Findings from two studies of fourth grade children learning the concept of mathematical equivalence are presented. The questions studied were: (1) Can knowledge conveyed in gesture but not in speech be tapped by a recognition technique; and (2) Does having implicit knowledge that is expressed in gesture but not in speech have implications for learning? The first question was addressed in a study of 17 fourth grade students who solved six pretest problems of the form $3 + 4 + 5 = \ldots + 5$ incorrectly. The students were presented six additional problems, one at a time, and were asked to rate the acceptability of six preferred solutions to each problem derived from strategies children often use. The second question was addressed in a study of 43 students who were unsuccessful on a pretest of six addition and four multiplication problems similar in form to the above example. These students were trained in the concept of equivalence and were retested. Results suggest that the knowledge a child expresses in gesture but not in speech reflects the first step in acquiring a concept and sets the agenda for future learning. (DE)
Implicit Knowledge Conveyed in Gesture
Sets the Agenda for Learning

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Paper presented at the Biennial Meeting of the

In the course of learning a concept, a child's knowledge undergoes many changes. One aspect of the child's conceptual knowledge that may change with learning is the child's ability to express that knowledge in words. In particular, at certain points in the course of learning certain concepts, children tend to have knowledge that they do not express in speech, and that they may be unable to convey in words (Goldin-Meadow, Alibali, and Church, 1993). In previous work, we have argued that one source of information about this unarticulated knowledge is the spontaneous gestures which children produce while speaking about their knowledge.

Based on the studies described in this paper, we argue that the conceptual knowledge that a child conveys in gesture and not in speech can be considered implicit knowledge. Specifically, we argue that this knowledge is implicit in the sense that it can be tapped by other behavioral techniques. In addition, we argue that there are consequences to having knowledge that can be expressed in gesture and not in speech. We suggest that the implicit knowledge conveyed in a child's gestures sets the agenda for future learning for that child. The knowledge that a child expresses in gesture and not speech captures what that child has just begun to learn, and what that child is ready to learn more explicitly.

In this paper, we present findings from two separate studies of children learning the concept of mathematical equivalence, or the idea that the two sides of an equation represent the same quantity. In previous work, we have found that fourth grade students typically are unable to correctly solve equations that have addends on both sides of the equal sign, e.g., 3+4+5=___+5 (Perry, Church, and Goldin-Meadow, 1988; Alibali and Goldin-Meadow, in press). Children have a variety of incorrect strategies for solving problems of this type. For example, many children add the numbers to the left of the equal sign, and ignore the addend on the right hand side. We call this procedure Add to Equal. Many other children add all the numbers in the problem. We call this procedure Add All.

In our previous work, we have asked children to explain their solutions to the problems they solve. Most of the verbal explanations that children produce convey a specific procedure for solving the problem. We have also found that most children spontaneously gesture while describing their solution procedures, and that these gestures also convey specific problem solving procedures.

Most of the time the procedure conveyed in speech and the procedure conveyed in the accompanying gesture are identical. For example, a child might explain her solution using the Add to Equal procedure in both speech and gesture. Such a child might say, "I added the 3, the 4, and the 5, and I put 12" while pointing to the 3, the 4, and the left 5. In this case, the child conveyed through her speech that she added the numbers to the left of the equal sign to get the solution. Similarly, in gesture, she indicated that she considered only the addends to the left of the equal
sign. Thus, both her verbal and her gestured procedures convey the same, incorrect reasoning about the problem.

However, in a substantial proportion of the children's responses, the procedure conveyed in gesture is not the same as the procedure conveyed in speech. For example, a child might explain her solution using the Add to Equal procedure in speech but the Add All procedure in gesture. Such a child might say, "I added the 3, the 4, and the 5, and I put 12" while pointing to the 3, the 4, the left 5, and the right 5. In this case, the child conveyed through her speech that she added the numbers to the left of the equal sign to get the solution. In gesture, however, she indicated that she considered all of the addends. Thus, both her verbal and her gestured procedures convey incorrect reasoning about the problems, but the reasoning conveyed in the two modalities is not the same.

In many cases, gesture conveys procedures that are more sophisticated than those conveyed in speech. Gesture may convey a correct procedure while speech conveys an incorrect procedure. One correct procedure which children sometimes express in gesture and in speech is the Equalizer procedure. A child who uses this procedure to solve the problem attempts to make both sides of the problem sum to the same total. Such a child might notice that the left side adds up to 12, and might then try to find a number to go with the 5 on the right side to make 12.

A child could explain her solution to the problem by describing the Add All procedure in speech, and conveying the Equalizer procedure in the accompanying gesture. Such a child might say, "I added the 3, the 4, and the 5, and then I added this 5," while sweeping a flat palm back and forth under the entire left side of the problem, bringing her hand down, and then sweeping back and forth under the entire right side of the problem. In this case, the child conveyed through her speech that she added all of the numbers in the problem to get the solution. In her gesture, however, she indicated that both sides of the problem have equal status. In our coding system, this is considered a gestured expression of the Equalizer strategy.

As these examples indicate, there are often strategies conveyed in gesture that are not conveyed in the accompanying speech. In fact, we have found that many children tend to express strategies in gesture that they never convey in speech (Goldin-Meadow, et al., 1993). This led us to question whether the knowledge conveyed in a child's gesture and never in that child's speech could be tapped by another behavioral measure.

One technique that has been used to in the past to assess implicit understanding is a recognition technique. We investigated whether we could use such a technique to tap knowledge that children convey in gesture and not speech. If procedures conveyed in gesture and not speech reflect implicit knowledge, children might recognize the solutions generated by those procedures as acceptable solutions to the problem, even though they never express those procedures in speech. With this in mind, we designed a study to investigate whether knowledge conveyed in gesture is implicit in the sense that it can be tapped using a recognition technique.
Study 1. Can knowledge conveyed in gesture but not speech be tapped by a recognition technique?

Method. Seventeen fourth grade children participated in the study. Each child completed a pretest in which he or she solved and explained a set of six math problems of the form $3+4+5=\_+5$ at the blackboard. Only children who solved all 6 of the problems incorrectly were included in the study. Following the pretest, each child was presented with 6 additional problems, one at a time. For each problem, each child was offered six potential solutions to the problem, and was asked to rate the acceptability of each of the solutions.

The solutions offered to each child were derived from the six canonical procedures which children typically use to solve the problems (see Table 1). As noted above, many children add all the numbers in the problem to get the solution. For the problem $3+4+5=\_+5$, a child using the Add All strategy would put 17 for the answer. Many children add the numbers to the left of the equal sign, and ignore the addend on the right hand side. For this problem, a child using this Add to Equal strategy would put 12 for the answer. Other children simply place one of the addends from the left side of the problem in the blank. We call this strategy Carry. For this problem, a child using the Carry strategy might put 4 for the answer.

Just as there is more than one way to get the wrong answer for this problem, there is also more than one way to get the right answer for this problem. A few fourth graders do in fact solve the problem correctly. For example, for the problem $3+4+5=\_+5$, a child may notice the fact that there is a 5 on each side of the problem, and might group the numbers that do not appear on both sides to get the solution. We call this the Grouping strategy. A child who uses the Grouping strategy would be likely to put 7 for the solution to the problem. In this part of the study, to differentiate this strategy from the other correct strategies, we offered each child the solution 3+4. Alternatively, a child might solve the equation by adding the numbers on the left side of the equation, and subtracting the number on the right side of the equation. We call this the Add-Subtract strategy. A child who uses the Add-Subtract strategy would be likely to put 7 for the solution to the problem. In this part of the study, to differentiate this strategy from the other correct strategies, we offered each child the solution 12-5. Finally, as mentioned above, a child might attempt to make both sides of the problem sum to the same total. Such a child might notice that the left side adds up to 12, and might then try to find a number to go with the 5 on the right side to make 12. We call this the Equalizer strategy.

Thus, for the problem $3+4+5=\_+5$, the solutions 17, 12, 4, 3+4, 12-5, and 7, corresponding to the six most commonly used solution procedures, were offered to each child. The children were asked to rate whether each solution was an acceptable solution to the problem. The children's response alternatives were "no," "maybe no," "maybe yes," and "yes." Each child responded to 6 comparable solutions for each of 6 problems, yielding 36 solutions, presented in blocks of 6. Note that, in this part of the study, children were asked only to rate the solutions, and not to explain them.

In coding the data, we started by looking at children's pretest explanations. Children's pretest explanations were coded in terms of the procedures children conveyed in speech and in gesture. For each child, each of the six canonical procedures was identified as expressed in both speech and gesture, expressed in gesture and not speech, expressed in speech and not gesture, or expressed in neither gesture nor speech. A sample coding sheet is presented in Table 2.
Our goal was to determine whether the children's ratings of the solutions were related to the modality in which the corresponding procedures were produced on the pretest. That is, we asked whether a child's ratings of a particular solution were related to whether that child produced the corresponding procedure in both gesture and speech, in gesture only, in speech only, or in neither gesture nor speech on the pretest. We predicted that children would be most likely to accept solutions generated by procedures that they expressed in both gesture and speech, and that children would be least likely to accept solutions generated by procedures that they expressed in neither gesture nor speech. Further, we predicted that, if the knowledge that children expressed in gesture and not speech was being tapped by this method, children would be more likely to accept solutions generated by procedures that they expressed in gesture and not speech, than solutions generated by procedures that they expressed in neither gesture nor speech.

For each child, a mean acceptance score for the solution generated by each procedure was computed. "No" responses received a score of 1, "maybe no" responses received a score of 2, "maybe yes" responses received a score of 3, and "yes" responses received a score of 4. Finally, a mean score for each modality category of procedures was computed. Thus, for each child, the average score for procedures produced in both gesture and speech, procedures produced in neither gesture and speech, and so on, was computed.

Results. Recall that each of the 17 children solved all of the pretest problems incorrectly. Each of these 17 children produced some procedures in both gesture and speech and some procedures in neither gesture nor speech. In addition, 9 of the children produced some of their procedures in gesture only, and 4 produced some of their procedures in speech only. Because there were so few children who had procedures in speech only, we were unable to analyze those data meaningfully.

We looked first at the subset of children who produced some procedures in both gesture and speech and other procedures in neither gesture and speech. These children did not produce any procedures in speech only or in gesture only. There were 8 children in this group. As seen in Figure 1, we found that the children were significantly more likely to accept solutions generated by procedures that they had produced in both gesture and speech on the pretest than solutions generated by procedures that they had produced in neither gesture nor speech on the pretest ($t(7)=2.44$, $p<0.05$). This result is not particularly surprising, but it does suggest that the acceptability ratings are a valid index of the children's knowledge.

The crucial comparison for our interests was whether children were more likely to accept procedures that they produced in gesture only than procedures that they produced in neither gesture nor speech. To investigate this question, we looked at the subset of children who produced some procedures in both gesture and speech, some procedures in gesture only, and some procedures in neither gesture nor speech. There were 9 children in this group. As seen in Figure 2, there were indeed significant differences in the acceptability ratings for the different groups of strategies ($F(2,16)=21.44$, $p<.001$). The children were in fact significantly more likely to accept solutions generated by procedures that they produced in gesture only than solutions generated by procedures that they had produced in neither gesture nor speech ($F(1,16)=5.44$, $p<0.05$). In addition, as expected, the children were significantly less likely to accept solutions generated by procedures that they had produced in gesture only on the pretest than solutions generated by procedures that they had
produced in both gesture and speech on the pretest (F(1,16)=17.08,p<0.001). Thus, solutions generated by procedures that children produced in gesture only were accepted at a level between that of solutions generated by procedures produced in neither gesture and speech and solutions generated by procedures produced in both gesture and speech.

Significance. These results indicate that the knowledge which children convey in their gestures can indeed be tapped by another behavioral technique. In this sense, then, the knowledge which children convey in their gestures can be considered implicit knowledge. This led us to question the potential consequences of having such implicit knowledge that can be conveyed in gesture. Does having implicit knowledge of a concept have implications for learning? We explored this question using data from a training study of mathematical equivalence.

Study 2. Does having implicit knowledge that is expressed in gesture but not speech have implications for learning?

Method. Forty-three children participated in this second study. Each child completed a paper and pencil pretest that included both addition and multiplication problems. Six of the problems were addition problems with equivalent addends, e.g., 4+5+7=___+7. Six of the problems were addition problems without equivalent addends, e.g., 6+3+8=___+7. Four of the problems were multiplication problems with equivalent multiplicands, e.g., 2x4x3=___x3. None of the children were successful on the pretest. Each child then explained his or her solutions to the first six problems to an experimenter at the blackboard. Next, each child was trained in the concept of mathematical equivalence by a second experimenter. The training focused on the principle of equivalence. During training, the experimenter told each child that the goal in solving the problems was to make both sides of the problem equal, and that the child needed to "find a number to put in the blank that makes both sides the same." The experimenter did not provide any specific procedures for "finding" the solution. Finally, each child completed a paper and pencil posttest comparable to the paper and pencil pretest.

Children's pretest explanations were coded in terms of the procedures children conveyed in speech and in gesture. Each child was then categorized according to whether or not he or she expressed any correct reasoning (i.e., any one of the three correct procedures described above) in gesture or speech on the explanation pretest. Eighteen of the 43 children expressed no correct reasoning in either modality on the pretest. Twenty children expressed correct reasoning in gesture and not speech on the pretest. Five children expressed correct reasoning in both gesture and speech on the pretest. Note that none of the children expressed correct reasoning in speech alone. That is, no child expressed correct reasoning in speech who did not also express correct reasoning in gesture.

We then explored whether children who expressed correct reasoning in gesture and not speech were more likely to acquire the concept through training than children who did not express correct reasoning in either gesture or speech. Note that these two groups of children can not be distinguished when their speech alone is considered. No child in either group produced any correct reasoning in speech.
Results. We looked at posttest success as a function of the modality in which correct reasoning was expressed on the pretest. To be considered successful, a child had to solve both the addition and the multiplication problems correctly on the posttest. Figure 3 presents the proportion of children who succeeded on the posttest for all three groups. The rightmost bar represents the children who expressed correct reasoning in both gesture and speech. Not surprisingly, of the three groups, children who expressed correct reasoning in both speech and gesture were most likely to succeed on the posttest problems ($x^2(1)=5.401$, $p<0.025$, comparing children who expressed correct reasoning in both speech and gesture to the other two groups).

We turn next to the two groups who did not express correct reasoning in speech. We found that the children who expressed correct reasoning in gesture on the pretest were in fact significantly more likely to succeed on the posttest problems than children who did not express any correct reasoning on the pretest ($x^2(1)=6.938$, $p<0.01$, comparing children who expressed correct reasoning in gesture and not speech to children who expressed no correct reasoning). Thus, children who expressed correct reasoning in gesture were more likely both to succeed on the addition problems and to generalize their understanding to the operation of multiplication than children who did not express correct reasoning.

Significance. In this study, children who expressed correct procedures in gesture on the pretest were more likely to benefit from training than children who did not. After training, children who had expressed correct procedures in gesture were then able to apply those correct procedures to solve the posttest problems. Thus, the correct reasoning conveyed in their gestures on the pretest was precisely the knowledge that they were ready to take in during training and that they then applied in solving the posttest problems. These results indicate that there are consequences to having knowledge that can be expressed in gesture and not speech.

General discussion. The studies described in this paper suggest that the implicit knowledge that a child expresses in gesture and not speech captures what that child has just begun to learn, and what that child is now ready to learn more deeply. In this sense, the knowledge that a child conveys in gesture may reflect knowledge that is currently within that child's zone of proximal development, as put forth by Vygotsky (1978). The knowledge a child expresses in gesture may convey to the child's communication partners what the child is ready to learn more explicitly (see Goldin-Meadow, Wein, and Chang, 1992, and Alibali, 1992, for evidence that adults are sensitive to information conveyed in children's gestures). In this indirect way, the implicit knowledge that a child expresses in gesture may have an impact on the child's learning environment. This knowledge may alert teachers or more advanced peers to what the child is ready to learn. The child's communication partners may then be able to adjust their input to the child so that the child receives input that would be especially beneficial at that particular moment (Church, Momeni, Williams, Garber, & Goldin-Meadow, 1992). In this way, the knowledge that a child conveys in gesture may even help to shape that child's learning environment.

Thus, the knowledge that a child expresses in gesture and not speech reflects the first step the child takes in acquiring a concept. Once that step has been taken in gesture, it guides the course of the child's development and sets the agenda for future learning.
References


Table 1
Solutions Corresponding to the Six Canonical Problem Solving Procedures

Sample Problem:
3 + 4 + 5 = __ + 5

<table>
<thead>
<tr>
<th>PROCEDURE</th>
<th>SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Incorrect Procedures</strong></td>
<td></td>
</tr>
<tr>
<td>ADD ALL</td>
<td>17</td>
</tr>
<tr>
<td>ADD TO EQUAL</td>
<td>12</td>
</tr>
<tr>
<td>CARRY</td>
<td>4</td>
</tr>
<tr>
<td><strong>Correct Procedures</strong></td>
<td></td>
</tr>
<tr>
<td>GROUPING</td>
<td>3+4</td>
</tr>
<tr>
<td>ADD-SUBTRACT</td>
<td>12-5</td>
</tr>
<tr>
<td>EQUALIZER</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 2
Sample Coding Sheet

PRETEST RESPONSES TO PROBLEMS

<table>
<thead>
<tr>
<th>Problem</th>
<th>Speech</th>
<th>Gesture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Add to Equal</td>
<td>Add to Equal</td>
</tr>
<tr>
<td>2.</td>
<td>Add to Equal</td>
<td>Add All</td>
</tr>
<tr>
<td>3.</td>
<td>Add to Equal</td>
<td>Add to Equal</td>
</tr>
<tr>
<td>4.</td>
<td>Add to Equal</td>
<td>Add to Equal</td>
</tr>
<tr>
<td>5.</td>
<td>Add to Equal</td>
<td>Add to Equal</td>
</tr>
<tr>
<td>6.</td>
<td>Add to Equal</td>
<td>Add All</td>
</tr>
</tbody>
</table>

ASSESSMENT OF REPERTOIRE

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Modality over the Repertoire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add All</td>
<td>GESTUPE ONLY</td>
</tr>
<tr>
<td>Add to Equal</td>
<td>BOTH SPEECH AND GESTURE</td>
</tr>
<tr>
<td>Carry</td>
<td>NEITHER SPEECH NOR GESTURE</td>
</tr>
<tr>
<td>Grouping</td>
<td>NEITHER SPEECH NOR GESTURE</td>
</tr>
<tr>
<td>Add-Subtract</td>
<td>NEITHER SPEECH NOR GESTURE</td>
</tr>
<tr>
<td>Equalizer</td>
<td>NEITHER SPEECH NOR GESTURE</td>
</tr>
</tbody>
</table>
Figure 1. Acceptance of solutions on the recognition test as a function of the status of the corresponding procedures on the pretest. Children in this group (N=8) were significantly more likely to accept solutions generated by procedures that they had produced in both gesture and speech on the pretest, than solutions generated by procedures that they had produced in neither gesture nor speech on the pretest (t(7)=2.44, p<0.05).
In Neither G Nor S  
In G But Not S  
In Both G and S

Figure 2. Acceptance of solutions on the recognition test as a function of the status of the corresponding procedures on the pretest. For children in this group (N=9), there were significant differences among the acceptability ratings for the different groups of strategies (F(2,16)=21.44, p<0.001). The children were significantly more likely to accept solutions generated by procedures that they produced in gesture only, than solutions generated by procedures that they had produced in neither gesture nor speech (F(1,16)=5.44, p<0.05). In addition, they were significantly less likely to accept solutions generated by procedures that they had produced in gesture only on the pretest, than solutions generated by procedures that they had produced in both gesture and speech on the pretest (F(1,16)=17.08, p<0.001).
Figure 3. Proportion of children successful on the posttest as a function of the modality of their correct procedures on the pretest. Not surprisingly, children who expressed correct reasoning in both speech and gesture were most likely to succeed on the posttest problems ($x^2(1)=5.401$, $p<0.025$, comparing children who expressed correct reasoning in both speech and gesture to the other two groups). In addition, children who expressed correct reasoning in gesture on the pretest were significantly more likely to succeed on the posttest problems than children who did not express any correct reasoning on the pretest ($x^2(1)=6.938$, $p<0.01$, comparing children who expressed correct reasoning in gesture and not speech to children who expressed no correct reasoning).