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ABSTRACT

This study investigated children's ability to interpret a natural social situation, depicted in a narrative story, and to use their understanding of that situation to generate and apply a mental model of debts and assets in solving problems including negative quantities. Fifty-one American students from a parochial school in a predominantly middle class, suburban community, and five Indian boys between the ages of 10 and 13 years old from Calcutta were subjects for the study. The American children were approximately half boys and half girls, 10 second-graders, 12 third-graders, 17 fourth-graders, and 12 fifth-graders. The Indian children only occasionally attended school and were all employed as houseboys. Each child participated individually in a two-part procedure. The first task was to reply to a series of questions about a story concerning the financial difficulties of a character named Sam. The second task was to solve a total of 16 equations by adding or subtracting negative numbers that paralleled the signed number problems presented in the story situation. Results indicated that children showed superior performance on problems posed in the context of the story, in contrast to their ability to solve isomorphic problems presented as formal equations. Those children whose performance was most enhanced were unschooled children from India. The debts and assets analogue appeared to encourage the use of a Divided Number Line model, resulting in difficulties when children had to perform calculations involving crossing over the zero amount from a debts to an assets status. (MDH)

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SOCIAL SENSE-MAKING IN MATHEMATICS; CHILDREN'S IDEAS OF NEGATIVE NUMBERS

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A narrative story-telling methodology was used to tap children's use of a debts and assets model to support calculations with negative numbers. Children showed superior performance on problems posed in the context of the story, in contrast to their ability to solve isomorphic problems presented as formal equations. Those children whose performance was most enhanced were unschooled children from India, who were very familiar with the social situations and problems depicted. The debts and assets analogue appeared to encourage the use of a Divided Number Line model, resulting in difficulties when children had to perform calculations involving crossing over the zero amount from a debts to an assets status.

This paper explores children's intuitions concerning negative numbers. There is a growing body of research indicating that people's intuitions about mathematical concepts are grounded in the goals, physical properties of entities, and social situations that they encounter in their everyday lives (e.g., Resnick, 1989), and that in many cases, their mathematical sophistication in concrete life situations may exceed their ability to solve isomorphic problems posed as "school math." However, in the case of negative numbers, it is not entirely clear what everyday experiences could serve as the basis for the development of relevant concepts. Historically, an arithmetic based on negative quantitites developed slowly and late. Although an early form of negative numbers was noted in China in the second century BC, it was not until the thirteenth century that modern bookkeeping was developed. Bookkeeping, the first systematic application of negative numbers, used Arabic numbers and double entry, and formalized a different status for debts and assets, with balance as a key concept and subtraction as the key operator. Practically, then, debts and assets served as the motivation for the development of negative and positive numbers, but little is known about whether and how children use an understanding of debts and assets to support arithmetic operations with quantities involving negative numbers. Peled, Mukhopadhyay, and Resnick (1989) interviewed first, third, fifth, seventh, and ninth grade children as they solved various forms of equations including negative numbers, and concluded that the models children had were quite abstract, rather than based on experiences in manipulating concrete objects or materials. The youngest children appeared to have no model at all. Among the older children, understanding of negatives progressed from a Divided Number Line Model, in which children computed by partitioning problems into moves toward and away from a balance point at zero, to a Continuous Number Line model that treated integers, both positive and negative, as coherent ordered entities. Neither the younger nor the older children spontaneously referred to natural analogues or practical situations like finances.

In this study, we investigate children's ability to interpret a natural social situation, depicted in a narrative story, and to use their understanding of that situation to generate and apply a mental model of debts and assets in solving problems including negative quantities. Our sample includes elementary school children who have not yet received formal instruction in negative numbers and a group of unschooled Indian children whose everyday experience includes considerable activity in buying and selling, owing and paying. To tap knowledge that children may have about this social context, we use a story-telling methodology, asking children to help reconstruct and interpret events in an extended narrative concerning the financial situation of a fictitious character.

Method

The U.S.A. subjects were 51 students enrolled in a parochial school in a predominantly middle class, suburban community; the Indian subjects were 5 boys in Calcutta. The American children were approximately half boys and half girls, 10 second-graders, 12 third-graders, 17 fourth-graders, and 12 fifth-graders. The Indian children attended school only occasionally (average schooling, two years) and were all occupied as houseboys. Each was between 10 and 13 years old (Indians not from the middle class are often uncertain of their exact age).

Each child participated individually in a two-part procedure. American children were interviewed in a schoolroom by one of two female interviewers; the Indian children were interviewed in a private home by a female Indian interviewer who spoke Bengali. The first task, the story interpretation, was to reply to a series of questions about a story concerning the financial difficulties of a character named Sam who lived on a farm and made a living by raising animals and crops. The story (which had an equivalent Bengali translation) was an extended narrative, approximately five pages of text, recounting a series of events that resulted in fluctuations in Sam's debts and assets. Children were told that the interviewer had heard the story a long time ago, and that they would be asked at various points to help reconstruct events.

The interviewer read the story aloud and asked the child's help in filling in "the missing parts of the story" by solving practical problems concerning negative numbers that were raised in the story context. The questions posed were deliberately ambiguous, that is, of the form, "What is Sam's situation?" rather than, "How much money does Sam owe?" This ambiguity elicited different kinds of answers, some more informative than others. As we will see, some children produced answers that primarily addressed the semantics of the story and the problems of the characters, whereas other children focused on and calculated quantities. The story presented four categories of target problems. Question 1 asked children to explain the consequence of a set of events leading up to the *creation of debt.* Questions 2 and 3 addressed the *cumulation of debt.* Questions 4, 5, and 6 concerned the *reduction of debt.* The

final item, question 6, concerned the *re-establishment of assets*.

The second task, equations solution, was to solve a total of 16 equations by adding or subtracting negative numbers. The equations paralleled the signed number problems presented in the story situation, and also included a few additional problems, including forms that have no real-life analogue. Each equation was presented in three mathematically equivalent notations, some including terms in brackets. The child was presented these equations one at a time in counterbalanced order and asked to solve each and explain the solution. Paper and pencil were provided. The Indian children were reluctant to try these problems, and therefore the equations solution task was not pursued with this sample. For the U.S.A. sample, order of presentation of the story situation and equations solution procedures was counterbalanced within grades.

Results

In this section we report results for each category of problems presented in the story situation and compare children's performance on these problems with their answers to the corresponding problems in the equations solution task.

Creation of Debt. The opening events in the story establish that Sam, starting with a given sum of money, receives some bills, including one from the carpenter, whom he cannot afford to pay. At this point, children were asked, "So, what's Sam's money condition?" Answering this question depends upon understanding, either intuitively or mathematically, that one can subtract a quantity from a lesser quantity to yield a negative quantity. Children's responses to this question, as well as all subsequent questions, were classified into a rough hierarchy ordered by the amount of semantic, quantitative, and mathematical understanding that they demonstrated. The least sophisticated responses mentioned only the immediate practical consequences of the story events. Somewhat more sophisticated were responses that focused on one or more of the relevant numerical quantities in the situation. The most mathematically sophisticated responses included calculations that combined these quantities as described by the events in the story. In Table 1 (and subsequent tables as well), categories of responses are indicated by the column headings; responses are arranged in order of sophistication from left (most sophisticated) to right.

The majority of the responses are in column 1, indicating both that children understood that Sam now owed money and that, in addition, they had performed a calculation to specify the amount of money owed. Smaller percentages of children, almost exclusively in the lower grades, gave answers that were correct but focused on only one of the relevant quantities. For example, responses in column 2 indicated that Sam did not have enough money (the word "enough" explicitly refers to the sum of [+a] - [+b], without calculating or estimating the debt). Responses in column 3 said that Sam needed more money (focusing only on -[+b]). Finally, responses in column 4 contained arithmetic errors and misinterpretations of the situation. Table

1 also establishes that although most children answered Question 1 fully and correctly, very few at any grade were able correctly to solve any of the forms of the corresponding equations. Notably, although none of the Indian children could perform the equations solution task, all five answered the story problems with responses tabulated in column 1, the most sophisticated type of answer.

Table 1: Responses to Story and Equations for Creation of Debts

	Story Question 1 (% each response)				Equations (% correct)		
Grade	1	2	3	4	a - b	[+a] - [+b]	[+a] + [-b]
Second (N = 10)	60	0	30	10	0	0	0
Third (N = 12)	83	8.5	8.5	0	17	17	17
Fourth (N = 17)	88	0	0	12	6	6	0
Fifth (N = 12)	92	8	0	0	8	8	17
Indian (N = 5)	100	0	0	0	-	-	-

Cumulation of Debt. In questions 2 and 3, children were asked about events that resulted in Sam's debt becoming greater. Responding to these questions involved understanding the addition of two negative quantities, or in the context of the story, the cumulation of money owed.

Table 2: Responses to Story and Equations for Cumulation of Debts

	Story (% each response)						Equations (% correct)		
	Question 2			Question 3					
Grade	1	2	3	1	2	3	- a - b	[-a] + [-b]	[-a] - [+b]
Second (N = 10)	40	50	10	50	50	0	0	0	0
Third (N = 12)	67	33	0	42	50	8	8	0	0
Fourth (N = 17)	65	25	10	47	41	12	6	12	0
Fifth (N = 12)	75	25	0	75	25	0	8	17	0
Indian (N = 5)	100	0	0	100	0	0	-	-	-

As Table 2 indicates, performance drops for both these questions, probably because children find it confusing to use the *add* operation to achieve a larger negative result. That is, the

arithmetic operation that is appropriate (adding) seems intuitively inconsistent with the objective (compile a greater negative quantity). In the most sophisticated responses to these questions (column 1 in Table 2), children point out that Sam owes more money and also go on to calculate the new quantity. Responses in column 2 mentioned the additional bills and noted that money was owed, but did not show any attempt to cumulate the debt. A few responses (under column 3) referred to unrelated issues in the situation. Once again, only the Indian children produced responses that were all classified as the most sophisticated. As in question 1, the U.S.A. sample did very poorly on the equation versions of these problems (recall that the Indians did not attempt them at all).

Reduction of Debt. Questions 4, 5, and 6 ask the children to explain Sam's situation after each of three events in which he receives money that he can credit toward reducing his total debt.

Table 3: Responses to Story and Equations for Reduction of Debt

	Story (% each response)											Equations (% correct)		
	Question 4			Question 5			Question 6							
Grade	1	2	3	1	2	3	1	2	3	4	5	- a + b	[-a] + [+b]	[-a] - [-b]
Second	90	10	0	70	30	0	0	50	10	20	20	0	0	0
Third	100	0	0	83	17	0	50	17	17	0	17	17	17	17
Fourth	100	0	0	100	0	0	5	18	24	41	12	6	0	6
Fifth	100	0	0	92	8	0	0	0	50	42	8	17	25	25
Indian	100	0	0	100	0	0	0	0	40	20	40	-	-	-

As Table 3 shows, even the youngest children performed very well on questions 4 and 5. To answer them, children had to realize that a negative quantity was being decremented by the introduction of a smaller positive quantity. Column 1 responses noted that Sam's debt had been reduced and, in addition, included a calculation of the amount of the reduction. Responses in column 2 mentioned only that Sam was still in debt or remained in debt. The younger children, in contrast to the older ones, were more likely to give column 2 responses, which summarized Sam's practical situation without trying to re-evaluate the quantity of the debt. In contrast to questions 4 and 5, children did very poorly on question 6, which is worded somewhat differently. Here, Sam has sold some farm animals and says, "At least this will help me out in my current crisis." The question asks, "What does Sam mean by 'help'?" Apparently, children found the word "help" misleading and believed that it implied a major change in Sam's circumstances. In contrast to questions 4 and 5, very few children responded to question 6 by trying to calculate the amount of debt reduction (column 1). Column 2 responses indicated that Sam's debt was almost erased; that he had only a little left to pay back.

However, the amount to be paid was not calculated. Many children mentioned only the (+b) quantity; that is, the amount of money that Sam received for selling his farm animals. Some children noted that this amount could be credited toward the debt (column 3); others simply noted the the quantity by pointing out that the sale had generated money without noting that this amount could be applied against the debt (column 4). Column 5 responses are particularly interesting; they represent misinterpretations of the story situation, generated, we believe, by the word "help" in the story question. These responses indicate that Sam had now erased his debt and had some money left over. Performance on this question, relative to questions 4 and 5, was most seriously disrupted for the fourth- and fifth-graders and the Indian children. The Indian children were especially skillful in using the story cues as a model for supporting their problem solution. In general, they were better than any of the American children at following the events of the story and keeping track of Sam's changing fortunes. However, in this case, when the semantic cue in the story implied a "happy ending," their performance, like that of the school children, also disintegrated; in particular, 40% of their responses were the misinterpretations in column 5. The school children's poor performance on the equations solution task confirms that they did not have sufficient formal training to override their misinterpretation of the story situation.

Re-establishment of Assets. Sam sells enough blankets to cancel his debt and have some money left over. Question 7, which asks about Sam's situation after this event, requires children to add a larger positive quantity to a smaller negative quantity, yielding a positive sum.

As Table 4 shows, children's performance in response to this question is approximately equivalent to their performance on the cumulation of debt. Column 1 responses indicate that Sam's debt has been erased and he now has money once again. Column 2 responses focus on the fact that he has money from the sale but do not mention that some of it must be credited to cancelling the debt. Column 3 responses are misinterpretations or misreadings. In contrast to results in the equations solution task that we have seen so far, children solved a substantial number of the equations that corresponded to this question, usually by simply inverting the positive and negative terms of the equation to form a simple subtraction problem. However, for no group did performance on the equations equal or exceed performance on the story problem.

Table 4: Responses to Story and Equations for Re-establishment of Assets

	Story Question 7 (% each response)			Equations (% correct)		
Grade	1	2	3	- a + b	[-a] + [+b]	[-a] - [-b]
Second (N = 10)	50	30	20	38	0	38
Third (N = 12)	67	25	8	50	17	58
Fourth (N = 17)	65	35	0	47	0	53
Fifth (N = 12)	58	42	0	42	17	42
Indian (N = 5)	80	0	20	-	-	-

Discussion

In this study, we find a sizeable difference in children's performance with the solution of signed number problems in two different contexts. When they encounter these problems embedded in a narrative story about the social context of debts and assets, their performance is far more complete and competent than when they encounter the isomorphic problems presented as formal equations with mathematical notation. Furthermore, those children who are most familiar with the particular social context presented--that is, the older American children and the Indian children--show the greatest enhancement in performance. This study, like other work in everyday reasoning (e.g., Carraher, Carraher, and Schliemann, 1985; Schliemann & Acioly, in press), finds that people show a superior ability to use and understand mathematical ideas when the relevant concepts and operations are introduced in a contextualized, familiar social situation.

A related methodological point is that we find story-telling an effective device for eliciting informal knowledge in elementary-school children. Although children's ability to solve short mathematics word problems has been much studied, the use of a narrative with an extended and believable storyline is an unexplored methodology for assessing mathematical understanding. Children's ability to perform with understanding in this assessment situation was enhanced, we believe, by their engagement in the story. We note here that story-telling is a particularly familiar and valued medium in the culture of nonschooled Indian children. The Indian children in our sample were deeply involved in the story of Sam's tribulations. Clearly they were not interpreting the interviewer's questions as a request for school answers. Instead, they often spent as much time thinking about practical strategies for solving Sam's problems as they spent on the calculation of quantities. For example, the Indian children worried about whether Sam should try to get a loan from his brother, and considered at length the merits and disadvantages of financial obligations with a family member. However, the story did not invariably enhance mathematical performance. When story cues were misleading, children--including the Indians--lost their track on the situation and made misinterpretations inconsistent

with the quantitative information being presented.

A final issue concerns the extent to which people engage in or avoid arithmetical calculation in their everyday reasoning (e.g., Lave, Murtaugh, & de la Rocha, 1984). Large percentages of the children answered the story questions not only by describing the social consequences of Sam's situation, but also by performing calculations to quantify Sam's current debts or assets. Children appeared to believe that calculating and tracking the amount of Sam's debt was relevant to the practical problem of helping him improve his situation. Some of these calculations appeared to be more difficult than others. For example, using the *add* operation to calculate a greater negative balance appeared confusing. Crossing the zero boundary to re-establish assets was also difficult relative to those problems in which Sam's finances fluctuated but remained within the debit or negative category. These results are consistent with those of Peled, Mukhopadhyay, & Resnick (1989), who described children as using a Divided Number Line model. It is likely that the use of this model is reinforced by the story context, since debts and assets are different kinds of financial entities with different social implications. We conclude that the effects of a meaningful social situation on mathematical performance may be complex, sometimes enhancing and sometimes disrupting performance, and perhaps differentially encouraging the use of particular mental models at the expense of others that children may have in their repertoire.

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