DOCUMENT RESUME

ED 356 231 TM 018 991

AUTHOR	Harris, Richard J.
TITLE	Structure Coefficients versus Scoring Coefficients as Bases for Interpreting Emergent Variables in Multiple Regression and Related Techniques.
PUB DATE	Apr 92
NOTE	19p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 20-24, 1992).
PUB TYPE	Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Correlation; *Discriminant Analysis; *Mathematical Models; *Multiple Regression Analysis; *Predictive Measurement; *Scoring; *Test Interpretation
IDENTIFIERS	*Structure Coefficients; Variables (Mathematics)

ABSTRACT

Interpretation of emergent variables on the basis of structure coefficients (zero order correlations between original and emergent variables) is potentially very misleading and should be avoided in favor of interpretation on the basis of scoring coefficients. This is most apparent in multiple regression analysis and its special case, two-group discriminant analysis. Six examples of real and hypothetical data illustrate the pitfalls in interpretation based on structure coefficients. Much of the current commitment to structure-coefficient based interpretations of regression variables and discriminant functions would disappear if researchers would take the additional step of computing a score for each subject on the linear combination of predictors or dependent variables implied by each of the verbal interpretations, and then computing the correlation between each simplified regression variate and "Y" or the "t" for the difference between the two groups with respect to each simplified discriminant function. Four tables are included, and four figures illustrate sample analyses. (SLD)

STRUCTURE COEFFICIENTS VERSUS SCORING COEFFICIENTS AS BASES FOR INTERPRETING EMERGENT VARIABLES IN MULTIPLE REGRESSION AND RELATED TECHNIQUES

Richard J. Harris
University of New Mexico

Prepared for K. R. Friedrich (Chair), "Interpreting beta weights as against structure and other coefficients: A semi-structured debate." AERA, San Francisco, 4/21/92

Structure Coefficients versus Scoring Coefficients as Bases for Interpreting Emergent Variables in Multiple Regression and Related Techniques

Richard J. Harris
University of New Mexico

Prepared for K. R. Friedrich (Chair), "Interpreting beta weights as against structure and other coefficients: A semi-structured debate." AERA, San Francisco, 4/21/92

Scope of Problem

The issue of how to interpret the emergent variables (regression variates, discriminant functions, canonical variates, components, factors) yielded by fully multivariate techniques (multiple regression, Manova, Canona, PCA, factor analysis) is a very broad one. If one does not interpret the emergent variable(s) yielded by your technique, you might as well have employed Bonferroni-adjusted univariate analyses (m Pearson rs between individual predictors and Y or p univariate ts on individual dependent variables or $p \cdot q$ correlations between X-set and Y-set variables, etc.), since such Bonferroni- adjusted critical values will be less stringent than the fully post hoc critical values, which must protect against an infinite number of tests involving linear combinations of independent or dependent variables.

Given the fact that this symposium is sponsored by a multiple-regression interest group, I'll focus on multiple regression and its special case, two-group discriminant analysis (Hotelling's T^2), but the points are quite generalizable to Manova, Canona, PCA, and factor analysis (FA). Another reason for focusing on MRA and T^2 is that the central point of this paper -- that interpretation of emergent variables on the basis of structure coefficients (zero-order correlations between original and emergent variables) is potentially very misleading and thus to be shunned in favor of interpretation on the basis of scoring coefficients -- is most clearcut in these two cases, where structure coefficients are directly proportional to the results of separate univariate analyses and totally unaffected by correlations among the original variables.

Arguments against Structure Coefficients

Core point:

An interpretation of an emergent variable as "that which distinguishes those who have high scores on X_1, X_3, and X_5 and low scores on X_2 and X_9" from those who show the opposite pattern" is equivalent to an interpretation of the emergent variable as "That which distinguishes those who have high scores on $W = X_1 + X_3 + X_5 - X_2 - X_9$ from those who have low scores on W." Similarly for an interpretation of the emergent variable as "that which distinguishes those who have higher scores, on the average, on X_1, X_3, and X_5 than they do, on the average, on X_2 and X_9 from those who show the opposite pattern", which is equivalent to interpreting it as "that which distinguishes those who have high scores on $W = (X_1 + X_3 + X_5)/3 - (X_2 - X_9)/2$ from those who have low scores on W". Where the X_js are physical variables, we could in fact place the physical objects along a straight line from highest to lowest score on W (even making the relative spacing proportional to W scores) and then try to see what configuration of physical properties determines the relative positions of the objects. Where the X_js are measures of psychological properties, we must instead think of the characteristics of people who have configurations of scores that lead them to have high, moderate, or low scores on W, but the basics of the concept-identification problem are essentially the same.

Thus we can convert any verbal interpretation of an emergent variable to selection of c_js such that $\sum c_j X_j$ orders the cases (usually people) very nearly the same as does the emergent variable itself. From this equivalence follow a number of properties, including:

1. If the c_js are chosen to match or approximate the scoring coefficients, a_j, that define the emergent variable as a linear combination of the X_js, then the relative spacing of the cases implied by our verbal interpretation will match their actual spacing on the emergent variable very well.

2. If the c_js are instead chosen to match or approximate the structure coefficients, r_j, that represent the correlation between the emergent variable and each X_j taken as the sole predictor of W, the match between the two rank orders will be poorer -- sometimes only slightly poorer, sometimes dramatically so.

3. As a consequence of (2), the properties of structure-coefficient-based interpretations (more compactly, "obfuscations") of emergent variables will mimic the properties of the emergants they supposedly interpret more poorly than will the properties of scoring-coefficient-based interpretations (more compactly, "revelations") -- sometimes dramatically so.

4. If we attempt to generalize the principle underlying structure-coefficient-based interpretations -- namely, that we should approximate any emergent variable as $\sum r_j X_j$, rather than as $\sum a_j X_j$ -- to path analysis, we would thereby forfeit any ability to uncover such phenomena as spurious correlation, indirect causation, and mediation.

5. Adopting structure-coefficient-based interpretations leads to the interpretation of a given variable depending (strongly and inappropriately) on whether it is measured directly as an original variable or uncovered in exploratory fashion as a linear combination of the original variables.

6. Closely related to Point 5, if there is a consistent relationship between a given set of variables and our outcome measure, that relationship will be uncovered by scoring-coefficient-based interpretation of our emergant, regardless of what other variables are included in our set of predictors; structure-coefficient-based interpretations lose this property of context-independence.

7. In MRA and 2-group discriminant analysis (T^2) structure coefficients are directly proportional to r_{iy}s and single-variable t_is, respectively. Structure-coefficient-based interpretations are thus completely independent of the correlations among the predictors or among the dependent variables. This is a very peculiar and undesirable property for a "multivariate" statistic.

8. Closely related to Point 7: For a given set of simple univariate results (r_{iy}s or t_is) there are a host of alternative optimal combinations of the predictors or of the dependent variables that could underly those univariate statistics. Basing our interpretation on scoring coefficients allows us to discriminate among these various underlying relationships, while relying on structure coefficients yields exactly the same interpretation for each of them.

I'll now try to demonstrate the above points with specific (though frequently hypothetical) examples.

Revelations vs. Obfuscations: Examples

A. The FFEAF study (Harris, Harris, & Bochner, 1982).

In this study subjects were presented with one of 8 stimulus persons. "Chris Martin" was described to 159 Australian psychology students as either a man or a woman in his or her late twenties who, among other things, is either overweight or of average weight and was either said to wear glasses or not to wear glasses. Each subject rated this stimulus person on 12 adjective pairs, two of which (masculinity and femininity) were replaced by the combined variable SEXAPP (extent to which masculinity/feminity matched gender). While the complete analysis was a 2 x 2 x 2 Manova, tests of the Obesity main effect constituted essentially a 2-group discriminant analysis. Overweight stimulus persons were rated less favorably on each of the 11 adjective pairs than were average-weight stimulus persons, thus suggesting a globally negative stereotype as the primary factor distinguishing our perceptions of the obese. However, the discriminant function (which yielded a much higher F for the Obes main effect than did any single adjective pair or the average of all 11) was quite close to Outgoingness and Popularity minus Attractiveness, Athleticism, and Activity, yielding the much more interesting finding that obese stimulus persons are generally seen as more outgoing and popular (relative to average-weight stimuli) than one would expect, given how unattractive, unathletic, and inactive they are thought to be.

Table 1

Scoring vs. Structure Coefficients, FFEAF Study

Dependent Variable	Univariate t	Scoring Coefficient	Structure Coefficient
X1 (Assertive)	1.552	.070	-.114
X2 (Active)	9.580	-.592	-.705
X3 (Intelligent)	1.024	.160	-.075
X4 (Hardworking)	2.890	.001	-.213
X5 (Outgoing)	.892	.313	-.066
X6 (Happy)	2.023	-.149	-.149
X8 (Attractive)	7.292	-.667	-.536
X9 (Popular)	2.074	.464	-.152
X10 (Successful)	1.598	-.052	-.118
X11 (Athletic)	7.988	-.373	-.588
SEXAPP	-.2.776	-.032	.204

Note: Scoring coefficient = weight by which raw score on dependent variable is multiplied in computing discriminant function.
Structure coefficient = zero-order correlation between dependent variable and discriminant function.

At least, we obtain that finding if we interpret our discriminant function in terms of the scoring coefficients. If, however, we instead examine the structure coefficients we would be led once again to the conclusion that there is simply a uniformly negative stereotype of the obsese, since the correlation of each original variable with the discriminant function has the same sign. This shouldn't be surprising, since these structure coefficients are directly proportional to the univariate t's. Thus, for instance, the ratio between the t's for Active and Popular is 4.62, which matches the ratio between the corresponding structure coefficients of .705/.1525 = 4.62. In terms of "performance" (ability to discriminate between ratings of overweight versus average-weight stimulus persons) the F_{Obes} computed on the discriminant function (i.e., the maximum possible F for any linear combination of the dependent variables) was 179.142, corresponding multivariate $F(11,136) = 15.170$, $p < .001$. The F_{Obes} computed on the simplified discriminant function described above (X5 + X9 - X2 - X8 -X11) was 143.29, while the same F was 102.22 (barely better than using X2 by itself) for a linear combination of the 11 variables using the structure coefficients as weights.

Thus had we been so foolish as to interpret our emergent variable in terms of its Pearson correlations with the original variables (i.e., the structure coefficients) we would have missed entirely a much more interesting and more powerful pattern of differences between perceptions of the obese and perceptions of average-weight individuals. This was considerably larger than the largest F on any single dependent variable, which was 91.77.

B. The Canonical Cautionary (Harris, 1989) was designed, as the title suggests, to examine the shortcomings of using structure coefficients as the basis for interpreting canonical variates. However, it is well known that a Canona is equivalent to a multiple regression analysis in which the outcome measure is the canonical variate for the other set of measures, so the generalization to MRA should be clear. The premise of this hypothetical example was that an ethologist (Prof. Bird E. Watcher) sought to find some combination of his physical measurements on happily-mated Beefy-Breasted Bowery Birds (BBBBs) that would provide reliable guidelines for selecting pairs for a captive breeding program to save this endangered species from extinction. Having selected two measures (total head length, from base of skull to tip of beak, and skull length, from base of skull to beginning of beak) to record for each bird, he obtained the following correlations among his four measures for each of the happily mated pairs:

Table 2
Correlations among Head-Length Measures

	MTL	MSL	FTL	FSL
MTL	1	.7071	.4564	0
MSL		1	0	0
FTL			1	.7638
FSL				1

Watcher was delighted to obtain a Canonical *R* of 1.0 between the set of male measurements and the set of female measurements. In addition, he computed the following scoring (canonical-variate) and structure coefficients:

Table 3
Scoring Coefficients and Structure Coefficients for BBBB Measures

Original Variable	Scoring Coefficient for		Structure Coefficient for	
	Male	Female	Male	Female
MTL	.7071		.7071	
MSL	-.7071		0	
FTL		.7071		.6455
FSL		-.7071		0

On the advice of his statistical consultant, Watcher used the structure coefficients to interpret his canonical variates, coming to the clear conclusion that zoo-keepers need only match their BBBB males and females on the basis of total head length (MTL vs. FTL), since the structure coefficients for the other measure in each set (MSL and FSL) were zero. Unfortunately, this pair of obfuscations of the canonical variates correlated only .456 in his pilot sample, despite the canonical R of 1.0. Indeed,

> Our well-intentioned ethologist was deluged with calls from 80% of the zoos complaining that their BBBB pairs, though matched ... on the basis of total head length, ... simply refused to have anything to do with each other. (Harris, 1989, p. 21)

Figure 1 should help you understand why,

Figure 1 about here, showing male/female BBBB pairs skewering each other in beak-to-beak portion of their mating ritual.

---

especially when supplemented by examination of the scoring coefficients, which show that the canonical variate is in fact total head length minus skull length -- i.e., beak length. The BBBB mating ritual, if it is to be executed successfully (and safely), requires that the male and female have very nearly identical beak lengths.

At any rate, Watcher returned to his statistical consultant, Dr. Loh Ding.

> As soon as the results of the matching attempts were described to him, Ding cut to the heart of the problem. "I see I should have sent a bit more time discussing your Canona with you. It is indeed true that the only relationship worth talking about is between MTL and FTL, as clearly revealed by the structure coefficients. However, the only way to achieve this relationship is to match your couples on the basis of [MTL - MSL] versus [FTL - FSL], as the [scoring] coefficients clearly reveal."
>
> "In other words, we should match the couples on the basis of beak length?," interjected our naive ethologist.
>
> "No, no. The matching is on the basis of the true conceptual variable total head length. One simply computes scores on this conceptual variable by measuring to-

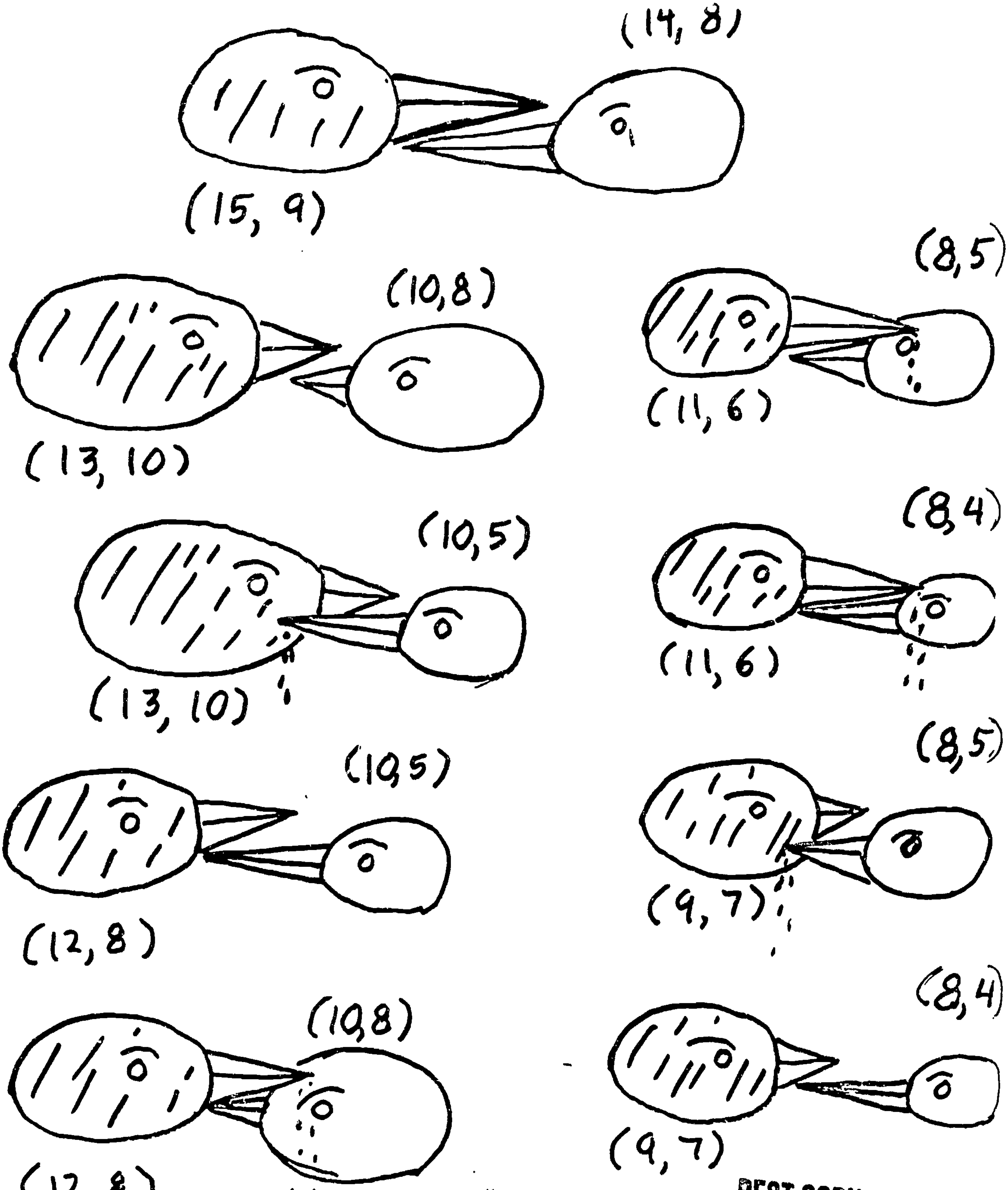
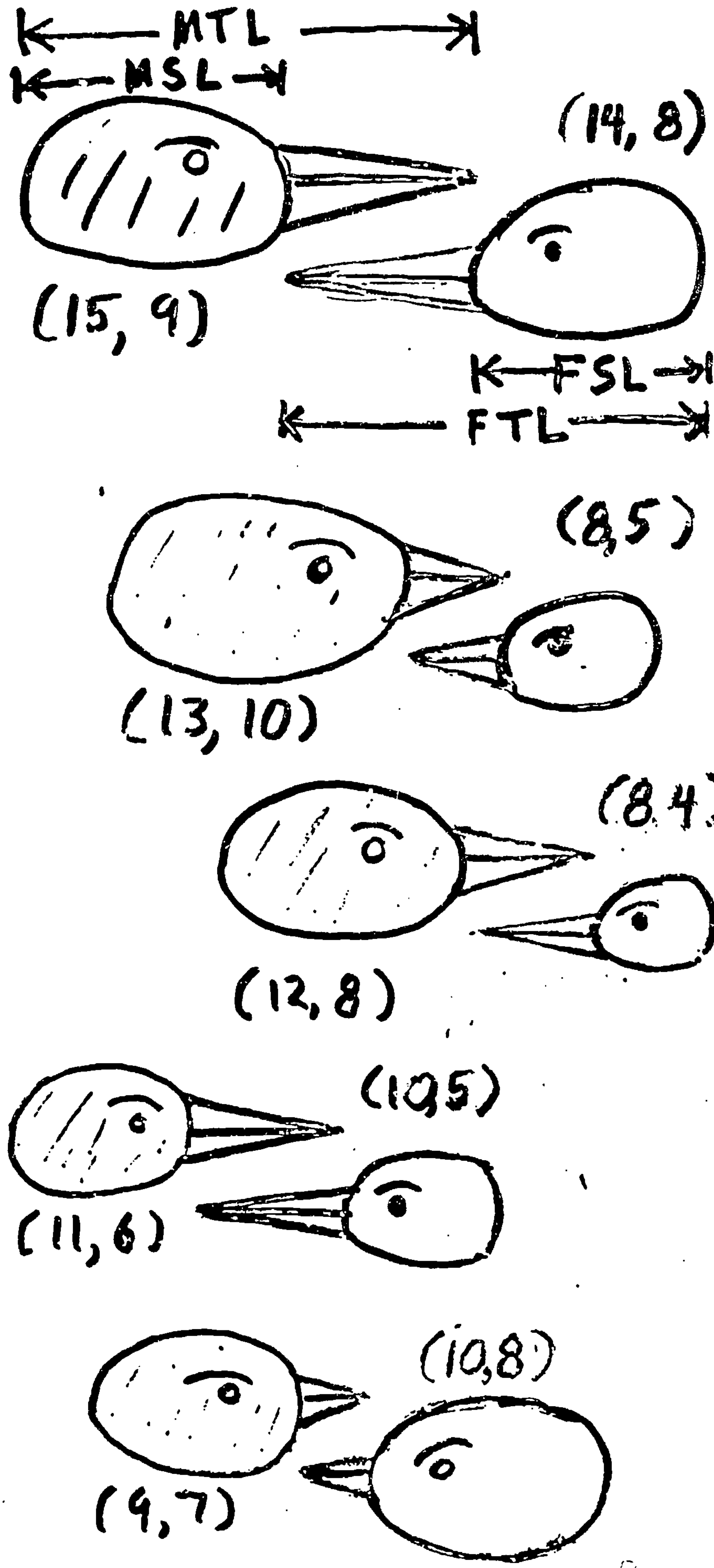


Figure 1. (See "Figure Captions" for explanation.)



R

	MSL	FTL	FSL
MTL	.707	.456	0
MSL	1	0	0
FTL		1	.764

C.V. COEFFS

MTL .707
MSL -.707

FTL .707
FSL -.707

STRUCTURE COEFFS

MTL .707
MSL 0

FTL .646
FSL 0

Figure 2. See "Figure Captions" for explanation.

tal head length and subtracting skull length, in accordance with the [scoring] coefficients. But you wouldn't want to describe what you're doing in terms of those misleading partial regression coefficients one finds in the table of [scoring] coefficients." (Harris, 1989, p. 22)

Armed with this information, Watcher was able to inform his zookeepers that they were indeed to match up BBBBs on the basis of total head length, but that they were to achieve this matching of total head lengths by making sure that the male and the female had identical "scores" on (total head length - skull length). As Figure 2 shows, this new procedure was successful at getting BBBB mating underway again.

Figure 2, showing male/female BBBB pairs with identical-length beaks happily conjoining beaks, goes about here.

Protests from zookeepers were reduced to occasional mutterings about the peculiarity of measuring beak length but calling it total head length, or musings about why birds matched on total head length seemed to differ so much on the corresponding physical measure while invariably having identical beak lengths.

The consequences of this rather schizophrenic (but frequently recommended) procedure for Watcher's career were somewhat more serious. First, try as he might he was never able to identify the role his conceptual variable of total head length played in BBBB courtship rituals. Worse, a junior colleague claimed to have conducted his own study of happily mated BBBB pairs, this time measuring beak length directly as one of his original variables. Naturally he found a perfect 1.0 correlation between male and female BBBB beak lengths (among those successfully mated, including the new pairs established by zookeepers around the world), and had no difficulty establishing the crucial role of HIS conceptual variable of beak length in BBBB mating. Watcher suspected this upstart of having actually COMPUTED "beak length" for each bird as total head length minus skull length, which would of course have invalidated beak length as the conceptual variable involved, but he felt constrained by the obligations of mentorship from exposing the fraud.

As you've no doubt noticed, the morals of this Canonical Cautionary include Points 1 - 3 (via the dramatic difference between the .45 correlation of the obfuscations as compared to the 1.0 correlation promised by Canonical R and delivered by the revelations), with the sad professional consequences of obfuscation-induced schizophrenia demonstrating Point 5 (why should whether we're allowed to discuss the perfect correlation between beak lengths depend on whether we initially uncovered it by making beak length an original variable or only happened upon it post hoc as the difference between two other variables?).

C. The Factor Fable (section 6.5.4 of Harris, 1985) was developed to help clarify the relative merits of obfuscations versus revelations in PCA and factor analysis, where the consensus among statisticians and researchers in favor of obfuscations is especially strong. However, factor scoring coefficients in PCA (and the most common type of factor-score estimate in factor analysis) can be obtained by carrying out a multiple regression in which the p original variables are predictors and a particular component or factor is the outcome variable. Thus the connection to MRA considerations should be clear.

This hypothetical case study involves an architect enamored of rectangular solids who seeks to uncover (via PCA) the fundamental dimensions underlying her heretofore intuitively generated house designs. She begins by taking three measurements on each of the houses she has designed over the years: F = perimeter of the frontal plane (the cross-section of the house parallel to the street), A = alley perimeter (the cross-section running perpendicular both to the street and to the ground), and B = basal perimeter (the distance required to walk around the outside of the house). For the houses she has designed, the correlations among these three variables are

F A B

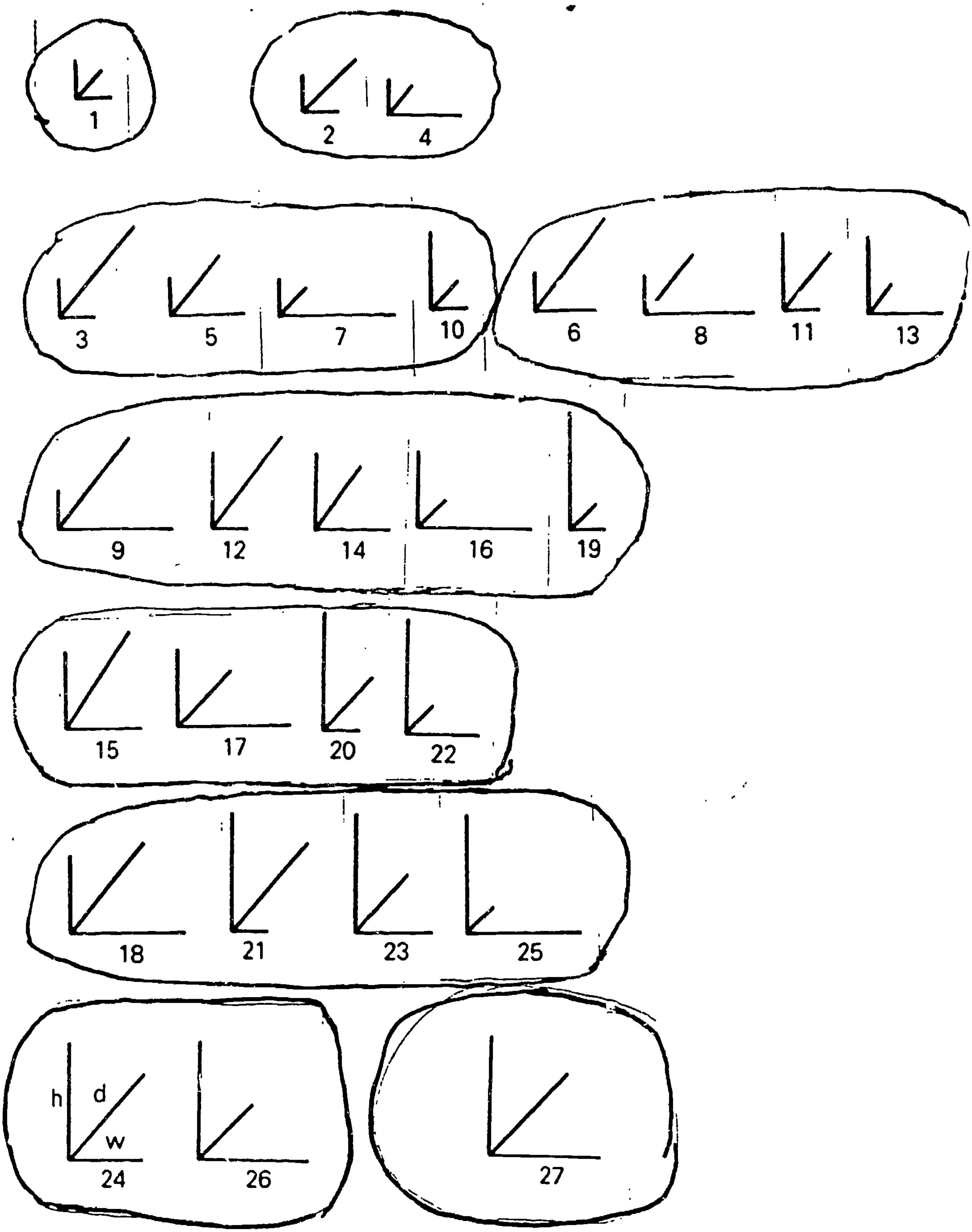
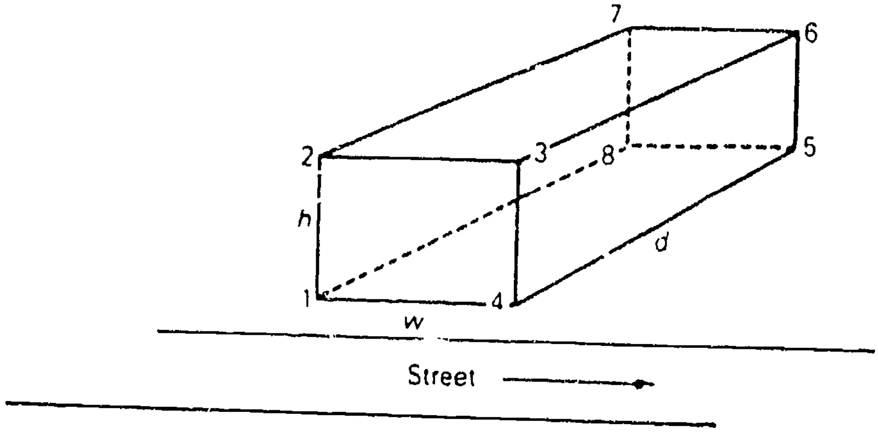


Figure 3. (See "Figure Captions" for explanation.)



Frontal size $(f) = \overline{1234} = 2(h + w)$

Alley size $(a) = \overline{3456} = 2(h + d)$

Base size $(b) = \overline{1458} = 2(w + d)$

Figure 6.3 Architectural dimensions of houses

(Harris, 1985)

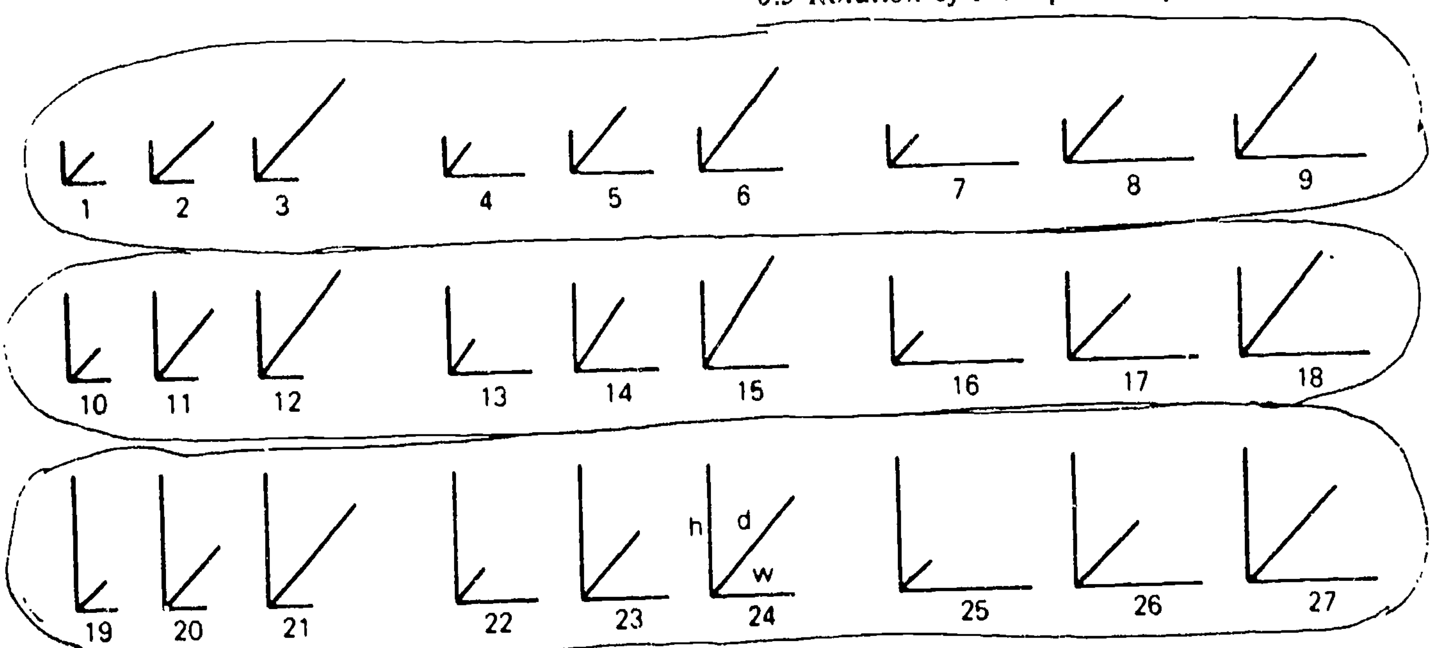


Figure 6.4 Schematic representation of 27 houses

Houses within each circled cluster have identical values (scores) on RPC1.

Figure 4. (See "Figure Captions" for explanation.)

	F	A	B
F	1	.5	.5
A	.5	1	.5
B	.5	.5	1

After conducting a PCA of this correlation matrix and then rotating the resulting components, she has settled upon a set of rotated components for which the structure matrix is

Correlations between Original Variables
and Rotated Components

	RPC1	RPC2	RPC3
F	.707	.707	0
A	.707	0	.707
B	0	.707	.707

We'll concentrate on coming up with an interpretation of the first rotated principal component (RPC_1). A very strong tradition in factor analysis dictates that she should base this interpretation on the structure coefficients (also legitimately called "loadings" when our factors are orthogonal) listed in the first column above.

Before proceeding with the usual approach of thinking about what physical property of a house would lead it to have high scores on F and A, let's consider an alternative approach more akin to confirmatory factor analysis: Try to think of a variable Q that would have a correlation of .707 with F, a correlation of .707 with A, and a zero correlation with B. I'm sure that a bit of introspection will convince you that this sort of indirect reasoning is not one even this statistically sophisticated audience is very good at -- at least not when working on an intuitive level, unaided by computation of actual correlations with various proposed variables.

Having failed at identifying RPC_1 as "that which would lead to the observed pattern of correlations with original variables," let's try instead the much more common route via examination of the obfuscation, i.e. via considering what would differentiate houses that have a high score on $F + A$ from those that have low scores on this linear combination. This, too, is not immediately obvious, but we can in this case supplement our search for such a variable by actually computing scores on $F + A$ and then arranging (schematic drawings of) the houses from low to high scores on $F + A$. This has been done in Figure 3.

Figure 3, showing houses ordered in terms of scores on F+A,
goes about here.

Still not a very easy concept-identification problem. However, we've made the problem difficult by constructing our ordering of the houses on $F + A$, which is NOT the linear combination of our 3 original variables that provides scores on RPC_1. To compute actual scores on RPC_1, so that we can construct an ordering of the houses on the dimension we're trying to identify rather than on an obfuscation thereof, we need only find the scoring coefficients by conducting an MRA in which the first column of the structure matrix provides the rxy vector and the matrix of correlations among F, A, and B provides Rxx. This yields a solution of

$zRPC_1 = .707zF + .707zA - .707zB$

with a corresponding R of 1.0.

Since the standard deviations of F, A, and B are all the same, our revelation (and thus RPC_1) is essentially F + A - B. Ordering the houses on this measure yields Figure 4.

Figure 4, with the 9 houses having a height of 1, the 9 having a
height of 2, and the 9 having a height of 3 forming 3 clusters,
goes about here.

Yes, Virginia, the dimension tapped by RPC_1 is height -- a very simple concept-identification problem when you base your interpretation on the scoring coefficients but a nearly impossible one if you insist on basing it on the structure coefficients.

D. Demo of Point 6 (Recovery/Masking of Relationship Imbedded in Different Contexts)

Consider a situation where there are 6 possible items to include in a test we're developing to predict job performance. The items might, e.g., be various job-related tasks tapping different skills needed in this occupation. Call these items *Q*1 through *Q*6. Assume, further, that all pairwise correlations among the 6 items are exactly .4. Finally, assume that in fact only *Q*1 and Q2 are truly necessary for job performance, so that *Q*1 + *Q*2 correlates perfectly with Y, an available and perfectly valid measure of actual performance on the job. Under these conditions the vector of correlations between Y and *Q*1 - Q6 will be rxy = [.837 .837 .478 .478 .478 .478]'. We're going to put together a test battery consisting of some subset of the 6 items (tasks) and interpret the relationship between that battery and *Y* by conducting an MRA of Y predicted from scores on the items in the battery, and then interpreting the regression variate either by examination of the regression coefficients themselves or by examination of the loadings of the original variables on the regression variate. (As we know, these will be directly proportional to the zero-order correlations with *Y*.) We measure the adequacy of each interpretation by computing the squared correlation between *Y* and the linear combination of items implied by our interpretation. Table 4 gives the results for four different test batteries.

Table 4
Validity of Scoring- vs. Structure-Coefficient Interpretations
of Test Battery/Job Performance Relationship

			r**2 between *Y* and	
Battery	Revelation	Obfuscation	Revelation	Obfusc
Q1,Q2	.5976(Q1+Q2)	.837(Q1+Q2)	1.00	1.00
Q1,Q2,Q3,Q4	.5976(Q1+Q2)	.837(Q1+Q2) + .478(Q3+Q4)	1.00	.89
Q1 thru Q6	.5976(Q1+Q2)	.837(Q1+Q2) + .478(Q3+Q4+Q5+Q6)	1.00	.82
Q1,Q3 thru Q6	.690Q1 + .092(Q3+...+Q6)	.837Q1 + .478(Q3+...+Q6)	.7531	.65

So long as the test battery includes the 2 crucial items, revelation of the regression variate correctly identifies the underlying relationship and yields the 1.0 correlation with _Y promised by _R. The obfuscation, however, (a) fails to tap the correct combination of items and (b) yields a correlation with _Y that is well below 1.0. Even when only one of the crucial items is included among the tests selected, the reve- lation yields a noticeably higher correlation with _Y than does the obfuscation.

E. Demo of Points 7 and 8 (Insensitivity of Structure-Coefficient-Based Interpretations to Differences in Underlying Relationships Yielding a Given Set of Zero-Order Correlations with Y)

Consider the very simple situation where we have two correlated predictors of *Y*, one of which has a substantial (say, .7) correlation with Y while the other correlates zero with Y. As the correlation between the two predictors varies from close to -1 through 0 to close to +1 the optimal combination of our two predictors for predicting scores on *Y* also changes from close to the simple sum of the two z-scores through $z1$ by itself to $z1 - z2$. Our scoring-coefficient-based interpretation will track these changes in the relationship between the predictors and *Y*, while our structure-coefficient-based interpretation will remain fixed at X1 by itself. Specifically,

Table 5
Squared Correlations between Interpretations and *Y* with Zero-Order
Correlations Fixed, Inter-predictor Correlations Varying

			Squared Correlation with			
r12	bz'	Obfuscn	Reveln	Obfuscn	z1 + z2	z1 - z2
0	z1	z1	.49	.49	.245	.245
.5	.933z1 - .467z2	z1	.653	.49	.163	.49
-.5	.933z1 + .467z2	z1	.653	.49	.49	.163
.7	1.372z1 - .961z2	z1	.961	.49	.144	.817
-.7	1.372z1 + .961z2	z1	.961	.49	.817	.144

Note that when $r_{12} > .5$ a simplified version of the regression variate (i.e., either $z1 + z2$ or $z1 - z2$) outperforms the structure-coefficient-based interpretation of that emergent variable as a predictor of Y, so that basing one's interpretation on $z1$ alone (i.e., on the structure coefficients) would lead to considerably worse prediction than basing it on a simplified interpretation, based on the scoring coefficients, of the underlying emergent variable as either the simple sum or the simple average of the two predictors.

F. Demo of Point 4: Extension of the Principle of Structure-Coefficient-Based Interpetation to Path Analysis Leads to Disastrous Interpretations

Path analysis of recursive models involves a series of MRAs, each involving prediction of a given endogenous variable from (a) all variables with causal arrows directed into it in our model and (b) all variables that precede it in the proposed causal order, whether or not our model predicts a nonzero path between a given exogenous variable and this endogenous variable. MRA (a) is used to estimate and test for statistical significance the paths our model specifies to be nonzero, while MRA (a) is used to test for statistical (non)significance the paths our model claims to be nonzero. (These correspond, respectively to James, Mulaik, & Brett's, 1982 Condition 9 and Condition 10 tests.)

Let's consider how a commitment to obfuscations might affect our interpretation of these two kinds of path-analytic MRA's. First, let's assume that we believe that any causal effect of $X1$ on Y is entirely mediated by $X2$ -- i.e. that

$$X1 \dashrightarrow X2 \dashrightarrow Y,$$

with no direct path from $X1$ to Y. We measure the relevant correlations, finding that $r_{1y} = .49$ (statistically significant for any reasonable N), $r_{2y} = .7$, and $r_{12} = .7$. For this model r_{12} and r_{2y} are estimates of the corresponding paths. so that we pass the Condition 9 tests (predicted paths significant) with flying colors. However, the crucial test of mediation is the sign and magnitude of the path from $X1$ to Y, which is estimated by conducting an MRA of Y predicted by both $X1$ and $X2$. In this MRA the regression coefficient for $X1$ and $X2$ will be zero and .7, respectively, so it would appear that our mediation hypothesis is fully supported by the data. (It also helps that the predicted correlation between $X1$ and Y under this model is .7*.7 = .49, as observed.)

UNLESS, of course, we have accepted the principle that regression variates are to be interpreted in terms of structure coefficients, which are in this case .7 and 1.0 for $X1$ and $X2$ in the present case. Thus our interpretation of our regression variate is that

$$z_y \text{ is best estimated by } .7z1 + z2$$

or some simplification thereof -- most likely z1 + z2, but certainly not a linear combination that omits $X1$ altogether. But the path coefficient from $X1$ to Y is by definition that weight it receives when it is used (together with other variables exogenous to Y. to predict (estimate) Y, so basing our interpretation of the regression variate from this MRA on the obfuscation amounts to asserting that the $X1$ to Y path is very NONzero, and our mediation model is thus wrong. This is of course an inevitable consequence of not allowing ourselves to look beneath the zero-order-correlation level of analysis, as interpretation of structure coefficients indeed does not permit. In fact, it can readily be shown that NO set of correlations among three variables can ever support a complete-mediation hypothesis if we insist on interpreting regression variates in terms of structure coefficients, since our structure coefficient will only be zero if the corresponding zero-order correlation with the endogenous variable is zero, and in that case we can hardly talk about mediation when there's nothing to be mediated.

A precisely parallel argument shows that interpretation of regression variates in terms of structure coefficients can never provide support for an hypothesis that the correlation between two variables is entirely spurious. (All we need do is to change our model for the above data to

$$X1 \leftarrow X2 \rightarrow Y$$

and carry out the same MRA as before.)

As a final example, consider a path model involving the variables F(emale gender), E(ngineering, rather than Education College faculty status), and S(alary). Female gender necessarily precedes the other two variables in the causal order, and we choose to hypothesize that which college you join affects your salary, rather than the other way around. Thus a complete path analysis of this fully recursive model would require that we take the correlation between F and E as our estimate of the path from F to E and obtain our estimates of the other two paths by regressing S on F and E jointly. If $r_{FE} = -.7$, $r_{ES} = .357$, and $r_{FS} = 0$, this MRA yields the equation

$$\text{estimated zy} = .7\text{ze} + .49\text{zf}$$

indicating (if we interpret our regression variate in terms of scoring coefficients) that the zero correlation between Female gender and Salary could be the result of a positive direct effect of F on S (once you're hired into a particular college being females has a moderate tendency towards getting you a higher salary) that is exactly counterbalanced by the negative indirect effect of being considerably less likely to be hired into the higher-paying college.

Such subtleties cannot, however, survive interpretation of regression variates in terms of structure coefficients, since the zero correlation between F and S guarantees a structure coefficient for F (a correlation of F with the regression variate) of zero. Thus a claim that structure coefficients are the appropriate basis for interpreting regression variates requires that we conclude that gender plays no role whatever in determining one's salary. (I started to add "within the context of these two predictors," but of course no such qualification applies to structure-coefficient-based interpretations, since the structure coefficient for a variable whose zero-order correlation with Y is zero will always be zero, no matter what other predictors are included.)

The general principle underlying this particular example is that interpreting structure coefficients rather than scoring coefficients will never permit a conclusion that an observed zero correlation could be the result of competing direct and indirect effects.

Of course researchers who prefer the schizophrenic approach of computing one's emergent variable from scoring coefficients but interpreting it on the basis of structure coefficients would feel no discomfort in accepting both the usual (and correct) path-analytic conclusion that S is affected positively both by F and E AND the obfuscation-based conclusion that S is solely determined by E.

Finally, as an aside, note that this example fits the classic prescription for the presence of enhancement (a variable whose individual correlation with the criterion is near zero nevertheless has a moderate to high regression coefficient when a second predictor -- the enhancer -- is added to the equation), but this enhancement effect would be very difficult to interpret as due to unreliable or invalid variance in F being suppressed by E. Rather, the addition of E to the equation has allowed us to identify an indirect source of negative influence of F on S (through E) that cancels out the positive direct influence of F on S. Insisting that any finding of regression coefficients whose pattern doesn't match that of the zero-order predictor-criterion correlations must be disregarded because of the likely presence of "suppression" would be just as effective at keeping us from uncovering competing effects of the sort involved in this (hypothetical) example as insisting that emergent variables be interpreted on the basis of structure coefficients.

Innoculation: Discussing the Arguments in Favor of Structure Coefficients

As the format of this symposium indicates, the position taken in this paper -- that interpretation of emergent variables should be based on the scoring coefficients that "operationally define" them, rather than on the more indirect and potentially misleading structure coefficients -- is hardly the modal position among multivariate statisticians. I can leave it up to my colleagues on this panel to document and demonstrate the ubiquity with which interpretation of emergants in terms of structure coefficients -- or at least co-equal consideration of structure coefficients and scoring coefficients -- is recommended and practiced. However, the power of recency effects and McGuire and Papageorgis' (1961) demonstration that they can be forestalled by approppriate "innoculation" techniques dictate that I rehearse for you a few of the counterarguments I suspect you're about to be exposed to, and point out to you some of the weaknesses of those counterarguments. (My comments in this section will be quite familiar to those of you who've read my 1989 *MBR* "Canonical Cautionary".)

1. "Your examples deal only with physical (or biological or rather abstract) variables. Surely structure coefficients would be more relevant than scoring coefficients for educational applications."

The lower validity of obfuscations (structure-coefficient-based interpretations of regression variates or discriminant functions) as compared to revelations (scoring-coefficient-based interpretations) is a function of the numerical properties of the data, not of the particular substantive application. Numerous examples of this shortcoming of obfuscations could be generated for any substantive area of education or psychology or sociology or ....

2. "Scoring coefficients and structure coefficients seldom differ as drastically as they do in your examples; if they aren't very different in a given situation, why not use the structure coefficients?"

Admitted: the closer the structure coefficients are to the scoring coefficients, the less mistaken an interpretation based on structure coefficients will be. But, why tolerate any misinterpretation when the correct basis for interpretation is so readily available? However, an ecological study of the degree of mismatch between scoring coefficients and structure coefficients found in "typical" data sets wold be useful in a number of ways, as would more explicit guidelines as to the properties of a data set that make it likely that the discrepancy between obfuscations and revelations will be minimal versus large and important. As a starting point, there is of course no discrepancy between the two in MRA if all predictors are mutually uncorrelated, since in that case the scoring coefficients are identical to the zero-order predictor-Y correlations and thus directly proportional to the loadings of original variables on the regression variate. By the algebraic equivalence of 2-group discriminant analysis (T^2) and MRA with a dichotomous outcome variable, there is also no difference between structure-coefficient-based and scoring-coefficient-based interpretations of discriminant functions when all dependent variables are mutually uncorrelated. How rapidly obfuscations and revelations diverge as the correlations among predictors or among dependent variables increase, and how sensitive the degree of divergence is to *differences* in these correlations, are appropriate objects of further empirical and analytic investigation.

4. "There are Monte Carlo results showing that scoring coefficients are less stable than structure coefficients."

This relative lack of stability of scoring coefficients is well established for MRA and 2-group discriminant analysis. Our esteemed chair, among others, has demonstrated, however, that the two sets of coefficients do not differ greatly in stability in Canona applications, which would of course include 3-or-more-group Manova. Nonetheless, even in applications where the greater stability of structure coefficients is firmly established, this simply demonstrates anew the fact that answering a univariate question (e.g., How well can a single variable approximate scores on the discriminant function?) "chews up" fewer degrees of freedom than does answering a multivariate question (e.g., What linear combination of measures defines the discriminant function?). If you are really that much more concerned with reliability than validity, why not just measure height (which is very stable for adults) and label it *paranoia* or *locusofcontrol* or *verbalintelligence*, etc., as needed?

As pointed out earlier, moreover, there is a much more serious kind of instability to which structure coefficients are more susceptible than are scoring coefficients: If there is a particular linear combination of variables that is an excellent predictor of *Y* or that very strongly differentiates between your two groups, interpretation of scoring coefficients will uncover that linear combination so long as all the variables included in the linear combination are included in your analysis. However, the interpretation you come up with based on structure coefficients will vary greatly as a function of the particular other variables included in your analysis.

4. "Regression coefficients (and, by analogy, 2-group discriminant-function coefficients) often behave in strange, unnatural ways, as when two variables that both have moderate correlations with a criterion variable are given very different weights in the regression equation."

This is a myth. Persistent, certainly, but a myth nonetheless. For the two-predictor case, it is readily shown that (a) two predictors, i and j, will receive identical z-score weights whenever $r_{iy} = r_{jy}$, and (b) the ratio of the squares of their z-score regression coefficients will equal the ratio of their ΔR^2 measures (the drop in R^2 that would follow upon dropping that predictor from the regression equation), so that the discrepancy in regression weights is exactly matched by the discrepancy in how much it would "cost" to drop one versus the other from the equation.

The situation is slightly more complicated in the general case, where

$$\frac{b_{z_i}^2}{b_{z_j}^2} = \frac{\Delta R_i^2}{\Delta R_j^2} \cdot \frac{1 - R_{j\cdot oth}^2}{1 - R_{i\cdot oth}^2},$$

where $R_{i\cdot oth}^2$ is the squared multiple corrlation between X_i and the remaining m-1 predictors. In other words, in the general case of $m \geq 3$ predictors, the discrepancy between z-score regression weights may partially reflect differences in how much of the explanatory weight a given predictor is "carrying" in the regression equation could be "absorbed" by other predictors in the equation -- hardly an arbitrary, coin-tossing allocation.

5. "It is very difficult to come up with substantive interpretations of sets of regression weights or discriminant function coefficients. It's much easier to come up with interpretations of structure coefficients. As an anonymous referee of the 'Canonical Cautionary' put it, 'Maybe my interpretations are delusional, but I prefer them to having none at all'."

This claim that regression weights and discriminant function coefficients are almost impossible to interpret is difficult for me to accept at face value, but it is a claim that I have heard repeated often by researchers I respect very much. My difficulty with the claim is that there are so many possibilities for post hoc interpretation of a set of p scoring coefficients as either

1. The sum or average of some subset of the p original variables (a total of $2^p - 1$ possibilities).
2. The difference between the unweighted average of one subset of the p original variables and the unweighted average of another subset of those variables (a total of $[e^p - 2^{p+1} + 1]/2$ possibilities).
3. The sum of one subset of the original variables minus the sum of another subset of those variables -- namely a (1,0,-1) simplification of the emergant -- namely a (1,0,-1) simplification of the emergent variable (same number of possibilities as for the difference between two unweighted averages of subsets).

Harris (1989) provides some explicit guidelines for coming up with interpretations of this sort. I also offer some suggestions as to the source(s) of the persistence of this perception of uninterpretability of scoring coefficients. Prominent among these is the fact that loadings of original variables on the regression variate are totally independent of the correlations among the predictors, so that examination of the vector of predictor-Y correlations usually "confirms" a structure-coefficient-based interpretation. Regression coefficients, however, do take these interpredictor correlations into account and thus aren't likely to be immediately obvious from inspection of the zero-order correlations between predictors and Y. Multivariate statistical procedures have the darndest habit of doing what they are designed to do (considering context in finding optimal linear combinations), rather than simply confirming what we already think we know from examining univariate statistics taken out of context, one variable at a time. The strongly entrenched feeling that multivariate results ought to match simple univariate results (as indicated, for instance, by the tendency to dismiss any lack of fit between r_{iy}s and b_{z_i} as due to "suppression" and by the common practice of restricting Monte Carlo analyses of MRA properties to population correlation matrices where, in essence, such lack of fit is ruled out) is analogous to asking a coach to select athletes for an all-star basketball team that can score the maximum possible points in (say) Olympic competition and then chastising her because the ten athletes selected aren't simply the ten with the highest individual points-per-game statistics.

(McFatter, 1979, demonstrates that any given situation in which a variable's regression coefficient is considerably higher than its zero-order correlation with the outcome variable is subject to a host of alternative explanations, only one of which involves suppression of irrelevant or invalid variance. Thus to dismiss a scoring-coefficient-based interpretation because the data are consistent with a suppressor-variable interpretation is fundamentally to engage in the circular reasoning that any set of regression weights that don't "match up" with the corresponding zero-order correlations are invalid because they don't match up with the corresponding zero-order correlations.)

Conclusion

I suspect that much of the current commitment to structure-coefficient-based interpretations of regression variates and discriminant functions would disappear if we more often took the additional step of computing a score for each subject on the linear combination of predictors or dependent variables implied by each of our verbal interpretations and then computing the correlation between each such simplified regression variate and Y or the t for the difference between our two groups with respect to each such simplified discriminant function. This would at once make our interpretation(s) more concrete and reveal how dramatic the shortfall between the performance of an obfuscation (its correlation with Y or its independent-means F) and the multiple R or T^2 yielded by the emergant it is supposedly interpreting, can be.

On the other hand, if you still find multiple univariate analyses much easier to deal with than truly multivariate analyses (in which case you will have plenty of company) go ahead and report your individual r_{iy}s or t_is (preferably with Bonferroni-adjusted alphas) and skip the MRA or T^2 step altogether (since that step will just add some computational labor to the process of reproducing your pattern of zero-order correlations or univariate ts in the guise of structure coefficients). Just don't mislead your readers by claiming that you're providing an interpretation of the (truly multivariate) regression variate or discriminant function.

References

Harris, M. B., Harris, R. J., & Bochner, S. (1982). Fat, four-eyed, and female: Stereotypes of obesity, glasses, and gender. *Journal of Applied Social Psychology, 12*, 503-516.

Harris, R. J. (1985). *A Primer of Multivariate Statistics*, Second Edition. San Diego: HBJ.

Harris, R. J. (1989). A canonical cautionary. *Multivariate Behavioral Research, 24*, 17-39.

James, L. R., Mulaik, S. A., & Brett, J. M. (1982). *Causal Analysis: Assumptions, Models, and Data*. Beverly Hills: Sage.

McFatter, R. M. (1979). The use of structural equation models in interpreting regression equations including suppressor and enhancer variables. *Applied Psychological Measurement, 3*, 123-135.

McGuire, W. J., & Papageorgis, D. (1961). The relative efficacy of various types of prior belief-defense in producing immunity against persuasion. *Journal of Personality and Social Psychology, 26*, 446-456.

Thompson, B. (1991). Need to find my copy of the paper Bruce kindly sent me a copy of demonstrating that canonical-variate weights are not necessarily less stable than structure coefficients in Canona.

Figure Captions

Figure 1. Male/female BBBB pairs skewering each other in beak-to-beak portion of their mating ritual.

Notes--Numbers in parentheses beside each bird represent the total head length and skull length, respectively, of that bird -- the original measures upon which the canonical analysis was performed. Matching of couples was on the basis of the structure-coefficient-based interpretation ("obfuscation") of the first canonical variate, namely skull length by itself.

Since a crucial phase of the mating ritual involved mutual stimulation of base of beak by tip of beak, mismatched beak length (difference between total head length and skull length -- the actual first canonical variate as reflected in the canonical variate scoring coefficients) led inevitably to skewering of one or the other bird, as depicted by drops of blood.

Figure 2. Male/female BBBB pairs with identical-length beaks happily conjoining beaks.

Notes--Again, numbers in parentheses represent total head length (MTL for the male bird, FTL for the female bird) and skull length (MSL and FSL). Matching of couples was on the basis of the actual first canonical variate, beak length = MTL - MSL for males, FTL - FSL for females, as indicated by the canonical variate scoring coefficients.

When matched on this basis, beak-to-beak mutual stimulation can be carried out safely.

Figure 3. Houses ordered in terms of scores on F + A.

Notes--For compactness, each house is represented by its left front edge. The vertical line represents height of the house (h); the horizontal line, width (w); the diagonal line, depth (d). Houses with identical values of F + A = 4h + 2w + 2d -- the structure-coefficient-based interpretation ("obfuscation") of the first rotated principal component -- are encircled by a solid line.

Attempting to identify the physical property that each cluster of houses has in common, and that leads to clusters towards the top of the figure sharing low values of that property while clusters towards the bottom share high values, would be very nearly impossible, and would certainly not lead to the correct interpretation of the first rotated principal component as simply the height of the house.

Figure 4. Houses clustered on the basis of F + A - B.

Notes--The clustering in this figure is based on the scoring-coefficient-based (i.e., correct) interpretation of the first rotated principal component. F + A - B = 4h, so encircling the 9 houses with a common value on this variable leads to 3 clusters of houses, with each of the 9 houses within the cluster having the same height but differing widths and depths. Identifying the first rotated principal component as simply the height of the house thus becomes a trivial concept-identification problem when one's interpretation is based on the scoring coefficients, rather than on the structure coefficients.