The appropriateness and feasibility of the "Curriculum and Evaluation Standards" published by the National Council of Teachers of Mathematics were examined with respect to the mathematics education of students with disabilities. Eleven experts from the areas of mathematics education, special education, and assessment were surveyed using a five-point forced rating scale. To investigate the response patterns among the experts, their responses were clustered using a hierarchical agglomerative technique. Most of the experts reached consensus that the Standards should be addressed through curriculum and evaluation in elementary level math education. All respondents perceived that the basic skills were adequately and feasibly addressed in the elementary curriculum. Some experts who believed in the visions provided by the Standards also approved the feasibility of the Standards in evaluation. In contrast, others who were concerned about individual educational needs of students with disabilities did not readily accept the propositions of the Standards. Little agreement on the appropriateness and feasibility of the Standards was found beyond the elementary grade level. A discussion of the implications for the math education of students with disabilities is included. (Contains 13 references.)

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Experts' Opinions about the Appropriateness and Feasibility of National Math Standards

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Experts' Opinions about the Appropriateness and Feasibility of National Math Standards

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Abstract

The appropriateness and feasibility of the Curriculum and Evaluation Standards published by the National Council of Teachers of Mathematics were examined with respect to the mathematics education of students with disabilities. Eleven experts from the areas of mathematics education, special education, and assessment were surveyed using a 5-point forced rating scale. To investigate the response patterns among the experts, their responses were clustered using a hierarchical agglomerative technique. Results indicated that the major factor determining opinions about the Standards was the respondents' basic belief about the appropriate form of math instruction for students with disabilities. Little agreement on the appropriateness and feasibility of the Standards was found beyond the elementary grade levels. A discussion of the implications for the math education of students with disabilities is included.
Experts' Opinions about the Appropriateness and Feasibility of National Math Standards

High national standards ... while critically important, are not panaceas for the Nation's educational problems. (National Council on Education Standards and Testing, 1992, p. 7)

Setting standards for schools and students has gained the attention of much of the educational community, and the Department of Education and other federal agencies have funded projects for the development of standards in science, history, the arts, geography, and civics (Viadero, 1992). These grants follow the publication and widespread acceptance of the Standards published by the National Council of Teachers of Mathematics (NCTM) (1989).

The activities of NCTM and other projects such as the New Standards Project (O'Neil, 1992) have been cited as evidence of the feasibility of setting national standards for all students. Recently, the National Council on Education Standards and Testing (NCEST) stated that "the experiences of NCTM demonstrate that standards setting is feasible -- it is being done ... states and local districts are responding to the NCTM Standards by changing the curriculum and style of teaching to reflect the challenging new standards" (National Council on Education Standards and Testing, 1992, p. 5). In fact, a survey by the Council of Chief State School Officers (Blank & Dalkilic, 1992) found that no less than 41 states have realigned or are in the process of realigning their state math frameworks with the NCTM Standards.

NCEST also stated that standards are "desirable and feasible mechanisms for raising expectations, revitalizing instruction, and rejuvenating educational reform efforts for all American schools and students" (National Council on Education Standards and Testing, 1992, p. 8, emphasis added). In contrast to this view is the argument that there is a difference in the feasibility of setting standards and the feasibility of student attainment of standards that have been set. Because the setting of standards is arbitrary, there is no policy in place that requires that standards "will challenge the most able students as well as those with special learning needs" (Council for Exceptional Children, 1992, p. 2).
Careful consideration of the range of student abilities is a formidable task, one that does not meld easily with the task of standard setting. As other curricular areas begin to develop standards, the NCTM Standards are being used as a guiding model. This has been the case in many states. Numerous professional and administrative groups have accepted the NCTM Standards as the "standard for standards." As this acceptance is translated in implementation efforts, it is expected that many issues will surface.

With this scenario in mind, there is a need to examine what the education community thinks about the NCTM Standards, with particular attention to the needs of students with disabilities. We examined response patterns to questions about the appropriateness and feasibility of the NCTM Standards provided by experts in the areas of mathematics education, special education, and assessment. The responses were clustered in order to investigate similarities and differences among the respondents. This paper summarizes these experts' opinions.

**Method**

**Participants**

Experts in the areas of mathematics education, special education, and assessment were nominated by representatives of professional associations in special education or math education: the Council for Exceptional Children (CEC), the National Association of State Directors of Special Education (NASDSE), the Office of Special Education Programs (OSEP), and the National Council of Teachers of Mathematics (NCTM). Twelve people agreed to serve as respondents. All recipients of the survey were university faculty members. Two respondents specialized in mathematics education. Five special education experts were heavily involved in mathematics education for students with disabilities. Six of eleven respondents were working with school-age students (grades K-12).

**Instrument**

A survey instrument was developed to examine experts' opinions on a variety of aspects of the Curriculum and Evaluation Standards developed by NCTM. Of interest here is the section
on the appropriateness and feasibility of the NCTM Standards. Appropriateness was defined as the degree to which the Standards should be addressed in the curriculum and evaluation for students with disabilities. Feasibility was defined as the overall likelihood that the Standards can be adequately addressed (see Shriner, Kim, Thurlow, & Ysseldyke (1992) for the complete survey). In this section, quantitative data were gathered through forced ratings. The ratings were on a 5-point scale (e.g., 1=very inappropriate or unfeasible, 5=very appropriate or feasible). In addition to rating the appropriateness and feasibility of the Standards with respect to students with disabilities, respondents were asked to comment on the extent to which their responses would change for different disability categories or degrees of disability severity.

Responses were obtained from 11 of the 12 persons to whom the survey was sent. Respondents were asked to indicate in writing whether they would allow their answers to be quoted. Two respondents did not grant permission for identification.

Data Analysis

The respondent ratings were subjected to a cluster analysis technique. This technique was selected because of the nonrandom sampling of respondents, which makes probabilistic statistics inappropriate. In addition, cluster analysis, as a quasi-statistical procedure, is superior to inferential visual inspection of data which tends to be based on a priori decisions for further validation or replication (Satz & Morris, 1981).

Cluster analysis represents a family of empirical techniques designed to identify homogeneous subgroups of subjects within a heterogeneous sample (Everitt, 1980). A hierarchical agglomerative cluster technique was used in this study. Three decisions are made in using this technique (Morris, Blashfield, & Satz, 1981). The first decision is to select an appropriate association measure. In this study, the elevation (high vs low feasibility, for example) of ratings was considered more important than their similarity across different standards. Thus, the squared Euclidean distance between ratings was used. With this measure, identical patterns of responding (i.e., r=1.00), are still considered different if they are at different levels (e.g., one high and the other low).
The second decision when using cluster analysis techniques involves the choice of a specific method for defining the similarity between groups of respondents. The average linkage method, which averages the similarities between all members of each cluster, was employed in this study. In the average linkage method, a cluster is defined as a group of entities in which each member has a greater mean similarity with all members of the same cluster than it does with all members of any other cluster (Blashfield, 1976).

The third decision is to identify the number of clusters in the results. There are several criteria to consider, such as the researcher's hypothesis and interpretability. One popular method is to find breaks in the curves of cluster coefficients plotted against the number of clusters.

Results

In order to facilitate interpretation of the clusters, the mean profile points of the respondents in each cluster were plotted. Comments made by the respondents also are provided to illustrate opinions about altering the Standards as a function of different disabilities.

Eight of eleven respondents indicated that they were very familiar with the NCTM Standards before they received the survey. Three special education experts without mathematics specialization indicated that they were aware of the NCTM Standards but had not read or discussed them in depth.

Evaluation Standards: Student Assessment

Eight respondents rated seven student-assessment evaluation standards in terms of their appropriateness and feasibility for students with disabilities. Three experts did not respond. The two-cluster solution indicated by these ratings is shown in Figure 1. Most of the experts who responded were categorized into Cluster 1 in which respondents generally agreed on the appropriateness of the Evaluation Standards-Student Assessment. Cluster 2 contained only one expert, who did not fit within the larger cluster. This respondent indicated that the ideas of communication, reasoning and mathematical concepts were "somewhat inappropriate."

Following the recommendation of Everitt (1980), this respondent was considered an "outlier" and dropped from the major trend. Outliers may be viewed as resulting from measurement error.
Figure 1. Comparison of Expert Ratings on Appropriateness of the Evaluation Standards - Student Assessment
or as being unique individuals with whom there are few comparable individuals in a given sample. Yet, it is interesting to see the response pattern of this expert since the person is a math educator for students with mild disabilities, and has had substantial influence on research and practice in special education.

Figure 2 is an illustration of a two-cluster solution for feasibility ratings. Cluster 1 reflects the belief that the Evaluation Standards are feasible for students with disabilities. Four experts fit within this cluster. Cluster 2, containing four experts, reflects the belief that all standards (except for "mathematical procedure") are not feasible. The fairly even two-cluster split indicated disagreement among the experts in the current survey. While the respondents generally accepted the Evaluation Standards as a vision, they showed a sharp division on whether the Evaluation Standards can be readily used in practice.

Curriculum Standards

Grades K-4. Cluster analysis of the appropriateness ratings for the Curriculum Standards for Grades K-4 indicated a three-cluster solution in Figure 3. Cluster 1 (n=6) was similar to the large cluster for overall ratings of appropriateness of the Evaluation Standards. The analysis also revealed two outliers. These distinct patterns of ratings showed a contrast between the "constructivist" and the "reductionist" views. Outlier 1 reflected the "constructivist" view for the most part, and sided with the emphasis of NCTM stressing the process-oriented standards. This person gave low ratings to the appropriateness of whole number computation, whole number operations, or fractions and decimals. On the other hand, Outlier 2, who reflected the "reductionist" view, gave low ratings on appropriateness of communication, reasoning, patterns, and relationships in math education for students with disabilities, while emphasizing numbers, measurement, and computation.

In Figure 4, which shows the clusters on feasibility ratings, there were two distinct clusters. Cluster 1 included four experts who indicated that mainly "basic skills" (whole number computation, whole number operation, number sense and numeration, and fractions and decimals) are feasible. Cluster 2, which included three math education experts, showed a profile
Figure 2. Comparison of Expert Ratings on Feasibility of the Evaluation Standards - Student Assessment
Figure 3. Comparison of Expert Ratings on Appropriateness of the Curriculum Standards (K-4)
Figure 4. Comparison of Expert Ratings on Feasibility of the Curriculum Standards (K-4)
of "very appropriate" across all standards. The outlier in Figure 4 was also the outlier in Figure 3. This person believed that basic skills are the only feasible portion of the *Standards*.

**Grades 5-8.** Five respondents did not rate the Curriculum Standards for Grades 5-8. The cluster analysis results of the appropriateness ratings of the remaining six respondents are shown in Figure 5. Cluster 1 shows a neutral position (rating of about 3) by four experts toward patterns and functions, algebra, statistics, and probability. Cluster 2 indicates appropriateness of the standards across all categories. On the feasibility of the *Standards* for Grades 5-8, three clusters emerged (see Figure 6). The first cluster had lower profiles on feasibility than it had on appropriateness. In addition, one outlier believed that all standards are very unfeasible, except for "computation and estimation" which was seen as somewhat less unfeasible. In contrast, the two experts in Cluster 2 rated all standards for grades 5-8 as very feasible, just as they had rated them as very appropriate.

**Grades 9-12.** Only two special education experts responded to the Grades 9-12 Curriculum Standards (Figures 7 and 8). Expert 1 rated the standards as "somewhat appropriate," but several were viewed as not feasible, such as trigonometry, calculus, and so on. Expert 2 gave low ratings on appropriateness to most standards (see Figure 7), and low ratings on feasibility to all the standards (see Figure 8).

**Non-responses.** It is interesting and instructive to examine the patterns of nonresponse on the appropriateness and feasibility ratings. The pattern of nonresponses was hierarchical; experts who did not respond to the lower grade level standards also did not respond to them at higher grade levels. Thus, the reasons for not responding seemed to be differentiated and added as the grade levels increased. For the elementary and intermediate grades, the nonresponders made comments about the complexity of the issues. One expert said that he did not feel qualified to respond to a question that addressed different disabilities and math education. Another expert contended that these "narrow, one dimensional" rating tasks were not sufficient. Some of the respondents perceived the issues in a different way. One expert noted:
Figure 5. Comparison of Expert Ratings on Appropriateness of the Curriculum Standards (5-8)
Figure 6. Comparison of Expert Ratings on Feasibility of the Curriculum Standards (5-8)
Figure 7. Comparison of Expert Ratings on Appropriateness of the Curriculum Standards (9-12)
Figure 8. Comparison of Expert Ratings on Feasibility of the Curriculum Standards (9-12)
The idea of moving away from the decontextualized rote learning of mathematics is exactly what should happen in education. Students should be presented with meaningful environments in which to learn and use mathematics. In our own research we have demonstrated that this can be done. Unfortunately, I feel that many of the examples that are provided in the Standards do not result in teaching students useful and meaningful math. In sum, I would rate each of the stems as being "very appropriate" and "very feasible". However, I do not think that the way the Standards are defined will allow that to happen for students with disabilities. (Ted Hasselbring, Vanderbilt University)

For the higher grade level standards, most of the respondents believed that these standards are unrealistic or the grade levels (9-12) are beyond their interest.

**Differentiation by Disability Category**

Respondents were also asked about whether their ratings would differ as a function of different disability categories or levels of disability severity. Five out of the nine who responded expressed confidence about the changes. For example:

- Of course. Cognitive level would be a determinant of the appropriateness of a particular learning. Physical input modification would be required for students with sensory problems or output difficulties. (Stanley Vitello, Rutgers-State University of New Jersey)

- Yes. My prediction for levels 5-6 would be significantly reduced for students with more severe cognitive disabilities-somewhat-reduced perhaps for K-5. But an assessment-based approach would be the only way to go - not to set predetermined disability-label ceilings. (Martha Snell, University of Virginia)

Four respondents said they would leave their ratings intact (some with reservations):

- No. I believe the process standards are exactly what students of all disability categories and levels need. I would, naturally, adapt these to appropriate curriculum activities and types of learning experiences. (John Cawley, SUNY at Buffalo)

- One has to be realistic in relation to expected outcomes for any population of students. The overall thrust of the standards is correct and appropriate for all students however. The mathematics programs for special education students based on paper-pencil calculation have always been inappropriate and are in need of massive overhaul. The Standards reaffirm this need. (a math education professor)

- No. There may be specific students for whom part of a specific standard may be non attainable, thus limiting the feasibility of that Standards but I do not have sufficient knowledge about disability categories to speak to this issue in general. (Pat Tinto, Syracuse University)
Discussion

The aim of this study was to obtain experts' opinions about the appropriateness and feasibility of the NCTM Standards for students with disabilities. Although the number of respondents was small (11 respondents) and there were no further external validations for the cluster analysis, the rating results and the responses to the open-ended question were informative. It must be noted that the "experts" who responded to the current survey consisted of a select group of professionals. Although they have little direct contact with students with disabilities for math education, they have influence over teacher training and often provide direct input to policymakers and professional organizations. Thus, each individual's viewpoint, whether it is incorporated into the current consensus or not, deserves careful attention.

The quantitative forced rating data reflected the same agreements and disagreements as a qualitative analysis in another report (Shriner, Kim, Thurlow, & Ysseldyke, 1992). The sample of "experts" in the current survey indicated their perspectives on present practice in math education and the Standards.

Most of the experts included in the current survey reached consensus that the Standards should be addressed through curriculum and evaluation in elementary level math education (grades K-4). All respondents perceived that the basic skills, such as whole number computations and operations or fractions, were adequately and feasibly addressed in the elementary curriculum. These are the basic skills and drills that characterize traditional math education. In special education, especially, research and practice have concentrated on the basic skills.

There was a distinct contrast in the experts' views of the overall likelihood that the Standards can be adequately addressed in practice. Some experts who believed in the visions provided by the Standards also approved the feasibility of the Standards in evaluation. In contrast, others who were concerned about individual educational needs of students with disabilities did not readily accept the propositions of the Standards. Even the present study,
which employed a very restricted sample, showed the distinction between "What to do" and "How to do it."

At the intermediate level (grades 5-8), there were two different attitudes toward the Curriculum Standards for both appropriateness and feasibility: overall approval and conditional approval. For instance, for many of the special education experts in the current survey, "algebra" was not very appropriate or feasible for students with disabilities. Yet, this middle school mathematics curriculum was presented by NCTM as "a bridge between the concrete elementary school curriculum and the more formal mathematics curriculum" (National Council of Teachers of Mathematics, 1989, p. 102); one critical transition is that between arithmetic and algebra. Many respondents seemed to be concerned about the difficulties of abstraction involved in algebraic concepts.

Not only was there a clear difference in the cluster profiles at the different grade levels, there also were fewer respondents as the grade level increased. Nine respondents had difficulty rating appropriateness and feasibility of the Standards for Grades 9-12. This trend suggested several important considerations. First, the "traditional" grade level standards (K-4, 5-8, and 9-12), even though they are "broad enough," do not fit very well for students with disabilities. A set of standards based on each student's capabilities and needs, not on the grade levels, might be more appropriate for students with disabilities.

Second, almost all "traditional topics" (11 out of 14 standards in the 9-12 curriculum) provide a two-stage curriculum (all students can...; college-intending students can...). The authors of the Standards contend that "college-intending" is not used in an exclusive sense, but as a means by which to identify the additional mathematical topics that should be studied by all students who demonstrated interest and achievement in mathematics (p. 124). Yet, few of the respondents to the current survey held out much hope that the secondary curriculum could be realistically implemented for high school students with disabilities. This viewpoint is troubling, given that over 140,000 students with disabilities entered college in the 1991-92 school year (Henderson, 1992). Of these students, 34,920 were students who identified themselves as having
a learning disability. The danger exists that the definition of "college bound" in the NCTM Standards is not inclusive of almost 9% of the college freshman population. It is important that the challenges faced by students with disabilities whose educational and career goals are similar to those without disabilities not be exacerbated by the wholesale adoption of standards and curricula.

Presently, standards are not adequately articulated, and curricula cannot be effectively implemented without significant changes to current teacher preparation and classroom practices. At present, the NCTM Standards are loosely coupled and filled with promise for establishing "process as outcomes." However, in relation to students with disabilities and students who are at-risk for academic difficulty, there is a treacherous schism between the perceived appropriateness and feasibility of the Standards. Recognizing this gap can be viewed as a motivating factor toward providing both appropriate and feasible opportunities to learn for all students.
References


