"Becoming Successful Problem Solvers" is a 12-episode video series designed for use with middle school students. The materials are intended to create new assessment tools for teachers, to develop instructional devices that are useful in promoting problem solving in middle school classrooms, and to promote classroom discourse about problem solving and misconceptions about mathematics. From a pool of volunteer teachers in 3 counties, 28 pairs of fifth- and sixth-grade classrooms were chosen. In each pair, one classroom was randomly assigned to an experimental group, with the other a control. Sections of this paper discuss the importance of classroom discourse and the nature of the video series; describe one of the 12 episodes (on fitting as many 3" x 5" pictures as possible into a 14" x 21" frame); examine students' and teachers' behaviors while utilizing the series; and report results of an evaluation study of the series. The study compared the misconceptions, beliefs, and attitudes about problem solving of students from 14 classes that viewed 6 episodes of the videotape series with students from 14 classes that viewed 1 episode of the series. Results indicate that: (1) the videotape series offers teachers an easy-to-use method for promoting classroom discourse; (2) a connection was confirmed between problem-solving performance in mathematics and students' beliefs about problem solving, communication, reasoning, and connections; (3) use of the series materials has beneficial effects on those beliefs and students' problem-solving performance; and (4) there is a plausible causal link between changing students' beliefs and improving their problem-solving performance through discourse. Sample surveys and test questions are included. (MDH)
PROMOTING DISCOURSE IN MATHEMATICS CLASSROOMS
USING A NEW VIDEO SERIES FOR MIDDLE SCHOOLS

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This paper is part of a symposium entitled "The Potential of Video-Based Materials to Promote Classroom Discourse in Mathematics," presented at the annual meeting of the American Educational Research Association (AERA) in Atlanta, GA (session 54.02, April 16, 1993). Other participants at the symposium: Shalom M. Fisch, Children's Television Workshop, and Betsy McNeal, University of Pennsylvania.
PROMOTING DISCOURSE IN MATHEMATICS CLASSROOMS
USING A NEW VIDEO SERIES FOR MIDDLE SCHOOLS

Introduction

There are currently 12 episodes in the video series called *Becoming Successful Problem Solvers*, which is designed for use with middle school students. The video series, together with accompanying Resource Books for teachers, has been under development since October 1990, supported by grants from the U.S. Department of Education and the National Science Foundation.¹

There were several goals to be achieved in developing the *Becoming Successful Problem Solvers* materials. First, we intended to create new assessment tools for teachers, focusing especially on students' beliefs about and attitudes toward mathematics and problem solving. Second, we set out to develop instructional devices that would be useful in promoting problem solving in middle school classrooms. Third, we specifically wanted to promote classroom discourse about problem solving, with particular attention to students' misconceptions about both mathematics itself and the nature of mathematical problem solving. Lastly, to help reach these goals, and to help middle schools move ahead with current reform efforts, we have carefully tied all materials in the series to the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (NCTM, 1989).

Our approach explicitly links assessment and instruction and also incorporates current views of mathematical problem solving -- as delineated, for example, by Polya (1957) and Schoenfeld (1985). Notably, *Becoming Successful Problem Solvers* permits teachers to assess students' beliefs about problem solving, which are critical to the development of their capacity to solve mathematics problems (see, e.g., Schoenfeld, 1983).¹

¹ The project has been supported by ED Grant No. R215A00456 (the Secretary's Fund for Innovation in Education) and NSF Grant No. MDR-9054652 (Instructional Materials Development program).
The Importance of Classroom Discourse

Promoting classroom discourse was identified as a central goal for the series for several reasons. For one thing, many studies show that teacher talk dominates students' talk in all subjects. This may be especially true in mathematics classes. One study across subject disciplines found that mathematics was most heavily weighted toward a narrow spectrum of instructional techniques, especially teacher lecturing and seatwork (Stodolsky, 1988). The National Assessment of Educational Progress (NAEP) reached similar conclusions, and found innovative forms of instruction (including small group activities) "disappointingly rare" in mathematics (Dossey et al., 1988, p. 10). Therefore, we wanted teachers to have students explain their reasoning more often, and listen carefully to the students' responses.

Another reason for focusing on discourse is that we hypothesized that many students have serious misconceptions about mathematical problem solving that are not adequately addressed in most classrooms (such misconceptions as: mathematics is essentially identical to arithmetic computation; all problems have only one answer; "real problems" come from books and teachers). If students' thinking could be made more apparent to teachers through discourse, we believed, then teachers would be more likely to work to change those misconceptions. We also felt that some teachers' own thinking about mathematical problem solving would grow, if misconceptions were brought out into the open more clearly through discourse.

Fortunately, the NCTM's Professional Standards for Teaching Mathematics (NCTM, 1991) also emphasize the importance of classroom discourse. In that volume, standards 2, 3, and 4 for teachers are all concerned with mathematical discourse. For example, NCTM asserts that an important responsibility of mathematics teachers is to "orchestrate discourse" in the classroom, which means, among other things: posing stimulating questions, listening carefully to students' ideas, and asking students to clarify and justify their ideas orally and in writing. "Teachers should regularly follow students' statements with, 'Why?' or by asking them to explain," according to the Professional Standards, in order to focus attention on mathematical reasoning, to better understand and assess students' thinking, as well as to
prod them to extend their thinking. NCTM adopted standards promoting classroom discourse in part because "the kind of discourse [described in the Professional Standards] does not occur simultaneously in most classrooms" (NCTM, 1991, p. 35). Developing new tools for promoting discourse, such as Becoming Successful Problem Solvers, therefore seems a valuable and timely undertaking.

The Nature of the Series

Each 15-minute video episode shows a pair of carefully scripted child actors engaged in an effort to solve an interesting, real-life, nonroutine problem. Accompanying each episode are paper-and-pencil instruments that students complete while viewing that episode.

The episodes and the instruments focus attention on two dimensions of problem solving that are often ignored, namely, students' beliefs about problems and ways to solve them, and their attitudes toward mathematical problem solving. Most items on the instruments ask students to mark whether they agree or disagree with a statement (or to mark neutral if they cannot decide whether they agree or disagree). Exhibit 1 shows the format of some typical items.

Students' answers to the questions provide the basis for a classroom discussion. One form of discourse that is specifically promoted by the series is that teachers ask students to explain why they have marked particular items in the way they have. Different points of view are elicited in a typical classroom discussion and, as we shall show, oftentimes students' discourse illustrates important misconceptions about mathematics and problem solving. From the outset, we hypothesized that the process of talking about students' misconceptions would provide an important vehicle for changing them, and evidence that we have collected suggests this is the case.

The problems shown in each episode are left unsolved. After showing a tape, students work on an open-ended question based on the video. The question, which can be worked on individually or in small groups, typically asks students to explain their reasoning as well as to present solutions.
These open-ended questions provide another opportunity to promote discourse, including discussions among small groups of students, written answers to the questions (preferably in full sentences), and, possibly, presentations or explanations given by students to the rest of the class.

Our intent today is to illustrate several kinds of discourse promoted by use of the series, and to present some data from an evaluation of the first ten episodes. However, before we do so, it will be useful to describe a few additional features of the series.

Each episode incorporates content from at least two of the following strands (taken from the NCTM Standards): number, geometry, statistics, probability, and measurement. (In one episode, we have also included algebra.) The strand that NCTM calls "patterns and functions" is represented in every episode.

In addition, each episode focuses on students' beliefs about one aspect of problem solving (either (a) problem formulation, (b) problem treatment and heuristics, or (c) problem follow-up) and about one other key strand in the NCTM Standards (either (a) communication, (b) reasoning, or (c) connections). Also, each episode focuses on students' attitudes or dispositions relating to either (a) the usefulness and importance of mathematics, (b) its appropriateness for the student, or (c) his or her enjoyment of the subject. In all, then, each episode focuses on what we call three "key areas." (See Exhibit 2.)

The Club Photographs

One of the 12 episodes is called The Club Photographs. In this episode, Gloria is trying to fit as many 3" by 5" portrait photographs as possible into a 14" by 21" picture frame. She hasn't taken the pictures yet, but she demonstrates to her friend Edwin that she can hold the camera horizontally or vertically.

The mathematical content of The Club Photographs involves measurement and geometry -- two of the areas identified above. The items on the
instrument for students focus particularly on problem treatment and
heuristics (how should this problem be approached?), reasoning (what place
does reason have in solving the problem, as opposed to mechanical methods or
formulas?), and enjoyment (do students enjoy solving problems?). These are
the three "key areas" for this particular episode.

In part of the episode, the conversation between the characters goes as
follows:

Edwin: So you can make the pictures come out horizontally or
vertically.
Gloria: Or I could do some of each.
Edwin: Well, it seems to me that you’ve gotta decide if it’s an area
problem, or a perimeter problem.
Gloria: Whaddya’ mean?
Edwin: Well, if it’s an area problem, you multiply. And if it’s a
perimeter problem, you add.
Gloria: [Puzzled]. You lost me.
Edwin: If it’s a perimeter problem, you’d do 21 plus 14, that’s 35.
And then another 21 plus 14. That’s another 35. So your
total is 35 plus 35 or 70.
Gloria: You’re not saying that I could fit 70 pictures into this
frame, are you?
Edwin: No, all I’m saying is, that if it’s a perimeter problem, the
answer is 70 inches.
Gloria: But what does that have to do with my problem?
Edwin: Well, look Gloria, for every math problem there’s one method
you’re supposed to use. You tell me what kind of math problem
it is, and I’ll tell you the method for doing it.
Gloria: Are you sure? I was just sort of thinking about the problem
and trying things out.
Edwin: But what’s your method?

Discourse About Items on the Instrument

Several of the most interesting items on The Club Photographs
instrument for students focus on reasoning and problem treatment and
heuristics, and build upon this dialog. (See Exhibit 3.) In particular, Edwin's statement that,

"For every math problem there's one method you're supposed to use. You tell me what kind of math problem it is, and I'll tell you the method for doing it."

seems a kind of caricature of rigid thinking, and provides a basis for exploring one important type of misconception. Edwin imagines that the world of mathematics problems is neatly divided into "types," such as "area problems," "perimeter problems," and so on.

We have encountered groups of teachers who simply could not believe that their students would agree with such a statement, but our data show that large numbers of them did. In fact, fully forty percent of nearly 500 fifth-and sixth-grade students who used the episode agreed with this statement! This may indicate that the way we teach mathematics in school too often moves along from one "method" to another, clustering together lots of problems of a given type, instead of providing students real opportunities to think about the approach needed. Whatever the reason for the large number of "agree" responses to this question, it provides a surprising example of a serious misconception about problem solving.

In this episode, Gloria's more flexible thinking and her willingness to experiment are what create progress toward solving the problem. Thus, the episode also provides an excellent opportunity for class discussions in that Gloria's approach can be explicitly contrasted with Edwin's.

The fact that the students are discoursing in an organized way about their beliefs and attitudes toward mathematics, in the context of solving nonroutine problems, is -- from all we know about middle school classrooms -- extremely unusual. Teachers in the trials certainly expressed this opinion. The series is based on the assumption that listening to and reflecting on other points of view is a key element for students. The intent is to slowly lead those with misconceptions, or negative attitudes toward mathematics, to change their beliefs and attitudes -- recognizing that such change seldom happens because of a single experience.
It is usually not difficult to find students willing to comment on why they have marked items in a particular way. The fact that many different students generally volunteer to talk, including ones not usually identified by the teacher as good problem solvers, also seems important.

**Discourse About the Open-Ended Questions**

Following a discussion of some or all of the items on the instruments (the Resource Book suggests limiting discussion to a subset of all the items) students work on the open-ended question for that episode. In *The Club Photographs* this involves students in the classroom deciding for themselves the maximum number of photos that will fit. Gloria and Edwin find arrangements for 16, then 17, and finally 18 pictures. Will 19 fit? How about 20?

The kind of discourse that is promoted by the open-ended questions is somewhat different from the discourse about the items on the instrument. It often involves more student-student interaction, for one thing, since the teacher usually circulates among several groups of students. Also, it provides greater room for creativity and real problem solving in a less structured format than discussion of the items. This can provide emotional rewards to students: one excerpt of students working on the open-ended question for this episode shows genuine excitement and satisfaction as the group discovers a way to fit 19 photos in the frame.

**The Importance of Discourse**

We believe that both types of discourse (about the items and about the open-ended question) are useful to students. It is true that the discourse about the items usually is of a limited form, typically involving one student after another explaining to the group or to the teacher why he or she answered in a particular way. However, the intentional and repeated focus on beliefs and attitudes, including misconceptions that are often hidden from or unknown to teachers, more than compensates for the limited form of the discourse, we believe.

We observe one teacher who virtually monopolized discussion of the
instrument, explaining to students one after another what "the right answer" should be to every item! Fortunately, this was an extremely rare event in the many observations we made. The Resource Book now warns against this approach, and encourages teachers to listen to students and provide "right answers" sparingly. However, there is no magic formula that we know of to guarantee all teachers will be good leaders of discourse. Indeed, we devised this approach in part because so few teachers apparently even try to promote discourse about beliefs and attitudes.

Comments from many teachers also suggest that the kind of discussion the series promotes is unusual and welcome. The great majority of teachers in one extended study valued the videotapes as a stimulus for orchestrating student discourse about mathematical problem solving. That such discussions were not always familiar to teachers beforehand is illustrated by the following comments:

"I’ve really enjoyed and realized the value of questioning and thoroughly discussing problem solving; before I emphasized the charting of information and writing an equation."

"Attitude is essential. The series attacks ways children perceive problem solving, which changes a core of attitudes, making them better. I always approached problem solving with strategies only."

"The discussions that the tapes produced were of such quality. The students reflected and then communicated their views on particular areas while others were led to see the 'light' from their peers’ perceptions. Students learned to be more accepting of one another and appreciate other viewpoints."

**Evaluation Studies**

Two carefully designed studies of the first 10 episodes in the series (and the accompanying print materials) were conducted during the first six months of 1992, involving 74 classrooms in a total of six school districts. One study tested whether the series could be effective as a large-scale assessment device to distinguish between curricula. The second study examined its effectiveness as a tool for teaching problem solving and for informal classroom-based assessment. Several aspects of the second study are discussed below.
Study Design

Subjects. From a pool of volunteer teachers in three counties, 28 pairs of fifth- and sixth-grade classrooms were chosen, matched on school SES and teachers' estimates of their classes' overall mathematical abilities. In each pair, one classroom was randomly assigned to an experimental group, with the other a control, subject to the constraint that if a teacher taught more than one class that was involved in the study then all of that teacher's classrooms were assigned to the same condition. Thus no teacher was involved with both an experimental and a control class. (Most of them, in fact, taught only one class.)

Treatment. The experimental classes saw a total of 6 episodes (of the 10 then available) of Becoming Successful Problem Solvers, and engaged in the classroom activities (the "Agree-Neutral-Disagree" instruments, the post-viewing discussion, and the open-ended question) that the Resource Book calls for. This treatment took place over a three-month period in the spring of 1992. The episodes were to be used in a fixed order, in a design that assured that each episode was used in either 1/2 or 3/4 of the experimental classrooms and that the first and last episodes used by each classroom was one of four episodes. The assignment of episodes to teachers also took into account balances of mathematical content and of the "key areas" of each episode (e.g., problem formulation, reasoning, appropriateness -- as described earlier).

The control classes saw just one episode of Becoming Successful Problem Solvers, which in each case was the same episode that the matched classroom viewed as its sixth (and final) episode. These students completed the "Agree-Neutral-Disagree" instruments and the open-ended question for that episode, but the teachers were told not to engage in any discussion of the tape or the questions.

Instruments. All participating teachers answered questionnaires regarding their professional backgrounds and teaching practices, and they provided estimates of the mathematical abilities of individual students in their classrooms. After each episode the experimental teachers filled out a
short questionnaire about their reactions and their students' reactions to the
episode -- the appropriateness of the difficulty level, the students' apparent interest in the tape, and so forth. At the end of the study, the experimental teachers completed another questionnaire, which asked (in part) for their opinions on various of aspects of the tapes as a whole -- their effectiveness as instructional materials, ease of use, perceived effect on the teacher's views of problem solving, and so on.

Near the end of the study, all children completed a standardized vocabulary test (one of two halves of the CTBS vocabulary subtest, level 16). Each student also completed an independent mathematics test consisting of two non-routine mathematics problems (Appendix 1).

Scoring of the A-N-D items. Recall from the earlier description of the Becoming Successful Problem Solvers materials that each episode concentrates on three "key areas": one related to problem solving; one related to communication, reasoning, or connections; and one concerned with dispositions (enjoyment, perception of mathematics as useful or important, or perception of mathematics as an appropriate activity for the student to engage in); see Exhibit 2. Each "Agree-Neutral-Disagree" instrument contains 12 statements, 4 for each of the 3 key areas. Assigning a score of +1 to the choice (agree or disagree) that the "mature" problem solver would make, and -1 to its opposite, one creates a score, ranging from +4 to -4, for each student's responses, for each key area. (In some cases a choice of "Neutral" is defensible as a mature response, and appropriate adjustments were made in those instances.)

Scoring of the independent mathematics test. The two items of the independent mathematics test (Appendix 1) were scored from 6 (highest) to 1 according to a detailed scoring rubric. Two raters were involved; interrater agreement was high.

Training. The experimental teachers attended a workshop on the use of the materials. The workshop was given three times, once for each of the school districts participating in the study, and lasted approximately 2.5 hours. A similar, somewhat shorter, workshop was provided for the control
group teachers, but only after all the teacher and student instruments had been completed.

All experimental teachers were given a copy of the version of the Teacher's Resource Book that was current at the time. This included, for each episode, a verbatim copy of the script with sidebar notes on any character's lines that are significant in any way; detailed notes on the A-N-D statements, providing some guidance for discussion, and explaining the rationale behind the "mature" response(s) for each one; information on the open-ended question; and, in some cases, suggestions for extended follow-up problems that could be used at some other time following the class session in which the taped episode was used.

Once the classroom part of the study began, however, there was no additional interaction between the teachers and experimenters, aside from our keeping track of their progress as needed. The experimenters did not visit the classrooms, and videotapes and instruments were sent by mail (in both directions).

Non-Experimental Findings

Students' Misconceptions. As noted earlier, we hypothesized that the paper-and-pencil instruments, used in conjunction with the videos, would be able to detect a number of misconceptions about the nature of mathematics and problem solving. This hypothesis was confirmed in various ways.

One confirmation came through the development process, which involved repeated trials of the episodes in classrooms (even before they were videotaped). Some of this work was done with very small audiences, with follow-up interviews to confirm that in fact the children understood the A-N-D statements and to check what they really believed to be true. We found, for example, that Edwin's rigid approach appeals to many students, while Gloria's flexible thinking can be disconcerting to some students. In small groups, some students had this to say about Gloria:

"There always has to be a method and Gloria didn't have a method."
"She’s not good at math, but she’s trying to be."
"Trying things out isn’t really math."
"Gloria seems confused."

A further confirmation of the fact that children hold misconceptions was described earlier, based on Edwin’s statement that, "For every math problem, there’s one method you’re supposed to use." Other misconceptions held by large numbers of fifth- and sixth-grade students were also identified in this way (and several are described below).

We noted above that it seems significant that many teachers are surprised by such findings. There is some evidence that mathematics supervisors are more accurate in their estimates of students’ responses.

Experimental Findings

Matching of groups. Overall, the matching process yielded two groups that were well matched, although there were a couple of individual pairs of classes that turned out to be markedly different in general ability (as measured by the vocabulary test).

Effects of viewing six episodes. At the end of the study, students in the experimental group had significantly fewer misconceptions about problem solving (formulation, treatment and heuristics, and follow-up) than the comparison classes (p < .05), as shown by their higher Area 1 scores. (See Exhibit xx.) An example of the differences in beliefs between control and experimental students is that 59% of the control students (as opposed to only 46% of the experimental students) agreed with the statement "If you do the calculations right when you do a problem, the answer will be right," in a context where calculation had nothing to do with the problem at hand. Additionally, the experimental classes had more positive beliefs (p < .01) about communication, reasoning and connections (Area 2). (The differences between the two groups was about 0.2 standard deviations for Areas 1 and 2.)

-- Insert Exhibit 3 about here --
There were no significant differences in Area 3 scores (dispositions) between the two groups.

Not only were the experimental students' beliefs more positive overall than the control students' beliefs, the differences were greater for lower-ability students, minorities, and girls -- groups that are often a special target for improved instruction in mathematics.

Misconceptions and problem-solving performance. Data from the evaluation indicate that students who perform better as problem solvers are less likely to hold misconceptions about problem solving and mathematics. In many cases there is a plausible linkage between belief and performance. If, for example, one really believes that "You should always know what kind of problem you have before you start to work on it" (and 34% of the students responding to that item agreed with the statement), then one is unlikely to tackle an unfamiliar, nonroutine problem with much tenacity or enthusiasm. In summary, we believe that changing students' beliefs about mathematics and mathematical problem solving is one way to enhance their capacity to solve problems.

Students in the experimental classrooms, which had used six episodes of the series, performed better on the independent problem solving test than students in the comparison classrooms (F = 8.37, p < .05), with a difference of about one-fifth of a standard deviation. (This finding is based on analyses made at the student level. Data at the classroom level also favor the experimental classrooms, but the difference is not statistically significant.)

Taken together, these findings demonstrate that using only six episodes of Becoming Successful Problem Solvers can make modest positive changes in students' beliefs about mathematical problem solving, and in their performance on a mathematics problem solving test. We expect that using more episodes, over a longer period of time, would lead to even stronger results.

Teachers' opinions about the series. Teachers in the experimental classrooms expressed a high degree of enthusiasm about the series. All 20
experimental teachers reported that the materials were easy to use. Based on 174 classroom uses, teachers reported 78% of the time that the difficulty level was about right (neither too easy nor too hard). They also reported that students were "interested" or "very interested" in 90% of the sessions. More than 90% of the teachers agreed that the series "helped many students to see how mathematics is used in the real world," more than 80% felt that use of the materials was valuable for their students, and 70% agreed that using the series helped them, as teachers, to "understand how students really think about problem solving in mathematics." These findings suggest that the materials are likely to "travel" well, and may be disseminated widely.

**Limitations**

Several limitations of the study should be pointed out. First, no attempt was made to measure the teachers' behaviors during the course of the study, apart from being sure that they used the right episode at the right time. Hence we are not sure to what extent, if any, the teachers incorporated the underlying messages of *Becoming Successful Problem Solvers* into their teaching. For example, one would hope that, because they used the series, teachers more often discussed with students multiple approaches to problems, or presented problems that have multiple solutions, to discourage some of the common misconceptions about problem solving. Also, teachers might begin to focus on students' beliefs and attitudes regarding mathematical problem solving in other contexts, besides when they use these episodes. However, we have no way to know if these sorts of changes took place or not.

Second, the experimental group used only 6 episodes, over a fairly short period of time (3 months). We were pleased to find significant effects with only 6 episodes, but the effect sizes are modest. We would expect stronger findings with more intensive use of the series (more episodes, used over a longer period of time); however, the design did not permit us to test variations in intensity (aside from the comparison classes that used only a single episode).
Summary

Evidence from a great many earlier studies documents the need to promote more discourse in mathematics classrooms at all levels, including the middle grades. The NCTM Professional Standards endorses such changes in current practice, and provides a detailed rationale for them. New tools that encourage teachers to stimulate discourse are therefore timely and appropriate, as are alternative classroom-level student assessments, and new approaches focusing attention on students' beliefs about and dispositions toward mathematical problem solving. Becoming Successful Problem Solvers was created to help teachers change their practice in these ways.

A carefully designed study of classroom use of the first 10 episodes in the series suggests the following conclusions:

(1) *Becoming Successful Problem Solvers* offers teachers an easy-to-use method for promoting classroom discourse, focusing especially on students' beliefs about and attitudes toward mathematical problem solving;

(2) A connection was confirmed between problem-solving performance in mathematics and students' beliefs about various aspects of problem solving identified as important by the NCTM (notably beliefs about problem solving, communication, reasoning, and connections);

(3) Use of the *Becoming Successful Problem Solvers* materials has beneficial effects on those beliefs, and also on students' problem-solving performance; and

(4) There is a plausible causal link between changing students' beliefs and improving their problem-solving performance, and student discourse is the most likely means by which use of the series brings students' beliefs to light in the classroom and influences them to change.
Directions: For each statement, check whether you AGREE, are NEUTRAL or DISAGREE.

1. Gloria's idea of using the cards to see how the pictures would fit was a good one.

   AGREE | NEUTRAL | DISAGREE

2. When Edwin said, "Why multiply? That's just the way you do it!" it shows he's probably pretty good at math.

   AGREE | NEUTRAL | DISAGREE

3. I feel discouraged when I can't get an answer to a math problem right away.

   AGREE | NEUTRAL | DISAGREE

4. Edwin was basically right when he said, "For every math problem, there's one method that you're supposed to use."

   AGREE | NEUTRAL | DISAGREE

5. I liked the surprise when Gloria showed how to fit 17 photos in the frame.

   AGREE | NEUTRAL | DISAGREE

6. When Gloria said, "I was just sort of thinking about the problem and trying things out," it shows she's probably pretty good at math.

   AGREE | NEUTRAL | DISAGREE

Directions: Circle the best answer to the question.

7. At the end of Part 1, Gloria and Edwin had the frame partly filled in, like this:
   If they leave those photos as they are, how many more can they fit inside the frame?

   a. 0   b. 1   c. 2   d. 3   e. 4   f. 5

   STOP
THE CLUB PHOTOS, Part 2

NAME ___________________________ Date ___________________________

Directions: For each statement, check whether you AGREE, are NEUTRAL or DISAGREE.

<table>
<thead>
<tr>
<th>Statement</th>
<th>AGREE</th>
<th>NEUTRAL</th>
<th>DISAGREE</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. A math problem shouldn't take as much time as Gloria and Edwin are taking</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Sometimes, when I do a problem, I feel the same way Gloria did when she said, &quot;Yeah, that is neat!&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Edwin was right when he said, &quot;So it's better to have more of the pictures horizontal.&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. I'd like to find out if there really is a way to fit 19 photos in the frame.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>12. Overall, Edwin's way of figuring out how many photos will fit was good.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Overall, Gloria's way of figuring out how many photos will fit was good.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Directions: Circle the best answer to the question.

14. Each picture is 3" by 5". How much area, in square inches, would 20 pictures cover (if they don't overlap)?

   a. 15   b. 17   c. 28   d. 150   e. 300
Exhibit 2
THE "KEY AREAS" THAT ARE ASSESSED

Beliefs about problem solving (Area 1)
1. PROBLEM FORMULATION
2. PROBLEM TREATMENT / HEURISTICS
3. PROBLEM FOLLOW-UP

Other beliefs (Area 2)
4. COMMUNICATION
5. REASONING
6. CONNECTIONS

Dispositions
7. USEFULNESS & IMPORTANCE
8. APPROPRIATENESS
9. ENJOYMENT
### Exhibit 3

**ADJUSTED MEAN SCORES FOR THE THREE KEY AREAS, BY TREATMENT**

**A-N-D Items, Adjusted Mean Scores**

<table>
<thead>
<tr>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>+.18</td>
<td>+.17</td>
</tr>
<tr>
<td>Control</td>
<td>-.04</td>
<td>-.05</td>
</tr>
<tr>
<td>F-stat</td>
<td>4.75*</td>
<td>8.79**</td>
</tr>
</tbody>
</table>

Note: All scores were transformed into z-scores.

Note: These figures are for 1,53 df, with vocabulary as a covariate. The results are virtually the same whether or not the vocabulary and pairing are taken into account.

* p < .05  
** p < .01
REFERENCES


Becoming Successful Problem Solvers is a project of SRI International (1611 North Kent Street, Arlington VA 22209). Principal investigators are Andrew A. Zucker and Edward T. Esty. Support has been provided by the U.S. Department of Education (ED Grant No. R215A00456) and the National Science Foundation (Grant No. MDR-9054652). The series is distributed by HRM Video, 175 Tompkins Avenue, Pleasantville NY 10570. Information about obtaining the series is available by calling HRM at 1-800-431-2050.
Maria and her friend made up a game. They want it to be a fair game. They put four cards labeled

3, 5, 7, and 8

in a bag, like this.

They take turns. Without looking, a player takes two cards out of the bag. Then she adds the two numbers.

Maria gets a point if the sum (the answer) is more than 12. Otherwise, her friend gets a point. Then they put both cards back in the bag and draw again.

Whoever gets 10 points first WINS the game.

1. Suppose a player happens to pick 5 and 8. Does Maria win the point? ______ Why or why not?

2. Is Maria’s game fair? ______ Please explain why it is or isn’t.
Here's a new problem:

Pat wants to make a garden in the shape of a rectangle. She wants it to have at least 200 square feet.

1. She thought of making the garden 15 feet long and 11 feet wide. Why wouldn't that work for her?

2. Describe a garden that would work for her.

3. Pat wants to put a fence around her garden. Fence costs 50 cents a foot. How much would it cost to put fence all around the garden that you described in question 2?