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ABSTRACT

Promoting community awareness and understanding of the issues encompassed within the teaching and learning of school mathematics, and involving parents in their children's mathematics education are the first steps toward the goal that every child must gain mathematical power. This document helps to answer some of the most frequent questions that parents and community members ask about the teaching and learning of mathematics. The questions/answers posed here deal with the following topics: (1) student attitudes toward the utility of mathematics and toward self-efficacy with respect to mathematics; (2) the everyday relevance of the mathematical content of the curriculum, and its development; (3) the relationship and the integration of mathematics with other content domains of the curriculum; (4) the effectiveness of cooperative learning within mathematics instruction; (5) the development of higher-order, mathematical thinking skills; (6) active instruction versus information transfer in the school mathematics classroom; (7) issues concerning gender equity in mathematics instructional practices; (8) special problems and needs of minority students, learning disabled students, gifted students, and students with limited English proficiency; (9) the influence on mathematics instruction of textbooks, manipulatives, worksheets, calculators, and computers; (10) the effects of standardized tests and alternative assessment methods upon mathematical performance; (11) international comparisons; (12) the role of subject matter knowledge and other priorities in the development of teacher expertise; (13) departmentalized instruction at the elementary school mathematics level; (14) the impact of parent attitudes on mathematics achievement; (15) the role of homework; (16) reinforcements for mathematical learning found in the home; (17) the relationship between television viewing and mathematics achievement. (154 references) (JJK)

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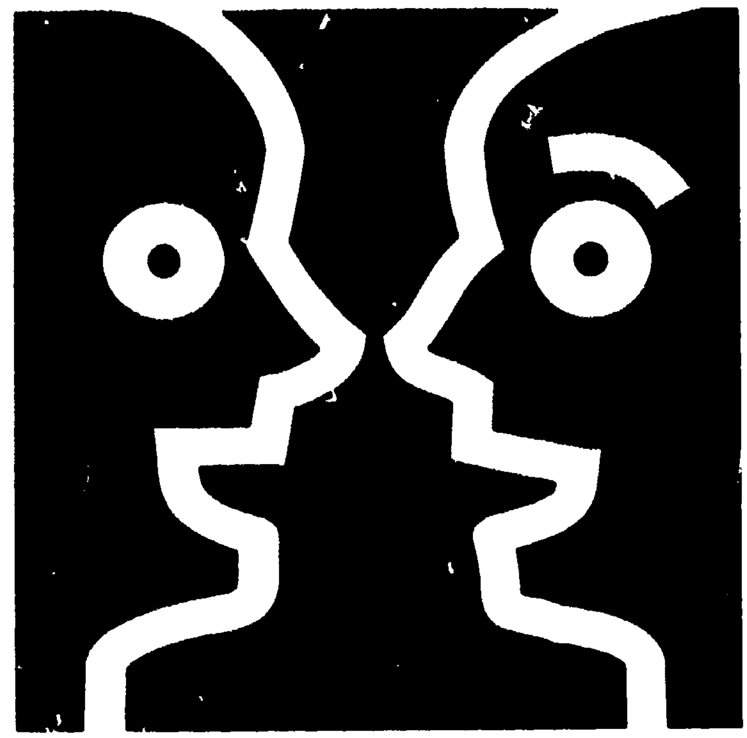

What We Know About Mathematics Teaching And Learning

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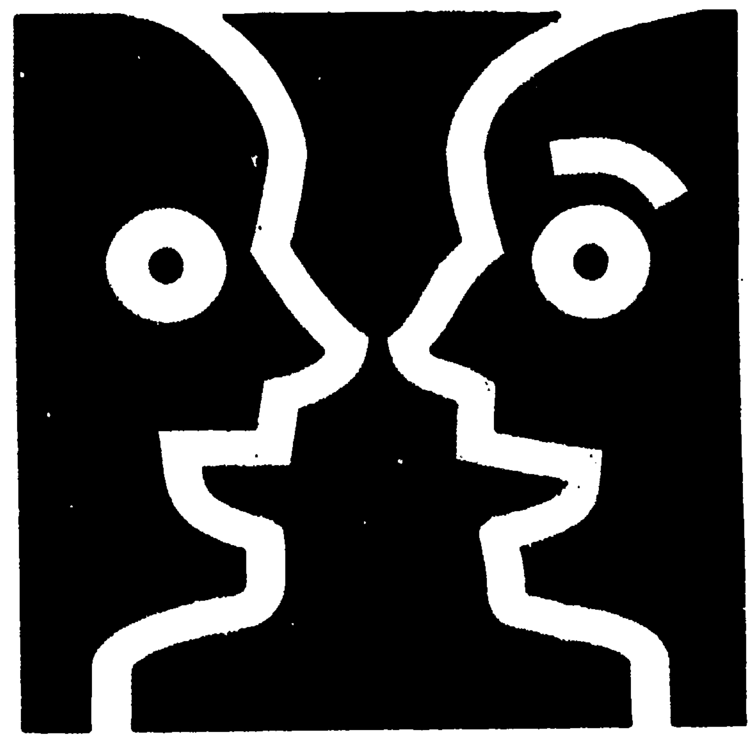

What We Know About Mathematics Teaching And Learning

by Nancy Kober

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PREFACE

We have prepared this document to help answer some of the most frequent questions parents and community members ask about the teaching and learning of mathematics. Many of the questions deal with math teaching and learning in the classroom: Should every elementary teacher be expected to teach math? What kinds of professional development strengthens teachers' math instruction skills? What instructional practices in mathematics build students' higher order thinking skills?

We have also included some common questions parents have about math learning in the home. What can parents do to make sure that their daughters get the same math education as their sons? Does watching television influence youngsters' math achievement? Do parents' attitudes contribute to mathematics achievement? How can parents tell if their child's teacher is teaching math well?

Understanding the issues involved in teaching and learning mathematics and involving parents in their children's math education are, we believe, the first steps in truly embracing the goal that every child receives a quality mathematics education.

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INTRODUCTION

Mathematics, says Webster's Dictionary, is "the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions; and of space configurations and their structure, measurement, transformations, and generalizations."

But that functional definition fails to capture the importance of mathematics to world culture and human endeavor. Some of history's great thinkers have tried to communicate their attraction to mathematics:

. . . "the queen of the sciences" (Karl Friedrich Gauss)

. . . "the most original creation of the human spirit" (Alfred North Whitehead)

. . . "the science which draws necessary conclusions" (Benjamin Pierce)

. . . "supreme beauty — a beauty cold and austere, like that of sculpture . . . yet sublimely pure" (Bertrand Russell).

Scholars and leaders have cited many reasons why students should study math: its contribution to the development of reasoning and independent thinking; its fundamental role in other disciplines, such as science and philosophy; and its practical uses in everyday life.

As society has become more complex, the reasons for becoming competent in math have multiplied. Workers can no longer get by with a single, well-practiced assembly line skill and a few reading and computation skills. Businesses want capable people with broad, transferable knowledge who can learn quickly on the job, master new technologies, and adapt to career change. They are demanding higher order, problem solving skills that cut across disciplines.

All of society is concerned about a generation of disadvantaged students, especially in the inner cities, being unemployable because they have dropped out of school or lack vital skills. In addition, a larger proportion of the future workforce will be made up of women and minorities — groups that are underrepresented in math, science, and technology.

To prepare students to cope with the technological, information-based society of the 21st century, schools will have to raise the level of education in general, and math instruction in particular. No longer can solid mathematics preparation be considered the domain of only gifted students. Virtually everyone will need mathematics to function well in work and in society.

But as many national reports have pointed out, American students trail their international counterparts in math performance, especially in higher order reasoning and analytical skills. In too many schools, mathematics is taught in a fragmented, dry, and watered-down way that saps it of power, beauty, and practicality. These deficiencies in mathematics education are threatening the nation's economic competitiveness and research leadership, as well as its supply of scientists, engineers, technicians, and math and science teachers.

The means exist for turning this situation around. More than a decade of educational research and development in mathematics education has yielded a wealth of information about what works in teaching and learning mathematics. This research is already being applied in many classrooms to good effect.

Several mathematics professional associations have elaborated upon recommendations from research and issued specific plans for upgrading mathematics curriculum, content, and methods. The National Council of Teachers of Mathematics, for example, has developed core mathematics curriculum standards for all students, which have garnered broad support inside and outside the profession. These standards call for a shift away from routine skills and toward higher order skills and conceptual understanding; greater use of educational technology in mathematics instruction; more active learning; a broad range of content; and an emphasis on practical applications.

Math programs around the country are incorporating these standards. One curriculum that has been in the forefront of these reforms is the Comprehensive School Mathematics Program (CSMP) developed with federal funds within the network of regional educational laboratories. CSMP is housed at the Mid-continent Regional Educational Laboratory outside Denver, Colorado. The program focuses on student thinking and problem solving and weaves together components such as probability, arithmetic, and geometry to give students a "big-picture" of math.

Moreover, several education reform panels are examining a restructuring of the teaching profession. Their recommendations are aimed at improving the quality, supply, and professional status of mathematics teachers, as well as helping teachers involve parents more actively in the education of their children.

All of these practices are grounded in what we know about mathematics teaching and learning. But sweeping change of the type needed in mathematics education will not occur without something else that is vital — public understanding and commitment.

EDTALK

MATH IN THE CLASSROOM

How do students' perceptions of math influence their math learning?

Attitudes and beliefs are powerful forces that work beneath the surface to enhance or undermine students' math performance. Students who like math and consider themselves competent at it are more apt to achieve highly and persist in advanced math courses. Students who dislike or fear math and doubt their own competencies are likely to achieve below their capabilities. Attitudes create a self-perpetuating cycle: children with positive beliefs about math perform well, which makes them like math and feel good about themselves; students with negative beliefs fall farther behind, which reinforces their low expectations and sense of failure.

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One such negative belief is that only a few talented students — the "geeky" misfits, usually white males — can attain excellence in math, and that having no head for math is socially acceptable. Another common negative belief is that math is an assembly of disconnected, fixed, and absolutely correct facts and formulas, which teachers are supposed to pass on to students. Students with this view tend to be passive learners, directing their energies toward memorization, without ever questioning what they're taught or pausing to figure things out for themselves.

This is not to say that all students with positive attitudes toward math will pursue advanced study and careers in math and science. Although the majority of seventh and eleventh graders polled by the National Assessment of Educational Progress (NAEP) in 1986 claimed to like math, considered it important, and had positive images of themselves as math students, fewer than half the students in either grade said they wanted to take more mathematics. Although most of the students believed math was a factor in getting a good job, fewer than half expected to work in careers that required math. Moreover, the correlation between attitude and achievement is not a simple, straightforward one. Although an overwhelming proportion said that they wanted to do well in math and were willing to work hard, their performance on the NAEP math assessment failed to reflect this. As the NAEP survey implies, regardless of student attitudes, other factors, such as the way math is taught, strongly affect students' math competency and career goals.

Why are some students turned off by math? When does this usually happen?

Children become turned off to math at different ages and for different reasons. Experiences inside and outside school, along with negative or stereotyped messages from teachers, classmates, parents, the media, and society have enormous influence.

Numbers fascinate young children — just witness youngsters' delight in counting objects around them. And research shows that preschoolers can solve addition and subtraction problems, usually by counting or arranging objects, well before they grasp the underlying concepts. Yet somewhere along the line, many children lose their natural exuberance about math.

Children become turned off to math at different ages and for different reasons. Experiences inside and outside school, along with negative or stereotyped messages from teachers, classmates, parents, the media, and society have enormous influence on student attitudes.

In elementary school, the majority of students say they like math and are good at it, with few differences between girls' and boys' attitudes. By about sixth grade, however, negative attitudes set in, especially among girls. By the time students face critical choices about enrolling in higher math courses, negative attitudes disproportionately keep girls out of the math pipeline.

For other children, the joy goes out of math soon after starting school. Children who experience difficulty and possibly failure as they struggle with math problems in a classroom may come to associate mathematics with feelings of frustration and anxiety. Children who sense that teachers, parents, and other authority figures do not expect them to answer hard questions, solve challenging problems, or take risks in the classroom may adapt their behavior to fit the adult expectations. In these ways, students develop the syndromes known as math anxiety, math avoidance, and learned helplessness.

Even children who seem to be doing well can be turned off by the way most schools teach mathematics. An emphasis on drill and practice, paper-and-pencil exercises, and standardized multiple-choice tests makes mathematics seem dull and sterile. Negative messages from outside reinforce unstimulating classroom experiences: parents who boast of their own incompetence in math; television programs that depict high-achieving math students as "nerds"; adults who cling to stereotyped notions about math being mostly for boys. Taken together, these experiences can quash student interest in math by the time students reach secondary school.

How can schools build positive student attitudes and beliefs toward math?

Negative perceptions are difficult to overcome. But research suggests that teachers and other adults can foster positive attitudes, even when there is little support from the home. As a starting point, teachers need to project that they themselves think math is challenging, fun, and useful. Teacher enthusiasm can set the tone for the whole classroom.

In addition, teachers must back up positive words and feelings with stimulating and enjoyable math activities. Most children enter school with a range of problem-solving skills learned from everyday situations and continue to acquire new procedures on their own outside of school. Rather than squelching these methods, schools can start in the earliest grades with math experiences that build on children's intuitive sense and natural curiosity. Concrete objects — calculators and computers, for example — can inject excitement into the classroom. At the same time, they promote student learning. Giving children chances to discuss, reflect and even write about their attitudes toward math can help them confront and conquer math anxiety.

Teachers need to project that they themselves think math is challenging, fun, and useful.

Teachers can counteract the perception of math as a fixed set of rules by providing some math activities that provoke thought, such as open-ended problems or problems in which students must weigh a variety of solutions. High-achieving students who might otherwise be bored can be given more challenging problems to solve.

Perhaps most importantly, schools need to create a climate of expectation that every child can learn math. Teachers must stress the importance of effort and persistence over innate ability and structure opportunities for every child to have some early, successful experiences.

Finally, students need reassurance that errors are a part of learning. Teachers can bolster positive attitudes by praising success and pointing out examples of sound reasoning and partially correct answers, instead of just marking answers right or wrong. Discussing why answers are right or wrong is a vital part of feedback, for even incorrect answers can exhibit good logic.

How can we make mathematics more relevant to students' lives?

The soundest way to convince students of the relevance of math is not by saying so, but by offering a curriculum that is relevant to today's job and life demands, one that builds students' problem-solving skills, challenges their powers of reasoning, and incorporates technology.

Students are more apt to view mathematics positively when they see its relevance to their everyday lives and futures. The soundest way to convince students of the relevance of math is not by saying so, but by offering a curriculum that is relevant to today's job and life demands, one that builds students' problem-solving skills, challenges their powers of reasoning, and incorporates technology. Students must see how the skills they are learning can be applied to other situations, and how the knowledge they already have leads to new knowledge.

Here are a few other things that teachers can do to make mathematics more relevant:

- Use problems and examples from everyday life, such as those with a consumer or sports bent, ratner than taking contrived problems from textbooks or worksheets. Personalize problem statements; for example, students can measure their height in different units, including nonstandard ones such as pencil lengths. Use problems with practical significance, that are written in natural language and reflect students' culture. Or ask students to create their own problem situations.

- Develop class projects that connect math to the real world, such as running a class store.

- Bring in parents and community people to discuss how they use mathematics in their jobs. Show that even rock stars and professional athletes use math.

What has research found about content coverage in the math curriculum?

Content is the meat of instruction, the mathematical understanding that teachers and parents hope students will acquire (as distinct from the methods by which they acquire it). Content is the sum of decisions about which topics teachers teach and in what order, how much time teachers devote to different math topics altogether and individually, and what levels of mastery schools expect from their students. Teachers are the main arbiters of content, but school policies, districtwide objectives, federal and state requirements, mandated textbooks, and testing policies also influence content.

According to research, most students do not receive enough math content, particularly advanced content, to prepare them for further study and the workplace. At the elementary level, too much math content focuses on computational skills — including long division and square roots that are seldom used in adult life — at the expense of higher order skills, such as problem-solving, conceptualization, applications, and reasoning. According to one study, computational skills receive ten times the emphasis in elementary schools that conceptual understanding or applications do. At the secondary level, despite the dramatic correspondence between advanced course-taking and student performance, a majority of the eleventh graders surveyed by the National Assessment of Educational Progress in 1986 reported taking no advanced math courses beyond geometry and Algebra I. One quarter were taking no math at all. This lack of higher-level content may help explain poor U.S. performance in higher order skills.

School districts and schools vary greatly in what they cover. In general, however, students do not cover enough content to become proficient in math.

Research also confirms the common-sense observation that the more time spent on the math content to be learned — as opposed to time spent on other subjects and on non-instructional distractions such as getting organized, passing out papers, and disciplining students — the better students perform. Many studies have examined the total amount of math content covered in a student's school career, the school year, and the school day. These studies conclude that school districts and schools vary greatly in the the amount of content they cover. In general, however, students do not cover enough content to become proficient in math.

... studies conclude that the mathematics curriculum in American schools tends to be a mile wide and an inch deep.

At the broadest level — the sum of a student's elementary and secondary education — it is clear that many students don't get an adequate amount of mathematics content simply because they stop taking math courses as soon as they can. Most school districts establish a strict sequence of math content — one can't study geometry until one has first mastered arithmetic — with accelerated classes reserved for the most talented students. However, coordination across grades is rare.

Within the school day, the allocation of time to math varies enormously among classrooms. Generally, though, mathematics receives a small amount of time compared to reading. Depending upon the school and the teacher, the amount of math instruction a student receives may differ by as much as 50 hours during a school year, according to one study of third and fifth grades. Some teachers in this study did not even teach math every day.

Separating out the achievement effects of time spent on specific topics is more difficult because one cannot readily distinguish effects due to the quantity of content instruction as opposed to those due to quality. In general, though, studies conclude that the mathematics curriculum in American schools tends to be a mile wide and an inch deep. Large numbers of topics receive only the briefest exposure at the elementary level, with no expectation of mastery. In fact, according to one study, 70 percent or more of the topics teachers covered received less than thirty minutes instructional time, with some receiving only five or ten minutes. Moreover, the specific topics covered vary greatly by school. This practice of superficial coverage runs counter to the lessons of research, which advocate more in-depth study of fewer topics. Teachers give many reasons for cursory coverage—preparing students for "the test," introducing topics that will be picked up in later grades, and reviewing topics that were not mastered the first time they were presented. The upshot is that much of what is taught at one grade is covered again in the next.

In addition, content coverage differs for high and low-achieving students. High-achieving students spend more time on concepts and applications and cover more topics. (Of course, this applies only when

the high-achieving students have been identified and served—a rare event in many schools.) Students who are working below grade level receive a curriculum heavy on review and drill, with a minimal amount of new content. By secondary school, these differences become especially pronounced. Students in advanced math classes receive significantly more content, with greater exposure to complex concepts and higher-order skills; students in lower level and general math classes encounter a curriculum that is essentially a review of elementary math.

Research has found that good teachers focus their lessons on clear and meaningful content, using demonstrations and examples relevant to the content being taught. Their lessons have an understandable pattern, and their transitions to new topics are well defined. These teachers also establish classroom routines that are predictable to students and take care of non-instructional business with minimum fuss.

At the broadest level, math needs to receive higher priority in schools, with a regular time allotted to it. At the individual classroom level, classes should be managed to maximize time on task and minimize distractions.

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What is the relationship between reading and writing and math achievement?

By high school, there is a strong correlation between verbal aptitude and achievement in mathematics.

Literacy is crucial to the acquisition of math skills. By high school, there is a strong correlation between verbal aptitude and achievement in mathematics. For this reason, math experts view communication as an essential component of the mathematics curriculum.

In the early grades, it may be hard to distinguish whether children's weaknesses in math stem from reading problems or conceptual problems. Young children develop language skills and mathematics skills at different rates. Some children compute well until they are faced with word problems. For these children, it is the language of math instruction that presents difficulty. Math textbooks have their own peculiar format, structure, vocabulary, narrative, and graphics, unlike any other type of children's book. They are among the most difficult reading for all students. With the emphasis most schools place on written answers, writing is another variable that can get in the way of young children learning math.

When students' reading and writing skills are weak or not fully developed, teachers must make special efforts to teach the language of math. Direct instruction of math symbols, graphics, and vocabulary can help students over the hump. In addition, teachers should communicate problems in a variety of ways — for example, using diagrams or symbols instead of words, or reading aloud — so that reading comprehension does not always factor in the solution.

There is a positive side to the relationship between reading and math. Mathematics can help children read better by providing opportunities to practice reading and to make inferences.

Research also supports the use of writing activities to improve math skills and help lighten math anxiety. Writing problems or keeping journals help students communicate about math and order their thoughts. Writing is an especially effective way to develop conceptual and higher order skills. One cannot assume, however, that good writers necessarily are high achievers in math.

How can math instruction be better integrated with the rest of the curriculum?

Real jobs and everyday situations are not easily compartmentalized, proponents of interdisciplinary learning argue, and neither should school be. Dividing the school day into thirty or fifty-minute chunks of subject matter leaves students unaware of the interdependence of such subjects as science and math.

A natural starting point in a more integrated approach to learning math is to link math and science. Integrated math and science programs, such as Project AIMS (Activities to Integrate Math and Science) and the Unified Science and Mathematics for Elementary School program, emphasize the role of mathematics as the primary language of other sciences and the chief analytical tool for technology.

Multidisciplinary programs are not limited to science. Social studies, reading, writing, and even art, music, and physical education can be partnered with math. One program, for instance, brings together math, social studies, art, and writing by teaching students a unit on number systems in other cultures. Moreover, curricular integration need not always begin with the math class. Mathematics lessons can be embedded in classes whose primary focus is another subject.

Multidisciplinary programs are not limited to science. Social studies, reading, writing, and even art, music, and physical education can be partnered with math.

Elementary schools that use less traditional approaches to instruction, such as thematic units and whole language, are natural environments for curriculum integration. One elementary program, for example, had the class design an African roundhouse as a way of learning geometry, measurement, social studies, and language arts.

The limited research on interdisciplinary programs suggests that students achieve as well as in traditional instruction, while student awareness of the relationships between subjects and the relevancy of math increases. There is also some evidence that the integrated approach increases communication and sharing among teaching staff.

One limitation of integrated programs is the lack of interdisciplinary materials. Textbooks discourage curriculum integration, and many teachers end up developing their own materials. Another potential obstacle is the tendency of inductive lessons, such as those used in integrated programs, to require more time for both preparation and execution.

How effective is cooperative learning for mathematics instruction?

Research shows that cooperative learning can result in higher achievement, greater self-confidence, better group relations, more cross-cultural integration, improved acceptance of mainstreamed children, and enhanced social skills.

The traditional image of a math class — students working problems alone at their desks or listening to teachers lecture — is becoming less common. In part this is due to recent studies that show cooperative learning approaches and small group work are effective alternatives to whole class instruction for students of all ages and achievement levels.

Cooperative learning is well suited to a variety of instructional purposes: working with concrete objects, solving problems (especially non-routine and open-ended ones), proving theorems, practicing skills, reviewing and sharing data, and using computers, to name a few.

In a typical cooperative learning situation, groups of three to five children work on a problem. They exchange views, discuss the merits of different approaches and solutions, and persuade each other of the soundness of their arguments. In this way, students' thinking processes are brought into the open andthey are exposed to different approaches. By explaining their views to others, children clarify their own logic and order their thoughts.

The teacher plays a critical role. He or she forms groups, establishes guidelines for work and cooperation, observes and interacts, ties ideas together, assigns homework or in-class work, answers questions and clarifies confusion, checks solutions, moderates and synthesizes. Most importantly, the teacher must guide the interaction by encouraging the flow of ideas, drawing hesitant children into the discussion, modeling skills and logic, and encouraging students to think out loud. The teacher should also convey the message that helping peers is expected.

Research shows that cooperative learning can result in higher achievement, greater self-confidence, better group relations, more cross-cultural integration, improved acceptance of mainstreamed children, and enhanced social skills. A review of eighty small-group cooperative learning studies revealed that in all but two, students in cooperative learning arrangements performed the same as or better

than students in traditional math settings; in over 40 percent of the studies, cooperative learning students showed significantly higher achievement.

Cooperative learning has several other benefits. Small-group cooperative learning gives all students the chance to be successful. Children who tend to be anxious about mathematics sometimes feel safer in small groups. In addition, groups can handle more challenging assignments and can complete tasks that students would not have time to tackle alone.

Cooperative learning is most positive when teachers interact frequently with the groups. Cooperative learning also works best when teachers have short-term goals for each group, regroup frequently, and are flexible about moving from small group to whole class instruction.

Heterogeneous groups that mix students of different achievement levels, gender, and race are usually advisable, according to research. Homogeneous grouping is not usually recommended; if used at all, it should be based on students' math achievement, not their overall academic ability, or on their conceptual needs rather than their skill levels. When students are grouped for remedial work, school professionals must take special care to assess students' needs appropriately.

Small-group cooperative learning gives all students the chance to be successful.

Cooperative learning is not without drawbacks. Teachers who are not used to group work may have initial trouble forming workable groups, mediating occasional conflicts, and handling the shift in their role. In addition, there are some tasks for which whole class instruction is more appropriate — introducing new topics, for example, or synthesizing small group work.

Grouping students with different achievement levels is a delicate task. Some teachers pay close attention to students' personalities and learning styles. After some experience with grouping, teachers can develop keen judgment about which students work comfortably

together and about the level of difficulty each student can tolerate. Other cooperative learning specialists simply use random groupings and emphasize group tasks over group structure.

The key lesson from research is to keep groups flexible and rearrange periodically. The size and makeup of groups should vary as subject content changes. A teacher might consider regrouping at the beginning of each topic or task. When groups are static, subtle hierarchies emerge. According to one study, the lowest ranked students in each group can develop anxiety toward math if groups stay fixed for too long.

The key lesson from research is to keep groups flexible and rearrange periodically.

Which math instructional practices enhance students' higher order thinking skills?

If one point is clear from over a decade of national reports about the state of mathematics education, it is this: U.S. students are not developing the higher order skills they need to compete in the modern workplace.

Most analysts trace these weaknesses to the fact that students have been taught math as a collection of disjointed facts and formulas to be practiced, memorized, and volleyed back in the form of right test answers. Research corroborates that most of the time in elementary math classes — perhaps as much as three-fourths — is spent teaching lower level skills, often through paper-and-pencil drill and seatwork. Problem solving, when it is taught at all, is most often done using end-of-chapter or workbook story problems. One study found that only 15 percent of the time in fourth grade math class is spent learning higher level math content.

Whether the teacher is using cooperative learning, whole-class instruction, or another approach, he or she can challenge students to search for patterns and connections; to create representations and explanations; to use a variety of strategies to work problems and check solutions; and to apply prior knowledge to new information.

Mechanics without meaning does not enable children to visualize relationships that make mathematics interesting, elegant, and logical. If students forget the mechanical procedure, they have nothing to fall back on. By contrast, when students understand the "why" behind something, they remember facts longer, use them more readily, and apply their knowledge to learning new tasks. Acquiring higher order skills and thinking strategies fuels lifelong learning.

Research suggests ways to address higher order skill deficiencies. One way is to use instructional techniques, such as cooperative learning, that encourage students to think for themselves — exploring topics, developing and refining their own ideas, thinking deeply about concepts, and discussing strategies and procedures.

In addition, teachers can explicitly teach thinking strategies through such procedures as modeling, class discussion, and encouraging students to think aloud. Whether the teacher is using cooperative learning, whole-class instruction, or another approach, he or she can challenge students to search for patterns and connections; to create representations and explanations; to use a variety of strategies to work

problems and check solutions; and to apply prior knowledge to new information. For example, a teacher might introduce a problem that can be solved in a variety of ways and have students discuss it. Or ask students to restate problems in their own words. Or have students make up their own problems.

An emphasis on higher order skills does not mean the elimination of computation, step-by-step procedures, and basic arithmetic operations. An effective program will teach these basic skills in a way that builds a foundation for students to later acquire more sophisticated understanding and problem solving skills.

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What constitutes active instruction in mathematics?

American education has traditionally viewed the student as a kind of jug into which one pours information. But finally, some schools are rethinking this premise and more actively engaging students in the learning process.

Active instruction is based on research that recognizes learning as a dynamic process in which a child brings to bear the knowledge he or she already has. Rather than trying to transfer information to the child in a ready-made package, active instruction builds on each child's level of understanding and conceptual development. The teacher becomes less of a transmitter and more of a facilitator and organizer. The classroom becomes a lively place where students freely exchange ideas and learn concepts by talking, exploring, and discovering.

Active instruction encompasses a range of instructional approaches — small groups and cooperative learning, class discussion, teacher interviewing, and the use of hands-on experiences and concrete objects. It is well-suited to teaching higher order skills.

Communication, including listening, speaking, reading, and writing, is a major part of active instruction. Teachers ask students to justify their answers, think aloud, weigh different options for solving problems, and sometimes put their thoughts into writing. In this way, students organize their thinking and confront incomplete understanding; teachers can tell whether students have grasped important mathematical concepts. The goal is to create an atmosphere in which independent thinking flourishes.

Studies have shown that instruction that features active and hands-on experiences has a positive effect on mathematics achievement and improves student attitudes toward math.

Active instruction encompasses a range of instructional approaches — small groups and cooperative learning, class discussion, teacher interviewing, and the use of hands-on experiences and concrete objects.

What produces gender inequity in math? How can these inequities be overcome?

Although research is divided about whether there are any meaningful gender differences in mental abilities, there is no evidence that females as a group have less aptitude for mathematics than males. At the elementary level there are few, if any, consistent gender differences. But by high school, even when the number of math courses taken is factored out, males score higher than females in math knowledge, skills, applications, and understanding.

Girls pay a high price for these achievement differences, which may prematurely foreclose their options for careers and higher education in math, science, and technical fields.

There is no evidence that females as a group have less aptitude for math than males.

A complex assortment of social forces produce or influence these gender inequities. Within the school, the attitudes of teachers and other staff and the interactions that occur in the classroom are key determinants. School counselors, for example, may subtly discourage girls from pursuing mathematics careers. Women teachers, who themselves may not be comfortable with mathematics, can foster inequities by sending out negative signals about math or treating males and females differently in the classroom.

One study found that fourth grade teachers were less likely to praise girls than boys for correct math responses and less willing to prompt girls who gave wrong answers. When teachers give feedback to girls on math work, they are more likely to focus on their intellectual abilities or work habits, whereas with boys, teachers are more apt to criticize such non-intellectual aspects as neatness and effort. Teachers in one study were eight times more likely to attribute boys' failure to a lack of motivation than they were girls' failure. Girls, moreover, tend to take this type of criticism to heart, as a true indicator of their talent.

Girls are particularly susceptible to a syndrome that has come to be called "learned helplessness." Learned helplessness sets in after a student has experienced what she perceives to be a "failure" in her early encounters with new material. Certain cues, such as the beginning of math class, resurrect the feelings of low self-esteem and ineptitude that accompanied those seminal experiences, and the student becomes unwilling or unable to adapt to new situations, even ones entirely

within her control. The student comes to feel that there is little sense putting forth effort because she will not be able to do the problem anyway. The feelings of helplessness can transfer across tasks and become indelibly associated with math itself.

The methods of instruction may influence girls' achievement as well. For instance, there is some limited evidence that boys learn more on computers than girls, which raises important, unanswered equity questions.

Educators can take several actions to reduce or eliminate gender inequities in mathematics. At the building level, the school administrator can put together professional development workshops to help the whole staff learn to create a fairer environment. At the classroom level, teachers can monitor their own behavior and attitudes more closely, taking care to eliminate sex bias. At the student level, teachers can initiate activities that confront sex bias head on. Discussing math anxiety or learned helplessness with girls who are having problems, assigning students to keep journals, conducting formal assessments of attitude, discussing the usefulness of math and the career opportunities for boys and girls, bringing in successful women scientists to serve as role models — these are just a few of the options.

New studies suggest the efforts to reduce inequities that have occurred over the last several years may be paying off. A very recent study shows that as late as tenth grade, girls and boys are scoring equally well on basic math tests. The new data suggest the gap between male and female students may be disappearing, most likely as a result of more girls taking advanced courses.

... teachers can initiate activities that confront sex bias head on.

What special problems do minority students face in math? What approaches can help overcome them?

Children of all races enter school with the capability to do math, but as they progress through the educational system, certain minority groups become underrepresented in mathematics classes and fail to fulfill their potential.

As with gender inequity, societal stereotypes, teacher attitudes and expectations, school climate, student attitudes, peer interactions, and classroom processes all come to bear on minority students' math performance.

Research has focused mainly on African-American and Hispanic students. As early as first and second grades, differences in achievement among African-American, Hispanic, and white students emerge. By the end of second grade, a greater proportion of African-American and Hispanic students have slipped below grade level, compared to white and Asian students. By the end of elementary school, many Hispanic and African-American students have fallen so far behind that catching up may seem almost out of reach.

At the secondary level, large differences in performance and course enrollments exist between minority and majority students. Minority students who attend schools with high minority enrollments achieve even less than their minority peers in other schools. African-American students take about one year less of high school mathematics than the national average and are much more heavily enrolled in lower-level math courses, such as general math.

It is difficult to consider racial and ethnic disparities in mathematics achievement separate from socio-economic status. A disproportionate number of minority students come from low-income families or families where parents are not well-educated. Both income and parental education are strong influences on student achievement and coursetaking patterns, and these factors most certainly affect the minority students' achievement.

As with gender inequity, societal stereotypes, teacher attitudes and expectations, school climate, student attitudes, peer interactions, and classroom processes all come to bear on minority students' math performance. For example, if a teacher has preconceived expectations about minority student performance, that teacher may call on minority students less in class. Often there are few minority role models among the teaching staff, especially in math. Research corroborates that

African-American students perceive differences in how they are treated in class. Research also indicates that minority students who do enroll in advanced classes tend to feel a sense of isolation and a greater need to prove themselves.

Many school districts have undertaken special efforts to increase the achievement and representation of African-American and Hispanic students, and these have led to noticeable improvements. As with gender inequity, school staffs need to become more aware of the influence of attitudes. Teachers need to monitor classroom processes such as the frequency and nature of their interactions with students of different races. Students must become more aware of their own attitudes and more sensitive to cultural differences of students from other racial and ethnic backgrounds. Techniques such as class discussions, student interviews, journal writing, and formal attitude assessments can help identify inequities.

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What are the special needs of disabled students in mathematics?

Research shows that educational technology, including computers and calculators, can be adapted for use by disabled students and can result in higher math achievement and improved self image.

Physicist Stephen Hawkins demonstrates that persons with disabilities can be among the most gifted of mathematicians. Structuring an appropriate math education for the physically handicapped child may simply be a matter of providing the right instructional aids. Research shows that educational technology, including computers and calculators, can be adapted for use by disabled students and can result in higher math achievement and improved self image.

Math information processing is especially difficult for learning disabled (LD) students. The vocabulary, text format, structure, and use of symbols in mathematics texts can seem like insurmountable obstacles to LD children, who tend to have difficulty with decoding, visual-spatial relations, directionality, and sequencing. Direct instruction in the language of math can help students recognize and get over these hurdles.

Teachers should also be sensitive to the fact that the settings of word problems are often far removed from the disabled child's everyday life. Personalized problems and student-written problems can help overcome these barriers.

Cooperative learning approaches have also been shown to be successful in mainstreaming disabled children in the math classroom. And as with any component of a disabled child's educational program, an individualized approach tailored to the child's needs is advisable.

What are the special needs of mathematically gifted students and how can schools help meet them?

If the repetitiveness and slow pace of the typical mathematics curriculum is a problem for the average student, it is even more of a problem for mathematically gifted students. While gifted students enroll in advanced math courses more frequently than other students, even then they are often not challenged to achieve to the best of their potential.

Moreover, gifted female, minority, low-income or disabled students are not always identified early enough or encouraged to pursue advanced courses. There is research evidence, for example, that stereotyped behavior of parents may impair the achievement of mathematically gifted girls.

It is usually not enough to provide a few enrichment experiences for gifted students; gifted students need opportunities, flexibility, and encouragement to move faster through their courses and tackle more challenging content. Successful math programs for gifted students focus on problem-oriented independent study, have dynamic teachers, use high-quality materials on a wide range of topics, and involve active recruiting and multiple measures to identify promising students. Gifted students also benefit greatly from interactions with intellectual peers through such activities as special competitions, contact with nearby universities, and summer programs.

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What special problems do children whose native language is not English face in learning math?

Not only is the language of mathematics hard to understand, but the cultural context of the problem may be unfamiliar to a child from the barrio or an Eskimo fishing village.

While the depiction of mathematics as a universal language of symbols sounds appealing, it is misleading. Language skills affect mathematics learning in many ways, and students with limited English proficiency (LEP) face a host of special problems in acquiring mathematics knowledge.

Mathematics has its own vocabulary, syntax, and format that present difficulties even for some English-speaking children. Imagine, then, the plight of the LEP child, confronted with unfamiliar terms ("quotient," "divisor"), words with double meanings ("times," "feet"), and a dense prose quite unlike that encountered in other books. According to research studies, language can be the hardest part of mathematics for LEP children; these students are more likely to make errors because of semantic problems than content processes.

Not only is the language of mathematics hard to understand, but the cultural context of the problem may be unfamiliar to a child from the barrio or an Eskimo fishing village. LEP children who are immigrants have additional obstacles: they may have been taught mathematics differently — some Latin American countries, for example, use different notations and steps in division — or may have had little or no formal schooling and be illiterate in their native language.

LEP students can make great gains when they're taught in a way that recognizes their special needs. Research appears to support the use of students' native language to reinforce math concepts and skills and discuss difficulties. When bilingual teachers or aides are not available, a monolingual mathematics teacher can still show sensitivity to the LEP student's special needs by systematically teaching math vocabulary, cutting down on the use of idioms, using culturally relevant problems and illustrations, and incorporating activities that teach reading and writing skills in a mathematics context. Hands-on experiences, such as the use of manipulatives, can help clarify meaning. Techniques that maximize communication among students and the teacher — such as letting children write their own word problems or establishing cooperative learning groups with English-dominant peers — also improve the achievement of LEP children. (There is evidence such groupings help native English speakers learn

better, too.) Further, teachers must recognize that not all LEP populations have the same needs, and that what works for a Hispanic child may not work for a Southeast Asian child.

The presence of chidren who speak different languages in the math classroom should not be viewed as a problem, but as an asset that will ultimately strengthen all children's educational experience. Providing opportunities for LEP students to teach their classmates number words in their own language or explain their monetary system, for example, can enhance the LEP student's self-confidence at the same time it exposes other students to a second language and culture.

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How do textbooks influence what teachers teach in mathematics?

Texts varied to a remarkable degree in the topics they covered, how much space each topic got, and how the topic was treated.

Most teachers use one textbook to teach mathematics. While they may deviate frequently and supplement that book with other materials, the text remains the prime determinant of topics that comprise math curriculum.

A number of studies, most from the early to mid-1980s, examined the quality, content, and influence of widely used mathematics textbooks and found them wanting in key respects. The most popular American math textbooks reviewed were found to be long, wordy, repetitive, and superficial in their coverage of topics. Most emphasized low-level computational procedures at the expense of thinking and understanding. Some were ambiguous and confusing.

Texts varied to a remarkable degree in the topics they covered, how much space each topic got, and how the topic was treated. The list of topics included in the most widely used texts was by no means standard, and at times seemed idiosyncratic. One 1983 study of four commonly used fourth grade texts, along with five popular norm-referenced standardized tests, found only six common topics, out of 385 total. Most topics received scant attention, and often a single topic was spread across many separate lessons. Compared with texts in other developed nations, American textbooks introduced topics at a slower pace. After about third grade, content tended to become repetitive, with the introduction of new material decreasing steadily until eighth grade.

Despite their length, most texts did not reflect the best of current research in their coverage or presentation of topics. For example, estimation, which research shows to be an effective learning tool, rarely appeared in textbooks before third or fourth grade. Chapters on measurement presented information without the hands-on exercises that research suggests is essential for learning this skill. Pictures were sometimes inadequate or misleading, especially in their presentation of three-dimensional volume. Many texts were written in a mechanical way that assumed teachers would follow them to the letter and in sequence, although this does not reflect common practice.

Perhaps the most serious weakness was the lack of emphasis on higher order skills. In the typical math text, according to one study,

over two-thirds of the exercises focused on skill practice; a small portion, usually less than one-fourth, focused on conceptual understanding. The problems in textbooks — usually in the form of long lists at the end of each chapter — were often nearly identical in format and dwelled on practice and review. Few opportunities existed for problem solving growth, and there was little recognition that problems could be solved in more than one way. Repetitive problem sets — even within a single topic — often were not sequenced to illustrate the way in which mathematical ideas build upon each other. Textbook story problems rarely resembled situations that students encounter in everyday life.

One must keep in mind that since much of the research is several years old, some of these problems may have been resolved by text publishers in their frequent revisions. Nevertheless, the question of how shortcomings in textbooks affect teaching remains a vital one. Some experts feel that textbooks contribute to the passive mode of learning and the emphasis on low-level skills that characterizes much math instruction. In addition, some researchers say, the long lists of similar problems may mislead students into thinking that computational speed and accuracy are more important than conceptual understanding.

This is not to overemphasize the importance of texts. Teachers still make many content decisions, and most teachers devote up to one-fifth of their class time to topics that are not in the book. Texts do not indicate how much time should be allotted to each topic, or how to address different learning styles among students; teachers make those decisions. In the hands of good teachers, a textbook can be a valuable guide for learning; but when relied upon exclusively, it is an insufficient, and sometimes unreliable, arbiter of content and method.

In the hands of good teachers, a textbook can be a valuable guide for learning; but when relied upon exclusively, it is an insufficient, and sometimes unreliable, arbiter of content and method.

What role do tangible objects, or manipulatives, play in mathematics instruction?

Mathematical learning in young children is strongly linked to sense perception and concrete experience. Children seem to move toward an understanding of symbols, and eventually abstract concepts, only after they have first experienced ideas on a concrete level. According to research, manipulatives are a good way — and a highly recommended way in the early elementary grades — of providing this tangible experience.

Manipulatives are objects with appeal to several senses. Students can touch, handle, and move them. They range from commercial products (unifix cubes, Cuisenaire rods, base-ten blocks) to everyday objects, such as nickels and dimes. Numerous studies have documented their positive effects on student achievement. Students who learn with manipulatives are better able to cross the bridge to the abstract world of mathematical oncepts and apply their knowledge to real-life situations. With their visual, auditory and tactile qualities, manipulatives promote active learning, build motivation, and counteract boredom. They are particularly helpful in introducing students to new concepts. Beansticks, for instance, can help students learn place value while reinforcing counting skills; an apple cut four ways can help children grasp fractions.

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Recent research also suggests that, contrary to some teachers' perceptions, manipulatives can be effective instructional tools for middle school students as well as elementary, and for children with low achievement as well as high. For example, studies have documented the benefits of using three-dimensional objects to help low-achieving youngsters understand geometric principles. Students in middle grades who might balk at handling unifix cubes can learn probability using a deck of cards or a pair of dice. There is some evidence from research that allowing students to move the objects themselves — young children learning to count by dropping marbles in a jar, for example — is preferable to having a teacher demonstrate the action or telling them to follow teacher directions. This is not necessarily true for all lessons, however; in some cases, a key math idea is conveyed better when the teacher controls the materials and directs students' attention to the main points.

A few caveats are in order. Not all types of manipulatives are right for all children, nor are manipulatives appropriate for teaching all topics. Children need opportunities to use (and even choose) objects appropriate to their developmental level and learning style. With young children especially, teachers must take care to provide objects that are not overloaded with distracting, extraneous details (such as irrelevant colors or brand names) and that suit students' degree of physical coordination. Nor do all children need to use manipulatives for the same amount of time. Prolonged use may unwisely encourage some children to stick with procedures that are overly simple or inefficient.

Success with manipulatives depends greatly on the teacher's ability to connect manipulation of objects with underlying mathematical concepts and choose appropriate objects. Despite the benefits of manipulatives, many teachers do not take full advantage of these important tools. While most teachers have access to a variety of manipulatives, they incorporate them into their lessons with varying frequencies, and some not at all. First grade teachers report frequent use, but the popularity of manipulatives declines from grade two on.

In summary, while manipulatives cannot do the job of the teacher, they are a key component of a balanced mathematics education program.

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What are the most common arguments for and against the use of calculators in the math classroom?

Hand held calculators are affordable, convenient, and endorsed by major education reform panels and mathematics teacher groups, yet they are not an everyday tool in most American classrooms. Some parents and educators think that is as it should be; adults who rely on calculators daily to balance their checkbooks or divide lunch tabs view the device, in a classroom setting, as a form of "cheating," a crutch, or a magical substitute for real math knowledge. Calculators, they claim, discourage students from acquiring basic arithmetic facts and developing the mental skills and discipline so crucial to mathematics.

Rather than replacing pencil-and-paper computation procedures, calculators are more likely to reinforce them; rather than substituting for independent thought, calculators are apt to sustain it.

These arguments don't stand up against a body of nearly 100 studies comparing students who regularly use calculators with those who do not. With very few exceptions, these studies, which cut across elementary and secondary grades, show either an achievement edge for the calculator group or no significant difference. They also put to rest some common misconceptions: rather than replacing pencil-and-paper computation procedures, calculators are more likely to reinforce them; rather than substituting for independent thought, calculators are apt to sustain it.

Studies also highlight many non-cognitive advantages of calculator use: students have more positive attitudes and enthusiasm about mathematics, more self-confidence in problem solving, greater persistence and willingness to seek alternative solutions, and greater motivation to work together. In summary, research reveals calculators to be a powerful, fun, and fascinating tool for helping children of all ages and achievement levels learn computation, estimation, problem solving, logical thinking, and other key math skills.

The crux, of course, is whether the calculator is treated as a labor-saving gadget or unique instructional tool. A program that asks students to mindlessly punch buttons and record results is not likely to engender much understanding of the concepts underlying the mechanics. Even seemingly sensible uses of calculators, like fact-drilling and answer-checking, are among the least productive. Moreover, there are some concepts for which the calculator is decidedly inferior. Although state-of-the-art calculators can solve

problems that involve fractions or division with remainders, they may not be the best tool for helping students grasp the concept of ratio that lies behind these operations.

The best instructional uses build upon the calculator's unique advantages—speed, for example, which allows students to work multiple problems and try out different approaches for learning the same mathematical principle. Calculators also present opportunities for students to solve challenging problems that would take too much time with paper and pencil, such as computing their age in hours or calculating the number of hot dogs it would take to make a line across Wrigley Field. With calculators, students of all aptitude levels can progress at their own pace and concentrate on problem-solving strategies without distraction. Calculators also expose students to new concepts, such as decimals, at an earlier stage. Research reveals that with a calculator, young children can solve multiplication and division word problems even before they are proficient in computation procedures.

Whatever the calculator's use, it's essential that teachers aid students with questions and discussions that help them think actively and reflect on the mathematical principles behind the calculator's operations — just as a good teacher does with paper-and-pencil exercises. Teachers should encourage children to estimate, retrace their steps, share their discoveries, and question the reasonableness of answers — important approaches in any mathematics program. The astute teacher will also make an effort to show students the calculator's limitations, perhaps by giving problems that are easier to solve on paper or that underscore what happens when an answer overflows the calculator's finite number of places.

Calculators also present opportunities for students to solve challenging problems that would take too much time with paper and pencil, such as computing their age in hours or calculating the number of hot dogs it would take to make a line across Wrigley Field.

Are computers good instructional tools ?

The clearest gains in achievement occur when computer-assisted instruction supplements, rather than substitutes for, other teaching and learning methods.

Students who enter the workforce without meaningful exposure to computers have been shortchanged by their schools. The role of computers in daily life, and their indisputable importance to scientific and technical fields, make them essential components of mathematics education.

According to research, computers are a valuable instructional tool, especially when used in conjunction with other instructional methods. For most children, computer-assisted instruction can improve achievement at least as well as traditional methods, and sometimes better, although this is not the case with all children. Research is unclear about why some children do better with computerized instruction and others do not. Low-achieving children tend to demonstrate marked gains when they use computers for individualized, remedial instruction. The clearest gains in achievement occur when computer-assisted instruction supplements, rather than substitutes for, other teaching and learning methods. In addition, research demonstrates that students who use computer-assisted instruction tend to have more positive attitudes about math than those who do not.

Computers are used more often in mathematics than in any other subject. And in math, the most common application of computers is for drill and practice. With the computer, students can learn more in a shorter time — up to 40 percent less time — which frees the teacher to concentrate on other instructional tasks. There is some evidence that computer-assisted instruction makes more sense in algebra than arithmetic because drill and practice in algebra is of a different order.

The computer can be a good motivator, as children generally prefer it over other drill techniques. In addition, the computer can make short work of certain procedures that are almost prohibitively tedious with paper and pencil — new microcomputer technology allows students to graph equations with the press of a button.

Some scholars have criticized drill programs as little more than flashy workbooks, concluding that drill is not the most efficient or promising use of computer time in mathematics. Although research shows that computers can be an excellent instructional aid for strengthening

higher order skills, only a limited number of programs exist that actually teach concepts or develop problem solving techniques.

In addition, computers have an abundance of exciting math applications — in geometry, simulations, and equation solving, for instance — that most schools have not begun to tap. One reason for this hesitancy is that many teachers lack the skills to implement more advanced computer applications. They may also be insecure about their own computer literacy. In the haste to acquire microcomputers and be on the cutting edge, many school districts skimped on teacher training and support services.

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Other key research findings about computers:

- Computer-assisted instruction in mathematics tends to work better when there is a high degree of guidance from the teacher.
- Students work longer on computer activities than they do on traditional activities, and talk more about the tasks, which may contribute to learning.
- Computers are good for group learning and increase interaction among students.
- Learning computer programming helps children develop problem solving skills, creativity, and self-direction. However, teaching programming is time consuming, and should not be the primary emphasis of a mathematics instructional program.
- Computer applications in testing can help teachers make immediate and fairly accurate diagnoses of student weaknesses.

Are workbooks and worksheets effective instructional tools?

Many experts view workbooks as an inferior substitute for hands-on learning, such as experience with manipulatives.

The primary goal of workbooks and worksheets — to get students to complete the pages or fill in correct answers — in some ways captures the essence of math as it is taught in many classrooms and judged by standardized tests. Workbooks and worksheets tend to be used for practice and review. They don't usually teach students higher order skills, although teachers can design their own worksheets to focus on whatever objectives they wish. Many experts view workbooks as an inferior substitute for hands-on learning, such as experience with manipulatives.

Workbooks are especially prevalent in remedial programs for low-achieving children and have been criticized for making some math compensatory education programs monotonous and ineffective.

The advent of more efficient methods of review and reinforcement, such as the computer, may diminish the use of workbooks and worksheets, although it is unlikely they will be phased out altogether because of their convenience. To parents, in particular, workbooks and worksheets are tangible evidence of their child's progress in mathematics.

What are the effects of standardized tests on mathematics teaching and learning?

Standardized tests play many roles in mathematics education. They're used to diagnose student needs, inform instructional decisions, evaluate student progress and judge teachers and schools. At the aggregate level, they provide a report card of local, state and national performance.

Because standardized tests are usually considered more objective and reliable than other evaluation methods, educators, students, parents, and the public view their results with gravity. Teachers are held accountable for scores on district wide standardized tests. Often scores are ranked and publicized. Schools that don't show gains may suffer a loss of reputation and other negative consequences.

When used well, which most often means in conjunction with other measures, standardized tests can help teachers pinpoint student weaknesses.

Testing at different points of time can tell teachers where they need to adjust their instructional approaches — for instance, if most students in the class continually miss questions about spatial relations. When tests are tied to the school's curriculum, as many locally-developed criterion referenced tests are, an item-by-item analysis helps a school assess whether its students are mastering local objectives. It can also provide useful information about the curriculum.

The results of standardized tests must be viewed with some caution, however. Even the most sophisticated tests can cover only a limited sample of what can and should be taught. Because answers to multiple choice items have limited utility in assessing higher order skills, an over reliance on standardized tests can perpetuate an inappropriate focus on lower order skills. Studies show — that some 30 percent in one study — place greater emphasis on basic skills than they otherwise would because that's what tests focus on. New forms of standardized tests, such as those designed for use with calculators, hold promise for alleviating some concerns about standardized testing.

Little evidence supports the contention that annual standardized testing drives teacher decisions about content. Teachers are more likely to base their instructional decisions on objectives, textbook tests and tests they develop themselves. Evaluation instruments that are closely aligned with the objectives of the curriculum are generally more useful for diagnosing instructional needs and modifying classroom instruction than standardized, norm-referencing tests.

Can alternative assessment methods effectively gauge student performance in math?

Major professional math organizations have endorsed alternative means of assessment such as teacher observations, interviews, student projects, and presentations.

Evaluation is a necessary part of instruction. In most school districts, however, evaluation remains synonymous with testing. A growing body of evidence shows that multiple and varied measures, often known as performance-based assessment or alternative assessment, offers a more valid picture of student performance. Major professional math organizations have endorsed alternative means of assessment such as teacher observations, interviews, student projects, and presentations.

In math, new forms of assessment ask students to manipulate concrete objects while teachers or other trained personnel observe and assess student skills. Manipulatives are especially useful for evaluating the development of young children, for whom paper-and-pencil tests are not very illuminating. Brief interviews and developmental checklists are also helpful for preschoolers and early elementary children.

Some school districts that use criterion-referenced tests produce mini-tests, cued to local math objectives, that students do on a computer. The computer automatically scores and reports on the results by objective.

Multiple choice exams seldom adaquately evaluate higher order skills. Some school districts use performance-based approaches, such as open-ended test questions and student demonstrations of problem solving, to assess mastery of more advanced concepts. Teacher observations, essays, math notebooks, portfolios of student work, skill mastery checklists, oral protocols, and group projects are other forms of assessment that let students show what they know and how they arrived at their conclusions.

Alternative assessments have some disadvantages. Reliability is a real concern, especially those forms of alternative assessment such as checklists and portfolios, that require teachers or staff to make individual judgments. To be most effective, alternative assessments must involve well-trained personnel at all stages of development, administration, and scoring. Because of this very need for trained administrators and scorers, alternative assessments are often considerably more costly and time consuming. In addition, their results are difficult to aggregate across districts and states.

Why do some nations consistently out-perform the U.S. in math?

The lackluster performance of American students on international assessments of mathematics achievement is a well-publicized and often lamented fact. In nearly every mathematics subject tested, U.S. students trail behind their peers from most other developed countries—a situation that at best constitutes a national embarrassment, at worst a threat to national security.

Researchers have studied other cultures intensively to figure out why foreign students out-perform ours. Most of these studies have examined Japan and other Asian countries, which consistently rank at the top of international comparisons, and focused on a combination of school and home factors to explain their success.

One key factor is time on task. Japan, for instance, has a longer school year and devotes a longer school day, and more hours of the average day to the study of math. By the time the average Japanese student enters seventh grade, according to some estimates, that student has taken the equivalent of 2.4 more school years of math than the average American student. Within the math class, Chinese and Japanese students spend more time engaged in academic activities and less in non-instructional distractions than their American counterparts.

Researchers also like to compare the math curriculum in Asian countries with that of American schools. Curriculum decisions are made by the national government in Japan. New topics are introduced according to a predictable schedule, and students are expected to master them when first presented. Asian students do not spend a lot of time on drill in school. Practice is supposed to be done in the form of homework, which is generally heavier than in the United States.

Teachers in Taiwan and Japan tend to be well prepared. Their lessons are lively and carefully organized, with clear transitions and cogent content. Unlike American teachers, they enjoy a high social status; they are not overburdened with administrative tasks and are seldom required to teach more than four hours per day.

By the time the average Japanese student enters seventh grade, according to some estimates, that student has taken the equivalent of 2.4 more school years of math than the average American student.

The Japanese culture places greater emphasis on effort over ability, and researchers cite this as a significant factor. Japanese parents demand a great deal from students. American parents, by contrast, are more likely to attribute success to ability. Japanese mothers spend more time directly helping their children with schc · rk and organizing their children's lives to minimize distracuons from education.

As this brief overview suggests, some facets of Japanese education could be adapted to American schools, while others would be inappropriate for our culture.

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What is the role of subject matter knowledge in teacher expertise?

Common sense suggests that teachers ought to have firm command of the subject they teach. While researchers have no quarrel with this premise, studies have been unable to clearly tie teachers' subject matter knowledge to either their classroom competency or student achievement.

Because so many factors contribute to teacher performance, it is difficult to isolate the effects of one variable. Studies have made clear, however, that teachers' knowledge of underlying math concepts is often fragmented and sometimes inadequate or distorted. Other studies have indicated that many teachers would like more opportunities to update their subject matter knowledge.

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Elementary teacher candidates can receive a bachelor's degree without ever having sat in a mathematics class with math majors. Instead, subject matter knowledge is transmitted through mathematics courses (generally less advanced) tailored to education majors. Some states do not require candidates for elementary certification to have taken any mathematics content or methods courses. While certification at the secondary level generally requires specific mathematics content courses, the common practice of granting emergency licenses to fill vacancies has meant that secondary teachers cannot always be assumed to have substantive subject matter background.

It is unrealistic to expect teachers who have not had in-depth preparation in mathematics concepts to explain these concepts to their students. In the absence of exposure to the most exciting and sophisticated dimensions of mathematics, many teachers teach math the same way they learned it — by rules and formulas, not concepts. Formal opportunities to acquire new knowledge are limited to occasional inservice workshops and, if the teacher is lucky, a couple of weeks in a summer program.

Most of the major reports on reforming mathematics education have called for significant changes in preservice and inservice education that will give teachers more solid grounding in subject matter. Strengthening the qualifications of teachers is a key step toward upgrading their professional status.

Should there be content specialists in elementary school?

According to research, one good way to organize an elementary mathematics education program is to appoint a mathematics resource teacher or specialist to oversee the entire math instructional program, work with students in small groups, teach more advanced math topics or special needs classes, and advise other classroom teachers. The benefits of this staffing pattern are manifold: exposure of all teachers to new materials and teaching techniques; expert handling of difficult topics; professional diagnosis and remediation for students who have problems; and special acceleration opportunities for gifted students, among others.

Several major mathematics professional groups have recommended placing a math specialist at the head of an elementary school program and fully-certified mathematics teachers in each secondary math classroom.

Based on these research findings and their own experience, several major mathematics professional groups have recommended placing a math specialist at the head of an elementary school program and fully-certified mathematics teachers in each secondary math classroom. The reform panels envision these specialists having more intense preparation in mathematics than most teacher education programs currently offer.

This ideal is far from prevalent. Most states have no certifications or endorsements for elementary math specialists. Changing the situation will require commitment of state certification agencies, state legislatures, schools of education, and the professional mathematics community.

What should be priorities in math teachers' professional development?

Research suggests several priorities for strengthening the skills of mathematics teachers. The first grows out of a philosophical change in the mission of mathematics education. A broad consensus exists that mathematics instruction should move away from a fixed set of routine skills and arbitrary rules, and toward challenging students' power to analyze, reason, and comprehend. With this change in mission comes a shift in the role of teachers — from "dispensers of knowledge" to "facilitators of learning," as the National Council of Teachers of Mathematics noted in its report, *An Agenda for Action*. Thus, a priority for professional development is to help teachers make instruction more active and student-centered, develop students' higher order skills, and encourage students to explore.

A priority for professional development is to help teachers make instruction more active and student-centered, develop students' higher order skills, and encourage students to explore.

A second priority is to help teachers keep up with changes in the fields of mathematics and education. Learning how to integrate calculators, computers and other forms of educational technology into mathematics instruction is one good example. Helping teachers incorporate additional hands-on experiences into the classroom and make assessment more useful are others.

A third priority is to help teachers communicate high expectations for all students — including female, minority, disabled, and disadvantaged students — and deliver instruction in a way that recognizes the special learning needs of each student.

School districts typically devote less than six hours a year to teacher inservice. To cover all their pressing staff development priorities, they will have to allocate more time and resources for inservice training and increase opportunities for staff collegiality. Sabbaticals, summer programs, computer conferencing with experts, interaction with mathematics professionals — all are viable ways to accomplish this.

A final lesson from research files: Reforms will work better if teachers themselves are actively involved in planning their professional development activities, and if these activities respect and recognize the beliefs and knowledge that teachers already have.

EDTALK

MATH IN THE HOME

How do parent attitudes affect children's mathematics achievement?

Children's early beliefs and attitudes about mathematics are forged in the home. Parent attitude toward mathematics is a reasonable predictor of children's math achievement at all grade levels. In fact, research shows, children's self-concept and confidence in their own math aptitude is more directly related to their parents' perceptions of their competence than to children's own achievement record. Parents have a lot more influence over expectations and perceptions than any single teacher.

Parents affect students' math achievement through the values they communicate about education, effort, long-term rewards of hard work and persistence, and personal responsibility. Parents who let it be known that success in math comes from effort are more likely to have children who do well in math and have positive feelings about it.

By the same token, negative parent attitudes about math can color children's perceptions and ultimately their achievement. One detrimental attitude is that many people are simply not equipped to succeed in math. Parents who had negative educational experiences in math themselves can pass along their fears to their children.

Parents also have great influence over their children's mathematics course decisions. High school students are more likely to improve their mathematics achievement if their parents help them choose the courses they take. Enrollment in advanced math courses often depends on parents pushing their children to take rigorous courses. This type of involvement in course selection is linked, to a notable degree, to parent education.

Children's self-concept and confidence in their own math aptitude is more directly related to their parents' perceptions of their competence than to children's own achievement record.

There are other things that parents can do, regardless of their own experiences and education levels:

- Talk to children about the relevance of math to future jobs and education; make clear that studying math has a long-term payoff.
- Let children know they can succeed.
- Set high expectations and keep tabs on children's progress.

How do parent attitudes contribute to girls' and boys' achievement in math?

Many parents have different beliefs about sons' and daughters' math achievement, even when sons' and daughters' grades and test scores are similar. In general, females receive less encouragement to pursue math from both parents.

As already stated, parents influence student course selections, attitudes, and self-concept in math. Research shows that many parents have different beliefs about sons' and daughters' math achievement, even when sons' and daughters' grades and test scores are similar. In general, females receive less encouragement to pursue math from both parents.

Mothers and fathers view their own math abilities differently and express different attitudes about math to their children. Mothers who don't see themselves as competent in mathematics may transmit this message to their daughters. Fathers of children with average achievement tend to set lower math standards for daughters than for sons. (This differentiation disappears with high-achieving students.) Moreover, parents' career aspirations for their children still reflect sex stereotypes about math and science careers being primarily for men.

Parents also hold different opinions about the role that ability and effort play for girls and boys. As a rule, parents think math is more difficult for daughters and assume that daughters must work harder to succeed. By contrast, they credit sons with more math talent. By focusing more on effort in the case of a daughter, parents can undermine a girl's confidence in her math talent and raise doubts about continued success. Because of these gender-related attitudes, girls must be more self-motivated to pursue math.

What is the relationship between math homework and achievement?

Some studies have found that math achievement is higher when students have regular homework. Just about as many studies have found no significant correlation between homework and achievement gains. No studies have revealed negative effects from homework. Therefore, it seems reasonable to conclude that while time spent on homework is not a potent predictor of student math gains, homework doesn't hurt and may very well help.

There is also some research evidence that positive effects of homework may accumulate over time and that homework helps students become more independent learners.

Homework appears to be most useful in improving students' computational skills. Short daily assignments seem to be the best form of practice. According to research, however, the practice of grading homework may be counterproductive. The most beneficial approach is for teachers to make home drill-and-practice activities risk-free; to credit students for doing homework and not punish them when they have difficulty. From the teacher's standpoint, homework should be a primary source of information about a child's progress, which the teacher can use to adapt instruction to student needs.

While time spent on homework is not a potent predictor of student math gains, homework doesn't hurt and may very well help.

Is there an optimal amount of homework?

Studies have found that one to three hours per night of homework can make low-achieving students perform as well as average students who do no homework.

Studies have found that one to three hours per night of homework can make low-achieving students perform as well as average students who do no homework. There is a marked correlation between the amount of homework students do and their percentile scores on standardized achievement tests. However, it is not clear whether this is because higher-achieving students are already more motivated. What is clear is that homework increases the amount of time spent learning, and that in itself, bodes well for student performance.

Teachers say they assign ten hours of homework a week, on average. But many students say they spend considerably less time than that on homework, and about ten percent say they do none at all.

The quality of homework is as important as the quantity. Students are more willing to do homework when teachers treat it as an integral part of the curriculum and give back written comments. Students also take homework more seriously when they perceive assignments as useful — for instance, when assignments require students to think instead of just fill in worksheets.

In assigning homework, teachers should attempt some individualization, so that students who have clearly mastered a skill are not consistently required to do "busy work" at home.

Does achievement increase when parents help children with math homework?

One study concluded that when parents monitor homework, students complete more assignments, have higher test scores and higher math grades. Another study of second graders found that when students and parents do homework together, children have better attitudes and achievement in math. Parent involvement alone, however, does not ensure that homework is useful. Teacher feedback on homework is essential.

When parents monitor homework, students complete more assignments, have higher test scores and higher math grades.

In cases where parents lack the skills to help their children with homework, schools and community groups sometimes sponsor homework hotlines, tutoring, parent workshops, and family math programs. One warning: Parents should never use math homework as punishment.

Even when homework is not assigned, parents can encourage children to devote time to math study at home. A number of exciting math materials exist that children and parents can use together at home. The learning experiences in these materials are less prescriptive than homework but still reinforce the value of study.

What is the relationship between math achievement and reading materials in the home?

The presence of reading materials in the home is significantly related to children's math achievement and attitudes, whether or not the books have anything to do with math.

The presence of reading materials in the home is significantly related to children's math achievement and attitudes, whether or not the books have anything to do with math. An even stronger reinforcement occurs when children read books at home themselves or when parents read books aloud to younger children.

In fact, books and children's literature are an excellent home supplement to traditional mathematics materials. Storybooks can be used to teach such mathematical concepts as counting, estimation, and ordering (What happened first? What happened second?).

Reading lies at the heart of education. It exposes children to ideas, cultures, and experiences outside their everyday lives; triggers creative thought processes; stimulates interest in new concepts and subjects; builds capacity for deep understanding and rational thought; and provides opportunities for discussion, sharing, laughing, and imagining.

American children do not spend much time reading on their own — just an average of seven or eight minutes a day in elementary school. This is unfortunate, especially since the amount of leisure time spent reading is a good predictor of school achievement.

If books are not available at home, trips with children to the library can be a meaningful alternative.

What types of activities can parents do at home to reinforce children's math skills?

Parents are a child's earliest and most enduring teachers. Parents, not teachers, supervise about 85 percent of a child's waking hours from birth through age eighteen. Unsurprisingly then, the home environment is a major force affecting student learning.

This era of single parent families, large numbers of children living in poverty, and families with two breadwinners places many stresses on the home environment. Some of these stresses can't be changed. But regardless of how much money parents make, or how much education they have, there is one thing all parents can do to make a difference in their child's school performance: show active support for and involvement in their children's education.

Parents who are involved in their children's schooling report more satisfaction with the education their children are getting and a better relationship with the school. Their children have fewer absences, greater motivation, and improved achievement, regardless of their aptitude levels.

At the most active levels, [parent involovement] can mean getting course outlines from the teacher, visiting the classroom, and going over report cards, tests, and assignments with children.

Parents can fortify a child's math education in a number of ways. First, they can become more involved in and concerned about their children's math achievement. This means questioning children about what happened in math class each day, how they are doing, and whether they are having any difficulties in math. It also means becoming familiar with general school policies about attendance, behavior, and academic performance and helping to enforce those policies. It can mean talking to teachers and other parents about a child's math courses, progress, and options. At the most active levels, it can mean getting course outlines from the teacher, visiting the classroom, and going over report cards, tests, and assignments with children.

Second, parents can promote solid educational values and habits by monitoring study; establishing homework, reading, and television time; and emphasizing the benefits of education, effort and discipline. Parents should also stand ready to praise their children's successes.

Third, parents can carry out educational activities outside of school.

Here are some examples of activities parents can do at home to reinforce math learning:

- Take children to science museums, become involved in family math programs run by schools and community groups, and participate in other community educational activities.
- Find math relationships in everyday life. Encourage young children to sort and measure while cooking, count passing objects while riding in the car, add up items while shopping, point out tables and graphs in the newspaper.
- Incorporate math enrichment and fun activities into the family routine. Use dominoes, checkers, dice, and other games to build computation and logic skills. Make wall charts of children's height, physical activities, reading — anything that is fun and motivating.
- Take a break when math enrichment becomes math drudgery.

Finally, parents can convey the message that devotion to study, whether through formal activities such as homework or less formal learning experiences such as outside math reading and family math projects, is a fulfilling and worthwhile use of leisure time.

Take a break when math enrichment becomes math drudgery.

What is the relationship between math achievement and television viewing?

Research is divided on the persistent question, as old as television itself, of whether TV viewing negatively affects student learning. Most studies focus on the relationship between reading achievement and television; however, since reading and math achievement are related, this research provides some indications of the effects of television on math learning.

A study by the Corporation for Public Broadcasting found that average viewing time for all children was about 25 hours per week. This means that by the time a student graduates from high school, he or she will have spent considerably more time watching television than attending school.

Several studies have tried to sort out the relationship between viewing time and achievement. One study found that students who watch less than two hours a day have higher achievement in reading. Another found adverse effects on achievement only when television viewing exceeds ten hours a week. Yet another study concluded that students who view more than six hours daily have sharply lower achievement scores in reading, writing, and math. An examination of the effects of television on student motivation found no significant correlation between the two.

Parents should remember that time spent watching television is time not available for other tasks that have clearly positive effects on achievement, such as reading.

One of the difficulties in assessing the influence of television is trying to sort out causes and effects. Students with higher intelligence tend to watch less TV than those of average or lower intelligence. Children with lower achievement tend to engage in activities that are counter-productive to education, such as doing homework in front of the TV and watching TV late on school nights. Heavy unchecked television viewing by children may be indicative of other problems in the family. In addition, the types of programs students watch seems to be relevant; certain programs have strong negative associations with achievement.

Parents should remember that time spent watching television is time not available for other tasks that have clearly positive effects on achievement, such as reading. If students spent just half the time reading that they spent watching television, they could finish over one hundred additional books every year.

How can parents use television to influence math education in a positive way?

When parents and children watch TV together and discuss it afterward, children are more likely to think about what they are watching and become more discriminating viewers.

Recognizing that children enjoy watching television, parents can monitor what their children watch, guide children toward educational programming, and set TV limits and schedules.

Quality television can be an effective and exciting way to teach math facts and concepts and stretch children's understanding and imagination. A range of educational programming with math content exists — *Square One* and *Sesame Street*, to name a few — and research indicates that these programs can help learning. Parents can redirect their child's attention away from purely recreational shows and toward higher quality programs. Many schools are also using television programming in math to supplement classroom instruction.

In addition, parents can get involved in their children's viewing. When parents and children watch TV together and discuss it afterward, children are more likely to think about what they are watching and become more discriminating viewers. In addition, parents can use television shows to initiate "spin-off" reading, writing and math activities. Moreover, when children of different ages watch quality programs together, the older children can help develop the understanding of younger ones.

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