This paper presents three experiments examining competence in the domain of proportional reasoning in several age groups (adult, and children in kindergarten, first, third, and fifth grade). In the three experiments, subjects indicated which of two flower boxes had the greater density of flowers. In experiments 1 and 2, the flowers were presented inside the flower boxes. In experiment 3, the flowers were presented as uniform rows of dots outside the flower boxes. In experiments 1 and 2, all subjects performed well, using density rather than number or area to make decisions. In experiment 3, however, only the older children performed well. Younger children based their judgments on the dimensions of number, revealing that when an explicit integration is required to be correct, younger children perform poorly. These results provide initial support for a two-process model for dealing with proportionality. (MM)
REASONING ABOUT DENSITY: THE INFLUENCE OF TWO PROCESSES

Janice Singer
Learning Research and Development Center
University of Pittsburgh
Pittsburgh, PA 15260
email: singer@unixd.pitt.edu

ABSTRACT

This poster explores 5 year olds' through adults' competence in the domain of proportional reasoning. In three experiments, subjects indicated which of two flower boxes had the greater density of flowers. In Experiments 1 and 2, the flowers were presented inside the flower boxes. In Experiment 3, the flowers were presented as uniform rows of dots outside the flower boxes. In Experiments 1 and 2, all subjects, including the kindergartners performed well. They used density, and not number or area to make decisions. In Experiment 3, however, only the older children performed well. The younger children based their judgments on the dimension of number. Thus, when an explicit integration is required to be correct, the younger children perform poorly.
Do children have greater competence than Piaget believed, or do newer tasks showing early competence tap processes other than the ones Piaget originally investigated? Recently, developmental psychologists have looked at children's early competence in certain domains with surprising results. In areas such as object permanence, conservation, and number knowledge, younger and younger children are exhibiting at least a basic level of understanding. However, these newer findings are not unequivocal (see Fischer & Bidell, 1991). Contradictions still exist: children do show competence in certain tasks, while in other more traditional tasks, children’s performance more closely matches Piaget's predictions.

This paper investigates this contradiction in one domain: proportional reasoning. Several recent studies suggest that young children can reason proportionally (Kohn, 1991; Spinillo & Bryant, 1991; Acredolo, O'Connor, Banks, & Horobin, 1989). However, another body of research finds that it is not until about 13 years of age that children acquire this skill (Noelting, 1980; Karplus, Pu'os, & Stage, 1983). These conflicting results may be reconciled by positing that each set of studies elicited different processes: one process (*P1*) allowing for the direct apprehension of proportions and another process (*P2*) involving the explicit representation and integration of variables. Early competence is shown when experimental tasks tap the first process (P1) and later competence is shown when the second process (P2) is necessary for successful performance. This thesis was examined in three experiments which are detailed below.

**Experimental Problem**

- Design a set of experiments so that in certain cases children show competence and in others they do not
- The main difference being whether P1 or P2 is necessary to perform well
- When the first process can be used young children should perform well
- When the second process is the only method available for successful performance, young children should perform poorly

**Experimental Solution**

- 3 Experiments were conducted - all use paired comparison format
Reasoning about density

- All looked at children's understanding of density (dots/area)
- First two experiments, children made decisions about the crowdedness of flowers in flowerboxes. The dots were displayed inside the boxes. Here P1 can be used, and hence children should have performed quite well.
- The Ex. 3 stimuli were a subset of the Ex. 2 stimuli. However, the dots were displayed outside, instead of inside the boxes. Here P2 was necessary for success, and hence the young children should have performed poorly.

Experiment 1

Subjects
81 subjects - 16 K, 16 1st, 17 3rd, 16 5th, and 16 adults.

Design
A set of nine boxes was defined by 3 densities [high, medium, low] crossed with 3 areas [large, medium, small]. The 36 unique combinations of two boxes from this set of nine were used as the paired-comparisons.

Procedure
The children were told a story about a bug who liked to jump in crowded flowerboxes. They were told that the bug only cared about how crowded or full the flowerboxes were, and did not care about which flowerbox was bigger or had the greater number of dots in it. The subjects were instructed, for each trial, to pick the more crowded flowerbox for the bug or to tell the bug if the flowerboxes were equally crowded. The computer presented all displays and collected correctness scores and reaction times. See Figure 1 for examples.

Results (overall correctness)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>69%</td>
</tr>
<tr>
<td>1</td>
<td>81%</td>
</tr>
<tr>
<td>3</td>
<td>85%</td>
</tr>
<tr>
<td>5</td>
<td>86%</td>
</tr>
<tr>
<td>A</td>
<td>98%</td>
</tr>
</tbody>
</table>

4
Reasoning about density

Figure 1 (Examples from Ex. 1). In the first example the left box is more crowded with flowers, in the second example the flowers are equally crowded.

Summary of Experiment 1

Subjects can make decisions about density. Even the kindergartners performed significantly above chance in this experiment. Other analyses showed that children were focusing on density, and neither area nor number.
Reasoning about density

**Experiment 2**

Subjects' performance in Ex. 1 could be explained by asserting that with displays such as those used, young children automatically attend to density as an integral-type perceptual dimension. If this were the case, the subjects would not necessarily have a concept of density as existing separate from other dimensions.

Thus, Experiment 2 further explored children's knowledge of density by requiring them to make decisions about density and numerosity for identical displays.

If subjects could switch their focus between density and numerosity, regardless of the display, then they must have a distinct knowledge of density as existing separately from numerosity, and can selectively attend to density as a unique dimension.

**Subjects**

80 subjects - 16 K, 16 1st, 16 3rd, 16 5th, and 16 adults.

**Design**

Six problem types: Small, Big, =Number, =Density, Conflict < 50 dots difference, Conflict > 50

Two Area Relationships: S/M, S/L

Two Judgment Types: Density and Numerosity Decisions

24 problems, two trials for each problem

**Procedure**

Subjects were told about two types of bugs, a eating bug, and a jumping bug. The jumping bug wanted crowded flowerboxes to jump around in, while the eating bug wanted boxes that were full of flowers for him to eat. For each trial, a jumping or a eating bug was centered between the two boxes. For a jumping bug, subjects were required to make a density decision. For a eating bug, subjects were required to make a numerosity decision. Trials were blocked in groups of 4 (e.g., 4 eating trials then 4 jumping trials, etc.). See Figure 2 for example problems.
Figure 2 (Examples from Ex. 2 - the first example shows the eating bug, the second example shows the jumping bug)

(correct density and number)

(correct number)  (correct density)
Reasoning about density

Results

(strategy data)

Subjects were categorized according to the dominant strategy they used for both types of problems. The following table shows the percentage of Ss classified according to either a density, a number, or an other strategy separately for number and density judgments.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Density</th>
<th>Number</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number Judgments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>31</td>
<td>32</td>
<td>37</td>
</tr>
<tr>
<td>1</td>
<td>19</td>
<td>56</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>75</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>62</td>
<td>32</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>94</td>
<td>6</td>
</tr>
<tr>
<td><strong>Density Judgments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>62</td>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>94</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>A</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The following table shows combined strategies for both problem types (e.g., ND refers to subjects who used number for number judgments, and density for density problems; DD to subjects who used density for both number and density judgments, NN to subjects who used number for both number and density judgments, NO to subjects who used number for number judgments and other for density judgments, etc.)

<table>
<thead>
<tr>
<th>Grade</th>
<th>ND</th>
<th>DD</th>
<th>NN</th>
<th>NO</th>
<th>CO</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>25</td>
<td>32</td>
<td>13</td>
<td>0</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>19</td>
<td>0</td>
<td>6</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
<td>13</td>
<td>0</td>
<td>18</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>62</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>A</td>
<td>94</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

Discussion

For the most part, subjects (especially above kindergarten) were able to distinguish the concept of density from that of numerosity. When problems were difficult subjects erred. This could be for one of two reasons:

1) they switched the basis of their judgments from density to number, and vice versa.

2) they were miscalibrated on the appropriate scale; i.e. a scaling error.

Using other analyses, it appears that the first explanation fits the kindergarten data and the first graders for numerosity judgments. That is, the younger children tended to switch from
Reasoning about density to number, and vice versa for hard problems. The second explanation fits the first grader's density judgments and the other grades errors. That is, the older children appeared to focus on the correct dimension, but were miscalibrated as to the correct scaling function.

**Summary of Experiments 1 and 2**

For the most part, subjects did make decisions based on density. They were sometimes affected by other variables, and by the difficulty of the problem. Nonetheless, their judgments did appear to be based on a conception of density.

**Experiment 3**

However, it is still not clear whether subjects were integrating information from two different dimensions to make decisions.

Experiment 3 explores this issue by presenting area and number information separately. In this case, an integration is required to be successful. If subjects were integrating in the previous two experiments, they should be able to do so here.

**Subjects**

82 subjects - 16 K, 8 1st, 8 2nd, 16 3rd, 16 5th, and 18 adults.

**Design**

Six problem types: Small, Big, =Number, =Density, Con < 50, Con > 50

One Area Relationship: S/M

One Judgment Type: Density

6 problems, four trials for each problem

**Procedure**

Task was explained, subjects completed all 24 comparisons. Comparisons were made for same stimuli as Ex. 2. See Figure 3 for examples.
Reasoning about density

Figure 3 (Examples from Ex. 3)

(correct density)

(correct density)
When an integration is required, below the third grade, young children perform poorly. They rely on number as the primary cue to make judgments.

At and above the third grade, subjects can integrate information from two different dimensions.

General Discussion

Subjects can make decisions about density when dots are displayed inside boxes. However, when dots are displayed outside boxes, and thus an explicit integration is required, below the third grade, subjects perform poorly.

See graph below
Reasoning about density

This set of results provides initial support for a two-process model for dealing with proportionality. Using P1, subjects can directly apprehend the relevant features of displays as they relate to proportions. Using P2, a proportion is computed via the separate representation and inter integration of different variables.

References


