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ABSTRACT

Children begin school with the ability to use their informal and implicit conceptual knowledge to guide their problem solving, but shift to the use of superficial strategies in their attempts to solve word problems as they progress through school. This paper describes a study designed to investigate the effects of an instructional sequence that emphasized conceptual understanding of numerical operations using part-whole concepts and the integration of these understandings with students' problem solving knowledge. The study involving 384 fourth grade students from a large urban school system examined: (1) students' conceptual knowledge of addition, subtraction, multiplication, and division; (2) students' reasoning in the solution of one-step and two-step word problems; and (3) the effect of instruction using part-whole concepts on students' abilities to solve a variety of one-step and two-step word problems. Data were collected through written tests, interviews, and attitude surveys. Results indicated that instruction using part-whole concepts with work problems produced long-term achievement for all ability levels. Interviews indicated that after instruction, low and average ability students in the part-whole group exhibited concept-driven strategies during problem solving and improved ability to communicate their reasoning, whereas the practice and control group students exhibited little change in their approaches to problems. (Contains 30 references.) (MDH)

EFFECTS OF INSTRUCTION USING PART-WHOLE CONCEPTS WITH ONE-STEP AND TWO-STEP WORD PROBLEMS

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Children begin school with the ability to use their informal and implicit conceptual knowledge to guide their problem solving (Carpenter, Hiebert, & Moser, 1981; De Corte & Verschaffel, 1987; Moser & Carpenter, 1982), but as they progress through school many children shift to the use of superficial strategies in their attempts to solve word problems (Fischbein, Deri, Nello, & Marino, 1985; Sowder, 1988a, 1988b, 1989). These superficial strategies are based on the syntactic structure of word problems, key words, and procedural knowledge. Instead of relying on such strategies, children need to connect and extend their informal and implicit ideas to formal and explicit mathematical knowledge which they can use in problem solving.

This paper describes a study designed to investigate the effects of an instructional sequence that emphasized conceptual understanding of the operations using part-whole concepts and the integration of these understandings with students' problem solving knowledge (Huinker, 1990). More specifically, this study (1) examined students' conceptual knowledge of addition, subtraction, multiplication, and division; (2) examined students' reasoning in the solution of one-step and two-step word problems; and (3) investigated the effect of instruction using part-whole concepts on students' abilities to solve a variety of one-step and two-step word problems.

BACKGROUND

Models of word problem solving have highlighted the importance of constructing cognitive representations of problem situations that involve the quantities and relations among the quantities (De Corte & Verschaffel, 1981; Kintsch & Greeno, 1985; Riley, Heller, & Greeno, 1983). These representations allow children to concentrate on the underlying mathematical structure of the problem situation while disregarding irrelevant information. Additionally, children who are successful in learning mathematics appear to be those who continue to link their formal mathematical knowledge and use of symbols back to referents and informal knowledge, whereas weaker students allow this formal knowledge to become permanently disassociated from their informal knowledge (Resnick, 1986, 1988).

The development of a part-whole schema is a promising approach for connecting children's informal and emerging understandings of addition, subtraction, multiplication, and division to real-world situations and formal mathematical knowledge. Initially, children understand number through the "one more than" relationship based on counting. Eventually, children acquire a part-whole understanding of number based on the idea that a specific number can be thought of as the whole amount which can be separated into parts, as well as the parts can be combined to form the whole amount. Resnick (1983) asserted, "Probably the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole relationships.... This enrichment of number understanding permits forms of mathematical problem solving and interpretation that are not available to younger children" (p. 114).

This assertion is reflected in the most advanced level in the Riley et al. (1983) word problem solving model which involves the conceptual understanding of part-whole relations. They posited that "once children understand part-whole relations, they can use this knowledge to understand all change problems" (p. 181). Kintsch and Greeno (1985) agreed with this idea and asserted that children will also use their part-whole schema to solve comparison problems when their more-than and less-than schema is not sufficient. Morales, Shute, and Pellegrino (1985) have found that fifth and sixth grade students understood the difficult compare problems in a way that included part-whole relations. This knowledge of part-whole relations appears to be a key in advancing students' word problem solving ability because it provides a structure for interpreting the information given in word problems.This structure also allows children to broaden their conceptual knowledge of each operation to include many problem situations.

The value of using a part-whole approach to number has been observed with kindergarten children (Fisher, 1988). The part-whole schema for number provided a stronger conceptual base for emerging addition and subtraction strategies as demonstrated in a transfer of their part-whole knowledge to the solution of simple addition and subtraction change and combine word problems. Their mean score improved from 0.68 on the pretest to 5.63 on the posttest. The children in the other group scored 0.98 on the pretest and only 2.83 on the posttest. Dean and Malik (1986) have also found evidence of first and second grade students successfully using part-whole strategies.

In studies with third and fourth grade students, the author has also found that children can learn to successfully apply part-whole strategies to solve change and compare word problems (Huinker, 1989). The part-whole model appears to be effective for understanding and solving a variety of addition and subtraction word problems. In these same studies, students were also successful in using part-whole concepts to understand and solve multiplication and division word problems. The specific part-whole concepts which these students learned are listed in Figure 1. The development of these concepts appears to facilitate the linking of informal and formal mathematical knowledge during problem solving.

Addition:	Combining two parts to find the whole amount.
Subtraction:	Separating the whole amount into two parts.
Multiplication:	Combining equal parts to find the whole amount.
Division:	Separating the whole amount into equal parts.

Figure 1 Part-Whole Concepts for the Operations

It was hypothesized that the part-whole schema would provide a general knowledge structure for understanding addition, subtraction, multiplication, and division in a wide variety of problem situations, and that this schema would provide students with conditional knowledge for setting and evaluating goals and subgoals while problem solving. It was also hypothesized that emphasizing conceptual knowledge and mathematical language (both oral and written) with connections among real-world, concrete/pictorial, and symbolic representations would result in superior word problem performance. Further, it was hypothesized that the practice of just giving students problems to solve without this explicit instruction would not result in improved problem solving performance.

METHOD

Subjects

The subjects in this study were 384 fourth grade students from a large urban school system. Thirteen intact classes from seven schools participated in the study. Two classes were in self-contained settings and the others were in departmentalized settings which resulted in eight teachers being involved in the study. The socioeconomic status of the school population was characterized as low- and low-middle. The student population in this study was 99% African-American.

Procedures

This study used a pretest-posttest-retention test design with three treatment groups. One group received intervention based on a part-whole instructional sequence of 18 lessons developed by the author. Another group received practice in solving word problems without direct instruction. This group received 16 word problem practice worksheets that contained the same word problems used with the Part-Whole group. The other group acted as a control by continuing with their regular curriculum. The treatments were randomly assigned to schools. This resulted with four classes and three teachers in the Part-Whole group, five classes and three teachers in the Practice group, and four classes and two teachers in the Control group. The regular mathematics teacher for each class provided the instruction and administered the evaluation instruments. The teachers were given the materials for their treatment group, such as lesson plans, worksheets, transparencies, counters, and evaluation instruments and were inserviced by the investigator who met with them individually during one of their planning periods. The effects of the interventions were evaluated through a series of tests, interviews, and attitude surveys. The interviews were conducted by the investigator.

The study began in November and continued through the end of the school year. The addition/subtraction lessons and practice were presented over two weeks followed by a three to four week retention period. The multiplication/division lessons and practice were taught at the beginning of the second semester in February. These lessons and practice worksheets were presented over two to four weeks. The multi-step word problem lessons and practice were presented over two weeks following completion of multiplication and division. The final retention test was given three to four weeks later, in May, and included problems on all areas. Observations of the Part-Whole and Practice groups were made throughout the study with each teacher being observed at least twice.

Evaluation Instruments

Written tests. Students were given one-step and two-step word problem tests. On the one-step word problem tests, a response was scored as correct if the student was able to identify the relationships among the quantities in the word problem by writing a number sentence or expression that could be used to obtain the correct answer. Therefore, canonical and noncanonical number sentences and expressions were accepted as correct responses. Students were not required to figure out the answer even though many did. On the two-step word problem tests, students were asked to find the answer.

The semantic structure of the word problems and the numbers used in the problems were held constant across all tests, but the names of people and the type of objects in the word problems were varied as well as the order of presenting the problems. The numbers used in the problems were chosen carefully to prevent an inflation of correct responses from children using the immature strategy of letting the size of the numbers determine the operation (Sowder, 1988b). The number pairs used in the one-step word problems contained a one-digit number and a multiple of it. Some of the number pairs included: 8 and 24, 3 and 27, 5 and 30.

A pretest, posttest, and two retention tests were given on addition and subtraction. These tests also contained multiplication/division distractors. The pretest contained five addition/subtraction problems and the posttest and retention tests contained 11 addition/subtraction problems. Each combine, change, and compare problem was of a different type (Riley et al., 1983). Reliability coefficients were calculated with the Kuder-Richardson formula 20 for each test and ranged from 0.47 on the pretest to 0.70 on the final retention test.

A pretest, posttest, and retention test were given on multiplication and division. These tests contained addition/subtraction distractors. The pretest contained two multiplication, two partitive division, and two measurement division problems. The posttest contained nine problems, three of each type. The reliability coefficients were 0.67, 0.81, and 0.79 respectively.

An initial test, pretest/transfer test, posttest, and retention test were given on two-step word problems. The initial test and pretest/transfer test contained four problems. The posttest and retention test all six problems. The initial test was given at the very beginning of the study along with the addition/subtraction pretest. Following instruction and practice on all four operations the transfer test was given to determine if the instruction using part-whole concepts or practice with one-step word problems would improve performance on two-step word problems. This test was also used as a pretest for the work on multi-step word problems. The reliability coefficients ranged from 0.56 on the initial test to 0.82 on the retention test.

Interviews. The interviews examined the nature of the students' conceptual knowledge of addition, subtraction, multiplication, and division and their reasoning when solving one-step and two-step word problems. Individual interviews were held with three students from each class, a total of 39 students, at the beginning and end of the study. These students were selected by the classroom teacher as having high ability, average ability, or low ability in mathematics. The same students, except for three, were interviewed again during the final retention period. Three of the students moved during the study so three other students of comparable ability were interviewed. Each interview was audiotaped and notes were taken on the students' physical actions, such as the use of counters and their fingers, and reactions that were not captured on the audiotape. The tapes were transcribed incorporating the interviewer's notes.

Attitude Surveys. An attitude survey was administered to measure students' disposition towards word problems, number problems, and math, and to measure students' self-assessment of their ability in math and their ability to solve word problems and number problems. Eight statements were read out loud to the class by their teacher, as students circled either a smiling face, a frowning face, or a sad face to reflect their reaction to the statement. The same attitude survey was given at the beginning and end of the study.

Instruction Using Part-Whole Concepts

Each lesson generally involved working through two or three word problems as a whole class followed by some independent work. The teacher displayed a word problem on the overhead projector and then used a three-step heuristic to guide the students' thinking for each word problem: (1) thinking about the structure (without numbers); (2) representing the situation; and (3) solving the problem (Rathmell & Huinker, 1989).

First, the teacher and students discussed the structure of the problem situation using general terms rather than the specific numbers given in the problem. For example, perhaps LaVaugh has a package of cookies and he is going to share them with his friends. Next the students would represent the problem situation using counters at their desks. This representation was then modeled on the overhead projector. The teacher and students would discuss the representation, noting that they had taken the whole amount and made equal parts. This representation was then connected to a mathematical operation, a number sentence was written, and a solution was reached.

The work with one-step word problems was then extended to multi-step word problems with explicit attention given to an adaptation of the word problem schema of one-step word problems (De Corte and Verschaffel, 1981). Instruction focused on the word problem schema of multi-step word problems: (1) an unstated "hidden" question needs to be found and answered before the stated question can be answered, (2) two or more computations may need to be performed to solve the problem, and (3) the result of the first computation is to be used in the next computation.

RESULTS

Written test data were analyzed separately for total scores on addition/subtraction, multiplication/division, and two-step word problems. Then results were examined further by partitioning the scores into three ability levels, high, average, and low, determined by the pre-test scores. Results were analyzed using analyses of variance and covariance with the pretest as a covariate and repeated measures with contrasts.

Addition and Subtraction

Figure 2 shows the percent correct on each of the four addition/subtraction tests. The trend in the graph shows the superior performance of the Part-Whole group.

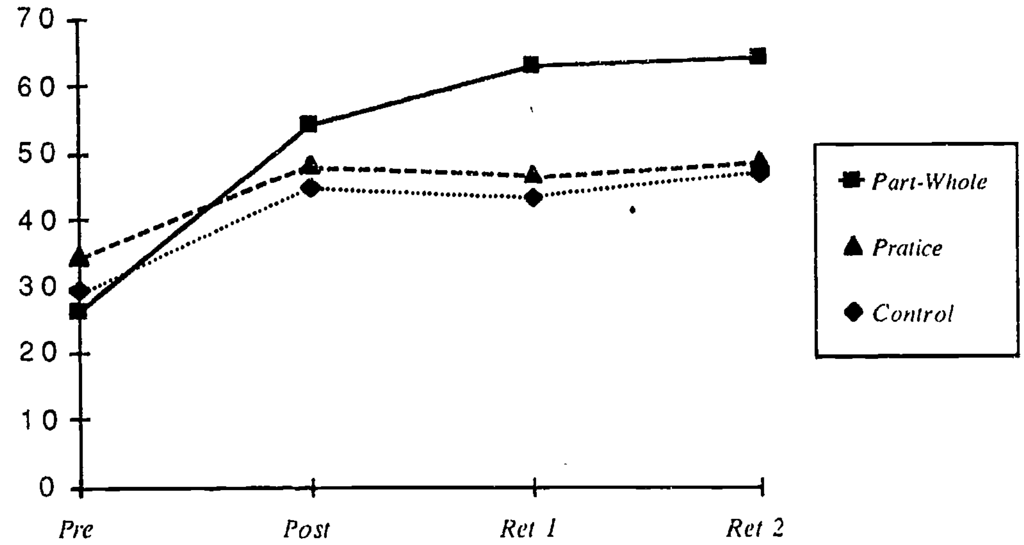


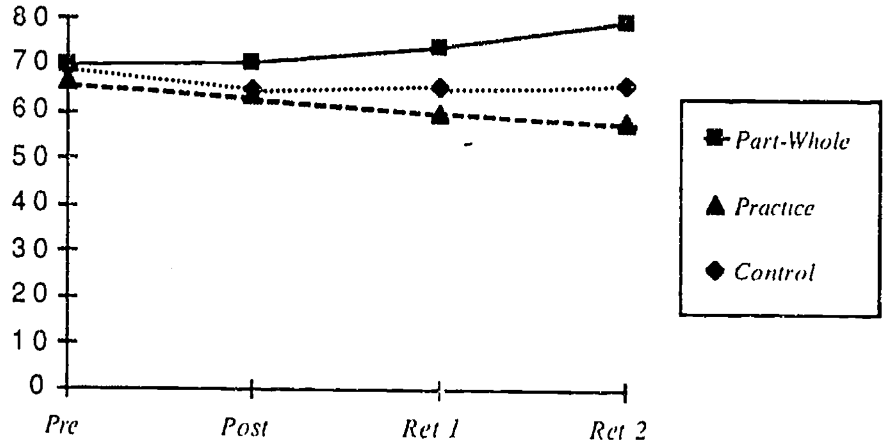
Figure 2 Addition and Subtraction Test Results (Percent Correct)

The mean number of items correct on each test are shown in Table 1. The pretest on addition/subtraction revealed no significant differences among the Part-Whole, Practice, and Control groups (F=2.41, p=.092). Significant differences did exist among the groups on the posttest and retention tests. The Part-Whole group significantly outperformed both the Practice and Control groups on the posttest (F=9.05, p=.003; F=11.62, p=.001, respectively), the first retention test (F=33.29, p=.000; F=34.02, p=.000, respectively), and the second retention test (F=26.23, p=.000; F=24.54, p=.000, respectively). There were no significant differences between the Practice and Control groups on any of the addition/subtraction tests.

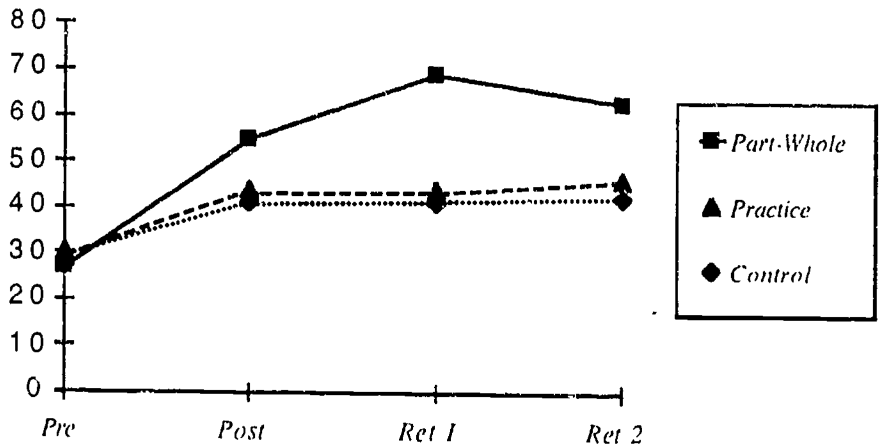
Table 1 Addition and Subtraction Test Results

	Pre (5 items) Mean (SD)	Post (11 items) Mean (SD)	Retention 1 (11 items) Mean (SD)	Retention 2 (11 items) Mean (SD)
Part-Whole (n=66)	1.303 (1.277)	5.985 (2.440)	6.393 (2.734)	7.061 (2.717)
Practice (n=121)	1.719 (1.253)	5.314 (2.429)	5.132 (2.598)	5.380 (2.395)
Control (n=83)	1.482 (1.310)	4.904 (2.382)	4.747 (2.749)	5.157 (2.796)

High Ability Students



Average Ability Students



Low Ability Students

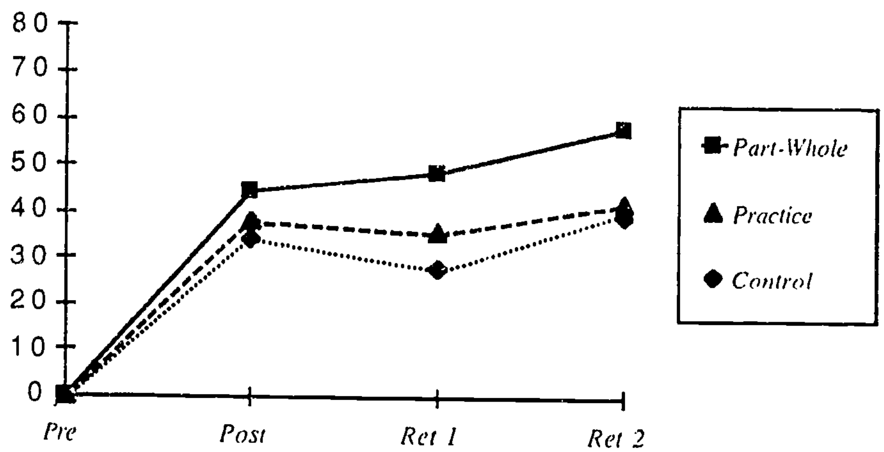


Figure 3 Addition and Subtraction Test Results by Ability Levels (Percent Correct)

The trends in achievement with percent correct among the three ability levels for the Part-Whole, Practice, and Control groups are shown in Figure 3. All low groups showed improvement from the pretest to the posttest, but only the Part-Whole group continued to show improvement on the Retention 1 test. The long-term effect of the part-whole cognitive structure for these students is evident from their high achievement on the Retention 2 test. The average Part-Whole group also made tremendous gains from the pretest to the posttest to the first retention test in comparison to the other average groups who showed a very small change in performance.

The mean number of correct items and standard deviations are displayed in Table 2. The achievement of the low Part-Whole group was higher than the Practice and Control average groups on the posttest and both retention tests, and the achievement of the average Part-Whole group was higher than the Practice and Control high groups on the posttest and first retention test.

Table 2 Addition and Subtraction Test Results by Ability Levels

	Pre (5 items) Mean (SD)	Post (11 items) Mean (SD)	Retention 1 (11 items) Mean (SD)	Retention 2 (11 items) Mean (SD)
High				
PW (n=12)	3.500 (0.674)	7.750 (2.417)	8.167 (2.038)	8.750 (2.221)
PR (n=35)	3.314 (0.471)	7.000 (2.544)	6.571 (2.392)	6.400 (2.725)
C (n=42)	3.474 (0.612)	7.158 (2.544)	7.211 (2.859)	7.263 (2.557)
Average				
PW (n=33)	1.333 (0.479)	6.030 (2.069)	7.545 (2.575)	6.879 (2.619)
PR (n=61)	1.508 (0.504)	4.803 (2.056)	4.803 (2.372)	5.098 (1.886)
C (n=42)	1.357 (0.485)	4.500 (1.991)	4.500 (2.244)	4.643 (2.526)
Low				
PW (n=21)	0.000 (0.000)	4.905 (2.488)	5.286 (2.648)	6.381 (2.837)
PR (n=25)	0.000 (0.000)	4.200 (1.915)	3.920 (2.597)	4.640 (2.644)
C (n=22)	0.000 (0.000)	3.727 (1.579)	3.091 (2.045)	4.318 (2.679)

The average Part-Whole group significantly outperformed both the average Practice and Control groups on the posttest (F=7.12, p=.009; F=10.28, p=.002, respectively), the first retention test (F=26.63, p=.000; F=29.88, p=.000, respectively), and the second retention test (F=13.28, p=.000; F=17.69, p=.000, respectively) giving evidence to the strength of the part-whole structure and its long-term effects for generating understanding. Significant differences also occurred on the first retention test between the low Part-Whole and Control groups (F=8.23, p=.005) and on the second retention test between the Part-Whole and Practice groups (F=4.69, p=.034) and between the Part-Whole and Control groups (F=6.20, p=.015). The low Part-Whole group scored higher than the other low groups on theses tests. The only significant difference which occurred for the high ability groups was between the Part-Whole and Practice groups on the second retention test (F= 3.73, p = 0.29). Practice in solving word problems without any explicit instruction did not produce any visible gains in achievement as there were no significant differences between any of the Practice and Control ability groups on any of the addition/subtraction tests.

Multiplication and Division

The percent correct on each of the multiplication/division tests and the trends in performance are shown in Figure 4. The dramatic improvement of the Part-Whole group from the pretest to the posttest contrasts sharply to the slight changes in performance of the Practice and Control groups.

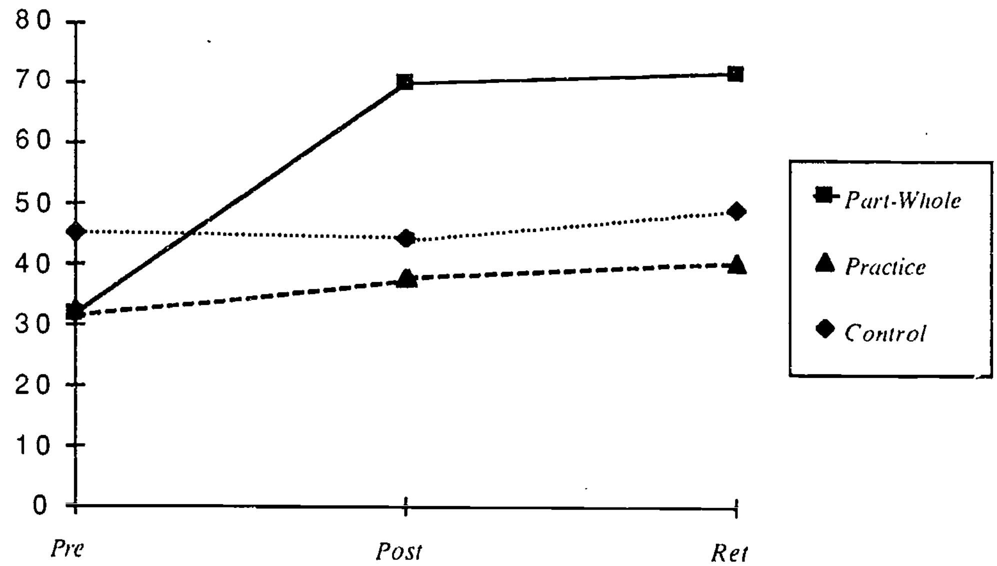


Figure 4 Multiplication and Divisio:ı Test Results (Percent Correct)

The mean number of items correct on each test are displayed in Table 3. The pretest on multiplication/division revealed significant differences among the Part-Whole, Practice, and Control groups. The Control group scored highest on the pretest and the Part-Whole group scored the lowest. Controlling for the pretest differences, significant differences occurred on the post and retention tests. The Part-Whole group exceeded the Practice group and the Control group in

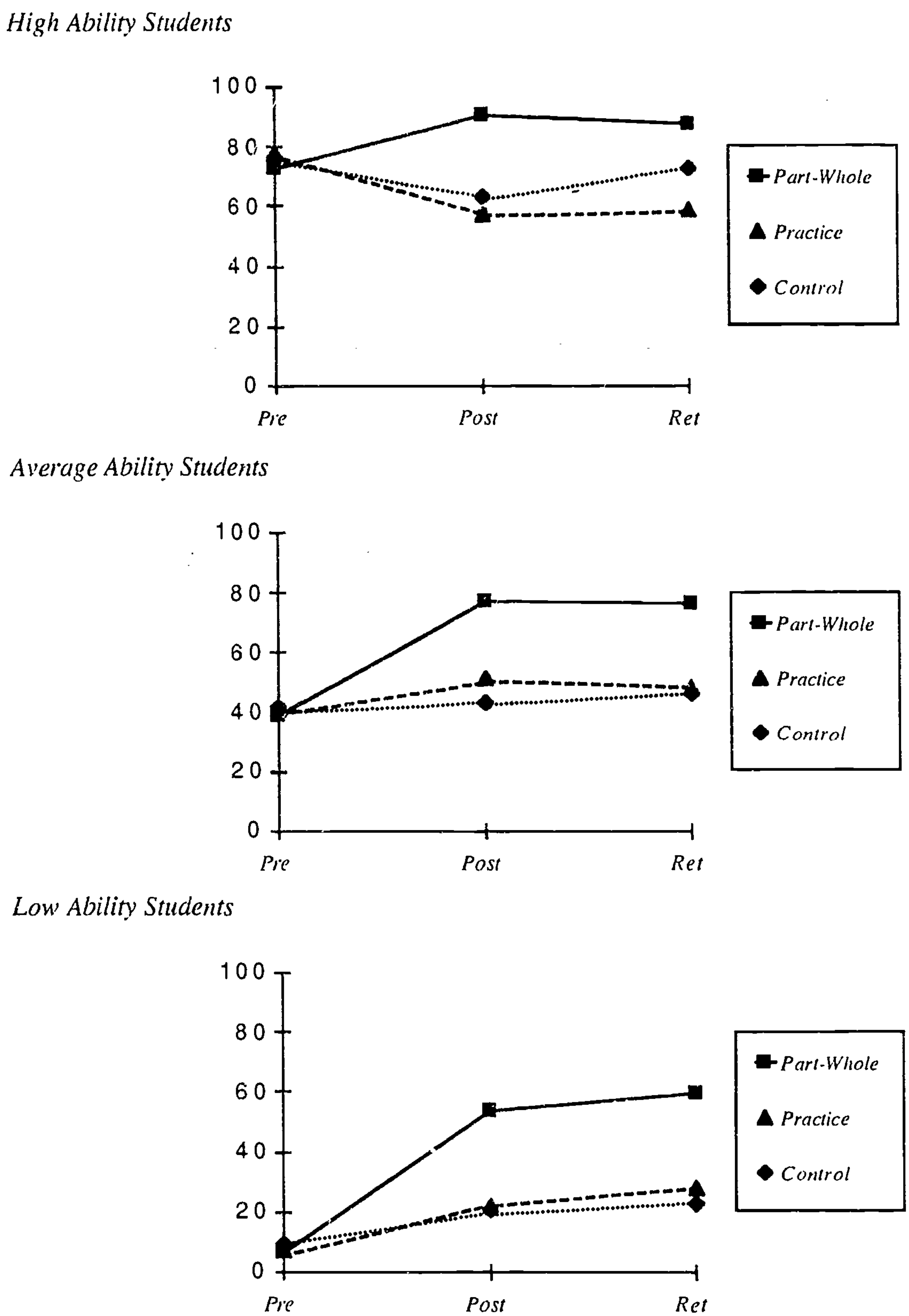


Figure 5 Multiplication and Division Test Results by Ability Levels (Percent Correct)

achievement on both the posttest (F=123.07, p=.000; F=114.61, p=.000, respectively) and retention test (F=122.15, p=.000; F=95.68, p=.000, respectively). While the Part-Whole group was the lowest on the pretest, they scored 32.1 percentage points higher than the Practice group on the posttest. No significant differences were found between the Practice and Control groups.

Table 3 Multiplication and Division Test Results

	Pre (6 items) M (SD)	Post (9 items) M (SD)	Retention (9 items) M (SD)
Part-Whole (n=104)	1.913 (1.559)	6.288 (2.157)	6.442 (2.158)
Practice (n=124)	1.976 (1.823)	3.403 (2.671)	3.661 (2.409)
Control (n=102)	2.706 (1.727)	3.980 (2.677)	4.441 (2.646)

The trends in achievement with percent correct among the three ability levels for the Part-Whole, Practice, and Control groups are shown in Figure 5. All Part-Whole ability groups made impressive gains from the pretest to the posttest, especially the low group. In comparison, the achievement of the Practice and Control ability groups improved slightly or even dropped.

The mean number of correct items are displayed in Table 4. The achievement of the low Part-Whole group was higher than the average Practice and Control groups on the posttest and retention test and was even higher than the high Practice group. The achievement of the average Part-Whole group was higher than the high Practice and Control groups on the posttest and retention test.

Table 4 Multiplication and Division Test Results by Ability Levels

	Pre (6 items) Mean (SD)	Post (9 items) Mean (SD)	Retention (9 items) Mean (SD)
High			
PW (n=19)	4.368 (0.684)	8.105 (1.243)	7.895 (1.729)
PR (n=30)	4.667 (0.758)	5.133 (2.700)	5.300 (2.292)
C (n=37)	4.568 (0.765)	5.703 (2.570)	6.541 (2.116)
Average			
PW (n=42)	2.357 (0.485)	6.952 (1.807)	6.881 (1.966)
PR (n=31)	2.452 (0.506)	4.613 (2.108)	4.355 (2.229)
C (n=36)	2.528 (0.506)	3.917 (1.888)	4.194 (2.266)
Low			
PW (n=43)	0.395 (0.495)	4.837 (1.864)	5.372 (2.012)
PR (n=63)	0.460 (0.502)	1.984 (2.091)	2.540 (1.958)
C (n=29)	0.552 (0.506)	1.862 (2.083)	2.069 (1.163)

No significant pretest differences existed among any of the ability groups. The high Part-Whole group significantly outperformed both the high Practice and Control groups on the posttest

(F=31.32, p=.000; F=21.27, p=.000, respectively) and retention test (F=27.37, p=.000; F=8.71, p=.004, respectively). There was also a significant difference between the high Practice and Control groups which occurred on the retention test (F=8.41, p=.005) and surprisingly it favored the Control group.

The average Part-Whole group significantly outperformed the average Practice and Control groups on the posttest (F=27.93, p=.000; F=51.27, p=.000, respectively) and retention test (F=27.91, p=.000; F=35.56, p=.000, respectively). The low Part-Whole group also significantly outperformed both the low Practice and Control groups on the posttest (F=59.07, p=.000; F=46.31, p=.000, respectively) and retention test (F=71.14, p=.000; F=68.73 p=.000, respectively). There were no significant differences between the average or the low Practice and Control groups.

An item analysis was conducted on all multiplication/division tests. The tests contained problems with small numbers (basic facts) and problems with larger numbers. Common errors were to add or divide for all groups. The multiplication problem with smaller numbers was much easier for all the groups on all the tests. On the posttest and retention test, almost 90% of the Part-Whole group answered the problem with small numbers correctly and over 50% responded correctly to the problems with larger numbers. These students had a good sense of multiplication in problem situations with small numbers while their operation sense with larger numbers was still developing. In the problems with larger numbers, the responses of the students from the Practice and Control groups appeared to result from guessing among addition, multiplication, and division.

The division problems on the tests involved both measurement and partitive situations with small and large numbers. The common errors were more varied for all groups on these problems with many students often selecting addition, subtraction, or multiplication. The problems with smaller numbers were easier than those with larger numbers.

Two-Step Word Problems

The initial test on two-step word problems given at the beginning of the study showed no significant differences among the Part-Whole, Practice, and Control groups (F=0.187, p=.830).

At the completion of work with the four operations and one-step word problems, a test on two-step word problems was given to measure the transfer to two-step word problem performance. This transfer test also served as the pretest for the instruction/practice on two-step word problems. Figure 6 shows the percent correct on each of the four-two-step word problem tests.

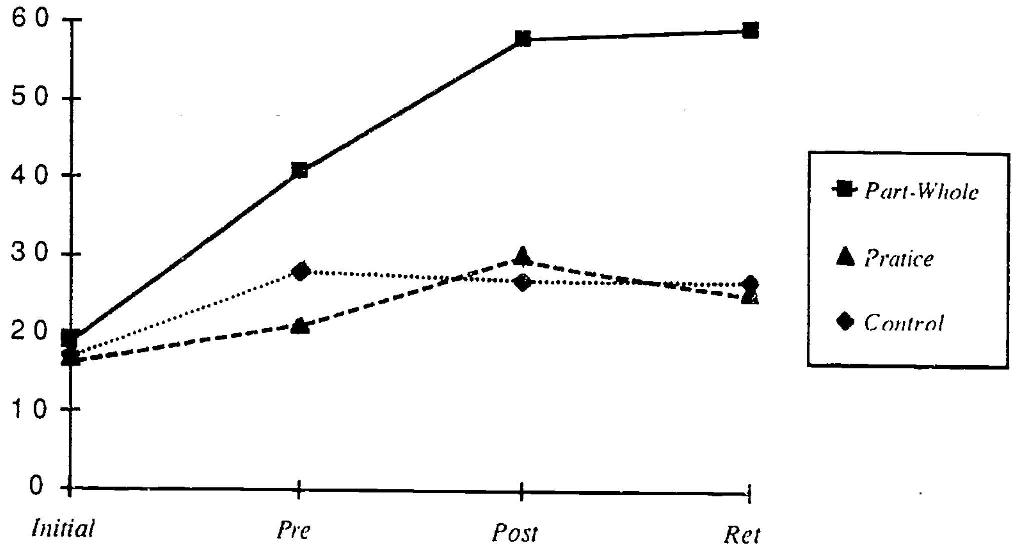


Figure 6 Two-Step Word Problem Test Results (Percent Correct)

Table 5 displays the mean number of items correct for each of the two-step word problem tests. The transfer test showed significant differences among the groups. The Part-Whole group significantly outperformed both the Practice (F=17.47, p=.000) and the Control (F=6.36, p=.012) groups on the two-step word problem transfer test. No significant differences were found between the Practice and Control groups, but the Control group scored higher on the transfer test than did the Practice group. This leads to the speculation that perhaps practice with one-step word problems hindered the performance of these students on two-step word problems. Their immature strategies may have been strengthened while their intuitive and informal problem solving ability may have been further suppressed.

Even with the increase in achievement for the Part-Whole group, the mean number of correct items was still less than 50 percent. The part-whole intervention did transfer to two-step word problem performance, but the intervention itself was not enough to produce a more desirable performance level. This established the need for explicit instruction on multi-step word problems.

The superior trend in performance of the Part-Whole group is clearly shown in the graph in Figure 6 for the post and retention tests. The Practice group made some improvement on the posttest but then their achievement dropped on the retention test.

Table 5 Two-Step Word Problem Test Results

	Initial (4 items) Mean (SD)	Pre/Transfer (4 items) Mean (SD)	Post (6 items) Mean (SD)	Retention (6 items) Mean (SD)
Part-Whole (n=93)	0.750 (0.946)	1.624 (1.510)	3.473 (1.874)	3.559 (1.942)
Practice (n=138)	0.667 (0.999)	0.848 (1.196)	1.826 (1.742)	1.522 (1.572)
Control (n=108)	0.685 (0.882)	1.111 (1.225)	1.620 (1.888)	1.620 (1.898)

Controlling for pretest differences among the groups, significant differences occurred between the Part-Whole and Practice groups and between the Part-Whole and Control groups on both the posttest (F=24.05, p=.000; F=52.81, p=.000, respectively) and retention test (F=51.00, p=.000; F=57.25, p=.000, respectively). A significant difference also occurred between the Practice and Control groups on the posttest (F=7.92, p=.005), but this difference no longer existed on the retention test. Practice with two-step word problems caused some short term improvement, but within three to four weeks the achievement of these students had already dropped to a mean score lower than the Control group. Gains were short-lived, providing added support for the cognitive structure built in the Part-Whole group.

The trends in achievement throughout the pretest, posttest, and retention test among the three ability levels for the Part-Whole, Practice, and Control groups are shown in Figure 7. The low and average Part-Whole groups showed great improvement from the pretest to the posttest. The low Practice group also showed good gains from the pretest to the posttest, while the average and high students from the Practice group showed a drop in performance.

The mean number of correct items are displayed in Table 6. The achievement of the low Part-Whole group was higher on the posttest and retention test than the average Practice and Control groups.

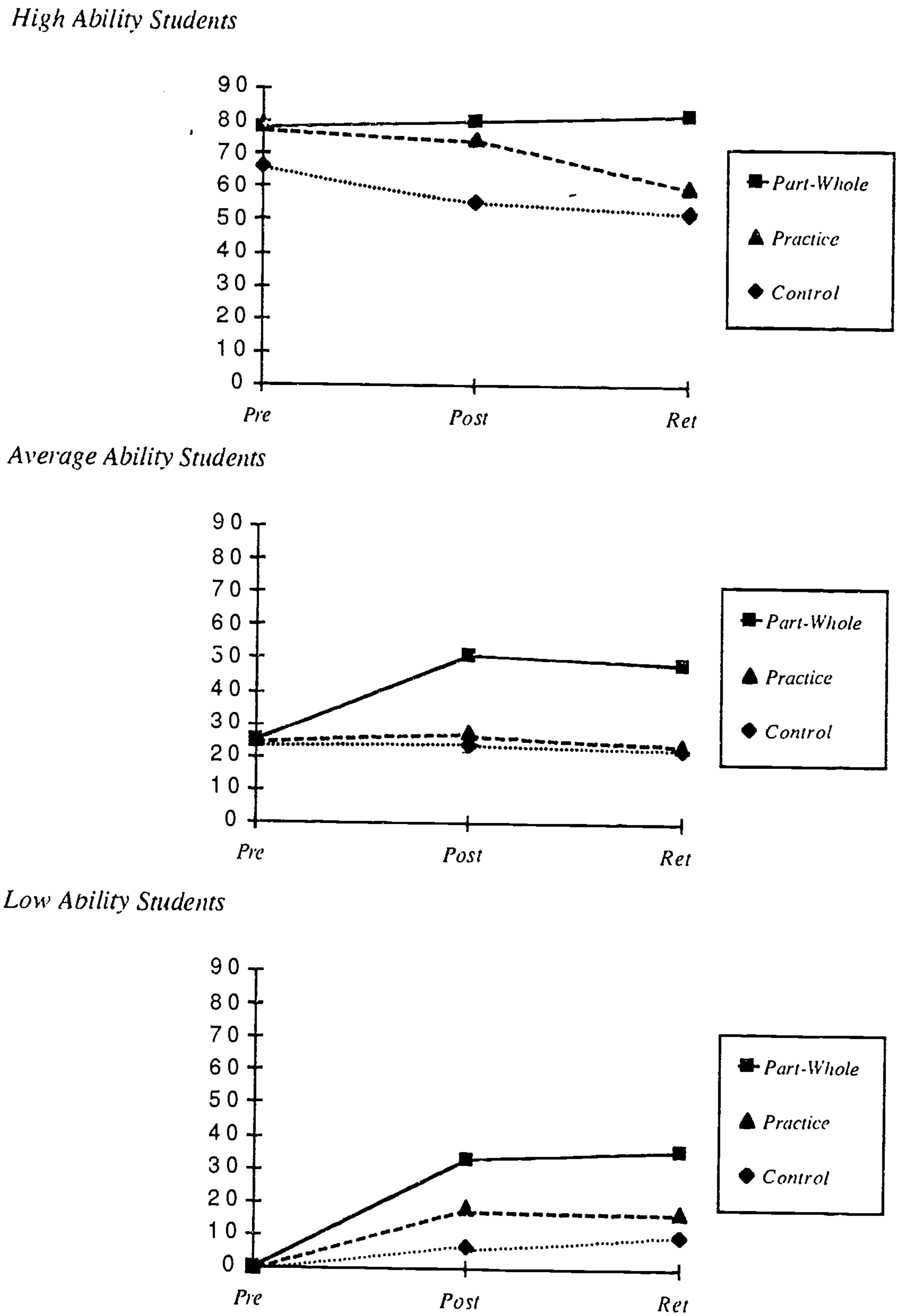


Figure 7 Two-Step Word Problem Test Results by Ability Levels (Percent Correct)

Table 6 Two-Step Word Problem Test Results by Ability Levels

	Pre (4 items) M (SD)	Post (6 items) M (SD)	Retention (6 items) M (SD)
High			
PW (n=42)	3.119 (0.803)	4.786 (1.298)	4.905 (1.226)
PR (n=24)	3.167 (0.761)	4.500 (1.532)	3.625 (1.789)
C (n=35)	2.657 (0.765)	3.343 (1.939)	3.143 (2.277)
Average			
PW (n=15)	1.000 (0.000)	3.050 (1.638)	2.900 (1.683)
PR (n=41)	1.000 (0.000)	1.659 (1.175)	1.415 (1.161)
C (n=27)	1.000 (0.000)	1.444 (1.368)	1.370 (1.418)
Low			
PW (n=31)	0.000 (0.000)	1.968 (1.378)	2.161 (1.695)
PR (n=73)	0.000 (0.000)	1.041 (1.111)	0.890 (1.021)
C (n=46)	0.000 (0.000)	0.413 (0.858)	0.609 (0.774)

Controlling for pretest variations, the high Part-Whole group significantly outperformed the high Control group on the posttest (F=9.29, p=.003) and retention test (F=12.50, p=.001) and the high Practice group on the retention test (F=9.22, p=.003). The average Part-Whole group significantly outperformed the average Practice and Control groups on the posttest (F=14.29, p=.000; F=16.26, p=.000, respectively) and retention test (F=15.75, p=.000; F=14.28, p=.000, respectively). No significant differences occurred between the high or average Practice and Control groups.

The low Part-Whole group significantly outperformed the low Practice and Control groups on the posttest (F=15.34, p=.000; F=36.76, p=.000, respectively) and retention test (F=27.44, p=.000; F=34.56, p=.000, respectively). A significant difference also occurred between the low Practice and Control groups on the posttest (F=9.14, p=.003), but this difference no longer existed on the retention test. Practice with two-step word problems caused some short term improvement for the low students.

An item analysis was conducted for the two-step word problem tests. The easiest problem involved the use of addition and subtraction. All other problems contained at least one multiplicative relationship. The hardest item involved two multiplicative relationships and contained one number written in its oral language form, "four", rather than as a numeral. The combination of these factors most likely caused its difficulty.

The error of simply adding all the numbers in the problem is often thought to be very common. This error did occur, but it was not the most common. Students were more likely to perform simply one operation with a pair of numbers or to perform two different operations. When students did perform two different operations it was common for students to execute two unrelated computations in which the result of the first step was not used in the second step. The reasoning of this Practice group student in the post-interview may explain some of these errors. The student shared the rule, "when there's three numbers you have to use two operations", that she either invented or learned from someone else which guided her problem solving.

S23: (Wrote 8+4=12 and then wrote 12–4=8.) Eight.

I: Tell me what you did here. Why did you add those numbers and then ...

S23: Because there's three of them [numbers] in the word problem and I used two [operations] and I had got 12 and I got 8.

I: So, when there's three numbers in a word problem you have to do a couple different things?

S23: (She nodded her head yes.)

I: Why did you add eight and four first?

S23: Because I didn't want to use subtract.

I: And then why did you subtract the four?

S23: Because that's the other one ... oh!

I: What? Go ahead if you want to change something.

S23: I used the four two times rather than the five. (She changed the 12–4=8 to 12–5=7.) Seven.

I: So why did you add the eight and the four, and not the eight and the five or the four and the five?

S23: ... (She shrugged her shoulders.)

The four lessons on two-step word problems positively affected the achievement of the Part-Whole group on the two-step word problem posttest and retention test, whereas the Control group showed very little change in performance. The Practice group did show some short-term improvement from the pretest to the posttest, but then their performance dropped on the retention test. Most of the growth shown by the Part-Whole group was a result of the gains made by the low and average ability students. These students can be successful problem solvers when given explicit instruction that helps them to develop a strong conceptual knowledge base and to understand the nature of multi-step word problems.

Interviews

Conceptions of the Operations. Students were asked to explain the meaning of each operation in the pre-interview. Children's lack of language with which to communicate their thinking about mathematics and problem solving was evident throughout the interviews. This observation was also made by Zweng (1979). Many of the student definitions for the operations revealed a link to procedural knowledge, either algorithmic or symbolic, rather than conceptual knowledge.

A student defined addition, "It's like, it's like a cross and number down here and a number up here." Another stated, "It means to put two things together and then add them like to see what the amount is at the end." Definitions for subtraction included: "Subtraction means borrow." and "Take the number at the top and the number at the bottom and subtract how many the number is at the bottom. And then put the answer down." Definitions for multiplication included: "Multiplication means to take the number at the top and the number at the bottom and then take the number at the top and take it how many ever times that the second number is." and "It's kind of like adding. You take that number and then you count that same number how many times it was times." and "Multiplication means when you have to carry a two or a one." Some definitions for division included: "Like there is times and the answer from the times you put first, and then you put a number from the times table and then you add it and it becomes the other number from the times table." and "It means to take the number and then however many times it can go into the other number and then put that at the top or at the end." and "It's to see how many numbers are in a number."

Most students referred to addition as add or plus, to subtraction as take away, to multiplication as times or as being like addition, and to division as either being like subtraction or being like multiplication. It is important for students to understand the relationships between and among the operations, but these students did not appear to understand these relationships. The students were using one operation to define another operation, almost like a circular definition, without having a good understanding of any of them.

Missing Connections. Children need to develop conceptual knowledge of the operations that is useful for problem solving. This knowledge involves connections among conceptions of the operations, real-world representations, concrete and pictorial representations, and symbolic representations. In the pre-interviews students were asked to pose problems that matched a given mathematical expression. Very few students were capable of posing an appropriate word problem or acceptable description of a situation. Most of the addition and subtraction responses showed a weak connection between symbolic and real-world representations, whereas most of the multiplication and division responses showed no connection. Some students were also observed getting multiplication and division confused. Explicit instruction needs to address the similarities and differences of these operations, as well as addition and subtraction.

Some of the students' stories for 13 + 8 = □ included: "There were 13 pencils in the store and 8 papers." "You have 13 pieces of candy and I have eight." "Well, you could have 13 balls and 8 pieces of candy." For 17 – 4 = □, some of the stories were: "There were 17 fish and 4 went back to the mother fish and... and how many in all, I mean, how many were left?" "John had 17 chips and Brandon has 4 chips. How many does John ... how many does Brandon ... how many does Brandon, how many chips he has?"

Some of the stories for 6 x 4 = □ included: "It's going to be six pictures, six pictures times four pictures is 24." and "John has 6 coins and Sarah had, Sarah had 4 ... how many more did Sarah have, how many more did Bill have than Sarah?" "I have... let me see, I have 6 pieces, sheets of paper and ... and I have four more in my folder. So, my friend she asked me to ... um ... how much was 6 times 4 and I told her it was 24." For 21 ÷ 3= □, these stories were told: "There were 21 slices of cake and each person had 3 slices of cake. How many slices of cake did they have in all?" "Well ... there's ... um ... I had 21 apples. I divided by three apples and I got 7 apples." "There was 21 pieces of gum in the bag and I ate three."

Students were also asked to make connections between symbolic and concrete representations in the pre-interview. Missing connections were observed. The student did not know how to concretely represent the problem even though the student was able to skip count by fours to find the answer.

I: With the chips, how could you figure out the answer to six times four? 6 x 4 = □

S21: I think I know that.

I: You know it in your head? What is it?

S21: Four, eight, 12, 16, 20, 24.

I: If you were going to use the chips to teach somebody in third grade how to do this problem, how would you do it?

S21: It's be like ... that's kind of hard!

The following student produced an incorrect memorized answer for the problem.. The student merely made a set of four counters, a set of six, and then a set of 12 to represent the numbers in the problem and the answer.

I: With the chips, could you show me how to figure out the answer to this? 6 x 4 = □

S26: Sixteen.

I: How did you figure that out? Did you just know it or did you figure it out?

S26: Just knew it.

I: Could you show me with the chips? Pretend that I didn't know what the answer was, how could you show me with the chips how to figure out the answer?

S26: (The student made a set of six chips and a set of four chips.) Six times four ... six times four ... (Then the student made a set of 12 counters.) This is the answer.

The following student was able to solve a division problem by skip counting, but was unable to represent the problem concretely.

I: Read this number sentence for me? 21 ÷ 3 = □

S1: Twenty-one divided by three.

I: Can you use these chips to figure out the answer?

S1: (The student made a set of three chips and then counted by threes to 21.)

I: What are you thinking?

S1: About ...

I: Well, you're counting by threes by going. 3, 6, 9, ...

S1: ... 12, 15, 18, 21.

I: So, what's the answer?

S1: Um ... seven.

I: How could you figure that out with the chips?

S1: (The student made a set of 21 chips. Then separated three chips from the set and then separated seven more of them from the set.)

Another student interpreted the division expression as calling for repeated addition. This student used the chips to show both numbers, 21 and three, and then made two more sets of 21. Finally the student counted all the chips, even the set of three, to get a result of 66.

I: Read this number sentence for me? $21 \div 3 = \square$

S9: Twelve divided by three. (The student read the number incorrectly.)

I: Can you use these counters to figure out the answer?

S9: Um ... I can try.

I: Okay.

S9: (The student made a set of 21 chips and a set of 3 chips.) I have to divide three times. Twenty-one divided by 3. (The student then made two more sets of 21 chips each and counted all of them by ones.) Sixty-six.

Word Problem Solution Strategies. Students were asked to solve one-step and two-step word problems during the pre- and post-interviews. They were allowed to use counters, their fingers, paper and pencil, or mental reasoning to solve the problems. Students in all three groups often used superficial strategies in the pre-interview. Many of the students in the Practice and Control groups continued to use these superficial strategies in the post-interviews months later, whereas most students in the Part-Whole group changed to the use of a mature strategy in which they choose the operation whose meaning fit the story. The students in the Part-Whole group also tended to spend more time per problem thinking about it and working on it.

This word problem was presented to the students in both the pre- and post-interviews: "Coleman bought 3 boxes of granola bars. Each box has 8 granola bars in it. How many granola bars does Coleman have?" S18 from the Practice group added the numbers in the pre-interview and still added the numbers in the post interview. The student responded as follows during the pre-interview:

S18: (Wrote 8+3.)

I: Why did you add?

S18: Because it says "how many granola bars does Coleman have?" and you have to add them up.

In the post interview the student responded this way:

S18: (Wrote 8+3=11.)

I: So, how did you know to add?

S18: To find out how many granola bars he has.

I: Well how did you know that you should add those numbers?

S18: You have to find out how many, and if you subtract it doesn't, um, it says how many and you shouldn't subtract because then it wouldn't tell you how many.

A student from the Part-Whole group used the counters during the pre-interview to make a set of 8 and a set of 3. Then the student counted all of them to get an answer of "Eleven." This student, however, clearly showed a strong understanding of the problem situation during the post-interview and could even explain why multiplication could be used to solve the problem as shown in the following excerpt.

S8: (The student made three sets of eight counters each.)

I: So, what is your answer?

S8: Twenty-four.

I: Now, how could you write a number sentence for that?

S8: (Wrote 3x8.)

I: Why times eight and not plus eight?

S8: Because it's not three granola bars.

I: What does this three tell you? That you have three ...

S8: boxes.

I: And what does that eight tell you.

S8: That you have eight, you have eight things in each box.

The following division story was presented to the students in both interviews: "Joel has 12 cookies. He put 3 cookies in each bag. How many bags did he use?" During the pre-interview, many students indicated the use of superficial strategies by stating key words or noting the action of using or taking which told them to subtract. Again, students in the Practice and Control groups tended to continue their use of superficial strategies, whereas students in the Part-Whole group demonstrate use of mature strategies based on the meaning of the division. This student from the Practice group solved the problem correctly in the pre-interview but was unable to write an accurate symbolic representation.

S26: Three ... three cookies ... twelve ... He used, I think he used 4 bags for each 3 cookies to put in.

I: How did you figure that out?

S26: I counted in my mind. (She kept track of the number of threes with her fingers.)

I: Can you tell me how you were counting in your mind?

S26: I was counting by threes.

I: So if you had to write the number sentence for this, how would you write it?

S26: ... plus or ... it's take away. (Wrote 12–7=.)

I: Why did you put take away?

S26: Because he put cookies in each bag and he took, he took, he took cookies out of 12.

However, by the post interview this same student merely added the numbers.

S26: Fifteen.

I: How did you figure that out?

S26: Add.

I: Why did you add those numbers?

S26: To see how many bags he used.

I: Why did you add and not subtract or multiply or divide?

S26: I don't know.

I: Is there anything about the story that helped you to figure out you should add?

S26: ... How many bags did he use?

The thinking of S4 and S5 from the Part-Whole group during the post-interview showed a change in thinking to the meaningful strategy of choosing the operation whose meaning fits the story. In the pre-interview, S4 reached a correct solution but was unable to connect it to a symbolic representation.

S4: (The student drew dots in groups of three and circled each group of three until there was a total of 12 dots. Then he counted the number of groups.) Four bags.

I: How would you write the number sentence for this problem?

S4: Um ... subtract ... (Wrote 12–3. Then he used his fingers to figure out that the answer to this was nine. He then realized this didn't match his other answer, so he thought for a while and then wrote 3x4.)

In the post interview, S4 responded as follows:

S4: Four.

I: How did you figure that out?

S4: Because three times four is 12.

I: Would you write down the number sentence for me?

S4: (Wrote 3x4=12.)

I: Could you write it any other way that would also be correct?

S4: (Wrote $12 \div 3 = 4$.)

I: Why would division be a correct way?

S4: Because it's 12 cookies and he put three of them into each bag. He divided them.

I: What does that mean, he divided them? ... If you're dividing something, what do you do to them?

S4: It means ... mm ......Oh. You divide them into groups.

I: Does there have to be anything special about those groups?

S4: They have to be the same amount.

In the pre interview, S4 subtracted the numbers.

S5: (Wrote $12-3=11$.) He used 11 bags.

I: How did you know to subtract?

S5: Because it says "how many did" and you always have to subtract to get the answer.

In the post interview, S5 responded as follows:

S5: (The student made a set of 12 chips and then separated it into four equal parts.) Four.

I: Can you write a number sentence to show me what you just did?

S5: Mm uh. (Wrote $12 \div 3 = 4$.)

I: So how did you know that this was a division problem?

S5: Because he put three cookies into each bag and he had 12 cookies.

Figure 8 illustrates the thinking of low and average ability students from the Practice and Control groups and Figure 9 presents the thinking of low and average ability students from the Part-Whole group as they solved two-step word problems during the post-interview. The strategies used by the Part-Whole group illustrated meaningful use of informal and formal mathematical knowledge, and the students from the other groups tended to quickly apply some operation or combination of operations to the numbers in the problem. The Part-Whole students could explain their reasoning as they solved the problems, whereas the students from the other groups usually could not. The conceptual knowledge of the students from the Part-Whole group became integrated with their problem solving knowledge throughout the duration of this study as indicated by their reasoning as they solved problems.

Holly had 18 balloons. Six of them broke. Then she gave the rest of the balloons to her 3 friends so that each friend got the same number of balloons. How many balloons did each friend get?

S21: You subtract three from 18.
I: Why?
S21: 'Cuz, um, she gave, she gave the rest of the balloons to three friends and you have to figure out how many balloons each friend gets.
I: And how do you do that?
S21: You subtract 18 from three.
I: And what do you get for an answer?
S21: Fifteen.
I: So, each friend got 15 balloons.
S21: Mm uh.

S23: (Wrote 18–3=5.) Five balloons.
I: Why did you go 18 minus three?
S23: Because three can't take away 18 and it says how many did each friend get.

S24: (Wrote 18–6=12 and then wrote 12–3=9.)
I: So, why did you decide to subtract, 18 minus six?
S24: Holly had 18 balloons and six of them broke.
I: And then why did you decide to subtract, 12 minus three?
S24: ... (No answer.)

Figure 8 Solution Strategies from the Practice and Control Groups for a Two-Step Word Problem

Holly had 18 balloons. Six of them broke. Then she gave the rest of the balloons to her 3 friends so that each friend got the same number of balloons. How many balloons did each friend get?

S7: (The student made a set of 18 chips.) I'm going to take away six.
I: Why do you want to take away six?
S7: Because six of them popped. (The student removed six of the chips.) And then she gave three of them away to her friends.
I: Is that what it says?
S7: Let's see, it says then she gave the rest of the balloons to her 3 friends so that each friend get the same number of balloons. (The student rearranged the chips to have three rows with four chips in each row.)
I: First, you subtracted this six, right?
S7: Yeah.
I: Then you gave or shared those balloon among your friends. What math operation is that?
S7: Divide.
I: Why is that divide?
S7: Because you put them in groups.
I: What's special about the groups you made?
S7: There's four in each group.

S11: Eighteen ... six ... (Wrote 18–6=12.) ... she gave ... there's three friends... (Then the student wrote 12+3= 5.) ... that equals ... fifteen.
I: How many balloons did each friend get?
S11: Fifteen.

I: The first part is perfect. Eighteen minus six. Then what happened?
S11: (The student reread the problem.) So she had to subtract it to get the same number of balloons.
I: You have the right idea. Use these 12 chips. Now give them to your three friends.
S11: (He separated the 12 into three equal parts of four chips each.)
I: How many balloons did each friend get?
S11: Four.
I: What math operation is this?
S11: Division.
I: Why is this division?
S11: Because you're making parts.
I: And what's special about all the parts that you made?
S11: You have to separate them.
I: Yes. You separated them into these special parts.
S11: Fours.
I: Yes. So what's special about all these parts.
S11: They're all even.

Figure 9 Solution Strategies from the Part-Whole Group for a Two-Step Word Problem

Attitude Surveys

The results of the attitude surveys are shown in Table 7. The survey was conducted at the beginning of the study in November and again in April or May. The pre and post results are the percent of students who circled a smiling face in response to the item. The change scores are the increase or decrease in percentage points from the pre- to the post-attitude survey. Most of the results show a decrease from the pre- to the post-attitude survey in the percent of students responding favorably. Overall, the students like to solve number problems more than story problems and think that it is easier to solve number problems than story problems. Most students also reported that math is fun and that they like it, in fact they like math better than school, but many fewer students reported being good at math.

Table 7 Attitude Survey Results (Percent Responding Positively)

	Part-Whole (n=106)			Practice (n=136)			Control (n=105)		
	Pre	Post	Change	Pre	Post	Change	Pre	Post	Change
I like doing story problems.	45.3	44.3	–1.0	58.1	44.1	–14.0	38.1	34.3	–3.8
Story problems are easy for me.	55.7	50.0	–5.7	64.7	47.1	–17.6	57.1	40.1	–17.0
I like doing number problems.	64.2	69.8	+5.6	61.8	54.4	–7.4	76.2	53.3	–22.9
Number problems are easy for me.	67.9	70.8	+2.9	67.6	60.3	–7.3	80.0	58.1	–21.9
I am good at math.	60.4	46.2	–14.2	51.5	47.8	–3.7	65.7	48.6	–17.1
Math is fun.	89.7	75.5	–14.2	69.1	64.7	–4.4	75.2	69.5	–5.7
I like math.	85.8	76.4	–9.4	69.1	71.3	+2.2	76.2	74.3	–1.9
I like school.	57.5	49.1	–8.4	63.2	58.8	–4.4	70.5	51.4	–19.1

The drop in the number of Part-Whole students who reported ease in solving story problems was not significant (X^2=0.023, $p > .05$) as determined by a McNemar Test for significance of change. In comparison, significant drops were found for both the Practice group with a drop of 17.6 percentage points (X^2=8.803, $p < .01$) and the Control group with a drop of 17.0 percentage points (X^2=9.091, $p < .01$). These drops most likely reflect the increased difficulty of the word problems which the students encountered through the interventions and the testing which included the more difficult addition/subtraction problem types and multiplication/division problems with larger numbers.

The drop in the number of students for each group who reported liking story problems was not significant. It is interesting to note the large drop for the Practice group. Their dislike for story problems grew much more than the other groups. On the other hand, the Part-Whole and Control groups had very small drops on this item.

DISCUSSION

The two major aspects of problem solving involve understanding the problem and solving the problem (Greeno, 1978; Mayer, 1985). Understanding problems requires constructing an accurate mental representation of the task environment (Greeno, 1978, 1980; Newell-Simon, 1972) which is facilitated when conceptual knowledge is integrated with problem solving knowledge (Greeno, 1978). The instruction using part-whole concepts with one-step and two-step word problems helped students achieve this integration of knowledge by emphasizing connections among meanings of the operations, real-world, concrete/pictorial, and symbolic representations. These connections were given explicit attention during instruction by using the three-step word problem heuristic: (1) thinking about the structure, (2) representing the situation, and (3) solving the problem.

This instruction also enhanced the development of the first, second, and third components of the problem-solving schema proposed by Marshall (1988). The development of the first component, the conceptual knowledge that describes general situations resulted from describing the operations using part-whole concepts and relating this to everyday experiences. The second component involves developing the conditional knowledge of knowing when and why to use specific conceptual and procedural knowledge. Instruction explicitly addressed this component as real-world and concrete representations were connected to and symbolic representations. The development of Marshall's third component, mechanisms for setting goals, was most evident in this study with the instruction on two-step word problems with the focus on the word problem schema of multi-step word problems. The fourth and final component, a collection of procedural rules, was not developed.

Marshall (1988) asserted that instruction must foster the development of long-term schematic knowledge structures. Fischer, (1988), Kintsch and Greeno (1985), Resnick (1983), and Riley et al. (1983) have noted the value of the part-whole schema in children's mathematics learning. The instruction using part-whole concepts produced long-term achievement on solving word problems for all ability levels of students in this study. This gives evidence of the strength and importance of helping students develop a part-whole knowledge structure. The students who were only given practice in solving word problems made small short-term gains that disappeared quickly once the practice was stopped. This gives further support to the findings of Wilson (1964) that practice alone does not improve problem solving performance.

The instruction using part-whole concepts and explicit discussions of connections also helped students link their informal and formal mathematical knowledge. The Part-Whole group students of all ability levels developed explicit conceptual knowledge (Greeno, 1987) as evidenced in their improved ability to express their reasoning with formal mathematical language (oral and written). These students demonstrated operation sense (NCTM, 1989) as they used their informal and formal mathematical knowledge to solve both one-step and two-step word problems.

Students from the Practice and Control groups often tried to use their formal mathematical knowledge to solve word problems, but many of them either could not explicitly express their reasoning or expressed reasoning that was linked to syntactic features of the problem rather than to their informal knowledge. This gives support to the position of Resnick (1986, 1988) that students who link their formal knowledge (oral and written) back to referents and informal knowledge will be successful in learning mathematics, whereas weaker students are those whose formal knowledge is not linked with their informal knowledge.

CONCLUSIONS

The instruction using part-whole concepts with word problems produced success on one-step and two-step word problems at the end of instruction and in the long-term for all ability levels of students, especially for low ability students. Providing students only with practice on one-step and two-step word problems was not effective. Practice produced small gains in the short run, but it was not effective in the long run This supports the position that the consequences of learning with understanding become evident in the long run, whereas learning by practice produces some small gains in the short run which disappear quickly once the practice is stopped (Hiebert & Lindquist, 1990).

In the interviews prior to instruction, most low and average ability students and some high ability students did not use conceptual knowledge of the operations to reason through the solution of word problems. Instead, they used many superficial strategies to solve one-step and two-step word problems, especially the use of key words, number size, and guessing. In the post-interviews, little change was noted in the approaches of Practice and Control group students, whereas Part-Whole group students of all ability levels tended to use the concept-driven strategy of selecting the operation whose meaning fits the story and to use both their informal and formal mathematical knowledge to solve problems. The Part-Whole group students also grew in their ability to explain their reasoning, whereas students from the other groups found it difficult to communicate their reasoning.

Instruction emphasizing the part-whole structure of the operations helped students develop a strong conceptual knowledge base with connections for real-world, concrete/pictorial, and symbolic representations, and helped students develop conceptual knowledge which they could express verbally. The approach taken in this study is an encouraging attack on a perennial troublesome topic in school mathematics. Instruction using part-whole concepts with one-step and two-step word problems shows promise for enhancing the teaching and learning of mathematics.

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