The purpose of this research was to understand fifth grade children's cognitive behavior as they wrote, solved and then, in small groups, shared original math story problems. Research questions examined children's: (1) beliefs about math in this problem-writing classroom, (2) math story problem-writing behavior, (3) difficulties with their self-generated problems, and (4) small-group problem solving behavior. Case studies were conducted in the context of a teaching experiment in one fifth grade classroom. Children were engaged to write, solve and then share math story problems three or four days a week during this one year study. There were three overlapping groups of participant children. Eight children were observed as they wrote and solved math story problems. Seventeen children, including the eight previously observed, were observed via audio-recordings as they shared story problems in small groups. The entire class of 25 children was interviewed or surveyed regarding their math-related beliefs. The findings indicate that these children tended to express problem-oriented and holistic beliefs about mathematics. Children showed a variety of planning behaviors during problem writing. Generally, children composed problems that they themselves had difficulty understanding or solving. Finally, children were extremely task-focused when sharing peer-generated problems in small groups. The outcome of small-group sessions was being shared. It was concluded that children's problem writing and solving behavior reflected the expectations and beliefs of this school math literacy community. The principal implication of this research for teachers is that children's original math story problems provide one important alternative source to textbook and teacher-generated math problems. Further research on the relationship between problem ownership and problem solving behavior is recommended. (49 references) (Author/MDH)
Writing, Solving and Sharing Original Math Story Problems: Case Studies of Fifth Grade Children's Cognitive Behavior

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Abstract

The purpose of this research was to understand fifth grade children's cognitive behavior as they wrote, solved and then, in small groups, shared original math story problems. Research questions examined children's (a) beliefs about math in this problem-writing classroom, (b) math story problem writing behavior, (c) difficulties with their self-generated problems and (d) small-group problem solving behavior.

Case studies were conducted in the context of a teaching experiment in one fifth grade classroom. Children were engaged to write, solve and then share math story problems three or four days a week during this one year study. There were three overlapping groups of participant children. Eight children were observed as they wrote and solved math story problems. Seventeen children, including the previous eight children, were observed as they shared story problems in small groups. These observations were collected primarily by the use of audio-recordings of children's individual and small-group activity. The entire class of 25 children was interviewed or surveyed regarding their math-related beliefs. Data analysis employed an analytic inductive procedure.

The findings indicate that these children tended to express problem-oriented and holistic beliefs about mathematics. Children showed a variety of planning behaviors during problem writing. Generally, children composed problems that they themselves had difficulty understanding or solving. Finally, children were extremely task-focused when sharing peer-generated problems in small groups. The outcome of small-group sessions usually depended on the didactic style of the child whose problem was being shared. It was concluded that children's problem writing and solving behavior reflected the expectations and beliefs of this school math literacy community.

The principal implication of this research for teachers is that children's original math story problems provide one important alternative source to textbook and teacher-generated math problems. Further research in the math story problem writing process and, particularly, the relationship between problem ownership and problem solving behavior is recommended.
Writing, Solving and Sharing Original Math Story Problems: Case Studies of Fifth Grade Children's Cognitive Behavior

This is a summary of a study that examined fifth grade children's cognitive behavior as they wrote, solved and shared original math story problems (Winograd, 1990). In this one year field study (1989-90), children wrote math story problems three or four days a week during the math period. The study entailed my (a) collaboration with one fifth grade teacher in the development of a story problem writing pedagogy and (b) observation of children as they were engaged in this activity, including their beliefs about mathematical literacy. Data collection entailed audio-recorded observation of children as they wrote, solved and then, in small groups, shared problems. I also interviewed children about their mathematical beliefs as well as their problem writing and solving behavior.

The study was exploratory in nature and intended to stimulate both research and pedagogic interest in a problem-writing pedagogy in elementary school math. The study raises questions and hypotheses in a largely unaddressed area of math education. The findings rendered at the end of this report are representative of the children in one fifth grade classroom.

Why a Study of Children Writing, Solving and Sharing Story Problems?

This study had its origins in several conditions surrounding the status of mathematics education as well as, of course, my biases related to these conditions. The first condition was a
sense of crisis in math education, particularly an unflattering perception of children's performance on school mathematics tasks (e.g., Kouba, Brown, Carpenter, Lindquist, Silver & Stafford, 1988; Stevenson, Lee & Stigler, 1986; Bishop, 1988; Schoenfeld, 1987; Romberg & Carpenter, 1986).

The second condition was the emergence in the 1980s of the whole language movement in elementary language arts teaching. Especially in the domain of writing, the idea of student ownership over writing topics and the writing process in general (e.g., Graves, 1983) made common sense to me. While Graves' (1983) work suggests that children do, indeed, have much to write about that reflects their real experience and interests, I wondered if the same could be true if we invited children to write about the math-related topics in their everyday lives.

A third antecedent condition to this research was the idea that problem solving activity is the basis of mathematical learning and should permeate the entirety of children's school math experience. This has been the consensus view among the math education community for some time (e.g., National Council of the Teachers of Mathematics, 1980).

Finally, the fourth condition was the idea that problem finding is the first step in the problem solving process, and the separation of problem finding and problem solving in school curricula has a negative influence on children's development as problem solvers (e.g., Getzels & Csikszentmihalyi, 1976; Dillon, 1982; Dewey, 1910; Flower & Hayes, 1980; Kilpatrick, 1987; Brown
Theoretical Background

The theoretical perspective of the present research is a social-interactionist one. This perspective regards children's accomplishment of academic tasks as an interpretive process that is inextricably linked to specific settings, partners and purposes. A discussion of the social-interactionist theories of Vygotsky (1978) and Doyle (1983) follows.

Vygotsky's Theory

Vygotsky's (1978) thesis is that psychological development reflects and emerges from social activity. The central context for learning from the Vygotskian perspective is one of joint activity between generally unequal partners, typically adult and child but also between more and less knowledgeable children. Vygotsky referred to this context of joint activity as the "zone of proximal development" (ZPD). The ZPD is the difference between the child's "actual developmental level as determined by individual problem solving" and a more complex level of "potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In common educational vernacular, the ZPD is the difference between the children's independent level and instructional level.

The metaphor commonly used in Vygotskian research to characterize the assistance an expert provides a novice is the "scaffold" (Wood, Bruner, & Ross, 1976). Scaffolding refers to the
support given to a novice by an expert through the use of dialogue to model and explain (also see Greenfield, 1984). Most studies using Vygotsky's ideas have considered only the adult as instructor. These studies include such diverse topics as classroom instruction in reading comprehension (Palinscar & Brown, 1984), apprentices in informal settings learning to weave (Greenfield, 1984), and mother-child dyads engaged in memory tasks (Rogoff & Gardner, 1984) and model building (Wertsch, 1979). Palinscar and Brown's work is unusual since it is a rare curriculum development based explicitly on Vygotsky's ideas.

Typically, in Vygotsky-oriented research, adults set the tasks for children, pose the initial questions, orchestrate the learning situation, participate in the scaffolding and, in effect, control the "context-setting" of the zone of proximal development. An important question in the present research is children's ability or inclination to orchestrate the content of their own ZPD by their writing of original math story problems.

Once this initial question is resolved (regarding children's initiating their own math problems), the research considers children's problem-solving activity. Specifically, to what extent can or do peers provide the necessary scaffolding, or support, for each other during math problem solving that is so typical of adult-child instruction, and what is the nature of this support?

Doyle's Task Theory

Vygotsky-inspired research typically considers children's
learning in natural and authentic settings in which their participation is volitional and uncoerced. Walter Doyle's (1983) task theory considers children's cognitive behavior in the evaluative and less volitional social setting of classrooms.

Doyle (1983) believes that the social conditions of academic tasks dramatically influence students' thinking on those tasks. According to Doyle, how students engage in academic tasks is a reflection of two conditions: ambiguity of the tasks and the risk-factor entailed in those tasks. Ambiguity refers to "the extent to which a precise answer can be defined in advance or a precise formula for generating an answer is available" (Doyle, 1983, p. 183). Tasks high in ambiguity are comprehension and opinion tasks. Memory and procedural tasks are lower in ambiguity. Risk refers to the "stringency of the evaluation criteria a teacher uses and the likelihood that these criteria can be met on a given occasion" (p. 183).

There are several aspects of Doyle's theory pertinent to the present research. First, tasks influence students' thinking in classrooms. How students attend to the teacher's lessons, how they respond to the teacher's questions, or how they engage in small-group activity, for example, depends on what needs to be done to accomplish those tasks. If the students have a worksheet calling for use of the dictionary to write definitions but the teacher, instead, offers a lesson on the origin of those words, children will not attend to this lesson as much as they would to a lesson that focused on how to use the dictionary. In effect,
students pay attention to information that is necessary for the accomplishment of tasks (also see Gauvain & Rogof, 1986; Tikhomirov & Klocho, 1981).

Second, students will pay most attention to the part of the task that is going to be subject to evaluation, and they attend primarily to those aspects of the task. One weakness of Doyle's theory here is that he does not take into account intrinsically motivating tasks, and that it is not uncommon for teachers to propose academic tasks that some children will find interesting and worthy of attention regardless of accountability.

Third, students manage the ambiguity of academic tasks to minimize the risk associated with them. If a task is very ambiguous and, at the same time, it has "stringent criteria for evaluation," students may act to minimize the ambiguity of the task (e.g., Doyle & Carter, 1982; Davis & McKnight, 1979).

Academic tasks take place in public setting and, argues Doyle, this has some important effects on children's cognitive behavior: answers and performance are public; peers are resources for task accomplishment; and because evaluation is public, the teacher is under pressure to adjust tasks to the level appropriate to the majority of students. Students "litcrate" in the ways of the classroom act in ways that, for themselves, seek to maintain an acceptable balance between task ambiguity and the risks of all forms of evaluation.

In all classrooms, one system of evaluation is the informal one of children comparing themselves to each other according to
perceptions of each other's academic performance and competence. The self-evaluation maintenance model, developed by Tesser and Campbell (1982), associates children's motivational behavior in classrooms with the need to maintain positive self-evaluations. The model maintains that children's self-evaluations emerge from their comparisons with other children. Although this is a narrow view of children's motivations, again discounting, for example, the role of intrinsically motivated activity, Tesser's model does appropriately recognize the competitive nature of traditional classrooms. In terms of children learning in the social environment of classrooms, the state of a child's positive self-evaluation, as it emerges from interactions and collaborative activity with peers, certainly does entail risks. How children cope with these social pressures may not always result in educative outcomes, and this competitive aspect of group activity may entail one important limitation in cooperative learning (for a related discussion, see Salomon & Globerson, 1989).

The writing and solving of math story problems is an ambiguous task that requires students to construct rather than reproduce knowledge, thus entailing quite a bit of unpredictability. In this inquiry, it was of interest to understand how children negotiated the ambiguity and risk of this task, both in their individual posing/solving as well as small-group activity.

**Peer Collaboration**

Research generally points to a positive relationship between peer teaching or peer collaboration and children's learning
Forman (1989) characterized child-child instruction as distinct from adult-child instruction in school settings according to the differences in the balance of authority and knowledge typical of each. In the adult-child scheme, the child is almost always the learner and the adult is almost always the teacher. In peer instruction, however, there tends to be more "reciprocity of interaction": the teacher and learner can alternate roles as the situation demands. It is the asymmetrical relationship between adult and child that led Piaget to dismiss the likelihood of cognitive change resulting from the direct interaction of adult and child (Tudge & Rogoff, 1989). He believed that adults, simply because of the asymmetrical balance of power and knowledge between child and adult, constrained children's inclination to question and raise new possibilities.

Peer collaboration from a Vygotskian perspective traces changes in the individual less to overt conflict of differing perspectives and more to the cooperation of peers, or the "co-construction of solutions" (Tudge & Rogoff, 1989). Forman (Forman & Cazden, 1985) studied the effects of peer collaboration with fourth-fifth graders engaged in a combinatorial task using chemicals. Forman analyzed the performance of student dyads and found that students who interacted cooperatively solved more problems using more systematic strategies than dyads that did not work together or inform each other of their thoughts and actions.
Forman suggests that the assumption of complementary roles by children serves to make complex tasks more manageable for individual children, thus paving the way for children to "master difficult problems together before they are capable of solving them alone" (p. 343).

The peer-teaching literature has considered the cognitive demands of teaching on the child teacher (Carrasco, Vera, & Cazden, 1981; Cazden, Cox, Dickinson, Steinberg, & Stone, 1979; Ellis & Rogoff, 1986; Mehan & Riel, 1982). Vedder (cited in Webb, 1989) suggested that the effectiveness of child teaching may depend on the following conditions:

1. The help must be relevant to the particular misunderstanding or lack of understanding of the target student,
2. It must be at a level of elaboration that corresponds to the level of help needed,
3. It must be given in close proximity in time to the target student's error or question,
4. The target student must understand the explanation,
5. The target student must have an opportunity to use the explanation to solve the problem. (Webb, 1989, p. 24)

In addition to Vedder's description of these "informational" aspect of teaching, Cazden et al. (1979) and Carrasco et al. (1981) also considered the interpersonal management demand on the child teacher. They viewed the teaching event as consisting of two inter-related aspects: "communication of information that the teacher knows and the learner does not; and the management of interpersonal aspects of the teacher-student relationship which ... is an asymmetrical relationship of more or less power" (Carrasco et al., 1981, p. 237). In addition to the management of the overt behavior of the learner, the child teacher (like the adult teacher) must also monitor and adjust her teaching to the
more subtle cognitive task engagement of the tutee (e.g., Is the learner merely acquiescing to my explanation without really understanding?). Particularly when the child is attempting to teach something that she has only recently learned or practiced, the interpersonal and informational demands of the teaching task do present a formidable challenge for children.

The behavior of child teachers becomes clearer when compared to the behavior of adult teachers on similar tasks. Ellis and Rogoff (1986) compared 8 and 9 year old teachers and adult teachers as they taught a sorting task to 6 and 7 year olds. One difference between the novice and adult teachers was in the changing patterns of their involvement during the teaching sessions. The adults were initially more directive and controlling, but as the learners developed greater proficiency with the sorting task, the adults became less controlling and allowed the learners more responsibility for accomplishing the task. The child teachers, however, maintained control throughout the sessions during which time they were either highly specific about what to do (without explaining their reasoning) or they played guessing games with the tutees. The child teachers were more concerned with completing the task than in insuring that the learners participated in an increasingly independent fashion.

The evidence suggests that one educative value of peer teaching is, perhaps, for the child teacher who, by actively teaching, processes the information at a deeper level and, therefore, learns it better (Allen, 1976; Hitano & Inagaki, 1987;
Webb, 1989). When considering the more active role learners tend to play with peers than with adults in formal instructional situations (Mehan & Riel, 1982), the value of peer teaching may also lie in the opportunity it provides for all children to participate in a more open and negotiable "construction of knowledge". As Barnes and Todd suggest, the "indeterminacy" in the understandings expressed by children is a condition of their developing new understandings. Children working together without adult interference are "able to explore alternative meanings rather than to rehearse (knowledge) taken over from the teacher" (1977, p. 127). While this is not to deny an important role adults have in participating in the learning experiences of children, or to deny some limitations of child-managed instruction, the literature does point to an important role for peer collaboration in children's learning.

Generally, the present research sought to understand children's cognitive behavior as they wrote, solved and then, with peers, shared original math story problems. Specifically, the following questions guided the inquiry:

1. Emerging from a regular experience of writing, solving and sharing story problems, what is the nature of children's beliefs and attitudes about school math literacy?
2. What is the nature of children's cognitive behavior as they composed math story problems?
3. To what extent do children write problems that they themselves have difficulty understanding or solving, and what is the nature of their difficulties with self-generated story problems?
4. When sharing problems in small groups, to what extent do children sustain their problem solving activity, and what is the quality of these interactions?
Methodology

Site and Participants

The site was a self-contained public school, fifth grade classroom located in a midwestern city. The classroom and school population were predominantly Anglo and middle-class. The classroom consisted of 25 children, 12 of whom were male. The teacher, Donna Strohauer, is a veteran of 23 years in the classroom.

Instructional Setting

Children wrote, solved and shared original math story problems three or four days a week during the math period for one entire school year. This problem-writing activity was accompanied in the math period by the use of the textbook; the two activities were not integrated by instruction.

Math class took place after lunch, from 1:15 p.m. to 2:30 p.m. The math class typically began with "mathematician's chair" (an adaptation of Graves & Hansen, 1983): one child sharing his/her problem (always written on the chalkboard) with the entire class. This opening phase of the math period usually lasted 15 minutes. The child-leader directed the class to "take a few minutes and see if you can do my problem." What usually transpired at this point included some of the following: (a) children called out answers; (b) someone asked for an explanation; (c) either the problem poser or volunteer went to the board to lead an explanation; (d) other children offered alternative explanations; (e) the teacher intervened to address some academic or behavioral issue; and (f) after some consensus was reached
regarding the solution, the problem poser asked the class, "What did you like about my problem?" and "How can I change the problem?", whereupon children responded for a few minutes.

At the conclusion of mathematician's chair, half of the class met with Donna in the back of the room for textbook-based instruction or a review of an old text assignment. The children not with Donna were at their seats working on one of two tasks: a textbook assignment or the writing, solving, and then sharing of a math story problem. Children here shared their story problems informally with peers of their choosing. After approximately 30 minutes, the two groups of children switched places.

During the final 15 minutes of the period, children formed into cooperative groups of three, and each child took turns presenting his/her problem to the other two. Donna tried to balance groups in terms of the children's problem-solving ability as well as their interest in this activity. This small-group activity was the most common vehicle for children to share their story problems. Finally, at the close of the 15 minute small-group activity, Donna asked groups to report to the whole class about what had transpired (e.g., "Was the problem truly a problem?" and "What strategies were used to solve the problem?").

Instruction in problem posing consisted primarily in immersing the children in models of each others' self-generated problems. Children encountered these models during mathematician's chair and also during small-group sharing time. Beyond this, the children were regularly encouraged by the adults to use topics
from everyday experience in their story problems. Particularly in the first six weeks of school, Donna and I shared with the students our own self-generated problems. However, while our topics reflected some real experience or interest, our questions were more hypothetical and speculative.

Generally, a portrait of this elementary math literacy community includes the following behaviors and events: much social interaction; children writing and sharing original math story problems; children receiving adult direct instruction of math concepts or problem solving strategies related to the textbook or their self-generated math problems; and children arguing, fighting, laughing, and collaborating with their original story problems as the focus of interaction. A substitute teacher once left Donna a note reporting the day's progress. Regarding math class, the substitute apologized for not completing the assigned lesson plan. But it was not her fault, she said, "The children spent 30 minutes arguing about one problem!" When Donna read this note, she was quite pleased.

Identification of Participant Children

There were three overlapping groups of participant children from this fifth grade classroom.

Individual children as they wrote and solved problems (n=8). The strategies used to select the eight children combined maximum variation and convenience strategies, recommended by Patton (1987). I was interested in selecting a variation of children according to their general academic status in school math:
(e.g., had they had a successful or unsuccessful school math experience; were they perceived by teachers as either high, low, or middle achievers in school math; did the children have a positive or negative self-image as math students). A review of the problem-solving literature led to the development of a list of behavioral criteria of effective and ineffective problem solvers (e.g., Flexer, 1987; Davis & McKnight, 1979). These criteria were used as a heuristic in identifying children as either high status, medium status or low-status.

Children in small groups (n=17). This group consisted of the same 8 children selected for problem posing and problem solving study, and then 9 additional children. Prior to small-group activity, Donna posted the names of those children who would be "teaching" their problem that day. If any of the focal eight children were teaching, I observed and audio-recorded their group. If two of these children were teaching, I taped two groups. Because they were included in small-group activity with the eight focal children, nine additional children were included in the analysis of small-group data.

Entire class' math-related beliefs (n=25).

Data Collection Procedures

During the period of data collection, I participated in Donna's fifth grade classroom two to five days a week, from the beginning of school in September to the end of school in May. Although I was present in the classroom at other times, most of my visitations occurred just before, during and after the math
period, from approximately 1:15 p.m. to 2:30 p.m.

I collected four kinds of observational data pertinent to the findings: (a) individual children writing and (b) solving original math story problems; (c) small groups of children (usually three) as they attempted to solve one of the group member's story problems; and (d) interviews, surveys, and observations of whole class discussion relevant to children's beliefs.

**Observations of Individual Children.** When it was time for the children to write story problems, I invited one of the case children to join me at the "research table." The table was a student's desk; a small tape-cassette player sat in one corner of the desk.

As the child wrote his/her problem, I used a hybrid of the clinical interview and talk-aloud procedure, referred to by Ginsburg, Kossan, Schwartz, and Swanson (1983) as "mixed cases." It is a variation of the talk-aloud procedure, during which the child is asked to say out-loud everything that comes to mind while solving problems; only now, occasional questioning by the researcher is used to clarify or check information emerging during the procedure. I also took field notes to "mirror" the child's writing and pertinent utterances during problem posing and solving to supplement my audio-record of the child's think-aloud and to facilitate the typed transcription of episodes.

**Observation of small groups.** After children wrote and solved their problems, they shared these with peers in small groups, usually consisting of two other children. Whenever
small-group sessions were scheduled, I took note of which one of the primary-case children was teaching, and taped that child's group. After placing the tape recorder in the middle of the group, I would situate myself just a few feet away with my back turned to feign non-interest and took notes to supplement taping.

**Interviews.** I collected data related to children's math beliefs and attitudes via (a) open-ended interviews, (b) an open-ended survey of the entire class, and (c) audio-recorded observations of children's interactions during whole-class discussion. The survey was administered orally to the entire class at the end of the study. The questions were open-ended in that the children were able to make any response they wished in their own words.

**Data Analysis**

The analysis of data followed an analytic inductive process recommended by Lincoln and Guba (1985). Data that described case children's posing, solving, and sharing activity were divided into episodes, or periods "of time during which an individual . . . is engaged in one large task" (Schoenfeld, 1985, p. 292). Based on the schedule of the alternative pedagogy as it was implemented in this classroom, the data were separated into the following episodes:

1. Problem posing, during which time the child began and completed the writing of an original math story problem.

2. Problem solving, during which time the child began and completed (though not always completing) the solving of the problem.
3. Problem sharing, during which time the child brought his/her problem to a small group of peers for their solution activity.

The main units of analysis, then, were the problem-posing episode, the problem-solving episode, and the problem-sharing episode. There were some data, such as the children's beliefs, that were analyzed at the comment, or utterance, level. Guided by my research questions, data were organized into categories and these categories led to the study's findings. For greater detail related to data analysis, see Winograd (1990).

Findings

Beliefs

These fifth grade children expressed holistic and problem-based conceptions of mathematical literacy. According to the children, the "good" math student generally worked hard, worked diligently to understand problems, solved problems quickly and was a problem poser; the "good" story problem had interesting non-math content and was slightly challenging; and a problem was made challenging when it contained non-routine characteristics such as extraneous information or new math content. Most children claimed that peer-generated problems were more difficult than textbook problems. Comments by some children suggested that it was the variety of math content and the presence of extraneous information in peers' problems that made them more difficult than textbook problems. Ironically, most children preferred doing story problems written by peers.
Problem Writing Behavior

Story problem topics. The topics for most of the children's problems did reflect their authentic experience or imagination. Categories of problem topics from 28 observed writing episodes include the following: personal interest/curiosity (n=10); personal reading (n=5); environmental cue (n=3); impersonal, not explicitly reflecting authentic experience or story (n=5); actual experience (n=4); and fiction (n=1). See Appendix for samples of student problems.

Writing process behaviors. When writing story problems, children tended to use the focal, or culminating, question to guide their writing behavior. In 57% of the episodes (13 of 23) in which problem writing was observed, children would engage in the following steps in problem composition: (a) identify the general topic of the problem; (b) generate, even if tentative, a culminating/end question related to the topic; (c) write information, or the problem text, that served to provide information needed to answer the question; (d) and, finally, write the final question. The process of story problem writing, in these situations, is more or less recursive: the child writer, during problem construction, is continually moving back and forth between the slowly emerging text as written and his/her idea of the culminating question.

In six episodes, children did not generate their final culminating question until after the problem text was completed. Five of the six episodes in which the focal question came after
composition reflected the work of low-status children. Except for one episode, all high and middle-status students were focal-question directed in their writing.

**Problematizing strategies.** During problem writing, children were observed to working purposefully to increase problem difficulty. However, children did not apparently consider how they could make problems more difficult for themselves; rather, their concern seemed to be how to increase problem difficulty for their peers. Problem writers endeavored to increase problem difficulty in the following ways:

1. adding extraneous numerical information
2. adding extraneous non-numerical information
3. adding pertinent information
4. using large or perceived difficult numbers (e.g., odd numbers were perceived by children to be more difficult than even numbers)
5. avoiding a standard question (e.g., not asking, "What the average", since writer senses that this is a routine question for most of class)
6. making a sub-procedure of the problem a potential problem (e.g., instead of writing, "There are six hours and 40 minutes in school day," child writes, "School starts at 8:40 and ends at 3:20.")

**Problem Solving Behavior**

I collected 27 episodes in which children attempted solution to self-generated problems. In 21 of these episodes, children experienced difficulty in either understanding or solving their problems. Problematic episodes were grouped according to the degree of assistance required by the problem solver. Episodes reflected one of four categories: (a) assistance very controlling/child's solution effort completely inappropriate/n=6; (b) assistance moderately controlling/child's solution effort par-
tially appropriate/n=4; (c) child primarily in control/child's solution effort mostly appropriate/n=5; and (d) child in control/child commits minor error/n=4. Two problematic episodes were not analyzed because the children chose to not engage in problem solving. An illustrative protocol and commentary from one episode follows. This episode is an example of the child primarily in control, and his/her solution efforts are mostly appropriate.

I met with the problem writer, Rachel, a few months earlier immediately after she had composed this problem. She was totally confused by the problem, and I advised her to put it aside for a few days and I would work with her then. It was not until four months later that we returned to the problem (see Figure 1).

Rachel immediately identified the "thirteen steps" information as extraneous and the "thirty minutes/fifteen minutes" and "stopping for a drink" information as pertinent to the solution. Apparently, she had thought about this problem since our last meeting four months earlier. After she thought about the relevance of the cul-de-sac (versus a conventional rectangular block of houses), I got her to attend to the central question.

30. K: Ok. Alright...so...how would you catch up with her? That's the question.
32. K: Would you?
33. R: I don't know. I think I have an idea of how to solve it...I think you would...do a picture or something. I'm not exactly sure how to do this but...you would say she stopped every three minutes for one minute, then you would see how many three minutes there were in 15, divided by three which there are five...and then...then I'm stuck (laughs)...but then maybe that five-no...ya draw a picture.
34. K: Why don't you try it?
35. R: Ok. (gets paper) See, you'd...(makes 15 marks) Ok, there's the 15.
37. R: But, I don't understand my problem is if when...if she's 15 minutes away does the minutes she stopped count in
the 15, I don't think so...do you? Would you think...that it
would count...'cause see if it was she walked for three
minutes and then she stopped for three minutes, that'd be
four minutes, that'd be four minutes, right?
38. K: Right.
39. R: But if she was 15 minutes away, if she was walking 13
steps a minute, when one minute she stopped to get a drink,
it would not count...
40. K: Right.
41. R: So she'd have three minutes walking, then a drink,
then three minutes then a drink, then three minutes walking
then have a drink-
42. K: Right. Ok...
43. R: OK?
44. K: Ok.
45. R: Ok...so then here's the three minutes (refers to 15
marks) and she has one drink right here (makes tiny mark
after third mark)-
46. K: Uh huh.
47. R: And then three minutes one drink right there, three
minutes right there, one drink right there, for one minute-
49. R: So, in other words, she did that...it would take
her...(counts 15 marks and five additional "drink" minutes)
20 minutes to get to my house...understand?

One source of indecision for Rachel was whether the drink-time
was to be part of the 15 minutes (utterance #37). Somehow, in
#39, she used information already identified as extraneous (13
steps a minute) to decide that the drink-time should be added to
the 15 minutes. When she reconciled this issue, her solution
process transpired virtually without interruption.

It appears as if Rachel conducted a dialogue with herself in
the accomplishment of the problem, with me functioning as her
alter-ego or, in Vygotskian terms, as her "other regulator". My
presence provided a social context for her to first reapproach
the problem and, then, follow her own intuitions about the solution. She repeatedly posed questions that she then tried to answer herself. They were perhaps the kinds of questions I might have asked her if she did not (Would you think...? Right? I do not think so, do you?), or perhaps the kinds of questions she anticipated I was thinking of or was about to ask. So, my presence, provided her with an audience to try out her ideas.

Small-Group Problem Solving Behavior

Thirty small-group episodes were studied. A list of related findings follows.

1. Problem writers tended to bring story problems to small groups that were difficult for one or more members of that group (in 23 of 30 episodes).

2. Children tended to maintain on-task focus of their conversations in small group. When off-task utterances did occur, it was the the child whose problem was being shared who usually worked to maintain the group's attention on the problem at hand. In 30 episodes, 1470 conversational turns were counted and 69 of these were coded as off-task. Only six of these off-task utterances were made by problem writers.

3. Approximately half of the small-group episodes were successful. A successful episode occurred when one or more of the children who were initially confused by the problem made any progress in understanding/solution (partial or complete) by the end of the episode.

In groups that were successful, there was a tendency for the
problem writer (or the child "in the know") to assume a non-directive didactic style that allowed the problem solvers the time to explore (e.g., reread, draw picture) and generate their own understandings of the problems. The problem writers assumed this non-directive style either purposefully or unintentionally by their own indecision about the problem.

In unsuccessful groups, there was a tendency for the problem writer to assume a directive didactic style, often oriented to solution procedure, and it was this style that inhibited problem solvers from exploring meanings of problems and the creation of their own understandings.

Bruce's heartbeat problem illustrates the adverse effect of a highly-directive didactic teaching style. The problem was a difficult one for Rachel (see Figure 2). After first reading the problem to her, Bruce spent 20 seconds in reconstructing the meaning himself. He then asked Rachel if she needed help.

7. B: Ok. Now 20 seconds. See, how many-gotta figure out what percentage of that is to...um, 60 seconds, to a minute, in other words...like, in other words, 20 divided by 60...(he sets up algorithm, 60 divided by 20). You need to figure out like, ok. I'll tell you the answer. It's gonna be 1/3. One-third-um...20 is 1/3 of 60—I mean 1/3 of 120. So...

After another minute of Bruce doing most of the talking, all of it connected to the solution procedure, Rachel blurted out, "Ok, now I get it." Bruce did not challenge or question Rachel's claim to comprehension. However, Rachel's tone of voice seemed, to me, unconvincing of understanding, so immediately after the episode, I interviewed her. Her sentiments were not unusual of
solvers in other unsuccessful episodes.

19. K: Rachel, did you understand it?
20. R: Not really.
21. K: Was it important to you that you understand his problem?
22. R: Well, I'd like to understand it but I don't see what good it would do to understand it. But I would like to understand it.

Bruce's teaching was oriented entirely to the solution. Once Rachel requested help, his assistance was unrelenting and quick-paced. Rachel was a child whose persistence in the face of difficulty was much greater when she worked her own problems than others'. So, with perhaps only a moderate motivation to grapple with other children's problems combined with Bruce's highly directive teaching, Rachel's bailing out in the end may not have unreasonable. In Rachel's mind, unbeknownst to Bruce, there was no pretense that she understood the problem.

Discussion

The preponderance of data indicates that these fifth grade children were capable and interested in initiating and then sustaining their own math problem solving activity. Fundamentally, children behaved as problem writers and problem solvers in ways that reflected the expressed and tacit expectations of this math literacy community. The community's influence was generally educative; that is, children endeavored to write interesting and challenging problems and then engage in sustained problem-solving activity. However, children's interaction with the literacy community sometimes influenced their behavior adversely.
Educative Effects of the Community

Children fundamentally behaved in ways that reflected the expectations of this math literacy community. As the good math student was someone who wrote interesting and challenging problems, children strived to write interesting and challenging problems. As the good math student worked diligently to understand and solve story problems, children behaved in ways that, at least outwardly, matched this image of the literate math student.

An assumption of this discussion is that all people, including children, seek to participate successfully in the social life of their communities (e.g., Borko & Eisenhart, 1980). It was the math literacy community that provided the rationale, expectation and substantive assistance for these children's problem writing and solving activity. And children's motivation to participate effectively in the community stimulated them to become sensitive, or attentive, to information related to problem writing or problem solving. For example, very early in the project, the children realized that extra information made story problems more difficult. Soon after this information was shared publicly (and it was a constant theme in children critique of each others' problems), most children began infusing their problems with all sorts of extra information, some of it sensible to the problem and some of it not. Another example is children's awareness of each others' topics. When one child's topic was well received by the group, there tended to be a flurry of similar topics the next few days. A third example entails the extent and variety of
children's planning behaviors during problem composition that was oriented, for the most part, toward the composition of challenging and interesting problems. The motivation to write interesting and difficult problems was clearly a reflection of the beliefs of this math literacy community. These beliefs, voiced publicly by children among themselves, served as evaluation criteria that children used to guide their activity as math students. As Doyle's (1983) theory suggests, the children did cue into those aspects of this problem writing/solving task that were subject to evaluation and, then, they worked to meet those expectations.

The rationale for children's writing of problems, for most children most of the time, was to pose problems that peers would find either interesting or difficult. Several findings support this interpretation: (a) the frequency with which children referred to peers when they wrote problems; and (b) the "taskmaster" behavior of posers during small group activity. The children who worked most diligently at maintaining task-focus of small group activity were those children who were sharing problems.

The community was also instrumental in sustaining children's problem solving activity. Children tended to show great personal interest in each others' problems, and small group interaction appeared to provide the scaffold that maintained children's attention and engagement in problem solving as long as it did. At times, small group activity provided children with problem-specific information that led to increased understanding or
solution of problems. Even when groups did not provide problem specific information for children in need of assistance, the group often served to maintain children attention to the problems, and it was this "scaffolded attention" that often led children to increased understandings or solutions. I think that the peer group sustained problem solving activity as long as it did partly out of an expectation that good math students strive to understand and solve problems, partly out of children's enjoyment of peers' self-generated work, and, finally, partly from the vested interest and effort of the problem writer to, perhaps, gain esteem through the sharing of his/her problem. Withdraw the social aspects of this task, and it is unlikely that children would have written story problems as interesting and challenging as they did. The educative function of small group activity in children's performance of academic tasks has been noted by others (e.g., Barnes & Todd, 1977). However, this study suggests that the potential of small group activity in children's learning is enhanced when children have some ownership of the problems at hand. And extending Vygotsky (1978), this study also suggests that children are capable of regulating (and thereby assuming ownership) the content of instruction in their zone of proximal development when invited to do so by instruction.

Adverse Effects of the Community

Children's interactions with the math literacy community did also adversely effect individual behavior during problem solving. There were two factors related to this negative effect: (a) the
risks involved in peer evaluation when combined with task difficulty and (b) the limitations of peer teachers.

Children sometimes reduced the complexity of the task in order to minimize the risks of negative peer evaluation. One strategy children used to cope with the risks of negative peer evaluation was the use of duplicitous public behavior when faced with apparently insoluble math problems. Many children were observed to indicate that they understood when, in fact, they really did not. It appeared that the public nature of problem solving activity, particularly in the context of a difficult task such as math problem solving, led children sometimes to change the task from problem understanding to impression management (see Doyle, 1983).

A second factor that impeded children's problem solving behavior was the limitations of peer teachers. Explaining the meaning and solution of difficult math story problems is certainly a challenging cognitive task itself for children (as well as adult teachers!). Child-teachers, engaged in the explication of math story problems, are engaged in a complex and non-routine task: explaining a problem or concept that they either were just learning, did not understand, or were verbalizing for the first time. Besides their own understanding of the problem, peer teachers had to effect the understanding of others. Specifically, the task for peer teachers was to first understand the content of the problem themselves, understand and explain the content in terms of their tutee's understanding, monitor their tutee's
changing understanding, make appropriate adjustments and, perhaps most demanding on the teaching task, manage their peers' cooperation and attention. Even on more routine tasks such as explaining worksheet directions, these types of task demands on child teachers have been observed to interfere with effective teaching (Cazden et al., 1979). In this study, when children attempted to explain the meaning of math problems that were still problematic for themselves, it was not unusual for their explanations to be incomprehensible. While some problem solvers' bail out behavior in these situations may have resulted from fear of negative peer evaluation, other children seemed to be motivated by a sense of, "Enough already! What's the use?" This appeared to be the attitude of Rachel when she bailed out of Bruce's heartbeat problem described earlier.

A second limitation of peer teachers was a didactic style that was often highly directive, quick paced and oriented exclusively to the solution and not understanding. As Cazden et al. (1979) also pointed out, it appears that children were oftentimes simply acting on their conceptions of good teaching. With some exceptions, children tended to share a conception of the good teacher: direct, telling and to the point. The objective for most of these children when they were teaching peers often appeared to be the accomplishment of the tasks and not necessarily understanding. The directive behavior of these fifth grade (age 10/11) in this study is similar to the behavior of eight and nine year old teachers in Ellis and Rogoff's (1986) study. While
not discounting developmental factors, several possibilities may explain the fifth graders directive teaching style: (a) a lack of strategic teaching knowledge; (b) an imitation of how they have seen adult teachers behave (i.e., teaching as telling); and (c) a lack of either interest, patience or confidence in teaching peers (so, therefore, get done with it quickly!). My hunch is children's pedagogic style could be significantly improved with some systematic in-servicing by an adult teacher who carefully models and explains good teaching.

Implications

The principal implication of this study for teaching is that children can collaborate with teachers in the construction of the math curriculum. Children's original story problems are a viable and easily accessible source of content for school math teaching and learning. Furthermore, the educative value of children writing and solving their own story problems appears to increase when they have the opportunity to share and teach these problems to peers. In terms of research, ethnographies are needed in the study of school math literacy communities and, specifically, the relationship between the activities of these communities, small-group dynamics, prevailing beliefs about appropriate behavior on academic tasks, problem ownership, teacher beliefs and individual children's cognitive behavior (see Eisenhart, 1988).
LIST OF REFERENCES


Ellis, S., & Rogoff, B. (1986). Problem solving in


Question: What makes someone a good math student?

<table>
<thead>
<tr>
<th>Category</th>
<th>N</th>
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<tr>
<td>On-task</td>
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</tr>
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<td>Problem Writer</td>
<td>7/33</td>
<td>21%</td>
</tr>
<tr>
<td>Answers Quickly</td>
<td>5/33</td>
<td>15%</td>
</tr>
<tr>
<td>Cooperative</td>
<td>5/33</td>
<td>15%</td>
</tr>
<tr>
<td>Understands Problems</td>
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<td>11%</td>
</tr>
<tr>
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Question: What makes a good story problem?

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<td>Interesting</td>
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<td>Challenging</td>
<td>8/32</td>
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<tr>
<td>Not Need Large Numbers</td>
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<td>9%</td>
</tr>
<tr>
<td>Elicits Positive Social Activity</td>
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</tr>
<tr>
<td>Moderately Difficult</td>
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<td>6%</td>
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Note. Number of children represented in belief data is 25. Some of children's responses contained more than one category; therefore, number of responses for each question varies.

Prompt: Compare Peer-generated and Textbook Story Problems

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<td>Textbook Problems More Difficult</td>
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<td>Equal Difficulty</td>
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Question: Which do you prefer: Peer or Textbook Story Problems

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<td>Equal Preference</td>
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Table 1. Children's Math Beliefs and Attitudes
I walk home from the bus stop almost every day. Every day my friend walks to my house. When I walk home we walk the same way. I don't live on a cul-de-sac. My friend stops for a drink every 3 minutes for one minute. I don't stop. I am 30 minutes away from my house if I walk 13 steps a minute. My friend also does. She is 15 minutes away from my house. She is very thin. Will I catch up with her?

Figure 1. Rachel's cul-de-sac problem.

My heart rate is 120 beats per minute. How many times will it be in 20 twenty seconds?

Figure 2. Bruce's heartbeat problem.
APPENDIX

Samples of Story Problems that Caused Difficulty for Problem Writer

Math

I am going on a trip, the pack pack cost $80. I need food and more warm clothes. It costs $200, I need an ice pick, two spiked shawls, I need a spiked gloves too, $80 each. I need a survival gun and knife for $80 each. I need a team of dogs to help for $80. I need a sleeping bag for $250 and a tent for $600. I also need cable for climbing. I need these at the price of $2.00 a foot. How much will this trip cost?

We have climbed 5 miles on a mountain each 2 weeks. How many miles do we climb in 25 weeks?
Mrs. Stahauer is my teacher. She can talk a mile a minute. School starts at 8:45 and ends at 3:20. She talks 3/4 of the day. How long do her pupils get to speak?

Oct 6, 1989

If each kid in the school had 2 books, how many books would there be if half of the kids put theirs together? There are 347 kids in the school and there are 34 teachers.

Jenny
Oct 10, 1989

Mom

We are going to buy Mrs. T a necklace for her Birthday. We get a $1.50 each week. The necklace costs $55.95. How many weeks will we have to save for the necklace?
An Alaskan Brown Bear is nine feet tall and the Polar Bear is eight feet, three quarters feet tall, the American Black Bears are five feet tall with the Alaskan Black Bear, Sloth Bear, Spectacle Bear, Giant Panda and the Sun Bear is three feet tall. If I shot three Alaskan Brown Bears, four Polar Bears, four Polar Bears, seven American Black Bears, one Sloth Bear and two Sun Bears, how many pounds of meat will you have to store away for the winter in your freezer? There’s 147 pounds per foot.

NBA basket ball hoops are 10 feet high. From the free throw line is 15 feet. Michael Jordan can slam dunk an NBA basket from the free throw line. If he did this 11 times how far is high would this be?
"I have seen all kinds of dogs. I have seen one with 73 spots, one with 29 spots, one with 30 spots. My dog has one spot. My dog is a year and seventy-three days. On January 20, 1990, the rest of the dogs are one year. When was my dog born?"

"I feed my dog every night. When Saturday comes, I get 1.00. How much money will I get each week? Equal to 1.00?"

11/30/90 Hootie's Money
Hootie is hiring me to count her money. I already counted 15.00 dollars. 5 seconds later I counted 10.00 more dollars. Hootie offered me 2.00 for the job or 10%. Which choice has more money?"