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PLANNED VERSUS UNPLANNED CONTRASTS: EXACTLY WHY PLANNED CONTRASTS TEND TO HAVE MORE POWER AGAINST TYPE II ERROR

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ABSTRACT

The literature is reviewed regarding the difference between planned contrasts, OVA and unplanned contrasts. The relationship between statistical power of a test method and Type I, Type II error rates is first explored to provide a framework for the discussion. The concepts and formulation of contrast, orthogonal and nonorthogonal contrasts are introduced. It is argued that planned contrasts are focused on thoughtful research questions of interest and reflect researchers' rational anticipation. An OVA test or unplanned contrasts, on the other hand, do not provide desired information in many situations. It is also explained that, to control for the possible inflated error rate for unplanned contrasts which usually test a large number of hypotheses, wanted or unwanted, some Bonferroni type of corrections are invoked. It is these corrections, usually built into statistical tables, that reduce the power of unplanned contrasts. This is demonstrated through a comparison of the critical values for planned contrasts and for some popular unplanned contrasts.
The classical analysis of variance (ANOVA) method developed by Fisher used to be a predominant analytical method favored by educational researchers (Willson, 1980). Historically, before the development of more powerful analytical methods like regression, general linear model or canonical analysis and the birth of modern high-speed computers, ANOVA was perhaps the only method that could be conveniently and effectively used to compare more than two (treatment or group) means. Somehow this method has become a sort of tradition and has ever since remained a popular analytical method in educational research (Daniel, 1989; Elmore & Woelhke, 1988; Goodwin & Goodwin, 1985a, 1985b). A related category of methods is generally known as multiple comparisons, although a variety of labels are readily available such as unplanned contrasts (used in this paper hereafter), a posteriori (or post hoc) comparisons, post-anova tests. This category includes methods such as LSD, Bonferroni, Tukey, SNK, Duncan, Scheffee, etc. Once the omnibus ANOVA F test, or OVA test (Thompson, 1985) detects some statistically significant difference, involving problems with more than two groups, at least one of those unplanned contrasts is needed, as is suggested in many statistics textbooks (Keppel, 1982; Kirk, 1968; Ott, 1989, to name a few), if researchers wish to find out which pair of means are different. These unplanned contrasts also seem to have been popular because they are known to have protection against Type I error and are easy to perform, especially with a computer package.

Despite the popularity of OVA and unplanned contrasts, many researchers have expressed their concerns with the technical problems and inappropriate applications of OVA and unplanned contrasts (Games, 1971; Hale, 1977; Jones, 1984; Rosnow & Rosenthal, 1989 ). One important issue raised involves the redundancy and irrelevance of OVA and unplanned contrasts in hypothesis testing in many situations: these methods test all possible hypotheses that are embedded in the combinations of mean comparisons, while researchers may be only interested in testing a few specific well defined research
hypotheses. An alternative method, planned contrasts, is then highly recommended in this situation (Hale, 1977; Keppel, 1982; Thompson, 1990).

Planned contrasts are analyses that are planned before the experiment even starts and are constructed from research hypotheses based on theory and the goal of the study (Keppel, 1982). Many researchers have argued in favor of planned contrasts (Hale, 1977; Rosnow & Rosenthal, 1989; Thompson, 1990; Tucker, 1991). One of the key arguments for this preference is that planned contrasts tend to have greater statistical power (power, for short hereafter) than OVA and unplanned contrasts (Hale, 1977; Hays, 1963; Keppel, 1982; Thompson, 1990). The treatment of this power issue, however, is anything but sufficient or informative, and usually is mentioned only with a passing comment in a chapter on planned contrasts in statistics textbooks. Thompson (1990) and Tucker (1991) render similar concrete discussions with small data sets to show that, for a given set of data, one of a set of planned contrasts can detect a significant difference between a pair of (complex) means while an OVA test fails to find anything statistically significant. Hale (1971) presents another case where planned contrasts are used for trend analysis. The planned tests give significant findings but the OVA test doesn’t. Little explanation is however available about why this is so. Few educational researchers understand why planned contrasts have more power than OVA and unplanned contrasts, even though a good understanding of the advantage of using planned contrasts can be rewarding in many research situations. This may account for the fact that planned contrasts have not been frequently used in educational research.

It is the aim of this paper to present an in-depth explanation to non-statistician educational researchers about why planned contrasts can have more power than do OVA and unplanned contrasts. The power issue in analysis is always related to the issue of significance testing of statistical hypothesis. While it is important to realize that significance testing is influenced by several factors, particularly sample size, and does not evaluate practical significance (Carver, 1978; Cohen, 1988; Rosnow & Resenthal, 1989;
Thompson, 1988), it remains important to see that lack of power due to inappropriate analytic methods causes even worse problems than only failing to get statistically significant results. With a significant finding, being it an artifact of sample size or something else, a report can get published. Lack of power, however, makes a test fail to detect a real difference in the data and consequently makes the researcher suffer the possible loss of many wonderful things (job, tenure and fame) that a significant finding at .05 level may offer (Rosnow & Rosenthal, 1989). In fact, in many educational research situations, such as research in special education, educational counseling or educational psychology, in innovative curriculum or instruction methods, factors like small and/or unequal sample sizes, and small effect sizes in the population are likely to reduce the power of statistical tests. It therefore becomes especially important for educational researchers to know how to select appropriate powerful analytical methods or tests. Experimental design plays a critical role, but this is not the issue in this paper.

This paper is intended to demonstrate that, where appropriate, use of planned contrasts can detect significant differences among means that OVA or unplanned contrasts can't. The relationship between power and two types of error is first examined and explained. This leads to the elaboration on the nature of contrasts, planned and unplanned contrasts, with regard to such important aspects as the rationale from using planned contrasts, the problem of error rate inflation in unplanned contrasts, and the required Bonferroni type correction. Two sets hypothetical data with one-way design will be employed to demonstrate that planned contrasts tend to have more power than OVA or unplanned contrasts and make the discussion concrete. This can be extended for more complicated designs and analyses (see Thompson, 1990; Hinkle et al, 1988). A relatively generalizable account is also presented to point out that error rate correction reduces the power of unplanned contrasts. The controversial issue of whether, and how, error rate correction should be applied to planned contrasts is introduced and discussed in the conclusion of the main body of this paper.
Type I error, Type II error and statistical power

There is an intricate relationship among Type I error, Type II error and statistical power of a test in hypothesis testing. A clear understanding of these concepts and their mutual influence helps an educational researcher in planning for a good research with an adequate design and analysis that promises maximum statistical power.

Type I error is defined as the error committed by falsely rejecting a true null hypothesis like $H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$. This means that the test finds a statistically significant difference between at least one pair of means of all the $k$ means while, in fact, there is none. Type II error is just to the opposite in that this error is committed when a null hypothesis is falsely retained. In other words, the test fails to detect a statistically significant difference among the $k$ means when at least one pair of means in the population are really different. The probability of committing an error is called an error rate and implies the amount of risk a researcher is willing to take. Since statistics is about probability and approximation, errors are unavoidable. The only thing researchers can do is to hope that this probability, i.e., error rate, does not get out of hand to become intolerably big. For some reasons, Type I error seems to be more of a concern to educational researchers and most other behavioral science researchers, and a .05 or 0.01 Type I error rate (denoted by $\alpha$) is conventionally regarded as an acceptable risk.

Type II error rate (denoted by $\beta$) is seldom explicitly expressed by researchers and has not been given due attention. Type II error rate is quantified as the complement to power: $\beta = 1 - \text{Power}$, where power is the probability of correctly rejecting a false null hypothesis, i.e., the probability of finding a significant difference when there is one. This indicates that power is a measure against Type II error and that only with sufficient power will a test be more likely to reject a false null hypothesis. Hence, the greater power, the lower is Type II error risk. However, since Type II error rate is inversely related to Type I error rate, smaller Type II error rate means higher Type I error risk. And higher power also implies a higher Type I error rate. It is therefore a challenge to the researcher to
strike a balance among these three factors when they select analytic methods. For the
discussion in this paper, it is enough to remember that Type I error rate can determine
both the Type II error rate and the power of a test. Readers interested in the power issue
are referred to the handbook on power by Cohen (1988) and to the article of McNamara
(1991) on the importance of power in educational research.

**Contrasts, planned contrasts versus unplanned contrasts**

"A contrast between two means is the difference between the means, disregarding
the algebraic sign," as (Kirk, 1968) explains. In this sense, all comparisons between
means are contrasts. A contrast is also understood as the linear function of the sum of all
weighted means such that the weights may sum to zero. The weights here are called
contrast coefficients and denoted as \( c_i \). Therefore a contrast can be expressed by the
formula: 
\[
C = \sum c_i X_i,
\]
where \( c_i \) is the assigned coefficient or weight for a mean \( X_i \) and
\[
\sum c_i = 0.
\]
A mean can be a simple mean like that of each group or a treatment, or can be a
complex mean which is the average of several group or treatment means, for example,
\[
\overline{X}_{123} = \frac{X_1 + X_2 + X_3}{3}.
\]
With three means \( X_1, X_2, X_3 \), a comparison between simple
means \( X_1 \) and \( X_2 \) is the contrast: 
\[
C = (+1)(X_1) + (-1)(X_2) + (0)(X_3),
\]
where the three coefficients add up to zero: \((+1) + (-1) + (0) = 0\). A comparison between \( X_1 \) and the
complex mean for \( X_2 \) and \( X_3 \) is the contrast: 
\[
C = (+2)(X_1) + (-1)(X_2) + (-1)(X_3),
\]
the three coefficients sum to zero.

In a set of contrasts where not all contrast coefficients are zero, if the cross
products of the contrast coefficients in any pair sum to zero, i.e., \( \sum c_i c_j = 0 \), where \( i \) and \( j \)
denote different contrasts within the set, this set of contrasts are called mutually
orthogonal contrasts. For \( k > 2 \) means, there can be several sets of orthogonal contrasts,
but within each set, there can be only \( (k - 1) \) mutually orthogonal (i.e., uncorrelated)
contrasts. Mutually orthogonal contrasts are equivalent to independent tests with each
contrast contributing a piece of non-overlapping information about the whole set of tests.
The sum of squares of individual contrasts add up to the total sum of squares for the contrast set. This total sum of squares is of the same value as the sum of squares of treatment in the corresponding OVA test. Some researchers have disagreed on whether to always use orthogonal or nonorthogonal contrasts (Huberty & Morris, 1988; Keppel, 1982; Lentner & Bishop, 1986; Thompson, 1990). The debate over this is beyond this paper. Both orthogonal and nonorthogonal contrasts will be used in this paper.

Planned contrasts, as was defined earlier, refer to comparisons of means (simple or complex) that are of the only interest to researchers and the researchers anticipate that these means might be different. This is usually the case in educational research because most studies are of theory-confirmatory in nature. Researchers usually derive research hypotheses from theories in the field, from their own work in the past and from the problems to be solved at hand. In Keppel's term (1982), planned contrasts are "the motivating force behind an experiment". Researchers know what they are looking for and they translate their research hypotheses into statistical hypotheses for testing. Huberty and Morris (1988) state that there are very few research situations where researchers are unable to specify all contrasts of interest before examining any outcome measures. They in fact even refute the effort to distinguish planned and unplanned contrasts and advocate that a single contrast test suffices in most contrast situations. The number of planned contrasts is usually small because experiments tend to be focused.

Although the term "unplanned" is said to sound "pejorative" (Thompson, 1990), "unplanned contrasts" is used in this paper merely to reflect the point that researchers don't need to formulate these comparisons before the experiment starts. In most statistics textbooks, it is said that when an OVA test is significant, that means something is going on or happening in the data, and further analyses are desired to find out what is going on. Hence one may use unplanned contrasts to comb through the data searching for significant differences. This is not to say that combing through data is a bad practice; in certain situations where researchers don't have much clue as to what is there in the data, this
might be the only sensible way to go. One serious concern with unplanned contrasts is the inflated Type I error rate and how to control this error rate. In almost all statistics textbooks and articles on unplanned contrasts, a discussion of this topic is inevitable. It is well known that, if the Type I error rate for one contrast is fixed at \( \alpha \) level, the total error rate for \( m \) has an upper bound of \( [1 - (1 - \alpha)^m] \). If the \( m \) contrasts are mutually orthogonal, i.e., independent, the total error rate reaches the upper bound or the maximum error rate. This total error rate is generally called experimentwise error rate and the error rate for each contrast is the comparisonwise or testwise error rate.

Unplanned contrasts make virtually all possible pairwise comparisons among means one way or another. For \( k \) simple means, there are \( \frac{k(k - 1)}{2} \) possible pairwise contrasts; there are also contrasts of complex means. For example, if the means for three groups are A, B and C, there are three contrasts of simple means: A vs B, A vs C, B vs C; there are also three contrasts of complex means: A vs (BC), B vs (AC), C vs (AB). Permutation and combination laws say that the number of contrasts grows quickly with every one more group mean added to the set. As a result, the experimentwise error rate can be extremely high. If \( \alpha = .05 \) for one test, the error rate for 5 independent tests is .23, and .40 for 10 tests! Various methods have been developed to exercise control over the inflation of error rate in unplanned comparisons and all the methods incorporate a Bonferroni type correction (Games, 1971; Thompson, 1990). These corrections are built into various tables available in statistics books and are also taken care of in computer packages like SAS for statistical analysis.

**Planned contrasts have more power than OVA tests and unplanned contrasts**

In planned, unplanned contrasts and OVA procedure, an \( F \) test is used. The calculated \( F \) statistic has to exceed a critical value that is determined by the specified \( \alpha \), and the degrees of freedom for both numerator and denominator. Given the same data set and the same \( \alpha \) level, but different test methods, logically, a method that yields a statistical significance is more powerful than a method that doesn't. In computer output,
the calculated $P$ value is another indicator. A small $P$ value can be taken as evidence that the null hypothesis can be rejected at a very low $\alpha$ level if this $\alpha$ level is chosen. The $P$ value, therefore, also suggests how powerful a test is.

The hypothetical data set in Table I contains scores for four treatment groups A, B, C and D) with four subjects each. The treatments are not structured and trend analysis is not considered here. Suppose the researchers want to investigate two research questions:

1) Is the effect of treatment A different from the effects of treatments B, C and D?

2) Does treatment B have a different effect than treatments C and D?

The researchers can use either planned contrasts or OVA and unplanned contrasts for analysis.

The OVA procedure tests the statistical hypothesis $H_0: \mu_A = \mu_B = \mu_C = \mu_D$. This would be answering the question, "Is there any different treatment effect between any pair of treatment groups?" This is, however, not what the researchers want to do because the test is not going to give any concrete information except that something happens or nothing happens in the data. The results from OVA for this hypothetical data are given in Table 2.

The test fails to reject the null hypothesis and one may think that the data wouldn't warrant the conclusion that there is any statistically significant difference between any treatment group effects. This is, however, somewhat counter-intuitive, for the gap between some group (A and D) means seems rather big (8 vs 4). The eta square is .37,
and this suggests a moderate effect size. The explanation is that an OVA in effect tests the average difference of all possible comparisons, and in so doing, the degrees of freedom for the numerator (treatment) is the number of treatment (k) minus 1, df = k - 1. Given a fixed effect size in the data, the mean square of treatment decreases as more treatments used in the OVA. On the other hand, the degrees of freedom for residuals, or error, also decreases and this leads to the inflation of the mean square residual. The F test statistic is then reduced, and so is the power of the test. This is exactly what Rosnow and Rosenthal (1989) has described.

All the while that a particular predicted pattern among the means is evident to the naked eye, the standard F-test is often insufficiently illuminating to reject the null hypothesis that several means are statistically identical. (p.

With planned contrasts, one complete set of three mutually orthogonal contrast is:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>3</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>C2</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>C3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

where the numbers are contrast coefficients. The statistical hypotheses tested by these three contrasts are:

- C1: $H_0(1): \mu_A - (\mu_B + \mu_C + \mu_D)/3 = 0$
- C2: $H_0(2): \mu_B - (\mu_C + \mu_D)/2 = 0$
- C3: $H_0(3): \mu_C - \mu_D = 0$

Contrasts C1 and C2 address the very questions the researchers are interested in. C3 is included a complete set of mutually orthogonal contrasts although this contrast is not of interest. And this is where use of orthogonal contrasts only is criticized. The argument is that contrasts should ask interesting research questions and not be dictated by orthogonality (Huberty & Morris, 1988). In practice, C3 can be dropped. Using the data
in Table 1, two sets of orthogonal contrasts are made with \( k - 1 \), or \( 4 - 1 = 3 \) contrasts in each set. Table 3 shows that, in either Set 1 or Set 2, each contrast has one degree of freedom, the \( F \) statistic for each contrast is the mean square of contrast divided by the mean square of the pooled variance which is the value of the mean square of residual in an OVA test. The three sum of squares of contrast \((30.83, 5.042, 1.125)\) add up to 36.25, the total sum of squares of the contrast set. This value is the same as the sum of squares of treatment in OVA test in Table 2. Each contrast is tested at a specified \( \alpha \) level, or error rate, no adjustment of the \( \alpha \) level is recommended by many researchers for reasons to be discussed later. In Set 1, the contrast between A and BCD is found significant and so is the contrast between AB and CD in Set 2.

Insert Table 3 about here

Unplanned contrasts present a complicated case for the sheer number of methods developed for these tests. Only a few popular unplanned contrast procedures in educational research are considered in this discussion. Included are Tukey's HSD, Bonferroni/Dunn, and Scheffe. Fisher's LSD, Duncan and SNK will also be mentioned. Although no unplanned contrasts should even be done here since the OVA test fails to reject the null hypothesis, Table 4 is presented to show that none of these unplanned contrasts are able to detect significant difference between treatment group means.

Insert Table 4 about here

If, in another similar experiment, another set of data were obtained as in Table 5. The researchers are interested in learning: 1) whether, among treatments A, B, C and D, the effect of A is different from B, C and D, and 2) whether the effects of A and B are
The contrasts both have significant results, as noted in Table 6.

Note, however, the two contrasts are not orthogonal in this example:

\[
C_{A,BCD} = (3)A + (-1)B + (-1)C + (-1)D
\]
\[
C_{AB,CD} = (1)A + (1)B + (-1)C + (-1)D
\]

The sum of the cross products of the contrast coefficients in the pair is:

\[
\Sigma CiCj = (3)(1) + (-1)(1) + (-1)(-1) + (-1)(-1) = 4, \text{ or } \Sigma CiCj \neq 0.
\]

The sum of squares of the two contrasts (28.521, 22.563) is 51.084, which is greater than 34.688, the sum of squares of treatment in OVA test in Table 7 below. This difference suggests that the two nonorthogonal contrast provide some overlapping information. The OVA test in Table 7 also rejects the null hypothesis for this data set. Now that the OVA test indicates that at least one pair of treatment means are significantly different, unplanned contrasts may now be performed to see which means are different.

The results of the different unplanned contrasts are summarized in Table 8.

Note that Bonferroni and Scheffe tests fail to detect any significant difference between the treatment effects. Of all the unplanned contrasts, Scheffe is the only method recommended for comparisons of complex means. The two research questions should be tested against the following two null hypotheses:

\[
H_0(1): \mu_A - \mu_B + \mu_C + \mu_D/3 = 0
\]
\[
H_0(2): (\mu_A + \mu_B)/2 - (\mu_C + \mu_D)/2 = 0
\]

The OVA test, though significant, gives no information about these two questions, and Scheffe indicates no significant difference from such comparisons. The planned contrasts,
however, unambiguously reject the two null hypotheses. Had the researchers used the OVA test or Scheffe test, they would have ended up in failure in this hypothetical research situation.

**Error rate protection accounts for the low power of unplanned contrasts**

The test results from the two hypothetical data sets have shown that planned contrasts are more powerful than either OVA or unplanned contrasts. It was explained earlier that the difference between OVA test and planned contrasts can be accounted for by the different degrees of freedom they use respectively. In the case of planned versus unplanned contrasts, the difference in power is in essence due to the fact that all unplanned contrasts invoke some protection measures to control for the possible inflation of Type I error rate because the likelihood of a large number of tests involved in unplanned contrasts.

For a specified Type I error rate, $\alpha$ level, the actual $\alpha$ level for each test is adjusted in various ways, depending on the type of unplanned contrasts, and is no longer the original $\alpha$ level. In general, the $\alpha$ level is reduced for each test and the critical value for a test is therefore bigger, making it more difficult to reject the null hypothesis. Most of the tests for unplanned contrasts between simple means, for instance, Bonferroni/Dunn, Tukey, SNK, etc., have been incorporated into statistical tables for easy reference. The critical values for these tests and planned contrasts for simple means are tabulated in Table 9 to illustrate the point that planned contrasts have lower critical values than unplanned ones. Note that a critical value for a test is used for planned contrasts, because, for $n$ observations and $k$ treatment groups, a planned contrast has an $F$ statistic with the degrees of freedom of 1 and $n - k$ for the numerator and denominator respectively. The square root of this $F$ statistic is a one-tailed $t$ statistic with $n - k$ degrees of freedom at the same specified $\alpha$ level. The critical values for both one-tailed and two-tailed $t$ test are provided in Table 9.
The critical values for all unplanned contrasts, because of the error rate protection adjustment, are greater than the critical values for the planned contrasts between simple mean comparisons. This holds true for complex mean comparisons where Scheffé test is used, as was shown earlier in the example with the second hypothetical data set. It can also be shown that the formula for Scheffé test is the same as for planned contrast (Hinkle et al., 1988, p. 378). However, while the critical value for planned contrasts is Ec with 1 and n - k degrees of freedom, the critical value for Scheffé test is Ec* and Ec* = (k - 1)Ec, where E has (k - 1) and (n - k) degrees of freedom. Therefore, the critical value for Scheffé test is inflated by (k - 1), or the degree of freedom for the numerator. Hence the Scheffé test is very conservative.

It is clear from the discussion up to this point that the planned contrasts tend to have more power than unplanned contrasts, especially in complex mean comparisons, and that the power for planned contrasts is gained because no adjustment is made for the error rate in the tests of hypotheses. This may appear unfair at the first glance to some people. In fact, some researchers believe that the same Bonferroni type of correction of error rate should be applied to planned contrasts (Huberty & Morris, 1988) or at least applied to nonorthogonal planned contrasts (Pedhuzur, 1991).

Most of the researchers writing on this issue, however, feel that no adjustment or error rate is necessary, although some conditions are necessary, such as only a small number of hypotheses (no more than the number of treatment minus one) are being tested (Keppel, 1982; Winer, 1971), or as Thompson (1990, 1991) recommends, "the multiple correlation between the planned contrast coding vectors and the vector designating assignment for a given effect does not exceed one".
The argument for no adjustment emphasizes that planned contrasts require that researchers have to think carefully about what they are looking for (Keppel, 1982; Tucker, 1991). If researchers believe that the hypotheses to be tested are well supported by theory or other research and they don't want to miss what they think exists in the data, they have legitimate reasons to have a more powerful test to guard against Type II error. Besides, from the practical point of view, well-oriented researchers wouldn't try to test hypotheses formulated from all possible combinations of treatments. In the hypothetical example and data in this paper, for instance, only two out of 12 all possible combinations (6 for simple means and 6 for complex means), have been tested.

Another interesting suggestion for handling error rate in planned contrasts is the idea of assigning different error rate to individual hypothesis test such that the tests of most interest have a higher \( \alpha \) level, say, .05 each, to ensure significant findings while the other tests of less interest or importance are given more stringent \( \alpha \) level, say .01 or even .001. The total error rate will then add up to less or equal to the specified \( \alpha \) level for the entire experiment. In Miller's words (1980), there is no law that insists on one \( \alpha \) level being equal to another one. Similar statements are also found in Kurtz et al. (1965), and Games (1971). There is also controversy over this idea (O'Neil & Wetherill, 1971).

This writer feels that the idea of differentially assigning error rate is acceptable. However, this seems to be appropriate only where a complete set of orthogonal contrasts are formed, but some of the contrasts are included solely for the purpose of obtaining the set of mutually contrasts, as in Table 3. After all, the question of what is an acceptable Type I error rate is largely a subjective consideration influenced by many non-statistical factors such as the convention in one's field, and the relevant graveness of committing a Type I error and a Type II error. Therefore, as Jones (1984) points out, this question "can only be answered in the context of a given experimental situation."

The controversy over plausible error rate for planned contrasts as well as the debate on the use of orthogonal versus nonorthogonal contrasts invites more research in
this area. Some empirical investigation and simulation experiments may be able to shed more light on the question whether or when researchers should become concerned with the outcome of analysis if different approaches are employed.

Summary

The literature was reviewed regarding the difference between planned contrasts, OVA and unplanned contrasts. The relationship between statistical power of a test method and Type I, and Type II error rates was first explored to provide a framework for the discussion. It was explained that a higher Type II error rate means lower power for a test; a lower desired Type I error rate (a small $\alpha$ value) also makes a test less powerful.

The concepts and formulation of contrast, orthogonal and nonorthogonal contrasts were introduced. It was argued that planned contrasts are focused on thoughtful research questions of interest and reflect the researchers' rational anticipation. An OVA test or unplanned contrasts, on the other hand, do not provide desired information in many situations. Planned contrasts, OVA and unplanned contrasts were compared and the results show that planned contrasts yielded statistically significant findings where neither OVA nor unplanned contrast did. For complex mean comparisons, in particular, planned contrasts always have greater power than unplanned contrasts. Two small sets of hypothetical data were employed to make the discussion concrete.

It was also explained that, to control for the possible inflated error rate for unplanned contrasts that usually test a large number of hypotheses, wanted or unwanted, some Bonferroni type of corrections are invoked. It is these corrections, usually built into statistical tables, that reduce the power of unplanned contrasts. This was demonstrated through a comparison of the critical values for planned contrasts and for some popular unplanned contrasts: Tukey, SNK, Bonferroni, Duncan and Scheffe. The issue whether planned contrasts should also be subject to Bonferroni type corrections and whether it is acceptable to assign a different error rate to each individual planned contrast in a set of contrasts was also briefly examined.
References


Table 1.

A hypothetical set of scores measuring the effects of four treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tr>
<td>9</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Mean: 8.00 5.75 4.75 4.00

Table 2.

Results from OVA using the data in Table 1.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>P</th>
<th>eta squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
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<td>36.250</td>
<td>12.083</td>
<td>2.360</td>
<td>.123</td>
<td>.37</td>
</tr>
<tr>
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<td>61.500</td>
<td>5.125</td>
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</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>97.750</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.

Results from planned contrasts with two complete sets of mutually orthogonal contrasts

<table>
<thead>
<tr>
<th>Contrast (Set 1)</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>P</th>
<th>eta squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>A vs BCD</td>
<td>1</td>
<td>30.083</td>
<td>30.083</td>
<td>5.87</td>
<td>.0322</td>
<td></td>
</tr>
<tr>
<td>B vs CD</td>
<td>1</td>
<td>5.042</td>
<td>5.042</td>
<td>.980</td>
<td>.341</td>
<td></td>
</tr>
<tr>
<td>C vs D</td>
<td>1</td>
<td>1.125</td>
<td>1.125</td>
<td>.220</td>
<td>.648</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contrast (Set 2)</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB vs CD</td>
<td>1</td>
<td>25.000</td>
<td>25.000</td>
<td>4.88</td>
<td>.0474</td>
</tr>
<tr>
<td>A vs B</td>
<td>1</td>
<td>10.125</td>
<td>10.125</td>
<td>1.98</td>
<td>.185</td>
</tr>
<tr>
<td>C vs D</td>
<td>1</td>
<td>1.125</td>
<td>1.125</td>
<td>.22</td>
<td>.648</td>
</tr>
</tbody>
</table>
Table 4.
Results from unplanned contrasts

<table>
<thead>
<tr>
<th>Test</th>
<th>( \alpha ) level</th>
<th>df</th>
<th>MSE</th>
<th>Min. Sig. Difference</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tukey's HSD</td>
<td>.05</td>
<td>12</td>
<td>5.125</td>
<td>4.7524</td>
<td>Non</td>
</tr>
<tr>
<td>Bonferroni</td>
<td>.05</td>
<td>12</td>
<td>5.125</td>
<td>5.0468</td>
<td>Non</td>
</tr>
<tr>
<td>Scheffe</td>
<td>.05</td>
<td>12</td>
<td>5.125</td>
<td>5.1799</td>
<td>Non</td>
</tr>
</tbody>
</table>

Table 5.
A second hypothetical set of scores measuring the effects of four treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>'9</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Mean: 8.00 5.75 5.00 4.00

Table 6.
Results from nonorthogonal planned contrasts for the data in Table 5.

<table>
<thead>
<tr>
<th>Contrast</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A vs BCD</td>
<td>1</td>
<td>28.521</td>
<td>28.521</td>
<td>8.83</td>
<td>.012</td>
</tr>
<tr>
<td>AB vs CD</td>
<td>1</td>
<td>22.563</td>
<td>22.563</td>
<td>6.99</td>
<td>.021</td>
</tr>
</tbody>
</table>
Table 7.
Results from OVA for the data in Table 5.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>P</th>
<th>eta squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>7</td>
<td>34.688</td>
<td>11.563</td>
<td>3.58</td>
<td>.047</td>
<td>.47</td>
</tr>
<tr>
<td>Residual</td>
<td>12</td>
<td>38.750</td>
<td>3.229</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>73.438</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.
Results from unplanned contrasts for the data in Table 5

<table>
<thead>
<tr>
<th>Test</th>
<th>α level</th>
<th>df</th>
<th>MSE</th>
<th>Min. Sig. Difference</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tukey's HSD</td>
<td>.05</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonferroni</td>
<td>.05</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheffe</td>
<td>.05</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9.
Critical values for planned and unplanned contrasts for simple mean comparisons based on the hypothetical data set in Table 1 or Table 2.

\[
\begin{array}{cccc}
    n = 16 & k (treatments) = 4 & df = 12 & \alpha = .05 \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Number of steps between ordered means</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>Test</td>
</tr>
<tr>
<td>Planned contrast (t)</td>
</tr>
<tr>
<td>Tukey</td>
</tr>
<tr>
<td>SNK</td>
</tr>
<tr>
<td>Bonferroni/Dunn</td>
</tr>
</tbody>
</table>