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ABSTRACT

The traditional format of mathematics instruction has not succeeded in providing the skills students need to work cooperatively to solve problems in industry. New models of instruction have been proposed to resolve this deficiency. Schoenfeld has used a technique that incorporates coaching, modeling, and fading strategies with college-level students. Treisman has improved minority student performance in calculus using a model based on collaborative problem solving. A hybrid model called cognitive apprenticeship merges the coaching-modeling-fading components of Schoenfeld's model and Treisman's collaborative workshop model to enable students to become better problem solvers while working together as members of a community of learners. Cognitive apprenticeship instruction was tested in community college industrial technology classes: two instructors each taught a traditional and an experimental technical mathematics class. Quantitative data from indicated students in the cognitive apprenticeship group scored slightly better than the control group on a problem-solving exam and the final exam, although not significantly. The scores of the cognitive apprenticeship students on a standardized exam were slightly lower than the control group, but not significantly. Two recommendations were proposed based on the results of the study: first, to explore the model further after certain suggestions were incorporated and second, to test it in other math-based classes. (Contains 61 references.) (YLB)

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PREFACE

This report is part of the National Center for Research in Vocational Education's (NCRVE) continuing effort to improve vocational and technical curriculum and instruction. This study is one in a series of investigations being conducted by researchers at the NCRVE that examine how people learn technical information and how that information can best be taught. This report describes a preliminary evaluation of an instructional method called cognitive apprenticeship. The results indicate that even when relatively untrained teachers use the cognitive apprenticeship approach, students can be expected to perform as well as students who are taught through a more traditional approach. However, the generalizability of the findings from this experimental study are limited due to the small sample size and the single data collection site. As such, the results of this study are presented in a very tentative fashion. It is hoped that this line of inquiry will serve as a pilot study for future research that includes larger samples, more extensive teacher training, and applications in diverse educational settings. It is also hoped that this exploratory study will be of interest to researchers, practitioners, and policymakers in both vocational and academic education who are interested in improving the quality and effectiveness of our nation's educational system.

EXECUTIVE SUMMARY

The future workforce will need to be better educated as the American economy becomes more global, as the workplace becomes more technological, and as management philosophies become more sophisticated. Greater mathematics and reasoning skills will be needed by the workforce of the future to competently co-exist with the complex technological workplace (Johnston & Packer, 1987). Unfortunately, the mathematics and problem-solving skills of the current workforce appear to be inadequate for many of today's technological jobs.

The traditional format of mathematics instruction has not succeeded in providing the skills students need to work cooperatively to solve problems in industry. New models of instruction have been proposed to resolve this deficiency. A technique that incorporates coaching, modeling, and fading strategies has been used by Schoenfeld with college-level students. In this paradigm, the teacher demonstrates problem solving to students by modeling how an expert might attempt to solve them and then provides challenging problems to the whole class to solve. Students work in small groups to solve similar problems, with the teacher acting as a coach, providing students with the necessary scaffolds to bridge their knowledge gaps. As students become more competent, the teacher provides less assistance.

Treisman has improved minority student performance in calculus using a model based on collaborative problem solving. Treisman studied the poor performance of minority students in calculus at the University of California at Berkeley. From this research, he proposed a model that incorporates a collaborative workshop environment in which students work together to solve difficult problems based on the course content. In this model, students are encouraged to become a member of a community of learners who work together to be successful in mathematics.

A hybrid model called cognitive apprenticeship merges the coaching-modeling-fading components of Schoenfeld's model and Treisman's collaborative workshop model. The cognitive apprenticeship paradigm provides students with the opportunity to become better problem solvers while working together as members of a community of learners.

Community college technology programs such as electronics, mechanical design, construction, automotive, diesel mechanics, and manufacturing are designed to provide students with the entry-level technical skills needed in the workplace. The technical mathematics courses included in these programs are designed to provide students with the mathematical skills and techniques that are needed in the technical fields. Problem solving must be, and is, a major component of this curriculum. Application problems based on the students' technical fields are stressed in technical mathematics courses.

Cognitive apprenticeship instruction that teaches students to be problem solvers, stresses application-based problems, and encourages collaborative problem solving may be a more effective way to teach technical mathematics than traditional instruction. This alternative method of instruction was tested in community college technical mathematics classes to improve student problem solving. Two instructors each taught a traditional and an experimental technical mathematics class. In the experimental class, forty percent of the class time was spent with students working in groups in a workshop environment to solve problems. During the lecture/discussion period, the teachers attempted to solve problems as experts solve them while relating their thought processes to the students. During the workshop sessions, the teachers acted as coaches, guiding the students while they worked to solve application-based problems.

The quantitative data from this study indicates that students in the cognitive apprenticeship group scored slightly better than the control group on a problem-solving exam and the final exam, although not significantly. The scores of the cognitive apprenticeship students on a standardized exam were slightly lower than the control group, but again the difference was not significant. This data indicates that the cognitive apprenticeship model of instruction supports student learning of technical mathematics and problem solving as well as the traditional model. From the analysis of hourly tests and student interviews, it appears that the cognitive apprenticeship model may work better for initial learning of mathematics rather than for review or reinforced learning.

While the cognitive apprenticeship model did not significantly improve student learning, it did have an impact on student attitudes toward mathematics and problem solving. The results from the SIMS (Second International Mathematics Study) attitude tests indicate that students in the cognitive apprenticeship group became more anxious about mathematics and problem solving than the control group. This may indicate that the students' belief systems have been challenged by the instructional method. The results also suggest that the students in the cognitive apprenticeship group gained self-confidence, were successful in establishing rapport with other members in their group, and, overall, seemed to enjoy the cognitive apprenticeship approach to teaching mathematics.

Two recommendations are proposed based on the results of this study. First, the cognitive apprenticeship method should be further explored after the following suggestions have been incorporated: (1) Student attendance at lab sessions should be required, (2) teachers need training in coaching and mentoring, (3) the technical knowledge of the mathematics teachers should be increased, and (4) more time should be allotted for the lab sessions. Second, the cognitive apprenticeship method should be tested in other math-based classes in science and technology.

INTRODUCTION

The mathematics and reasoning skills needed by the workforce in the future will rise substantially as the workplace becomes more technologically complex. It has been estimated that forty-one percent of the new jobs in the future will require high levels of mathematics and reasoning skill (Johnston & Packer, 1987). This is a large increase when compared to the fact that only twenty-four percent of the current jobs require those skills. Given the increased importance of mathematics and reasoning skills in the workplace, it appears that the problem-solving skills of the American workforce may be inadequate for many of the new complex jobs (National Council of Teachers of Mathematics [NCTM], 1989a).

While future jobs will require higher levels of mathematics and reasoning skills, it does not appear that the American educational system is preparing students with the desired capabilities. A recent study that examined the mathematical performance of students in thirteen nations found that American students performed below the international average (Crosswhite et al., 1986; McKnight et al., 1987). Poor performance in mathematics occurred at both the eighth grade and the twelfth grade levels. It appears that schools are not teaching students the mathematics and problem-solving skills they will need to compete in an international economy that requires complex technological skill. As a result, industry must retrain high school graduates to help them develop the skills needed to operate complex machinery, to diagnose malfunctioning equipment, and to be able to decide when intervention is necessary with automated systems.

Numerous instructional models for teaching problem solving have been developed to address the growing need for higher levels of mathematics and problem-solving skills. One model, proposed by Schoenfeld (1985, 1987), unites findings from cognitive science research with knowledge from the field of mathematics education. A second model emphasizes collaborative group learning in a workshop setting and has been successful for teaching calculus to minority students (McCaffrey, 1991; McCreary, 1991; Treisman, 1985).

While the above models of instruction have been used in a variety of ways to teach problem solving (Schoenfeld, 1987) and calculus (Treisman, 1985), they have not been used in vocational courses for teaching technical mathematics. Technical mathematics, by its very nature, provides experience in using mathematical techniques that are directly related to the future occupational needs of the student. The primary objective of technical mathematics courses is to provide technical-oriented students with experiences in the use of mathematical concepts and techniques that will be required in their future occupations. The purpose of this study was to develop and evaluate an instructional model for teaching technical mathematics to community college vocational students. This study used an approach called *cognitive apprenticeship* to answer the following research questions:

1. What effect does cognitive apprenticeship have on the problem-solving skills of community college vocational students?

2. What effect does cognitive apprenticeship have on the mathematics skills of community college vocational students?

3. What effect does cognitive apprenticeship have on the attitudes of community college vocational students towards problem solving and mathematics?

BACKGROUND AND CONCEPTUAL FRAMEWORK

Developing students' problem-solving abilities is a primary goal of both mathematics and vocational education. In fact, the National Council of Supervisors of Mathematics (NCSM) (1977) has stated that learning to solve problems is the principle reason for studying mathematics. This view was reiterated in a National Council of Teachers of Mathematics yearbook that was dedicated to the goal of teaching problem solving in mathematics (Krulik & Reys, 1980). Individual teachers have also been found to support the goal of helping students develop a systematic approach to solving problems. However, the fact that problem solving is a major goal of teachers and their professional associations does not correlate with what actually happens when teachers conduct their instruction (McKnight et al., 1987). The failure of teachers to teach problem solving in their classrooms may be due to their limited awareness of alternative instructional methods.

Teaching Problem Solving and Metacognition

Recent recommendations from mathematics educators suggest teaching problem solving by incorporating findings from research on how people learn (Greer & Mulhern, 1989; Johnson & Johnson, 1975; Peterson & Swing, 1983). The following section describes recent studies that have examined the impact of alternative instructional methods on metacognitive and problem-solving skills.

Developing Cognitive Skills Through Direct Instruction

Research on problem solving provides two perspectives that have implications for developing metacognitive and problem-solving skills (Bransford, Sherwood, Vye, & Reiser, 1986). First, one needs content knowledge to competently solve problems in a particular field. Second, one must be able to monitor and regulate cognitive processes (Alexander & Judy, 1988). The active monitoring and consequent regulation of these processes is called *metacognition* and is essential for problem solving in mathematics (Davis, 1984). Teachers have traditionally taught the factual knowledge that students need through direct instruction. Current research suggests that teachers should also directly teach metacognitive strategies to students (Brophy, 1986; Lester, 1985; Mathematical Sciences Education Board, 1990; Thomas et al., 1988).

A recent laboratory experiment with seven college students at the University of California at Berkeley explored whether direct teaching of problem solving would lead to improved problem-solving performance (Schoenfeld, 1985; Schoenfeld & Hermann, 1982). Two groups of students were selected and each was provided with the same treatment with the exception of one group being explicitly told what strategy was used to solve the problem. The data from this study showed that the strategy group's problem-solving performance improved much more than the non-strategy group's performance. Verbal protocols collected during the student problem-solving activities were analyzed to determine why the improvement took place. The protocols indicated that the strategy group used the strategies that had been taught to solve problems on their tests. The protocol data also suggested that simply providing problem-solving practice was not enough to improve students' problem-solving skills.

Other studies have shown the potential of explicit metacognitive training on the mathematics performance of college students. For example, David (1988) found that mathematics performance could be significantly improved through metacognitive skill training. Besides improving performance, studies have found that explicit problem-solving instruction can lead to improved attitudes of students towards problem solving (David, 1988; Esposito, 1983). The improvement of attitudes towards problem solving may in itself justify directly teaching problem solving.

Developing Problem-Solving Skills Through Cooperative Groups

In addition to directly teaching metacognitive and problem-solving skills to students, mathematical problem solving can be taught and learned in a cooperative group environment (Brophy, 1986; NCTM, 1989a, 1989b). Cooperative group environments can take several forms in the classroom. These forms include cooperative learning, peer tutoring, team discourse, and role modeling.

Cooperative Learning

Cooperative learning refers to "classroom techniques in which students work on learning activities in small groups and receive rewards and recognition based on the group's performance" (Slavin, 1980, p. 315). The effect of cooperative learning on computation skills tends to be positive (Maravech, 1985; Slavin & Karweit, 1985). In a study of fifth grade students, Maravech (1985) found that cooperative learning helped students build skills in mathematics. This study showed, however, that cooperative learning was less effective for higher order skills such as problem solving. In contrast to Maravech's study, Phelps and Damon (1989) performed an experiment with fourth grade students using peer collaboration. Their findings indicate that peer collaboration is an effective learning environment for tasks that require reasoning but not those that require rote learning.

Studies that incorporate cooperative learning techniques at the college level have recently shown promise. Schoenfeld (1985), Treisman (1985), McCreary (1991), and McCaffrey (1991) have reported improved levels of problem-solving skills for students in cooperative learning environments. Classroom studies have also shown that cooperative learning improves students' attitudes about themselves and school. Positive effects about race relations and cooperative concern for each other have also been found in several studies (Parker, 1984; Slavin, 1980; Slavin & Karweit, 1985).

Peer Tutoring

Peer tutoring is an example of cooperative learning that is not new to education. In the typical classroom of the early 1900s, older students were asked to teach younger students for the dual purpose of reinforcing the older students' knowledge and supplementing the small teaching staff (Bargh & Schul, 1980). Peer tutoring is also quite common in community colleges. Math labs use student tutors with some success. These students are selected to be tutors based on their math ability and their inclination to work with others.

Numerous advantages of peer tutoring have been documented. In an experiment that compared peer tutoring, adult tutoring, Computer-Assisted Instruction (CAI), and reduced class size, peer-assisted instruction was found to be the most cost-effective and academically beneficial approach (Levin, Glass, & Meister, 1987). This finding was counter to the expectation that CAI would be the best intervention. The effectiveness of peer tutoring has also been explored by Bargh and Schul (1980). Two groups of graduate students were provided with a set of materials to learn. One group was told to learn the material by themselves while the other group was told to teach the material to another person. The findings showed a significant learning gain for the students who taught the material to others. In a second experiment, Bargh and Schul explored the specific aspects of peer tutoring that result in positive cognitive effects. While acknowledging several limitations in the study, they found that the positive effects occurred during the time students prepared for the instruction. The initial presentation of the material and student feedback provided a negligible effect. The organization of material in preparation for teaching helps learners develop the links necessary to integrate the information into their existing knowledge structure (Champagne, Klopfer, & Gunstone, 1982).

Group Discourse

Cognitive theory supports the need for the learner to have task-related conversations with others (Schoenfeld, 1985; Skemp, 1987). Whimbey and Lochhead (1986) recommend that students work with someone who will listen to them as they think aloud while solving problems. Working with another student provides an interaction that helps in two ways. The interaction helps students learn how to modify another's thinking and helps students learn how to defend their own ideas (Krulik & Rudnick, 1980).

A strategy called *paired problem solving* emphasizes group discourse and has been effectively used in college-level classroom settings (Hall, 1987). Hall sent pairs of students to the blackboard to work on algebra word problems. One student was the recorder who wrote the other student's explanation of the problem solution on the blackboard. The recorder did not interfere in the problem-solving activity, but could ask questions such as "What should I do next?" or "Is that really what you want to do?" The students took turns solving the problems and writing on the board. The positive effects of paired problem solving may be because the dialogue helps learners integrate new knowledge into their existing knowledge base (Silver, 1985).

Skemp (1987) identifies four benefits of group discourse. First, the mere act of communicating our ideas helps clarify them because we have to attach words to the ideas which makes the ideas more conscious. Second, relating our ideas with those of others provides for the expansion of our own understanding and enables us to assimilate other ideas, and the explanation of our ideas to others enables them to assimilate our ideas into their knowledge structures. Third, discussion stimulates new ideas. The final benefit is the cross-fertilization of ideas: "Listening to someone else may spark off new ideas in us which were not communicated to us by the other, the result being a creative interaction, which at best, can be exhilarating to all concerned" (p. 89).

Role Modeling

Students need to see how mathematicians solve problems (NCTM, 1989b). Unfortunately in many mathematics classes, the problems that are solved are mere exercises for the teacher so students do not have the opportunity to observe the strategies that mathematicians use to solve difficult problems (Schoenfeld, 1983). Instead, teachers should act as a role model for students by solving problems in ways similar to those used by real mathematicians. Students need to observe how qualified practitioners behave and communicate to make sense of how expertise is developed through conversation and other activities.

Developing Cognitive Skills Through Situated Contexts

Cognitive science research indicates that the contextual information in problem situations is a major factor in learning. Situated learning is a term that describes the acquisition of knowledge and skills in an instructional context that reflects the way the knowledge and skills will be used in real life (Brown, Collins, & Duguid, 1988). This concept is not new to education. Dewey (1956) urged basing education in reality and suggested that a student should bring home from school each day something that could be used. Whitehead (1929) stated that "theoretical ideas should always have an important application within the pupil's curriculum" (p. 7).

Cognitive research provides many examples of learning that occurs in real life contexts. For example, Miller and Gildea (1987) taught vocabulary to students using dictionary definitions and a few exemplary sentences. After the instruction, a seventeen-year-old student used the word "stimulated" in the following manner: "Mrs. Morrow stimulated the soup." While this sentence does not represent common use of the word "stimulated," it does represent accurate word usage based on the dictionary definition. Learning vocabulary from a dictionary assumes that definitions and sentences are self-contained pieces of knowledge. In reality, those definitions rely on context. According to Brown et al. (1988), "All knowledge is, we believe, like language. Its constituent parts index the word and so are inextricably a product of the activity and situations in which they are produced and used" (p. 4). New situations and activities recast our understanding in new forms, and concepts continually evolve with each new occasion of use.

Another example of context was provided in a research project that attempted to see how people do arithmetic in the real world. People who were involved in a Weight Watchers program were observed doing computations (de LaRoche, 1985). Through analysis of the mathematics used by the dieters, it became evident that precision in measurement is one of the tenets of the Weight Watchers program. It was found that the preciseness of the measurement depends upon the specific situation in which the measurement is used. For example, the amount of time available and the type of substance being measured can determine the use of a precise or rough measurement. It was also found that some people in the study used measurement as a form of discipline for dieting which suggests that people do arithmetic for more than the customary means of calculation.

Schoenfeld (1985) provides yet another example of learning through situated contexts. Schoenfeld notes that students in mathematics classrooms often decide how to do a problem based on where the problem is located on a page, which is indicative of how difficult the problem will be. Consequently, students use this concept to design their solutions because the students have learned that the problems at the beginning of an assignment require few steps while problems at the end are more complex.

The Adult Mathematics Project (AMP) studied how people do arithmetic in the world. The project was an attempt to determine whether school-learned procedures in arithmetic are used after school (Lave, 1988). One example cited by Lave indicates how traditional story problems in school arithmetic differ dramatically from those used in life after school. The following example involves a shopper who is deciding how many apples to buy:

> I just keep putting them in until I think there's enough. There's only about three or four at home, and I have four kids, so you figure at least two apiece in the next three days. These are the kinds of things I have to resupply. I only have a certain amount of storage space in the refrigerator, so I can't load it up totally. (Murtaugh, 1985, p. 199)

If this was an example on a school test, the shopper might multiply two apples times four kids times three days minus the four apples at home plus one apple for the shopper for a total of twenty-one apples. Instead, the shopper bases the solution in the reality of the situation to come up with a realistic answer. It is interesting that this study found a large discrepancy in the accuracy of subjects doing arithmetic on tests (59% correct) and in a supermarket (98% correct). Apparently mathematics is a less difficult domain in the "real world" than in school!

Developing Skills Through Apprenticeship Instruction

School mathematics began as a need in the craft mercantile world (Cohen, 1982). Apprenticeship learning is still used today for the acquisition of skills ranging from language learning to the technical trades. Researchers have attempted to study the apprenticeship process to discover how learning takes place through this method of instruction. Examples of apprenticeship learning have been illustrated in Greenfield's (1984) study of young girls learning to be weavers in Mexico and Lave's (1982) study of tailors in Liberia.

Weavers in Mexico

Scaffolding is a process in which the expert provides bridges between what the apprentice can already do and what should be done in the future. Greenfield (1984) relates how scaffolding occurs through informal instruction of young girls learning to weave in Zinacantan, Mexico. Fourteen young girls were videotaped at various levels of learning how to weave. First-time weavers produced woven products, which, to the inexperienced eye, appeared to be done by expert weavers. Scaffolding was an important aspect of the process. The teacher aided the apprentice whenever she reached a point where she could not successfully continue. Initially, the teacher would take over part of the weaving process which provided an opportunity for the learner to observe the teacher as a role model. It also allowed the novice to complete the project and obtain a piece of woven cloth.

As the learners progressed, more of the activity is left to the apprentice. Greenfield's study indicated that the teacher took over the weaving sixty-five percent of the time in the beginning and only sixteen percent of the time later on. It was noted that the teacher was sensitive to the skill level of the learner and that the scaffold built on what the learner already knew or could do. The instructions from the teacher also varied as the apprentices became more proficient. At the beginning of the training, most of the instruction was by direct command; later, the instructions tended to be more indirect. When the teachers were asked how they taught the girls, they responded that they did nothing; the girls learned by themselves.

Tailors in Liberia

Lave's (1982) study of apprenticeship among Vai and Gola tailors in Liberia showed that in apprenticeship both the master and the apprentice are aware of the critical importance of the apprentice's learning. However, it is not just the mutual goal that organizes the learning process. Lave states,

> The tailors form a guild and the curriculum, at the highest level, can be seen as a concise summary of the articulation of the guild as a social institution with major dimensions of the social organization of Liberian society. Another major source of organization for learning comes from the work tailors do. Production processes have a logic and order to them, and these shape the apprentice's learning activities. Economic concerns also impose order on the learning process: it is more costly to make an error when cutting out a garment than when sewing it. Apprentices always learn to sew garments before learning to cut garments out. Add to this the practice that apprentices must purchase for themselves the material to make garments. From this emerges an ordering in which the apprentice works on small garments which can be made of scraps before items which take more fabric or more expensive fabrics. (p. 183)

In this Liberian society, learning was linked to an economic activity. It was not viewed as an end in itself.

Cognitive Apprenticeship

In response to the need to design better instructional methods, educators have proposed a new paradigm for instruction called cognitive apprenticeship (Brown et al., 1988). According to Brown and his colleagues, "cognitive apprenticeship methods try to enculturate students into authentic practices through authentic activity and social interaction in a way similar to that which is evident and evidently successful in craft apprenticeship" (p. 18).

Six teaching strategies comprise the cognitive apprenticeship model: (1) modeling, (2) coaching, (3) scaffolding, (4) articulation, (5) reflection, and (6) exploration (Collins, Brown. & Newman, 1987). Modeling involves the expert illustrating the required skill or activity, similar to the teacher in Greenfield's (1984) weaver study. The student has the opportunity to see how an expert in the field performs the task. In cognitive apprenticeship, the teacher provides examples and demonstrations to students and explains why the steps are chosen. The teacher then attempts to solve problems as an expert, rather than as a person just doing exercises. As students practice the new skills, the teacher provides hints to students.

Coaching consists of watching the students perform the task, providing hints, and aiding students to become more expert. This is done by both the teacher and peers. Scaffolding is the process of providing bridges for the learner, so that when a task cannot be done, the teacher performs part of the task. Just as the teacher of the weavers wove the more difficult parts of the cloth, allowing the novices to complete a quality product, teachers should provide assistance so students can continue and complete the problem-solving activity. As the students become more proficient, the teacher fades into the background, allowing the students to take on more of the project by themselves.

The cognitive apprenticeship model of instruction proposed by Brown and colleagues (1988) was developed, in part, from existing instructional models. Two of those existing models were used to design the treatment tested in this study. They include

Schoenfeld's (1983) teaching of problem solving and Treisman's (1985) Honors Workshop project.

Schoenfeld's Approach

Schoenfeld (1983) claims that teachers can build the scaffolding needed for students to integrate knowledge into their cognitive structure. In a problem-solving course that he teaches, Schoenfeld incorporates a set of problem-solving strategies for solving mathematical problems and teaches them to students. His teaching employs the concept of modeling by showing students how he, a mathematician, solves problems. He even invites the students to bring problems to class for him to solve. In attempting to model how mathematicians solve problems, Schoenfeld does not end the problem-solving activity once an answer has been obtained because mathematicians do not stop when a problem has been solved. Mathematicians will attempt to generalize, look for alternative solutions, and identify easier methods to solve the problem.

After teaching the appropriate strategy to the students and providing scaffolding by showing them how he and others use the strategy, Schoenfeld assigns students to groups and provides them with carefully selected problems to solve. He uses the group dynamics and discourse as an opportunity for the students to build their own schema. While the students work in groups on the problems, he acts as a roving coach, not evaluating what they are doing, but questioning them and checking to see if they are monitoring the problem-solving process. As the students become more proficient in solving the problems, Schoenfeld stays in the background, allowing them to be more independent in their activity.

Honors Workshop Project

Another instructional model that incorporates the concepts of cognitive apprenticeship was developed by Treisman (1985) at the University of California at Berkeley. In attempting to determine why Chinese students excelled in Freshman calculus and why African-American students performed poorly, Treisman did case studies on twenty students from each group. He followed the students around, built rapport with them, and observed the students' study habits. He found that the Chinese students combined academic study with social activities by studying together in groups. In these groups, they each critiqued the work of others. A friendly competition existed among them, but they shared information so they could all excel. Within these study groups, called "study gangs" by the students, they also shared information about how to resolve the difficulties imposed by campus life.

When he studied the African-American students to see what caused them to fail at Berkeley, he expected that weak home life, low motivation, and low socioeconomic status would explain their failures. Instead, he found that these were not factors in the failure of African-American students. The students were well-motivated and had been excellent students in high school. One major theme that emerged was the rigid separation between the academic and social life of the African-American students. Two components of the African-American students' culture seemed responsible for this. The first was a strong emphasis on self-reliance in the African-American community. Instead of seeking help when needed, the student would try to work harder. Second, separation of the academic and social aspects is what helped the student succeed in high school.

Treisman instituted the Honors Workshop to address the problem of minority students' poor performance in calculus. The workshop sessions are designed so students can work collaboratively in small groups. The workshop leader is a facilitator of the discussions among students. The role of the leader is that of a catalyst of learning, rather than a presenter of information.

One measure of the success of the program is reproducibility; that is, can these ideas be used in other institutions? Because the Berkeley student population is fairly elite, the effectiveness of the model has been tested with students of different characteristics. The model has been used in over twenty schools in California and has been studied at the University of Illinois at Urbana-Champaign (Peressini & McCreary, 1989). The model has also been adopted at the City College of New York which has a commuter student population. The improvement shown for students in the workshop at City College increased to a grade point average of 3.2 compared with that of 1.8 for calculus students not in the workshop (Conciatore, 1990). Treisman suggests that each institution must adapt the model to its own culture. Yet, he cautions that the success of the model is not due to individual components such as providing students with more difficult problems to solve or mentoring students, but, rather, to the entire concept of the model (Treisman, 1991).

METHOD

The purpose of this study was to develop and evaluate an instructional model for teaching technical mathematics to vocational community college students. This study used a factorial design to control for the internal validity factors of history, maturation, testing, and instrumentation. Two teachers were selected to participate in the study to control for differences in instructional ability. Each teacher taught one class using the cognitive apprenticeship method and one class using the traditional lecture/discussion technique. To control for the novelty effect that can result in better performance by the treatment group due to the teacher's enthusiasm generated by the newness of the treatment, the teachers were encouraged to maintain the same level of enthusiasm and instructional rigor in each class. To accomplish this goal, the teachers met regularly and discussed the progress of the experiment. An outside observer monitored the instructors' performances by visiting the four classes twice during the experiment. The participating teachers also visited the other classes to ensure that similar teaching methods were being used.

Subjects

The sample for this study was selected from the population of students enrolled in technical mathematics classes at a comprehensive community college in central Illinois. The Fall 1990 student population was thirteen-thousand students with a full-time equivalency of six-thousand students. The students were a heterogeneous group, varying in race, age, and socioeconomic status, which mirrored the population of the community college district.

The technical mathematics courses had been part of the industrial technology curriculum for twenty years. The technical mathematics courses were taught by full-time mathematics teachers who were members of the technology department rather than the mathematics department. The role of the technical mathematics curriculum was to provide students with the mathematical skills necessary to complete the technology program. These technology programs included electronics, robotics, mechanical design, manufacturing, diesel automotive, architectural construction, and numerical control.

Students who entered the program were expected to have had intermediate algebra skills. The technical math sequence included topics that were standard for such courses. Innovations in the course during the past eight years included the incorporation of calculators for computation and the use of computers for illustration and repetitive calculations.

Due to the scheduling difficulties at a community college, it was not possible to randomly assign students to either the experimental or the control group. Consequently, intact classes were identified and the treatment and teacher randomly assigned to the classes. A total of sixty-four students were enrolled in the technical mathematics courses during Fall semester of 1991. Thirty-five students were enrolled in the cognitive apprenticeship sections and twenty-nine students were enrolled in the control group sections. Eleven students officially withdrew from the technical mathematics courses during the semester. Eight of these students were in the cognitive apprenticeship group while the other three were control group students. The difference in attrition between the two instruction methods was non-significant, $t(62) = -1.32, p > .05$.

The students who did not complete the course were interviewed by telephone to determine why they withdrew. Nine of these students were successfully contacted and interviewed. None of the students who withdrew indicated that the instructional method was the reason they dropped the course. Three students indicated that they failed to complete the course due to family reasons, three students indicated that they had work conflicts, one student had health problems, one student had inadequate prior preparation, and one student had a personality conflict with the instructor. On the basis of these interviews, it does not appear that the type of instructional method influenced the decision of students to remain enrolled in the mathematics course.

Procedure

This experiment attempted to integrate the coaching-modeling-fading aspects of Schoenfeld's problem-solving course with Treisman's collaborative learning model. The Schoenfeld and Treisman models were combined to develop a cognitive apprenticeship approach that might be effective within the constraints of an existing community college technical mathematics course. The study took place during ten weeks of a fifteen-week semester. The classes met for five fifty-minute periods each week.

The experimental treatment called *cognitive apprenticeship* differed in structure from the traditional lecture/discussion approach to teaching technical mathematics. During the formal instruction, the teacher modeled problem solving sixty percent of the time and used the whole class for group problem solving. Most of the problems used in class were relevant to the technical areas in which the students were majoring. Since less time was available for teacher lecturing, fewer examples were illustrated by the teacher. Collaborative small group problem solving was then done by the students forty percent of the time. During this time, the teacher acted as a coach by providing support when needed.

Three of the five periods per week were allocated for teacher modeling of solutions to problems by acting as an "expert" who solves problems, rather than working through the problems as mere exercises. The problems to be solved by the students were selected because they could be solved using the appropriate strategy and they contained mathematical concepts relevant to the course material. The teacher presented problems to the class and modeled the solution of the problem using a feasible heuristic. During this stage, the teacher did not just solve the problem. The teacher, as an expert, attempted to describe the process used to select the appropriate heuristic or algorithm. Through this method, it was shown that there are many ways to solve each problem, that different solutions often exist, and that the context and constraints of the problem affects its solution. The teacher then presented similar problems to the class and the whole class worked together to solve the problem. Students were also encouraged to bring problems to the teacher so that they could see that even teachers are "stumped" and often need time and outside expertise to solve problems.

During the remaining two class periods, each week the students worked in teacher assigned groups in a collaborative work environment. Student groups sat together at tables and worked at solving problems that were more difficult and more application-oriented than typical math exercises. Students coached one another while attempting to solve the problems. Students encouraged one another to talk through solutions and thereby improve their knowledge of mathematical and technical language. Students were encouraged to demonstrate their group solutions to other student groups.

During the small group problem-solving sessions, the teacher acted as a coach, roaming from group to group, asking students questions. The teacher asked students what they were doing, why they had selected the method, and provided bridging or scaffolding when they lacked knowledge. When students were "stuck," the teacher would provide the information that was needed or direct the students to supportive resources. The amount of assistance provided by the teacher lessened as the students became more proficient.

The control group received the traditional lecture-discussion or template (Frandreyer, 1991) method of instruction. This method has been used for decades in mathematics classrooms at the college level. Class begins with the teacher asking whether the students have any questions on the previous homework. The teacher then presents new material, does sample problems, answers student questions, and assigns homework problems.

Data Collection

Pre and posttests of mathematical skill, problem-solving skill, and student attitudes towards mathematics were administered to the students in all four groups. Existing instruments in the form of final tests and the ACT college Mathematics Placement Exam (MPE) were used to determine the effect of the cognitive apprenticeship model of instruction on students' mathematical skill. The MPE is a standardized exam developed through a collaborative effort between high schools, community colleges, and universities in Illinois, with the assistance of the American College Testing Program (1990). The purpose of the test is to place students appropriately in college mathematics classes. Level Three of the exam, which tests college algebra and trigonometry skills, was used for this experiment. There are two parallel forms of this exam: one form was administered for the pretest and the other for the posttest.

To determine the effect of cognitive apprenticeship on students' problem-solving skills, a problem-solving test was developed that was relevant to the technical fields of the students. Problems were selected by the community college instructors from the various technical areas. The problem exam was administered as a take-home examination that allowed students more time to use outside resources and to solve more problems than could be completed during class time. An analytic scoring evaluation method was used to grade the problem-solving test. This technique is recommended by the National Council of Teachers of Mathematics (NCTM) when it is desirable to provide feedback to students on the key categories associated with problem solving. The Second International Mathematics Study (SIMS) attitude test was used to assess students' attitudes toward mathematics. This test was administered at both the beginning and the end of the experiment.

Three data sources were used to assess the qualitative aspects of the instructional method: (1) observations of classes, (2) student reports and interviews, and (3) teacher logs and interviews. These three sources of data allowed for triangulation of the qualitative data. An observation instrument was developed to capture the environmental context of the technical mathematics classrooms. The observation instrument was based on classroom observation instruments developed by Weber, Puleo, Kurth, Fisch, and Schaffner (1988) and Johnson (1990). An observer visited the experimental and control classes during presentation/lecture periods at the beginning of the experiment. The same mathematical topic was being presented during each of the four periods. During the observation period, the observer completed an observation form every ten minutes, for a total of five observations per classroom period. The observer also visited the two experimental labs immediately following the classroom observations. A second set of observations was made at the end of the experiment. The observer was an experienced observer in college classrooms.

A post-treatment interview was used to study students' perceptions of the teaching method. The interviews varied from fifteen to twenty-five minutes in length, depending upon the amount of discussion by the student. The interviewer, who was a community college student familiar with the experiment, asked a series of predetermined questions, allowing the opportunity for expanded comment on the various topics. Students were asked to meet with the interviewer individually in a confidential, comfortable setting on their own time. Thirteen students volunteered to participate in the interview. Students who did not want to be interviewed or whose schedules did not allow them to be interviewed were assured that their grade would not be affected. To encourage students to be candid, student confidentiality was guaranteed and the students were assured that the results of the interviews would not be reviewed by the researcher until after the course grades had been determined.

Reciprocal observations and teacher logs provided the teachers' perspectives of the actual implementation of the experiment. Teachers used reciprocal observations to determine differences in teacher mentoring styles and implementation of the cognitive apprenticeship model. The teachers periodically met with each other and discussed any noticeable differences. Weekly logs were kept by both teachers to record difficulties and positive features of the experimental method.

Data Analysis

Data was analyzed using both descriptive and inferential statistics through the SPSSx program (SPSS, 1983). Analysis of variance (ANOVA) was performed on the exam scores from the two treatment groups, with change in problem-solving skill as the dependent variable. This analysis allowed between teacher effects and within teacher effects to be analyzed. Graphs were drawn to aid in interpretation of interaction effects between teacher and method. Analysis of covariance was performed on the dependent variable of problem-solving skill, with the pretest mathematical skill score as the covariate to address the concern of statistical regression. Multiple analysis of variance (MANOVA) was used to analyze the dependent variables of problem-solving skill and technical mathematics skill. This eliminated the requirement that the correlation between the dependent variables be constant in ANOVA.

Qualitative data was collected to provide a clear picture of the process of implementing cognitive apprenticeship in a classroom environment. This data was obtained from the interviews, teacher logs, and observer forms. The aim of this analysis was not to determine the effectiveness of the strategy, but to provide formative evaluation data that could be used to enhance further implementations of the model. A technique called pattern analysis was used to analyze the qualitative data. Pattern analysis involves the search for key linkages between components of the data (Erickson, 1986). Multiple passes through the entire set of qualitative data were made to identify patterns. A descriptive system that built on categories drawn from the cognitive apprenticeship literature was used to code the qualitative data. Examples of these categories included student comfort, group dynamics, student roles in the group, teacher scaffolding, teacher coaching, teacher fading, metacognition, building understanding, students questioning each other, discourse providing synergism, student self-confidence, application problems, and problem solving. Due to the large number of coding categories, an attempt was made to combine the categories to focus the observations on specific aspects of cognitive apprenticeship. Anecdotal data provided a picture of the process of implementing the cognitive apprentice method of instruction. A holistic impression of the method as it is used was recounted.

RESULTS

This quasi-experiment was an exploratory study to determine the effectiveness of cognitive apprenticeship for improving student problem-solving ability in community college technical mathematics classes. Overall, the results indicate that the technique as implemented in this study was no more effective than traditional instruction.

Performance on Examinations

Following the completion of the course, all students were given a problem-solving examination, a final course examination, and a standardized mathematics knowledge examination (MPE). No significant differences were found between the test scores of the apprenticeship group and the control group on the post-treatment examinations (see Table 1). The lack of significant differences between the two groups on the mathematics and problem-solving exams was true even when prior ability, as determined by the MPE pretest, was used as a covariate. This finding indicates that the cognitive apprenticeship model, as used in this study, did not significantly improve student learning of either mathematics or problem solving beyond the level of learning achieved through the traditional approach.

Table 1
Comparison of Main Effect Differences on Mathematics and Problem-Solving Tests

Type of Exam by Group	*n*	*M*	*SD*	*F*	p^*
Problem-Solving Exam					
Control Subjects	24	63.35	25.09	.507	.481
Apprenticeship Subjects	20	68.67	26.65		
Final Exam					
Control Subjects	25	106.40	27.86	.218	.643
Apprenticeship Subjects	26	108.89	23.86		
MPE Posttest					
Control Subjects	23	16.74	6.20	.011	.917
Apprenticeship Subjects	24	16.63	4.77		

Note: *All exam score comparisons are non-significant at $p>.05$

The interaction effect of teacher and treatment was not significant on the problem-solving exam or the final exam. This finding indicates that both instructors in this study were equally effective in using the cognitive apprenticeship method. The interaction effect between teacher and the type of instructional method was significant on the MPE posttest (p=.019), even when controlling for prior student math ability (p=.047). The cognitive apprenticeship group means on the MPE posttest differed little (teacher 1=16.44, teacher 2=16.73), unlike the control group means (teacher 1=11.29, teacher 2=19.13). For the traditional method of instruction, the performance of students on the standardized test appeared to be dependent upon the teacher.

Throughout the course, hourly examinations were used to provide periodic updates of the students' progress in the course. Student performance on the hourly exams appeared to change after the experiment was in place. Further investigation indicated that differences between the two groups on the hourly exams occurred on trigonometry tests rather than the tests involving algebra concepts. The results of an ANOVA indicated that students in the cognitive apprenticeship group performed significantly better on the hourly tests involving trigonometry, $F(53)=4.749$, $p<.05$. This finding suggests that the cognitive apprenticeship method is more effective when new concepts are being taught, rather than when old concepts are being reviewed. Selected problems from the trigonometry tests were then examined to determine how the two groups differed in their test-taking approaches. Five tests from each group were selected and reviewed to determine how the students attempted to arrive at their answers. The students in the cognitive apprenticeship group drew more accurate diagrams and attempted more potential solutions than the control group. Students in the control group tended to erase information that they did not think was correct. Even when the students in the cognitive apprenticeship group neglected a key concept needed to solve the problem, they still proceeded to attempt to arrive at a solution.

Student Attitudes Toward Mathematics and Problem Solving

While the two methods for teaching technical mathematics appeared to be equally effective in developing mathematics and problem-solving skills, the cognitive apprenticeship method had a greater influence on student attitudes and anxiety towards mathematics, problem solving, and teamwork. Five questions from the SIMS attitude test were used to determine if there were attitude changes between the two groups (see Table 2).

Table 2

Attitude Questions Used to Determine Student Anxiety

	Strongly Disagree				Strongly Agree
1. It scares me to have to take mathematics.	1	2	3	4	5
2. I could never be a good mathematician.	1	2	3	4	5
3. I am not so good at mathematics.	1	2	3	4	5
4. When I cannot figure out a problem, I feel as though I am lost in a maze and cannot figure my way out.	1	2	3	4	5
5. No matter how hard I try, I still do not do well in mathematics.	1	2	3	4	5

Student Attitudes

Student scores on the five questions on the SIMS attitude test were examined to identify differences between the pre- and posttest. The results suggest slightly higher mathematics anxiety in the cognitive apprenticeship group while the control group showed less anxiety. While there is a slight difference in anxiety towards mathematics between the two groups, it was not statistically significant as shown in Table 3, $t(45)=2.27$, $p>.05$. Analysis of this variable indicated that the anxiety level of students in the cognitive apprenticeship group had increased during the treatment. As expected, the control group displayed less change in their anxiety since the instructional method was similar to what they had experienced before. That attitudes towards mathematics and problem solving did not improve for the students in the cognitive apprenticeship group may indicate that their attitudes were challenged by the instructional method. The slight increase in anxiety may have resulted because the cognitive apprenticeship students gained a more accurate view of how difficult it is to solve real problems using mathematics (Pobre, 1991).

Table 3
Comparison of Anxiety Differences Between Groups as Shown by the SIMS Attitude Test

Group	*n*	*M*	*SD*	*t*	*p*
Control Subjects	21	-1.52	4.61	2.27	.054
Apprenticeship Subjects	26	.42	3.06		

Changes in Attitudes

Student attitudes were also analyzed in terms of the individual statements on the SIMS attitudes test. Dependent *t*-tests were run on all statements for students who completed both the pre- and post-attitude tests. Sixteen statements that dealt with gender, technology, or change in mathematics were not included in the analysis since they were not directly related to the research questions. Of the thirty-eight remaining statements, only three questions showed a significant change in student attitudes at the p=.05 level for one group but not the other.

The cognitive apprenticeship group showed significant change on the statement "I would like a job that uses math." This group of students strongly agreed with that statement before the experiment (Mean=4.08, SD=.628), but had significantly less agreement afterwards (Mean=3.62, SD=.784). The control group showed no change in attitudes toward this statement. The cognitive apprenticeship group showed a significantly stronger agreement after the experiment with the statement "I feel good when I solve a mathematics problem in mathematics." The control group showed less agreement, but it was not statistically significant. Student interview data, teacher logs, and observation forms also suggested that students in the cognitive apprenticeship group enjoyed solving problems more and increased in confidence as the semester progressed. The control group showed a significant decrease in agreement with the statement "When I can't solve a math problem I feel I'm in a maze." The cognitive apprenticeship group showed an increase in their agreement with this statement, but the change was not significant.

In summary, the attitudes of the students in the cognitive apprenticeship group changed significantly in the following ways: (1) they have less desire to have a job using math, (2) they are less willing to work a long time to understand a math problem, and

(3) they have strong positive feelings when they solve a math problem. Students in the control group were found to be less willing to work a long time to understand a math problem, and they felt as if they were in a maze when they could not solve a math problem.

Enjoyment of the Cognitive Apprenticeship Method

Student interviews indicated that students enjoyed being taught through the cognitive apprenticeship method of instruction. While several students indicated discomfort at the beginning of the course, they stated that they became more sure of themselves as the semester progressed. Teacher logs reported that students in the cognitive apprenticeship classes seemed enthusiastic and often stayed after class to complete their problem exercises. The outside observer noted that students in the cognitive apprenticeship classes appeared very comfortable solving problems during the second set of observations. Only two of the thirteen students who were interviewed felt that the cognitive apprenticeship instruction was not preferable to traditional lecture discussion methods.

Instructor Impressions of the Cognitive Apprenticeship Method

Both instructors maintained teacher logs during the experiment. These logs were analyzed to identify patterns and themes that emerged from the instructor's perspective of the method. The themes that evolved from the instructor logs covered instructional difficulties, student dialogue, student problem solving, and student self-confidence. The following comments illustrate the prevalent themes in the instructor logs.

Instructional Difficulties Faced by the Instructors

- Some problems with student attendance in the lab sessions occurred as time progressed. Points should be assigned to encourage student participation in the labs.

- Sometimes pacing was a difficulty. Some student groups finished before others which meant that some students did not have a chance to complete all of the problems. Most students may need more time to complete the problems.

- The noise level of the classroom increased considerably in the cognitive apprenticeship class. The instructor seems to set the tone of the lab session, whether it is quiet or talkative.

- There appeared to be some difficulty getting some of the "standard material" covered during the lectures.

- Students sit in the same place during the lecture/presentation period as they did the first day of class. They sit in their groups during the lab.

- Students needed considerable "bridging" on physics problems.

- There is a tendency for the instructor to provide students with answers to the problems. Instructors may feel uncomfortable if they do not have a chance to work out the solutions to the problems in advance.

Comments Related to Student Communication

- Students began to communicate across groups.

- Students made comments such as "We figured . . . out by ourselves"; "3 heads are better than 1"; "What does this mean?"; and "Did I confuse you?"

- Students challenged the meaning of questions such as "How many days in a week, 5 or 7?"

- Students wanted to work together on the exams. They claimed that the instructor encouraged them to in labs so they should also be cooperative on examinations.

- Students often stayed after class to work on practice tests.

- Students talked about getting together before the next test to study.

- Students cheered other group members when they performed well on the hourly tests.

- One student stayed in class to work with other students even though he had taken the final exam already.

Comments Related to Student Problem Solving

- A student brought in a math problem that his sister had in her class. He wanted to solve it with some of the techniques that the class did not yet know.
- Students had great fun when making up their own problems.
- Students challenged the wording of problems because they seemed unrealistic (e.g., rivets on the edge of a plate).
- In a paired quiz, one pair of students searched for two different ways to solve the problem.
- Students wanted to find easier ways to solve problems than just grunting out the solutions.

Comments Related to Student Self-Confidence

- Weaker students showed insight into non-standard math problems such as the pendulum and helicopter.
- A group with five students performed much better when the two strong students were absent.
- One student told another "After I get my answer I always check . . . to make sure it is reasonable."
- One student made up his own theorem on triangles.

The teacher logs support the notion that the cognitive apprenticeship method helps students feel as though they have become a member of the mathematics community, enhances their understanding of mathematics, and improves their metacognitive, problem-solving, and application skills through student discourse, improved self-confidence, and teacher mentoring. The instructional difficulties of pacing and coordinating the activities of the experiment did not appear to have negatively affected student attitudes.

DISCUSSION

The cognitive apprenticeship model of instruction was neither validated nor invalidated by this study. Students in the cognitive apprenticeship classes performed as well (and as poorly!) as those in the traditional classrooms. The cognitive apprenticeship model did have a slight impact on student attitudes and anxiety toward mathematics and problem solving. While this study did not prove the claim that cognitive apprenticeship is effective for teaching technical mathematics at the community college level, it does show that the method is both plausible and as effective as the traditional method. Further refinement in the model needs to take place. What is clear is that this model requires very careful planning, extensive teacher training, and considerable effort to be truly effective.

Two major implications for instructional design and implementation arise from this study. First, because cognitive apprenticeship had little impact on student learning beyond the level achieved by traditional approaches, the cognitive apprenticeship model should be further refined and tested in other technical mathematics classes. The lack of a significant difference may be explained by the fact that both teachers in the experiment had twenty years of experience teaching using the traditional model. They had refined their traditional instructional methods until they were effective for teaching community college students. As the cognitive apprenticeship method of instruction becomes more refined, it is likely that student performance may surpass the performance of students taught through the traditional model. The following four recommendations are made to address the implementation concerns that arose during the study:

1. *Student attendance at lab sessions should be required.* From student interviews and teacher logs it was apparent that the different mode of instruction was initially uncomfortable for students. Student comments such as "The awkwardness of it the first time," "At first I wasn't comfortable because I didn't know the people very well," and "Other math classes are more structured" show that the student attitudes towards change in instructional method may require students to be oriented towards the value of the method. Since students felt uncomfortable at first, and since they were not experiencing the direct teaching with which they were familiar, some students did not attend regularly. Explicit attempts must be made to encourage students to attend and participate in the lab sessions.

2. *Teachers need in-depth training in coaching, scaffolding, and mentoring.* Student comments such as "The teacher doesn't help us at all" indicated that more training of teachers as coaches is needed. Although both teachers received training in the method, many years of using traditional teaching techniques makes it difficult to relinquish control of the class to the students. In the recent video conference entitled *Increasing Minority Participation in Math-Based Disciplines* (Treisman, 1992), the recommendation that experienced teachers be trained to use new methods was mentioned several times.

3. *The technical knowledge of the mathematics teachers should be increased.* In order to provide the scaffolding for students in application problems, teachers must have a deep understanding of the applications. If the teacher is just using a teacher's manual to solve application problems, then the teacher is just copying the solution and not modeling the technique to the student. NCTM (1989a) urges mathematics teachers to become more versed in the applications that students need for the workplace. The difficulty of not providing answers to the students bothered one of the teachers in this study. This concern may have been due to a lack of depth in the physics knowledge needed to provide the support needed to help students solve the problems.

4. *More time should be allotted for the lab sessions.* Solving complex problems can not be done in a short time. Teacher logs indicated that some students did not finish the problems. The following student comment illustrates that more time is needed to build student understanding. "We went through stuff real fast. And when we got together our discussions were at our own pace, kind of slow. We took our time understanding."

Implications for Further Research

The results from this and other studies demonstrate the need to continue studying the appropriate design and implementation of the cognitive apprenticeship model (Treisman, 1992). To guide future studies in this area, the following suggestions for impending research are offered:

1. *Will extension of the model to physics, chemistry, and other technical courses improve student problem-solving performance?* The model used in this study or a modification should be tested in math-based science and technology courses such as physics, chemistry, electronics, and mechanics. If students have the opportunity to experience instruction that encourages collaboration in several classes, they could become more skilled in the process. Courses in science and technology provide rich opportunities for students to apply mathematics, science, and technical concepts and seem like appropriate proving grounds for the cognitive apprenticeship model.

2. *Will changing student groups improve the instructional model?* As recommended by P. McCreary (Personal Communication, November 26, 1991) from the University of Illinois, the changing of groups during the treatment would allow students to change roles within the group. Due to student attrition, some groups were reduced to two or three members. Changing student groups would allow more interaction among students in the class. Additional studies in which the groups are changed should be attempted.

3. *How does closure affect the student learning process?* The concept of closure was mentioned in the student interviews and teacher logs. The following two quotes from students indicate varying levels of discomfort with not knowing the answer: "I wanted feedback," and "We didn't know if we had the right answer. I guess the default was, that if we heard nothing, we assumed it was right, and if she told us something else then, we went back." Students indicated discomfort in not knowing whether they had the correct answers. Since people in the workplace do not have an answer book to look at to confirm the answer to problems, always knowing whether the answer is correct is not realistic. Schoenfeld (1983) suggests that too often we stop when we get the "answer" and do not explore other possibilities. Further study should be done to resolve this closure issue.

4. *Will student's increased fear of mathematics persevere after continued cognitive apprenticeship instruction?* The results of the student attitude tests suggest that the cognitive apprenticeship model challenges how students feel about mathematics and problem solving. Further studies should be done to determine whether student attitudes continue to change when they have had more experience with the method in math, physics, chemistry, and technology.

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