflect that expressed in the *Standards*? Of course, such a comparison is problematic, since these tests were developed before the *Standards* were written.

The results of our analysis of each battery at each grade level were similar (Romberg, Smith, Smith, & Wilson, 1992). For example, the categorization of the Grade 8 SIMS battery is shown in Table 2. One can only conclude that the content coverage is out of balance.

Insert Table 2 about here.

Performance profiles based on these tests cannot be used to make a valid judgement about "world-class" mathematics achievement for American students, let alone those of any other country. A second approach to valid comparisons of outcomes is now being undertaken by the New Standards Project (Resnick, Nolan, & Resnick, 1994).

They have defined five **benchmarking questions** that serve to standardize their data collection:

- What are students in other countries expected to know and be able to do at key transition points in their schooling careers?
- What kinds of performances are used to demonstrate competence?
- What counts as "good enough" in these performances?
- What percentage of students are meeting the standards?
- What reform efforts are underway or on the horizon?

In order to answer these questions, they are collecting such material as national, regional, or school curricula; legislative or ministerial directives related to content; commonly used textbooks; examinations, especially national high-stakes examinations; graded student work; scoring rubrics or guides; data on pass/fail rates; and tracking information. They also seek out teachers and other education professionals who can provide the context and history within which these materials can be understood. At each phase of their data collection, they request commentary from these panels of experts in order to be sure that they understand how the materials are used in their home country.

Compare Expectations

For expectations one could examine the high-stakes examinations administered in different countries. For example, at Grade 12 one could examine the high school completion exams, such as those given in the different states in Australia, or in Norway, or in The Netherlands. But what would they be compared with in the U.S.? Chantal Shafroth (1993) recently made such a comparison. For the United States, she used the SAT Level I and Level II tests, and the AP Calculus Test. Her general findings are shown in Table 3 and the kinds of questions asked in Table 4. These analyses are followed with an examination of the different

---

Insert Tables 3 and 4 about here.

---

types of questions posed. Examples of these are given in Table 5. Even a cursory look at this report indicates that there are vast differences in mathematical expectations among these countries and between them and the United States.

Insert Table 5 about here.

A second type of high stakes examinations are college admission tests. For example, the Mathematics Association of America recently published a set of admission examinations from several Japanese universities (Wu, 1993). Two typical items are shown in Table 6. The reason for publishing the examinations was to demonstrate to American mathematicians and mathematics educators the fact that the Japanese ask entering college students mathematical questions that many American college majors in mathematics would have difficulty answering.

Insert Table 6 about here.

Similar comparisons could be made with respect to examinations administered in some countries at the other levels of schooling. In fact, the New Standards Project (Resnick, Nolan, & Resnick, 1994) is currently studying examinations administered in The Netherlands and France for 8th graders to compare with tests they are developing.

In summary, while very incomplete, the evidence from these reports indicates that there are quite different mathematical expectations for students in many countries. It should be noted that the differences in both terminal and university entrance examinations are not just that some countries expect their students "to cover more" mathematics. This assumes that the discipline is a

linear sequence of topics. The fact is that countries differ in what they consider to be the important mathematics all students should learn. Some of these differences may be overt, such as teaching transformation geometry or measurement with metric units; others are covert, such as a focus on number sense rather than on computational proficiency.

Compare Programs

To compare programs, one could examine curricular frameworks. Geoffrey Howson (1991) examined the national curriculum frameworks for mathematics for 14 countries. The United States was not one of these, since we do not have a centralized education system. Howson concluded "There is no 'easy' way of comparing what is done and what is achieved in various countries. Major differences in philosophy and structure make simplistic comparisons dangerous" (p. 7).

He goes on to state, "National school systems reflect social cultures and traditions, and are much influenced by economic considerations, past and present. Perhaps the simplest measure of the latter is the length of compulsory schooling. This can be as high as 12 years, but Portugal, for example, is only now moving away from a 6-year system" (p. 7). He also found that while politicians argue that there is a need to "forge a closer link between the national curriculum and assessment procedures than would appear to exist in any of the other countries below. In fact, there are references to assessment in a few national curricula. . . ." (p. 28).

Ken Travers and Ian Westbury (1989) reported on a more extensive, but perhaps less useful, analysis of mathematics curricula across the 22 countries that participated in SIMS. The focus of this study was on the relationships between the "intended curriculum," the "actual curriculum," and the "attained curriculum." The expanded model for gathering information for the study is shown in Figure 1. The utility of the analysis is hampered by the focus on variation across countries on specific features (making it difficult to get a sense of any one country's program), and on the relationships to the SIMS item pool.

Insert Figure 1 about here.

Compare Cultures

The differences in school cultures across nations with respect to the teaching of mathematics has been systematically studied by Stevenson and Lee (1990). They studied the "context of achievement" for a sample of American, Chinese, and Japanese children. They concluded that

> the poor performance of the American children in this study was due to numerous factors, many of which are neither elusive nor subtle. Some of the most salient reasons for poor performance appear to be the following: insufficient time and emphasis were devoted to academic activities; children's academic achievement was not a widely shared goal; children and their parents overestimated the children's accomplishments; parental standards for achievement were low; there was little direct involvement of parents in children's schoolwork; and an emphasis on nativism may have undermined the belief that all but seriously disabled children should be able to master the content of the elementary school curriculum. (p. 103)

In a less formal study, Jan de Lange (1992), whose staff at the Freudenthal Institute in The Netherlands has been working with mine to develop instructional materials for middle school students, has reported on the vast differences in the culture of schools in America and The Netherlands. These differences include governance, the role of administrators, the role of parents, the daily rites and rituals of schooling, scheduling, interruptions, athletics, and so on. While schools have been established by all societies to educate their children, there are vast differences in how different cultures have actually created and defined schooling.

THE MATHEMATICAL SCIENCES EDUCATION COMMUNITY APPROACH

First, we believe that comparative studies are very important. We can learn by understanding how different countries decide what mathematics their students should learn; how they teach that mathematics; how they expect their students to learn that mathematics; and how they determine student progress, proficiency, or achievement. Such comparisons can make our "commonplace" actions and beliefs problematic.

Second, we do not see the study of what other countries do as an attempt simply to keep abreast. We believe in the old adage: "Anyone who just wants to keep abreast is bound to be second best." Comparative studies should be seen as opportunities for us to learn and reflect on our actions, and not simply as an attempt to copy the ideas of others.

Third, for achievement, we recommend either the development of a more balanced examination system that is aligned with principles articulated in NCTM's *Standards*, or the selective adaptation of methods other countries use to judge achievement and compare our students using procedures based on the examination systems of those countries.

Fourth, we doubt that the overall effort of attempting to develop a common test battery similar to those in SIMS, IAEP, or NAEP upon which one can validly compare student achievement across countries at any grade level is worth the cost and effort. We are particularly concerned that future studies (including TIMSS) will compare student achievements across countries in a horse-race fashion on examinations that fail to reflect world-class aspirations for the students in any country, let alone those in the United States.

Finally, given the differences in schools and cultures, we doubt that there can be any agreed-upon criteria that could be called "world class." To strive for such is merely political rhetoric. As such, it may detract or undermine our efforts to make needed changes in schooling. Our work on mathematics and the teaching and learning of the subject in schools is only a small part of a much more serious need to restructure schooling in America.

REFERENCES

de Lange, J. (1992, August). Mathematics in American schools. Paper presented at the International Congress on Mathematics Education, Quebec City, Canada.

Educational Testing Service. (1990). International assessment of educational progress. 1991 mathematics assessment: Secured and publicly released test items.

Howson, G. (1991). National curricula in mathematics. England: The Mathematical Association.

International Association for the Evaluation of Educational Achievement. (1985). Second study of mathematics: Technical report IV, instrument book. Publicly released test item.

Jaffe, A. (1984). Appendix C. Ordering the universe: The role of mathematics. Renewing U.S. mathematics: Critical resource for the future. Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

Resnick, L. B., Nolan, K. J., & Resnick, D. P. (1994). Benchmarking education standards. Paper presented at the Conference on Uses of International Education Data. Washington, DC: National Academy of Sciences.

Romberg, T. A. (1993). NCTM's *Standards*: A rallying flag for mathematics teachers. Educational Leadership, 50(5), 36-41.

Romberg, T. A., Allison, J., Clarke, B., Clarke, D., Pedro, J., & Spence, M. (1991, November). School mathematics expectations: A comparison of curricular documents of eight countries with the NCTM *Standards* of the U.S. Prepared for the New Standards Project.

Romberg, T. A., Smith, M., Smith, S., & Wilson, L. (1992, June). The feasibility of using international data to set achievement levels for the National Assessment of Educational Progress (NAEP). Madison: National Center for Research in Mathematical Sciences Education, University of Wisconsin-Madison.

Shafroth, C. (1993). A comparison of university entrance examinations in the United States and in Europe. Focus, 13(3), 1, 11-14.

Stevenson, H. W., & Lee, S. (1990). Context of achievement. Monographs of the Society for Research in Child Development, 55(1-2) (Serial No. 221).

Travers, K. J., & Westbury, I. (1989). The IEA study of mathematics I: Analysis of mathematics curricula. New York: Pergamon Press.

U.S. Department of Education. (1991). America 2000: An education strategy. Washington, DC: Author.

Wu, L. E. T. (Ed.). (1993). Japanese university entrance examination problems in mathematics. Washington, DC: The Mathematical Association of America.

TABLE 1. THE NATIONAL GOALS FOR EDUCATION (U.S. DEPARTMENT OF EDUCATION)

1. By the year 2000, every child will start school ready to learn.

2. By the year 2000, the high school graduation rate will increase to at least 90 percent.

3. By the year 2000, American students will leave grades four, eight, and twelve having demonstrated competency in challenging subject matter including English, mathematics, science, history, and geography; and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning, and productive employment in our modern economy.

4. By the year 2000, U.S. students will be the first in the world in science and mathematics achievement.

5. By the year 2000, every adult American will be literate and will possess the knowledge and skills necessary to compete in a global economy and exercise the rights and responsibilities of citizenship.

6. By the year 2000, every school in America will be free of drugs and violence and will offer a disciplined environment conducive to learning.

Joint Statement by the President and the
Governors of the United States of America
February 26, 1990

TABLE 2. CATEGORIZATION OF THE 1985 SIMS 8TH-GRADE ITEMS

NAME OF TEST Year Grade Level	SIMS 1985 8
Number of Items	180

CONTENT AREA	n	%
Number	67	37
Measurement	28	15
Patterns and Functions	7	4
Algebra	23	13
Geometry	38	21
Statistics	16	9
Probability	1	1

PROCESS CATEGORY	n	%
Problem Solving	1	1
Communication	12	6
Reasoning	40	22
Connections	0	0
Computation/Estimation	120	67
Recall	7	4

LEVEL OF KNOWLEDGE	n	%
Procedural Knowledge	114	63
Conceptual Understanding	66	37

TABLE 3. GENERAL COMPARISON (SHAFROTH, 1993)

Country	Duration of exam	Types of exams	Description of exam	Time per question or part
France (F)	180 minutes	AB (Lib Arts) CE (Science)	16 to 20 parts grouped in 3 problems	10 minutes
Germany (G) (Hesse)	180 minutes 240 minutes	Basic Advanced	12 to 15 parts grouped in 2 or 3 problems	14 minutes 16 minutes
Netherlands (N)	180 minutes	A (applied) B (basic)	16 parts grouped in 4 problems	11 minutes
Portugal (P)	15 minutes + 30 minute tolerance	1 2	12 to 14 parts grouped in 5 problems	11.5 minutes
Scotland (S)	105 minutes 150 minutes	O or H	40 multiple choice & 28 parts grouped in 13 problems	2.6 minutes 5.3 minutes
United States	60 minutes 90 minutes 90 minutes	Level I, II AP Test	50 multiple choice, 45 multiple choice & 14-17 parts grouped in 6 problems	1.2 minutes 2 minutes 5.8 minutes

TABLE 4. PERCENTAGES OF QUESTIONS OF EACH TYPE IN TESTS WITH ANSWERS (SHAFROTH, 1993)

TESTS	FRANCE AB	FRANCE CD	GERMANY Bas	GERMANY Adm	NETHERLANDS App	NETHERLANDS Bas	PORTUGAL I	PORTUGAL II	SCOTLAND	UNITED STATES
Group 1 Comput.	50	18	54	38	55	62	42	53	64	83
Group 2 Show that	17	32	8	19	18	16	13	21		
Group 3 Proofs	10	31	21	29		13	26	20	7	7
Group 4 Graphs	17	14	8	9	5	25	5	13	4	3
Group 5 Open Ans.	6	5	8	5	22		10		4	7

TABLE 5.

PROBLEM FROM A FRENCH EXAMINATION (A)
(25% of total grade) (1988)

Archimedes has proved that:

> The area of the segment of a parabola is equal to 4 times the third of the area of a triangle, having the same base and the same height as the segment.

We want to illustrate this theorem with an example:

1) Given the parabola of equation: $y = -(x - 2)^2 + 9$, and the point S of coordinates (2,9), find the coordinates of A and B, the two points where the parabola intersects the x axis.

   Let AB be the parabolic arc passing through S.

2) The segment of the parabola is defined to be all the points of the plane between the arc AB and the x axis. Compute the area of the segment.

3) Compute the area of the triangle SAB.

4) Show how this example illustrates Archimedes' Theorem.

PROBLEM FROM AN APPLIED DUTCH EXAMINATION (1989)

A bridge is constructed on a river. One arch of the bridge is represented by the formula: $h = c - 10(e^{0.05x} + e^{0.05x})$, where h is the distance in meters from the point on the bridge above AB to the line AB, and x is the distance PO in meters.

T is the highest point on the arch AB (Figure II).

1) Show that given $AB = 40\ m$, $c = 30.9\ m$.

2) Given $c = 30.9$, find the maximum value for h.

3) Compute h for several values of x and sketch the graph of the arch, using a scale of 1 cm for 2 meters.

4) In the summer, usually the line AB is in shallow water. In the winter, the level is commonly considerably higher. Use your graph to determine approximately for what level under the bridge the distance between T and the water level is 4 meters.

5) Use a calculation to check your answer to 4.

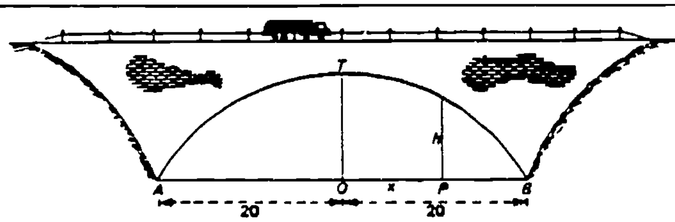

Figure II

TABLE 6.

TOKYO UNIVERSITY
HUMANITIES EXAMINATION (1991)

Given a regular pyramid V with a square base, there is a ball with its center on the bottom of the pyramid and a tangent to all edges. If each edge of the pyramid base is a, find the following quantities:

1) the height of V

2) the volume of the portion common to the ball and the pyramid

Note: A regular pyramid has a square base adjoined to four isosceles triangles, with each edge of the square making up the base of one of the triangles.

SHIGA MEDICAL UNIVERSITY
MEDICAL SCHOOL EXAMINATION (1991)

Let l be the line of intersection of two planes:

$\Pi_1: ax + y + z = a$

$\Pi_2: x - ay + az = -1$

1) Find a direction vector of the line l.

2) The line l describes a surface as the real number a varies. Let (x,y) be the point of intersection of the surface and the plane $z = t$. Find an equation which gives the relation between x and y.

3) Find the volume bounded by the two planes, $x = 0$ and $x = 1$, and the surface obtained in (2) as t varies.

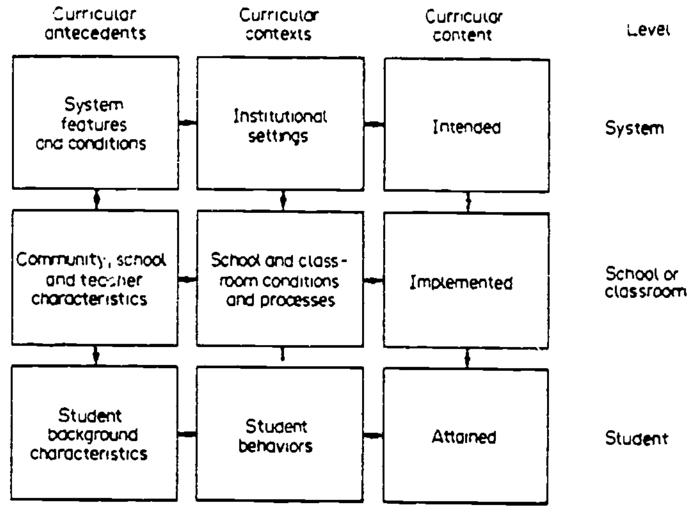

FIGURE 1. An Expanded Model for the Study.