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ABSTRACT

This study was conducted to determine whether the symbolic form of achievement test items influences student performance and whether teachers' use of different symbolic forms during instruction influences estimates of student achievement. The analyses conducted to examine these issues used student achievement test data and teacher responses to questions about their instructional methods from the Second International Mathematics Study (SIMS) of eighth-graders. Most items used traditional symbolic forms, but some used alternative forms such as diagrams, graphs, or verbal expressions. To investigate the effects of symbolic form, it would be necessary to hold other features of the test item constant (numerical complexity, for example); this was not possible with SIMS data. There were significant relationships between teachers' use of symbolic forms and student achievement. Results suggest that the use of alternative symbolic forms may be important for student learning of mathematics. A 14-item list of references is included. An appendix contains 29 tables of data from the analyses, and 16 figures illustrating symbolic forms. (SLD)

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Center for Research on Evaluation, Standards, and Student Testing

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CSE Technical Report 284

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Introduction

Cognitive psychologists have long recognized that mathematical problem-solvi ; often involves translating from the symbolic representation of the problem as given to another symbolic form in which the problem can be solved (e.g., Clement, Lochhead, & Monk, 1980; Hooper, 1981; Nesher, 1982; Shavelson, 1981; Shavelson & Salomon, 1985). Alternative symbolic representations of problems include, for example, words, numbers, algebraic symbols, tables, graphs, diagrams, and pictures (see Hooper, 1981). Lesh, Post, and Behr (1987) go so far as to claim that "the ability to do...translations are significant factors influencing both mathematical learning and problem-solving performance" (p. 7). Indeed, students able to solve mathematical problems do so by representing the problems not in a single symbol system, but in several systems, each corresponding to different parts of a word problem (Lesh, Landau, & Hamilton, 1983). Furthermore, it is well documented that many students have difficulty translating from one symbolic form to another (Clement, Lochhead, & Monk, 1980; Galvin & Bel, 1977; Hooper, 1981; Nesher, 1982; Paige & Simon, 1966).

By concentrating on few symbolic representations (numbers, algebraic symbols, words) for any particular concept, typical mathematics achievement tests give incomplete information about a student's ability to solve problems (see, for example, Cronbach, 1984; Frederikson, 1984; Messick, 1984). Thus, to obtain maximum information about students' ability to solve problems related to a mathematical concept, an achievement test should include items that explicitly and thoroughly test students' ability to translate between different symbolic representations: items that require students to (a) interpret information presented in different symbolic representations and (b) generate different symbolic representations from a given one.

Tests that systematically vary symbolic representation of the problem as given and the responses required do not yet exist. However, comprehensive mathematics tests that incorporate some variability in the symbolic form of the problem do exist. The one analyzed here is the 180-item eighth-grade test from the Second International Mathematics Study (SIMS), a test that was designed to cover the full range of topics taught in eighth-grade mathematics. Data from this test are available for large samples of students from the U.S. and many other countries around the world. The first purpose of the study reported here, then, was to explore the effects of symbolic form of achievement test items (both the problem as presented and the response required) on student performance on this large-scale standardized test. Examining the role of symbolic representation in achievement in the context of existing data is an important first step before tests and instruction that systematically vary symbolic form can be developed. Analyses of these data may reveal important questions that new tests can be designed to answer.

A closely related issue is whether students' ability to translate between different symbolic representations is linked to the instruction that students receive. Students may perform better with some symbolic forms than others because they have experienced them during instruction. The second purpose of the present study, then, was to explore the relationship between teacher use of various symbolic forms in instruction and student achievement. The data for this phase of the study came from an extensive questionnaire that asked SIMS teachers about their instructional methods. These data provided a unique opportunity to analyze the relationship between teachers' instructional methods and student achievement.

The Effect of Symbolic Representation of Test Items on Achievement

Classification of SIMS Items

The first set of analyses focused on the role of the symbolic representation of the test item on student performance. To determine the effect of symbolic representation, it is necessary to control other features of the problem as much as possible. Most importantly, it is important to control for the concept or content being tested. Comparison of performance across symbolic forms of items is sensible only for items measuring the same concept. Consequently, clusters of items on the SIMS test that measured the same concept or content but varied the symbolic form of the problem were identified. Item clusters were identified on the basis of the symbolic form of the item stem and of the response required. Altogether, six clusters of items that varied the symbolic form of the item stem could be identified on the SIMS test. The topics of the six item clusters were: (a) proportional reasoning (9 items), (b) distance-rate-time (8 items), (c) volume (5 items), (d) generating formulas or equations (5 items), (e) statistics (3 items), and (f) coordinate systems or graphs (4 items). All items required a numerical response; that is, students were asked to choose among different numerical alternatives. The items in each cluster appear in Figures 1 through 13.

As can be seen in Figures 1 through 13, the symbolic forms of the item stems were classified as either a word, table, diagram, graph, or algebraic/numerical algorithm problem. An item was classified as a word problem if it described the problem in words, rather than using words only to give directions or embedding words in a predominantly numeric/algebraic problem. Table problems gave information in a table that was needed to solve the problem. Diagram and graph problems gave information in diagrams and graphs, respectively. Algorithmic problems were posed in symbolic terms (algebraic/numeric) without the context being described in words. It should be noted that the symbolic forms of the problems are not "pure"; the table, diagram, and graph problems all had accompanying verbal text, although some problems could be solved without using the information given in the verbal text.

Additional clusters of items that were identified varied the symbolic form of the response required instead of the symbolic form of the stem of the item. For example, five items on the SIMS test included tables of information as part of the item. But two of these items required students to give a numerical answer, two required students to generate a formula, and one required students to locate a position on a bar chart (see Figure 14). Five other problems included bar charts and asked students to generate a verbal description or a numerical response (see Figure 5). Finally, two problems included geometric diagrams but asked students to select an algebraic expression or a number (see Figure 16).

The identification of these clusters of items showed that the SIMS test did vary the symbolic form of the problem for some concepts. The variability of symbolic form of the response required, however, was slim: The vast majority of items required numerical responses. Only a minority called for a response in algebraic, verbal, or other symbolic forms.

Patterns of Achievement Across Symbolic Forms

Tables 1 through 7 give the mean pretest performance, mean posttest performance, and pre-post gains for each item in the clusters identified above. In all cases, the results clearly show that the symbolic form of the item was not a critical factor in determining student performance on the items in the SIMS test. The variability of performance across items with the same symbolic form was typically greater than the variability between different symbolic forms. In Table 1

(proportional reasoning) for example, word problems, table problems, and diagram problems had comparable difficulty (on the average). The variability of items within a symbolic form, however, was striking. Taking an extreme case, the proportions of students correctly answering the two problems with tables were 23% and 78%, respectively, on the posttest.

Inspection of the items in each set suggests numerous factors that mav influence item difficulty more than does symbolic form. Items with the same symbolic form often showed substantial differences on many factors. The proportional reasoning problems, for example, differed in the complexity of the numerical relationships involved (e.g., a ratio of 1:4 verses 3:7), the use of whole integers or decimal numbers, whether the ratio was made explicit in the problem ("the ratio of 2 to 5") or had to be inferred from words ("a boy 5 units tall casts a shadow 3 units long") or from numbers given, the number of values to be calculated (1 verses 2), and whether metric units were involved. Since the items varied unsystematically on these factors, it is impossible to isolate the effects of any one. But it is quite likely that they overwhelmed any effects of the symbolic form itself.

Because it is not possible to isolate the effects of symbolic form on achievement for any concept area, the remaining analyses did not compare performance across items with different symbolic forms, but instead focused on the relationship between instruction and achievement within symbolic forms.

Relationship between General Teaching Styles and Achievement

The majority of the analyses in this study focused on teachers' use of different symbolic representations during instruction and the relationship between teaching style and achievement. Identification of teaching methods was carried out at two levels. At a general or macro level, teaching methods were identified that cut across large content domains (e.g., fractions or algebra). At a more specific or micro level, teaching methods were identified with respect to particular topics or concepts (e.g., proportional reasoning, volume of rectangular solids). This section describes the analyses and results at the general level (general teaching styles) and the next section describes the analyses and results at the specific level (topic-specific teaching methods).

Identification of General Teaching Styles

Two strategies were used to identify teaching styles: factor analysis and logical groupings of teaching variables. Teaching styles were identified for two general content areas defined on the SIMS teacher questionnaire: fractions and algebra. Because the factor analyses of the other three content areas—ratio, proportion, and percent, measurement, and geometry—were not interpretable, the results are not reported or discussed here.

Factor analyses. For each factor analysis, all teacher items that pertained to the use of a symbolic representation were included. These items either explicitly mentioned use of a particular symbolic representation (e.g., use of tables of data) or indirectly referred to symbolic representation in the context of a particular method (algebraic approach to solving proportional equations).

For both content areas, two factors emerged: symbolic flexibility and rules. Most of the teacher items for the symbolic flexibility factors concerned the use of alternative representations for fractions or algebraic expressions (e.g., tables, graphs, diagrams, manipulable aids or activities) or manipulations of information presented in alternative forms. The rules factors consisted predominantly of teacher use of numerical or algebraic procedures for manipulating fractions or algebraic expressions.

The teacher questionnaire items and factor loadings for fractions and algebra appear in Tables 8 and 9, respectively.

The questionnaires for the content areas also had a number of items soliciting teachers' opinions about aspects of teaching and student achievement. The factor analysis of the opinion items for algebra yielded two interpretable factors: perceived student difficulty (a high score indicating a strong belief that students have difficulty in solving algebra problems), and importance of conceptual understanding (a high score indicating a strong belief that it is important for students to conceptually understand problem-solving procedures). The opinion items and factor loadings appear in Table 10.

Analyses of the distributions of teacher variables show that most teachers used or emphasized the majority of instructional methods listed. For fractions, 65% of teachers used or emphasized the methods corresponding to symbolic flexibility and 56% used or emphasized the methods corresponding to use of numeric rules. For algebra, 58% of teachers used or emphasized the methods corresponding to symbolic flexibility and 86% of teachers used or emphasized the methods corresponding to use of algebraic rules. Furthermore, on the opinion items, teachers on the average moderately agreed with statements suggesting that students had difficulty with the material and moderately agreed with statements suggesting that conceptual understanding is important.

Logical groupings of teacher items. Inspection of the factor solutions showed that although the majority of teacher items corresponded to the general categories of symbolic flexibility and numeric/algebraic rules, some items did not. To produce "cleaner" groupings of items, items were grouped on a logical basis using the general categories of teaching styles suggested by the factor analyses. The resulting groupings of items for fractions and algebra appear in Tables 11 and 12, respectively. For algebra, an additional grouping of items was identified: use of applications in problems (e.g., story problems). (The fractions questionnaire did not ask teachers about use of applications.) For each grouping of teacher items, a composite was formed weighting items equally.

Identifying teaching style on logical grounds yielded somewhat different distributions than the teaching style factors emerging from the factor analyses. For fractions, teachers used fewer methods corresponding to symbolic flexibility (53%) and more methods corresponding to use of numeric rules (68%). For algebra, teachers used more methods corresponding to symbolic flexibility (64%) and fewer methods corresponding to use of algebraic rules (64%). For the new algebraic factor, use of applications, 6% of teachers reported using or emphasizing applications in their instruction.

Relationship between General Teaching Styles and Achievement

We expected that instruction emphasizing alternative symbolic representations would be beneficial for performance on items that were presented in non-traditional symbolic forms or contexts (e.g., word problem in which procedures to be carried out are not clearly specified or inferred; problems with tables or diagrams). We also expected that instruction emphasizing numeric or algebraic rules would be beneficial for items presented in "traditional" ways, that is, in which the procedures to be carried out are clearly specified or inferred (e.g., procedural problems or word problems of a clearly recognizable "type"). Specifically, we hypothesized that high scores on the symbolic flexibility factors would be positively related to performance on non-traditional items, and that high scores on the rules factors would be positively related to performance on traditional items. Furthermore, we expected that use of applications in instruction would be

beneficial for problems on the test phrased as applications (typically, word problems presented in a meaningful context).

To determine the relationship between teaching styles and student achievement, multiple regression analyses were conducted for each test item, using pretest scores and teaching styles as predictors. The unit of analysis was the class mean. For the fractions teaching styles, multiple regression analyses were conducted for 11 fractions items on the SIMS test; for the algebra teaching styles, multiple regression analyses were conducted for all algebra items on the test. The fractions and algebra classifications are those used by the SIMS test developers. To determine the relationship between teaching style and achievement, it was important to analyze only those classes that had an opportunity to learn the material necessary to answer an item. If students were not taught the material needed to answer an item, it would not matter what instructional method a teacher used. Consequently, the analyses for each item included only those classes whose teachers reported that they taught the material needed to answer the item during that year.

Fractions teaching styles. The results of the multiple regression analyses of fractions achievement appear in Tables 13 and 14. The results are sometimes quite different for the teaching style factors from the factor analysis and for the teaching styles resulting from logical groupings of instructional items on the questionnaire. The results using the logical groupings are stronger and more interpretable, so the discussion here focuses on them (Table 14).

Symbolic flexibility is a positive predictor of fractions achievement for 12 test items. However, most of these items were straightforward procedural items, asking students to carry out manipulations using fractions, decimals, or percents. Symbolic flexibility was not a significant predictor for most of the items involving alternative symbolic forms (e.g., number line, diagram, region of a square). Only three of the regression coefficients for use of numerical rules were statistically significant, only slightly more than would be expected by chance, so they are not interpreted here. The only noteworthy result is that they are all negative, which is true for over half of the nonsignificant coefficients as well.

There are at least two possible interpretations of the positive effect of use of alternative symbolic representations in instruction on performance on procedural test items. The question is whether the teacher's instructional style influences student achievement or whether the teacher's instructional style is a response to pre-existing strengths and weaknesses of students.

The first interpretation is that emphasizing alternative symbolic representations is beneficial for achievement on traditional types of fractions problems (those that emphasize procedures). Perhaps using alternative representations helps students understand the concepts behind the procedures. Increased conceptual understanding may help students remember and correctly apply the procedures.

Alternatively, teachers may have been adapting their instructional styles to pre-existing patterns of student achievement. That is, teachers who emphasized alternative symbolic representations may have had students who already showed mastery of procedures for manipulating fractions. Since the analyses controlled for pretest achievement, it is unlikely that the latter explanation is the predominant one. The only satisfactory way to resolve this issue, however, is through controlled experimentation in which comparable groups of students are given instruction in fractions that systematically varies the emphasis on alternative symbolic representations.

Algebra teaching styles. The results for the two regression analyses, teaching styles from the factor analysis verses logical identification of teaching styles, yielded quite different results, mainly because of the addition of the third teaching style, use of applications, in the latter analyses. The analyses using teaching styles from the factor analysis (Table 15) suggest a positive effect for symbolic flexibility for some items and a negative effect for use of algebraic rules for other items. Most of the items for which symbolic flexibility was a significant (positive) predictor were not straightforward algebraic procedural items. They were word problems (Items 013, 052, 152) or problems that asked students to do a non-traditional task, such as "choose between several number lines" (Item 082; see Table 15).

Interestingly, all of the significant coefficients for use of algebraic rules (from the factor analysis) were negative. The greater the teachers' reported use of algebraic rules, the lower was student achievement. Nearly all of these items were presented in strictly numerical or algebraic terms without any meaningful context.

When the use of the applications teaching style was added to the regression analyses (along with some modifications in the symbolic flexibility and algebraic rule groupings compared to the factor analyses), most of the significant effects for symbolic flexibility and use of algebraic rules disappeared (Table 16). Instead there appeared a positive effect for use of applications during instruction. Interestingly, the vast majority of the items with significant coefficients for use of applications during instruction did not involve applications. Rather, they were procedural problems asking students to solve or simplify algebraic or numerical expressions. For most of the applications test items (e.g., story problems), the use of applications teaching style was not a significant predictor of performance.

As was true of the fractions results, the interpretation of the algebra results is somewhat ambiguous. Use of applications in instruction may be beneficial for achievement; however, teachers who emphasized applications may have done so because students already showed mastery of algebraic procedures. Again, although controlling for pretest achievement lends credence to the first interpretation, only experimentation will resolve the issue.

Algebra teacher opinions. Although most of the regression coefficients for the algebra opinion factors were not statistically significant, they do form a consistent pattern (Table 17). First, nearly all of the coefficients for "perceived student difficulty" are negative. The more difficulty that teachers believed their students had, the lower the performance on the achievement test. As in the algebra teaching styles, two interpretations are possible. First, teachers' beliefs may shape their instruction and, consequently, student achievement. Or, teachers' beliefs may be a function of pre-existing student difficulties. These analyses control for pretest achievement, however, so the negative relationship for teacher beliefs holds even among classes with the same starting achievement level. This tends to support the former interpretation.

Very few of the regression coefficients for "importance of conceptual understanding" are statistically significant, although most are positive. This suggests that classes with teachers who believe that conceptual understanding is important tend to show relatively high performance on algebra items. One would expect that teachers who believe in the importance of conceptual understanding would emphasize it in their teaching. And conceptual understanding may give students a more varied repertoire of skills for solving problems than instruction that does not.

Identification of Topic-Specific Teaching Methods

The analyses and results described above deal with general content domains: fractions and algebra. A more specific approach to identifying teaching methods was also carried out. For these analyses, the focus was on four of the specific topics identified on the test: proportional reasoning, distance-rate-time, volume, and generating formulas or algebraic equations. These topics were selected because (a) items on the SIMS test were varied in symbolic form and (b) items on the teacher questionnaire specifically addressed these topics.

Two sets of analyses were conducted for the specific topics. First, questions on the teacher questionnaire dealing with a topic were grouped logically into instructional categories that seemed coherent. The relationship between these teaching categories and achievement was then explored. Second, the teacher items were not grouped into categories, but served as the basis for cluster analyses of teachers to identify groups of teachers with similar styles. Differences in achievement for these teachers were then explored.

Initially, we had hoped to create categories of teaching methods that would correspond to different symbolic representations. For example, use of word problems, problems with tables, problems with graphs. These dimensions could then be related to student performance on test items presented in different symbolic forms. Unfortunately, however, few items on the teacher questionnaire addressed the symbolic representation of instruction for a specific topic. So broader groups of items were formed that sometimes concerned symbolic representation but incorporated other elements. The resulting groupings of teacher questionnaire items for each topic appear in Tables 18 through 21. The items seemed to group naturally according to whether methods were based on conventional symbolic forms (numerical, algebraic) or alternative forms (tables, graphs), were presented in the abstract or in a real context, whether problems were open-ended or presented as a clearly-specified procedure, and whether manipulable aids were used. For proportional reasoning, for example, the teaching methods identified were: (a) use of numerical methods in abstract, symbolic terms, (b) use of numerical methods in a real context, (c) use of open-ended problems presented in a real context, and (d) use of construction and measurement (pertaining to similar triangles).

Clusters of teachers. Another analytic approach used to differentiate among teaching styles for the four specific topics (proportional reasoning, distance-rate-time, volume, generating equations) was cluster analysis. In these analyses, teachers were clustered according to their responses on the teacher questionnaire items pertaining to the topic. As in the analyses described above, only teachers who responded that they taught the material to enable students to answer all items corresponding to a specific topic were included.

The cluster analyses were conducted using the computer program CLUSTAN (a combination of the Ward method and an alternative procedure called K-Means produced the most interpretable and reliable cluster solution). The means and standard deviations of the teaching variables for each cluster are presented in Tables 22 through 25 for the four specific topic areas. For proportional reasoning and distance-rate-time, most teachers clustered in a single cluster, making it inappropriate to compare teaching styles across clusters. For volume, teachers in both of the two main clusters emphasized use of physical models, although the first cluster also emphasized measurement of objects. For generating equations, teachers in both of the two main clusters emphasized deriving formulas from verbal descriptions.

Relationship between Topic-Specific Teaching Methods and Achievement

Logical groupings. Only teachers who reported that they had taught material relevant to all of the items in a topic (e.g., proportional reasoning) were included. This produced smaller samples of teachers than the previous analyses. Because the samples were too small to use multiple regression analysis with all of the teaching methods as predictors, partial correlations between teaching methods and posttest achievement controlling for pretest achievement were calculated (Tables 26 through 29).

Although few of the partial correlations are statistically significant, there seems to be one consistent trend across topics. All of the significant results for solving problems in a real context are positive, suggesting that attaching meaning to problems may be beneficial for achievement. On the other hand, using methods that deal with the symbols themselves without meaning attached to the procedures shows mixed relationships with achievement—with positive, negative, and nonsignificant correlations with no clear pattern. No clear pattern of relationships emerged for manipulative methods or non-traditional symbolic forms (e.g., graphs, tables).

Differences between teacher clusters. To determine whether clusters of teachers produced different levels of student achievement, analyses of covariance between clusters were conducted using pretest scores (class means) as the covariate. These analyses included only the two main clusters for volume and the two main clusters for generating formulas or equations. For volume, no analysis produced statistically significant results. For generating formulas or equations, only one significant result emerged: Cluster 2 had higher achievement than Cluster 1 on item 052 (generate the equation for the following verbal problem: "The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed"). However, because the clusters of teachers were so similar on teaching methods, it is difficult to explain why they differed on this one item in particular. This may have been a chance results. In conclusion, then, teacher clusters basically did not differ on achievement.

Conclusions

This study set out to determine whether symbolic form of achievement test items influences student performance and whether teachers' use of different symbolic forms during instruction influences estimates of student achievement. The analyses conducted to answer these questions used student achievement test data and teachers' responses to questionnaires about their instructional methods from the Second International Mathematics Study (SIMS, eighth grade).

Most items on the SIMS achievement test used traditional symbolic forms—numbers or algebraic expressions. A minority of items used alternative symbolic forms (e.g., diagrams, graphs, verbal expressions). Identifying items using different symbolic forms for the same mathematical concept revealed that items varied substantially on a number of additional factors, such as the complexity of the numbers involved, making it impossible to isolate the effects of the symbolic form of the item per se. To investigate the effects of symbolic form on student performance, it will be necessary to hold constant features of the item other than symbolic form (such as numerical complexity, the exact concept being tested, metric verses non-metric units, the context of the problem). It is possible that such a test may reveal effects of symbolic form in a way that analysis of the SIMS test—which was not designed to answer this question—could not.

Concerning the relationship between teachers' use of different symbolic forms during instruction and student performance, the analyses performed here

showed some significant relationships. The strongest results were for the following teacher styles: symbolic flexibility during instruction (emphasis on or use of non-traditional symbolic forms), use of applications of mathematical procedures, and use of real contexts in math problems. All of these teaching styles were positively related to achievement for some groups of items. Teaching styles that emphasized use of mathematical procedures (rather than applications in a meaningful context) in traditional symbolic forms (e.g., numbers, algebra) consistently had little or no relationship with achievement.

These results suggest that use of alternative symbolic forms during instruction may be important for student learning of mathematics. A logical next step is to carry out experimental studies in which instruction is systematically varied and student performance is compared. In combination with specially designed tests that systematically vary symbolic form of the item, such studies have considerable promise for clarifying the role of symbolic form in mathematics instruction and achievement.

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Appendix

TABLE 1
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS:
PROPORTIONAL REASONING

	Total (n = 181)			Typical (n = 113)			Enriched (n = 49)			Algebra (n = 19)		
ITEM	Pre	Post	d [a]	Pre	Post	d	Pre	Post	d	Pre	Post	d
Words												
047	46	59	13	39	52	13	51	70	19	74	75	1
079	33	43	10	29	39	10	35	49	14	54	62	8
143	57	58	1	52	53	1	62	66	4	78	71	-7
190	50	55	5	45	50	5	55	63	8	80	80	0
026	33	44	11	30	38	8	31	50	19	57	67	10
Table												
142	17	23	6	13	17	4	22	27	5	21	38	17
152	66	78	12	61	71	10	66	83	17	84	92	8
Diagram												
156	36	51	15	33	44	11	39	62	23	52	79	27
197	36	47	11	32	41	9	39	56	17	53	56	3

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

Note:

OTL = 5

TABLE 2
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS:
DISTANCE-RATE-TIME

	Total (n = 147)			Typical (n = 97)			Enriched (n = 42)			Algebra (n = 8)		
ITEM	Pre	Post	d [a]	Pre	Post	d	Pre	Post	d	Pre	Post	d
Word												
078	16	23	7	16	20	4	14	27	13	32	41	9
141	46	52	6	44	46	2	47	58	11	51	85	34
192	45	49	4	37	44	7	48	51	3	77	67	-10
Table												
152	66	78	12	61	71	10	66	83	17	84	92	8
Graph												
066	61	59	-2	47	51	4	72	66	-6	88	77	-11
160	48	63	15	42	60	18	55	67	12	77	77	0
161	46	50	4	41	43	2	54	65	11	76	71	-5
Alogrithm												
017	60	71	11	53	63	10	61	79	18	88	89	1

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL - 5

TABLE 3
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS: VOLUME OF RECTANGULAR SOLID

	Total (n = 154)			Typical (n = 105)			Enriched (n = 42)			Algebra (n = 7)		
ITEM	Pre	Post	d	Pre	Post	d	Pre	Post	d	Pre	Post	d
Word												
039	38	57	19	33	52	19	48	66	18	57	77	20
104	36	48	12	33	43	10	43	56	13	40	69	29
136	18	13	-5	20	13	-7	15	12	-3	8	17	9
168	8	9	1	7	9	2	8	10	2	14	14	0
Diagram												
072	26	37	11	23	32	9	31	48	17	35	55	20

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL = 5

TABLE 4
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS: GENERATING EQUATIONS/FORMULAS

	Total (n = 70)			Typical (n = 28)			Enriched (n = 15)			Algebra (n = 27)		
ITEM	Pre	Post	d	Pre	Post	d	Pre	Post	d	Pre	Post	d
Word												
016	26	34	8	19	26	7	21	32	11	37	45	8
052	49	55	6	42	49	7	50	61	11	64	64	0
149	54	66	12	46	56	10	47	65	18	73	83	10
Table												
019	35	44	9	26	36	10	34	49	15	53	57	4
055	19	37	18	12	21	9	9	39	30	39	62	23

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL = 5

TABLE 5
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS:
STATISTICS

ITEM	Total (n = 154) Pre	Post	d	Typical (n = 102) Pre	Post	d	Enriched (n = 44) Pre	Post	d	Algebra (n = 8) Pre	Post	d
Word												
067	79	74	-5	77	71	-6	81	79	-2	82	75	-7
132	85	83	-2	86	83	-3	83	83	0	89	82	-7
Bar Graph												
099	39	45	6	38	42	4	39	51	12	48	48	0
Numerical												
035	32	45	13	27	38	11	41	60	19	62	65	3

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL - 5

TABLE 6
MEAN PERCENT CORRECT FOR ITEMS PRESENTED IN DIFFERENT SYMBOLIC FORMS:
COORDINATE SYSTEM/GRAPH

	Total (n = 85)			Typical (n = 43)			Enriched (n = 20)			Algebra (n = 22)		
ITEM	Pre	Post	d	Pre	Post	d	Pre	Post	d	Pre	Post	d
Word												
029	18	34	16	15	24	9	15	35	20	26	50	24
Graph												
028	35	64	29	30	53	23	37	72	35	50	84	34
Graph+Words												
126	26	50	24	18	35	17	22	53	31	50	82	32
169	32	35	3	31	30	-1	29	36	7	37	47	10

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL = 5

TABLE 7
PERFORMANCE ON ITEMS WITH DIFFERENT SYMBOLIC FORMS OF RESPONSE REQUIRED

	Total (n = 89)			Typical (n = 43)			Enriched (n = 25)			Algebra (n = 21)		
ITEM	Pre	Post	d	Pre	Post	d	Pre	Post	d	Pre	Post	d
INTERPRETING TABLES												
Table-->Formula												
019	35	44	9	26	36	10	34	49	15	53	57	4
055	19	37	18	12	21	9	9	39	30	39	62	23
Table-->Number												
142	17	23	6	13	17	4	22	27	5	21	38	17
152	66	78	12	61	71	10	66	83	17	84	92	8
Table-->Bar Chart												
032	63	66	3	59	62	3	72	74	2	73	78	5
INTERPRETING BAR CHARTS												
Bar Chart-->Words												
034	73	75	2	71	72	1	74	82	8	100	83	-17
Bar Chart-->Number												
099	39	45	6	38	42	4	39	51	12	48	48	0
130	37	43	6	31	38	7	49	50	1	61	60	-1
162	49	53	4	44	48	4	62	63	1	53	71	18
Bar Chart + Table-->Number												
098	58	69	11	51	63	12	73	78	5	81	95	14
Diagram-->Formula												
093	15	39	24	11	32	21	14	41	27	33	66	33
Diagram-->Number												
027	20	32	12	18	26	8	22	33	11	24	53	29

[a] d=difference between pretest and posttest

Note: Unit of analysis=class mean. Only classes whose teachers reported that they taught the material relevent to an item during the current year included in this table.

OTL = 5

Table 8
Factor Loadings for Fractions Teaching Style Items

Teacher	Item	Symbolic Flexibility	Numeric Rules
F019	Fractions as parts of regions	.71	
F023	Fractions as parts of a collection	.66	
F055	Fractions as comparisons	.61	
F060	The sum of two fractions as the combination of fractional parts of a collection	.56	
F065	The sum of two fractions as a combination of two measurements	.53	
F082	A decimal as part of a region	.52	
F094	A decimal as a comparison	.52	
F059	The sum of two fractions as the union of two regions	.51	
F100	Use concrete materials to illustrate operations with decimals	.35	
F035	Fractions as decimals	.34	
F043	Fractions as ratios	.32	
F078	A decimal as another way of writing a fraction	.31	
F086	A decimal as an extension of place value	.30	
F031	Fractions as quotients	.25	
F099	Relate operations with decimals to operations with whole numbers, teaching rules for placing the decimal point	.20	
F061	The sum of two fractions on the number line		.70
F027	Fractions as the coordinates of points on a number line		.64
F063	The sum of two fractions as the sum of two decimals		.58
F069	Using the formula		.55
F066	The sum of two fractions as joining two segments		.44

Table 8 (Cont'd)
Factor Loadings for Fractions Teaching Style Items

Teacher	Item	Symbolic Flexibility	Numeric Rules
F074	A decimal as the coordinate of a point on the number line		.43
F098	Relate operations with decimals to operations with fractions		.38
F090	A decimal as a series		.38
F067	Using the least common denominator in a horizontal format		.36
F047	Fractions as measurements		.36
F039	Fractions as repeated addition of a unit fraction		.35
F062	The sum of two fractions as the sum of two quotients		.32
F051	Fractions as operators		.30
F064	The sum of two fractions using fractions as repeated addition of the unit fraction		.28
F068	Using the least common denominator in a vertical format		-.20

Table 9
Factor Loadings for Algebra Teaching Style Items

Teacher	Item	Symbolic Flexibility	Algebraic Rules
A096	Having the students inspect graphs and find formulae to express the relationships portrayed by the graph	.64	
A097	Providing data from which formulae or equations are developed	.63	
A046	Use of physical situations	.57	
A072	Development by use of physical situations	.51	
A098	Having students collect data on related variables and formulate the relationship between the variables	.44	
A099	Having students create new formulae based on known, simpler formulae	.36	
A034	Using examples of physical situations	.36	
A058	Subtraction by rules		.54
A042	Addition by rules		.46
A074	No development--students were given rules		.39
A075	Using properties of equality with operations with numbers		.34
A073	Development by use of patterns		-.30

Table 10
Factor Loadings for Teacher Opinion Items: Algebra

Opinion Item		Perceived Student Difficulty	Importance of Conceptual Understanding
A145	Average students have difficulty in translating verbal and written sentences into mathematical sentences and vice versa	.78	
A144	Average students have difficulty in solving word problems involving linear equations	.78	
A146	Average students have difficulty with applications involving linear equations	.76	
A135	A great deal of practice is required in order for students to acquire competence in performing operations with directed numbers.	.34	
A139	Most students cannot be expected to master the use of letters for unknowns quickly; they have to become accustomed to this usage slowly over a long period of time	.34	
A138	Most students find it difficult to appreciate the significance of studying the structural properties (additive inverse, order relation, distributive law, etc.) of the set of integers	.25	

Table 10 (Cont'd)
Factor Loadings for Teacher Opinion Items: Algebra

Opinion Item		Perceived Student Difficulty	Importance of Conceptual Understanding
A137	Average students are usually not satisfied with knowing only the rules for performing operations with integers; they want to know why the rules work		.58
A141	In solving equations, it is important that students be able to justify each step in their solution procedure		.58
A136	It is important for students to understand how integers obey general laws like the distributive law, the associative law, etc.		.47
A149	The notion of equivalent equations is useful in helping students understand solutions		.46
A143	The notion "solution set" (those values of the unknown which make the relation true) aid the students' comprehension of linear equations		.42
A134	It is very important to justify the rules for multiplying integers		.38
A142	Solving linear equations by trial and error helps students understand the meaning of a solution		.30
A147	When solving problems, it is important for students to first identify the type of problem (age, digit, mixture, etc.) being solved		.27

Table 11

Fractions Teaching Styles: Logical Groupings of Teacher Variables

Teacher Questionnaire Item	Description	Not Used (%)	Used But Not Emphasized (%)	Emphasized (%)
ALTERNATIVE SYMBOLIC REPRESENTATIONS				
F019	Fractions as parts of regions	27	47	26
F023	Fractions as parts of a collection	45	41	14
F027	Fractions as the coordinates of points on a number line	21	49	29
F043	Fractions as ratios	24	38	37
F047	Fractions as measurements: this container holds	46	40	13
F051	Fractions as operators	88	10	2
F055	Fractions as comarisons	37	40	23
F059	The sum of two fractions as the union of two regions	50	37	13
F060	The sum of two fractions as the	72	21	7
F061	The sum of two fractions on the number line	48	40	12
F065	The sum of two fractions as a combination of two measurements	59	32	9
F066	The sum of two fractions as joining two segments	52	39	9
F074	A decimal as the coordinate of a point on the number line	28	43	29
F082	A decimal as a part of a region	36	49	16
F086	A decimal as an extension of place value	6	18	77

F094	A decimal as a comparison	72	22	6
F100	Use conciete materials to illustrate operations with decimals	88	11	1

NUMERIC REPRESENTATIONS

F035	Fractions as decimals	1	15	84
F039	Fractions as repeated addition of a unit fraction	41	41	18
F062	The sum of two fractions as the sum of two quatients	78	10	12
F063	The sum of two fractions as the sum of two decimals	30	53	17
F064	The sum of two fractions using fractions as repeated addition of the unit fractions	55	33	12
F067	Using the least common denominator in a horizontal format	33	30	36
F068	Using the least common denominator in a vertical format	2	12	87
F069	Using the "formula"	68	21	11
F070	Using any common denominator in a horizontal format	46	36	17
F071	Using any common denominator in a vertical format	16	36	48
F072	Using decimals	43	44	13
F078	A decimal as another way of writing a fraction	2	21	78
F090	A decimal as a series	38	41	21
F098	Relate operations with decimals to operations with fractions	25	51	24
F099	Relate operations with decimals to operations with whole numbers, teaching rules for placing the decimal point	3	12	85

Table 12

Algebra Teaching Styles: Logical Groupings of Teacher Variables

Teacher Questionnaire Item	Description	Not Used (%)	Used But Not Emphasized (%)	Emphasized (%)
ALTERNATIVE SYMBOLIC REPRESENTATIONS				
A018	Extending the number ray to the number line	4	25	72
A026	Using vectors or directed segments on the number line	56	27	17
A034	Using examples of physical situations	5	33	62
A038	Addition by number line	10	50	40
A046	Use of physical situations	10	52	38
A050	Subtraction as addition of opposites on the number line	38	36	27
A072	Development by use of physical situations	42	46	12
A096	Having the students inspect graphs and find formulas to express the relationships portrayed by the graph	66	20	14
A097	Providing data from which formulas or equations are developed	38	40	22
A098	Having students collect data on related variables and formulate the relationship between the variables	75	17	8
A099	Having students create new formulas based on known, simpler formulas	58	33	9
NUMERIC/ALGEBRAIC REPRESENTATIONS				
A022	Presenting integers as solutions to equations	17	40	43
A030	Defining integers as equivalent classes of whole numbers	84	10	6

A042	Addition by rules	5	14	81
A054	Subtraction of a number as the inverse of addition of that number	37	24	38
A058	Subtraction by rules	3	8	89
A062	Subtraction as a number of units	53	35	12
A066	Subtraction as "What must be added"	50	37	13
A070	Development by use of repeated addition	43	42	14
A071	Development by the extension of properties of the whole number system	71	18	10
A073	Development by use of patterns	24	40	36
A074	No development - students were given rules	27	22	51
A075	Using properties of equality with operations with numbers	12	14	74
A079	Using properties of inverses with numbers	27	25	48
A083	Using arithmetical reasoning	47	42	12
A087	Using trial and error	80	19	2
A091	Using rules	25	29	45
A095	Presenting formulas and explaining the meaning of the terms in the fomula	3	13	84
APPLICATIONS DURING INSTRUCTION				
A100	Age problems	28	38	34
A101	Digit problems	50	23	27
A102	Mixture problems	56	24	20
A103	Percent problems	20	26	54
A104	Distance-Rate-Time problems	16	32	52
A105	Interest problems	17	21	62
A106	Area-Volume problems	28	24	48

A107	Physical-Natural Science problems	72	16	12
A108	Energy or Ecological problems	62	30	9

Table 13

Multiple Regression Analysis Predicting Fractions Performance
From Teaching Style Factors

		Unstandardized b	
Test Item	Description	Symbolic Flexibility	Numeric Rules
Common Fractions			
003	2/5 + 3/8 is equal to	2.61	1.75
004	Which of the following is a pair of equivalent fractions?	.41	.87
043	Which of the points A,B,C,D,E on this number line corresponds to 5/8	-1.09	3.64*
044	There are 35 students in a class. 1/5 of them come to school by bus, another 2/5 come by bicycle. How many come to school by other means?	-2.93	1.56
075	In the figure the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to	.67	-.53

076	Four 1-liter bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many liters of ice cream had been EATEN?	-.97	.65
107	$1\,{}^{2}/_{5} - {}^{1}/2$ is equal to	1.14	-1.40
139	3/5 + 2/7 is equal to	4.06	.58
185	Which is the closest estimate for the answer to $5\ {}^{3}/7 + 6\ {}^{5}/8$	-1.98	-.10
186	1/2 x 1/4 is equal to	-1.02	2.85
187	3/8 - 1/5 is equal to	-.11	.87
188	The picture shows some black and some white marbles. Of all these marbles what fraction are white?	2.57	-.08
Decimal Fractions			
005	0.40 x 6.38 is equal to	-.31	.92
006	Alexandra walked from Riverview to Bridgeport, which are 3.1 kilometers apart. During her walk she lost her watch, went back 1.7 kilometer to find it, and then continued in the original direction until she reached bridgeport. How many kilometers		

	had Alexandra walked altogether when she arrived at Bridgeport?	2.67	1.96
007	(847.36) is the number in the box, the digit 6 represents	.68	1.63
045	The value of 0.2131 x 0.02958 is approximately	.96	2.46
077	The position on the scale indicated by the arrow is	-.10	4.45
078	A runner ran 3,000 meters in exactly 8 minutes. What was his average speed in meters per second:	4.2*	5.58**
108	.004 | 24.56 In the division above, the correct answer is	1.75	-.37
109	In the discus-throwing competition, the winning throw was 61.60 meters. The second place throw was 59.72 meters. How much longer was the winning throw than the second place throw?	1.23	-.34
140	7 $^{3}/20$ is equal to	2.47	2.41
141	The speed of sound is 340 meters per second. How long will it take before the sound of a car horn reaches your ears if the car is 714 meters away?	.67	3.16

182	Which of the following is thirth-seven thousandths?	4.43*	1.20
183	74.236 rounded to nearst <u>hundredth</u> is	1.51	-1.65
184	The large square has area 1 square unit. The area of the shaded part is	-.75	3.72

Ratio, Proportion, Percent

008	In a school of 800 pupils, 300 are boys. The ration of the number of boys to the number of girls is	1.22	.83
009	30 is 75% of what number?	1.38	3.36*
046	20% of 125 is equal to	1.61	1.21
047	If the ration of 2 to 5 equals the ratio of n to 100, then n is equal to	-2.30	4.31*
079	A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the color he wants. If he has 28 liters of green paint, how many liters of yellow paint should be added?	1.05	.43
110	In a school election with three candioates, Joe received 120 votes. Mary received 50 votes, and George received 30 votes. What		

	percent of the total number of votes did Joe recieve?	-1.22	1.85
142	The table shows the values of x and y, where x is propoiional to y. What are the values of P and Q?	-1.00	2.26
143	If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?	-1.37	1.74
163	There are five black buttons and one red in a jar. If you pull out one button at random, what is the probability that you will get the red button?	-1.01	2.22
177	Candidate A received 70 percent of the votes cast in an election. If 4200 votes were cast in the election, how many votes did candidate A receive?	-1.72	3.74
178	72% is equal to	4.83*	1.84
179	20 is what percent of 80?	2.46	2.43
180	$150 is divided in the ratio of 2 to 3. The smaller of the two amcuts is	-.56	3.16
181	A model boat is built to scale so that it is 1/10 as long as the original boat. If the width of the original boat is		

	4 meters,the width of the model should be	-.48	2.73*
189	1/5 is equal to	2.82*	-1.17
190	Cloth is sold by the square meter. If 6 square meters of cloth cost $4.80, the cost of 16 square meters will be	1.00	2.02
191	The price of an article was $100. The price was first raised by 10% and was then reduced by 10% of the new price. What is the price of the article now?	2.43	4.52**
192	A car takes 15 minutes to travel 10 kilometers. What is the speed of the car?	-.21	.25

Note: Each equation has pretest scores and teaching style factors as predictors.
Unit of analysis = class mean

*$P < .05$
**$P < .01$

Table 14

Multiple Regression Analysis Predicting Fractions Performance
From Teaching Styles Based on Logical Groupings of Variables

		Unstandardized b		
Test Item	Description	Symbolic	Flexibility	Numeric Rules
Common Fractions				
003	2/5 + 3/8 is equal to		6.23	4.51
004	Which of the following is a pair of equivalent fractions?		9.78	-10.72*
043	Which of the points A,B,C,D,E on this number line corresponds to 5/8		1.66	3.76
044	There are 35 students in a class. 1/5 of them come to school by bus, another 2/5 come by bicycle. How many come to school by other means?		-7.15	10.21
075	In the figure the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to		2.32	-3.36

076	Four 1-liter bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many liters of ice cream had been EATEN?	1.89	-1.90
107	$1\,{}^{2}/_{5} - {}^{1}/_{2}$ is equal to	2.37	-2.57
139	3/5 + 2/7 is equal to	12.00	-2.80
185	Which is the closest estimate for the answer to $5\,{}^{3}/7 + 6\,{}^{5}/8$	-8.50	8.56
186	1/2 x 1/4 is equal to	.02	3.21
187	3/8 - 1/5 is equal to	.13	2.23
188	The picture shows some black and some white marbles. Of all these marbles what fraction are white?	5.36	2.27

Decimal Fractions

005	0.40 x 6.38 is equal to	1.53	-.48
006	Alexandra walked from Riverview to Bridgeport, which are 3.1 kilometers apart. During her walk she lost her watch, went back 1.7 kilometer to find it, and then continued in the original direction until she		

	reached bridgeport. How many kilometers had Alexandra walked altogether when she arrived at Bridgeport?	11.62	-2.04
007	(847.36) is the number in the box, the digit 6 represents	1.64	2.23
045	The value of 0.2131 x 0.02958 is approximately	9.29*	-4.73
077	The position on the scale indicated by the arrow is	2.95	10.85
078	A runner ran 3,000 meters in exactly 8 minutes. What was his average speed in meters per second:	-2.32	5.21
108	.004 / 24.56 In the division above, the correct answer is	12.26*	-13.26*
109	In the discus-throwing competition, the winning throw was 61.60 meters. The second place throw was 59.72 meters. How much longer was the winning throw than the second place throw?	5.06	-2.85
140	7 ^{3}/20 is equal to	10.48*	1.05
141	The speed of sound is 340 meters per second. How long will it take before the sound of		

	a car horn reaches your ears if the car is 714 meters away?	14.22	-11.06
182	Which of the following is thirth-seven thousandths?	17.54**	-6.80
183	74.236 rounded to nearst <u>hundredth</u> is	4.49	-6.43
184	The large square has area 1 square unit. The area of the shaded part is	18.64*	-22.44**

Ratio, Proportion, Percent

008	In a school of 800 pupils, 300 are boys. The ration of the number of boys to the number of girls is	5.65	-1.23
009	30 is 75% of what number?	9.47	1.65
046	20% of 125 is equal to	11.83**	-9.53*
047	If the ration of 2 to 5 equals the ratio of n to 100, then n is equal to	2.92	-1.83
079	A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the color he wants. If he has 28 liters of green paint, how many liters of yellow paint should be added?	4.25	-1.27
110	In a school election with three candidates,		

	Joe received 120 votes. Mary received 50 votes, and George received 30 votes. What percent of the total number of votes did Joe recieve?	-3.11	6.30
142	The table shows the values of x and y, where x is propotional to y. What are the values of P and Q?	-1.24	3.44
143	If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?	-2.07	2.47
163	There are five black buttons and one red in a jar. If you pull out one button at random, what is the probability that you will get the red button?	2.97	-10.52
177	Candidate A received 70 percent of the votes cast in an election. If 4200 votes were cast in the election, how many votes did candidate A receive?	-2.05	4.72
178	72% is equal to	14.00*	-1.73
179	20 is what percent of 80?	10.96*	-1.19
180	$150 is divided in the ratio of 2 to 3. The smaller of the two amouts is	1.71	4.31
181	A model boat is built to scale so that		

	it is 1/10 as long as the original boat. If the width of the original boat is 4 meters,the width of the model should be	1.45	3.13
189	1/5 is equal to	9.35*	-7.40
190	Cloth is sold by the square meter. If 6 square meters of cloth cost $4.80, the cost of 16 square meters will be	8.97*	-1.57
191	The price of an article was $100. The price was first raised by 10% and was then reduced by 10% of the new price. What is the price of the article now?	13.55**	-2.7
192	A car takes 15 minutes to travel 10 kilometers. What is the speed of the car?	2.74	-5.06

Note: Each equation has pretest scores and teaching style factors as predictors.
Unit of analysis = class mean

* $P < .05$
** $P < .01$

Table 15

Multiple Regression Analysis Predicting Algebra Performance
From Teaching Style Factors

Test Item	Description	Unstandardized b Symbolic	Flexibility	Numeric Rules
Intergers				
012	(-2) x (-3) is equal to		1.20	-2.00
013	The air temperature at the foot ot a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?		2.39*	.72
049	-5 (6-4) is equal to		6.25**	-7.93**
082	The set of integers less than 5 is represented on one of the number lines. Which one?		5.44*	-2.41
113	(-6) - (-8) is equal to		5.85*	-1.46
Rational Numbers				

014	Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?	1.34	.60
Integer Exponents			
084	0.00046 is equal to	3.04	2.19
172	Find the value of N, $N = 10^3 + 10^1 + 10^0 + 10^{-2}$	.90	1.11
Formulas			
015	Simplify: $5x + 3y + 2x - 4y$	-.65	-.94
016	Soda costs a cents for each bottle, including the deposit, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?	-2.26	-4.67
052	The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of printing n cards?	4.73+	.49

085	If y dollars are shared equally among four boys, how many dollars does each boy receive?	1.50	-4.68
115	If x = 3, the value of -3x is	2.31	-7.83*
116	If x=y=z=1, then x-z/x+y is equal to	3.55	.90
148	Which of the following is FALSE when a,b,and c are different real numbers?	2.20	.14
149	A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing 2y kg. What weight of tea does he now have?	2.06	.18
195	A number x is multiplied by itself and the result is added to four times the original number. This can be expressed as	4.35	-2.64
Polynomials			
053	When x = 2, 7x+4/5x-4 is equal to	3.08	-2.31
088	a/15 - b/5 is equal to	-4.43	-3.96
Equations and Inequalities			
017	If P = LW and if P = 12 and L = 3, then W is equal to	1.11	1.22

018	If $6x - 3 = 15$, then, $6x = 15 - 3$ (i) $6x = 12$ (ii) $x = 12/6$ (iii) $x = 2$ (iv) the error in the above reasoning, if one exists, FIRST APPEARS in line	4.58	-3.98
054	Which equation is true for all values of n?	2.83	-6.28**
086	If $4x/12 = 0$, then x is equal to	1.58	-4.77***
087	The Davis family took a car trip from Anabru through Bergen to Chase. They then drove back to Bergen through Earlville, and then returned to their home in Anabru. If the total distance they drove was 115 dilometers, how far is it from Anabru to Bergen?	1.52	.65
117	"Six times a certain number (call it q) equals the sum of eight and twice the number." This can be written as	1.91	-5.11
118	$x/2 < 7$ is equivalent to	2.81	-4.75
151	If $5x + 4 = 4x - 31$, then x is equal to	-.34	-1.73
196	The sentence " a number x decreased by 6 is less than 12" can be written as the inequality	.42	-2.36

Relations and Functions			
019	The table compares the height from which a ball is dropped (d) and the height to which it bounces (b)	1.16	-3.99*
055	For the table, a formula that could relate m and n is	2.59	-4.77
152	A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds given by d = 4t. In the table, x is equal to	5.86***	2.13
Finite Sets			
120	The symbol P Q represents the intersection of sets P and Q and the symbol P Q represents the union of sets p and Q. Which of the following represents the shaded portion of the diagram below?	90	-9.15*

Note: Each equation has pretest scores and teaching style factors as predictors.
Unit of analysis = class mean, Test scores = precent correct

+ $P < .06$
* $P < .05$
** $P < .01$
*** $P < .001$

Table 16

Multiple Regression Analysis Predicting Algebra Performance From Teaching Styles Based on Logical Groupings of Variables

Test Item	Description	Unstandardized b		
		Symbolic Flexibility	Algebraic Rules	Applications
Intergers				
012	(-2) x (-3) is equal to	-5.83	8.92	7.28
013	The air temperature at the foot ot a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?	3.96	-.19	3.96
049	-5 (6-4) is equal to	4.67	3.92	9.27+
082	The set of integers less than 5 is represented on one of the number lines. Which one?	.91	7.51	6.55
113	(-6) - (-8) is equal to	-3.40	19.18*	13.16**

Rational Numbers				
014	Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?	.45	-11.26	13.84**
Integer Exponents				
084	0.00046 is equal to	-2.45	-4.41	7.94
172	Find the value of N, $N = 10^3 + 10^1 + 10^0 + 10^{-2}$	-3.43	7.84	-.72
Formulas				
015	Simplify: $5x + 3y + 2x - 4y$	-9.36	1.63	16.60*
016	Soda costs a cents for each bottle, <u>including the deposit</u>, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?	-9.92	-2.11	8.24
052	The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of -			

	printing n cards?	.74	13.44	5.44
085	If y dollars are shared equally among four boys, how many dollars does each boy receive?	-.26	22.57**	1.61
115	If x = 3, the value of -3x is	-7.93	14.32	13.96**
116	If x=y=z=1, then x-z/x+y is equal to	-5.34	14.71	13.88**
148	Which of the following is FALSE when a,b,and c are different real numbers?	-7.77	11.28	12.21
149	A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing 2y kg. What weight of tea does he now have?	3.77	-1.63	7.51
195	A number x is multiplied by itself and the result is added to four times the original number. This can be expressed as	3.57	7.91	6.24
Polynomials				
053	When x = 2, 7x+4/5x-4 is equal to	5.65	7.25	2.84
088	a/15 - b/5 is equal to	-22.40*	-14.73	17.28+
Equations and Inequalities				
017	If P = LW and if P = 12 and L = 3, then			

	W is equal to	.22	2.97	5.63*
018	If $6x - 3 = 15$, then, $6x = 15 - 3$ (i) $6x = 12$ (ii) $x = 12/6$ (iii) $x = 2$ (iv) the error in the above reasoning, if one exists, FIRST APPEARS in line	-1.29	6.33	14.85**
054	Which equation is true for all values of n?	-1.62	8.52	3.01
086	If $4x/12 = 0$, then x is equal to	-2.68	-6.46	10.45**
087	The Davis family took a car trip from Anabru through Bergen to Chase. They then drove back to Bergen through Earlville, and then returned to their home in Anabru. If the total distance they drove was 115 dilometers, how far is it from Anabru to Bergen?	2.70	-2.31	2.37
117	"Six times a certain number (call it q) equals the sum of eight and twice the number." This can be written as	-7.09	11.37	10.59*
118	$x/2 < 7$ is equivalent to	2.29	-8.33	7.19
151	If $5x + 4 = 4x - 31$, then x is equal to	-7.30	.75	6.78
196	The sentence " a number x decreased by 6 is less than 12" can be written as the			

	inequality	-6.69	7.08	8.18*
Relations and Functions				
019	The table compares the height from which a ball is dropped (d) and the height to which it bounces (b)	9.54*	-7.52	-2.22
055	For the table, a formula that could relate m and n is	-9.44	5.31	20.87**
152	A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds given by d = 4t. In the table, x is equal to	4.78	6.92	12.03**
Finite Sets				
120	The symbol P Q represents the intersection of sets P and Q and the symbol P Q represents the union of sets p and Q. Which of the following represents the shaded portion of the diagram below?	4.48	-21.98	6.22

Note: Each equation has pretest scores and teaching style factors as predictors.
Unit of analysis = class mean, Test scores = precent correct

+ $P < .06$
* $P < .05$
** $P < .01$

Table 17

Multiple Regression Analysis Predicting Fractions Performance From Teaching Opinion Factors

Test Item	Description	Unstandardized b	
		Perceived Student Difficulty	Importance of Conceptual Understanding
Intergers			
012	(-2) x (-3) is equal to	1.09	5.33**
013	The air temperature at the foot ot a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?	-.27	-.35
049	-5 (6-4) is equal to	-3.72	1.40
082	The set of integers less than 5 is represented on one of the number lines. Which one?	-3.79	1.88
113	(-6) - (-8) is equal to	-.41	4.32

Rational Numbers			
014	Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?	2.12	1.51
Integer Exponents			
084	0.00046 is equal to	1.26	1.15
172	Find the value of N, $N = 10^3 + 10^1 + 10^0 + 10^{-2}$	-2.70	-1.49
Formulas			
015	Simplify: $5x + 3y + 2x - 4y$	-7.76*	4.04
016	Soda costs a cents for each bottle, including the deposit, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?	.94	-3.22
052	The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of		

	printing n cards?	-3.60	2.59
085	If y dollars are shared equally among four boys, how many dollars does each boy receive?	-2.72	4.17
115	If x = 3, the value of -3x is	1.06	2.59
116	If x=y=z=1, then x-z/x+y is equal to	-2.98	4.61
148	Which of the following is FALSE when a,b,and c are different real numbers?	-2.35	1.60
149	A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing 2y kg. What weight of tea does he now have?	-.67	.79
195	A number x is multiplied by itself and the result is added to four times the original number. This can be expressed as	-1.78	3.77
Polynomials			
053	When x = 2, 7x+4/5x-4 is equal to	.60	3.63
088	a/15 - b/5 is equal to	-5.71	7.97
Equations and Inequalities			
017	If P = LW and if P = 12 and L = 3, then		

	W is equal to	-1.30	2.58+
018	If 6x - 3 = 15, then, 6x = 15 - 3 (i) 6x = 12 (ii) x = 12/6 (iii) x = 2 (iv) the error in the above reasoning, if one exists, FIRST APPEARS in line	-5.05*	2.90
054	Which equation is true for all values of n?	-4.68*	1.83
086	If 4x/12 = 0, then x is equal to	-3.22*	2.51
087	The Davis family took a car trip from Anabru through Bergen to Chase. They then drove back to Bergen through Earlville, and then returned to their home in Anabru. If the total distance they drove was 115 kilometers, how far is it from Anabru to Bergen?	.96	.07
117	"Six times a certain number (call it q) equals the sum of eight and twice the number." This can be written as	-3.64	1.68
118	x/2 < 7 is equivalent to	-3.27	1.82
151	If 5x + 4 = 4x - 31, then x is equal to	-2.20	4.87*
196	The sentence " a number x decreased by 6 is less than 12" can be written as the		

	inequality	-3.54*	3.85*
Relations and Functions			
019	The table compares the height from which a ball is dropped (d) and the height to which it bounces (b)	-5.44**	2.40
055	For the table, a formula that could relate m and n is	-8.22**	3.76
152	A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds given by d = 4t. In the table, x is equal to	-2.59	2.78
Finite Sets			
120	The symbol P Q represents the intersection of sets P and Q and the symbol P Q represents the union of sets p and Q. Which of the following represents the shaded portion of the diagram below?	3.59	5.03

Note: Each equation has pretest scores and teaching style factors as predictors.
Unit of analysis = class mean, Test scores = precent correct

+ $P < .06$
* $P < .05$
** $P < .01$

Table 18
Description of Teaching Methods: Proportional Reasoning

Teaching Method and Teacher Questionnaire Items	Not Used (%)	Used But Not Emphas. (%)	Emphas. (%)
NUMERIC METHODS (Abstract)			
R020 Ratio as a fraction	2	24	73
R021 Ratio as the quotient of two whole numbers	13	33	53
R032 Proportions as equivalent fractions	4	31	64
R036 Proportions as equivalent quotients	40	40	20
R040 Using multiplication or division to equate numerators and denominators	16	47	38
R041 Finding the cross products and then solving the resulting equation	2	11	87
R042 Dividing the terms of one ratio and then solving the resulting equation	60	22	18
NUMERICAL METHODS (Real Context)			
R018 Ratio as a rate	13	53	33
R019 Ratio as a comparison	2	36	62
R024 Proportions as equivalent ratios	7	33	60
R028 Proportions as equivalent comparisons	16	47	38
R044 Use proportional reasoning without an equation	24	63	13
R045 Use a proportional equation	4	13	82
R046 Use the unit method without an equation	38	60	2
OPEN-ENDED PROBLEMS (Real Context)			
R050 Calculating the size of a population from a sample estimate	[illegible]	33	22
R051 Problems involving buying decisions based on cost rates	4	36	60
R052 Mixture or recipe problems	18	55	27
R053 Real world problems using simila triangles	16	33	51
CONSTRUCTION MEASUREMENT (Similar Triangles)			
G122 Graph paper or tracing paper	45	24	31
G123 Measurement	10	40	50

Table 18 (Cont'd)
Description of Teaching Methods: Proportional Reasoning

Teaching Method and Teacher Questionnaire Items		Not Used (%)	Used But Not Emphas. (%)	Emphas. (%)
G124	Constructions with ruler and compass	40	26	33
G125	Geoboard	82	14	4
G126	Environment	34	51	14
G127	Dilations (stretching and shrinking)	57	14	29

Table 19
Description of Teaching Methods: Distance-Rate-Time

Teaching Method and Teacher Questionnaire Items	Not Used (%)	Used But Not Emphas. (%)	Emphas. (%)
SOLVING EQUATIONS (Algebraic)			
A075 Using properties of equality with operations with numbers	12	19	69
A079 Using properties of inverses with numbers	39	20	41
A083 Using arithmetical reasoning	37	56	7
A087 Using Trial and error	76	21	2
A091 Using rules	33	17	50
OPEN-ENDED ALGEBRA PROBLEMS (Real Context)			
A072 Development by use of physical situations	28	49	23
A104 Distance-Rate-Time problems	2	12	86
GRAPHS, TABLES (No Real Context)			
A096 Having the students inspect graphs and find formulas to express the relationships portrayed by the graph.	50	29	21
A097 Providing data from which formulas or equations are developed	24	40	36

Table 20
Description of Teaching Methods: Volume

Teaching Method and Teacher Questionnaire Items		Not Used (%)	Used But Not Emphas. (%)	Emphas. (%)
MANIPULABLE MEASURING AIDS				
M031	Rulers (meterstick, yardstick, 12 inch ruler, etc.)	67	33	0
M032	Measuring tape	9	49	42
M033	Trundle wheel	2	13	86
M034	Aids representing non-standard units of measurement	11	51	38
M035	Geoboards, graph paper, or grids.	24	46	31
M036	Aids representing standard units for area (centimeter squares, centimeter cubes or rods, etc.)	11	46	44
CONTAINERS				
M037	Graduated cylinders	6	31	64
M038	Containers (liter, gallon, etc.)	9	49	42
M039	Fillable models of geometric solids	9	38	53
OPEN-ENDED PROBLEMS (Manipulable)				
M042	I have my students estimate the size of real world objects	18	64	18
M043	I have my students identify objects whose measurement is as close as possible to a given number of units	31	46	24
FORMULA				
M106	I presented the formula $V = l \times w \times h$ or V= (area of base) x (height) and demonstrated how to apply it by means of examples.	0	18	82
FINDING VOLUME USING UNIT CUBES				
M107	I presented a physical model of a right prism (box) with its faces marked off in square units, as illustrated below...	26	46	29
M108	I provided my students with units cubes and asked them to build rectangular prisms of specified dimensions...	90	11	0

Table 21
Description of Teaching Methods: Generating Formulas of Equations

Teaching Method and Teacher Questionnaire Items	Not Used (%)	Used But Not Emphas. (%)	Emphas. (%)
DATA IN TABLE, GRAPHS (No Context)			
A096 Having the students inspect graphs and find formulae to express the relationships portrayed by the graph.	41	34	25
A097 Providing data from which formulae or equations are developed.	9	47	44
COMBINING VARIABLES			
A098 Having students collect data on related variables and formulate the relationship between the variables	72	9	19
A099 Having students create new formulae based on known, simpler formulae.	53	38	9

Table 22. Teacher Clusters Based on Teaching Methods for Proportional Reasoning

Cluster 1 (n=36)		Cluster 2 (n=5)		Cluster 3 (n=3)	
Teaching Item	Test Item	Teaching Item	Test Item	Teaching Item	Test Item
VP3	079	VP1	047	VP2	152
G126	142	VP4	143	VP7	197
G127	156	VP5	190	G125	
F008		VP6	026	F104	
R050		G015		R040	
R053		G157			
		R028			
		R042			
		R043			
		R044			
		R046			
		R051			
		R052			
		R066			

Note: This table lists teaching and test items on which a cluster had the highest mean among all clusters.

Table 22a Teaching Variables for Proportional Reasoning Problems

Variable Numbers	Variable Contents
VP1	
G122	Graph paper or tracing paper
G123	Measurement acitivities were used to study properties of similar triangles, e.g., proportionality of sides
G124	Constructions with ruler and compass
VP2	
R020	Ratio as a fraction
R021	Ratio as the quotient of two whole numbers
R032	Proportions as equivalent fractions
R036	Proportions as equivalent quotients
VP3	
R041	Finding the cross products and then solving the resulting equation
R045	Use a proportional equation
R068	The proportion method of solving percent problem
R072	The proportion method of solving percent problem
R076	The proportion method of solving percent problem
VP4	
R047	Scale models (airplanes, automobiles)
R048	Finding distances from map
R049	Scale drawings
VP5	
R018	Ratio as a rate
R019	Ratio as a comparison
R024	Proportions as equivalent ratios
VP6	
R008	The concept of proportion
R009	Solving proportional equations
VP7	
R081	Activities related to developing the concept of ratio
R082	Activities related to developing the concept of proportion
R083	Activities related to solving proportional equations
R084	Application/problem solving activities related to ratio and proportions (textbook word problems, problems related to real world situations, recreational problems, challenging problems, etc.)
F008	Finding equivalent fractions - including reducing fractions
F104	Activities related to finding equvalent fractions - including reducing fractions

(to be continued)

(continue)

G015	Similarity of geometric figures (including similar triangles)
G125	Geoboard for teaching similar triangles
G126	Environment for teaching similar triangles
G127	Dilations for teaching similar triangles
G157	Activities related to similarity of geometric figures (including similar triangles)
R028	Proportion as equivalent comparisons
R040	Using multiplication or division to equate numerators and denominators
R042	Dividing the terms of one ratio and then solving the resulting equation
R043	Techniques for solving proportions (numerically or symbolically)
R044	Use proportional reasoning without an equation
R046	Use the unit method without an equation
R050	Calculating the size of a population form a sample estimate
R051	Problem involving buying decisions based on cast rate
R052	Mixture or recipe problems
R053	Real world problems using similar triangles
R066	Application or problems from real world sources, such as newspapers or individuals involved in the use of mathematics

Table 23. Teacher Clusters Based on Teaching Methods for Distance-Rate-Time

Cluster 1 (n=36)		Cluster 2 (n=3)		Cluster 3 (n=4)	
Teaching Item	Test Item	Teaching Item	Test Item	Teaching Item	Test Item
VD1	066	A106	078	VD2	141
A104	017	A072	152	VD3	192
A084		A104	160	A096	161
				A097	

Note: This table lists teaching and test items on which a cluster had the highest mean among all clusters.

Table 23a Teaching Variables for D-R-T Problems

Variable Number	Variable Content
VD1	
A127	Activities related to evaluation formulae (for given values of the variables)
A129	Application/problem solving activities related to use of formulea (textbood word problems, problems related to real world situations, recreational problems, challenging problems, etc.)
A130	Activities related to solving literal equations
A131	Activities related to solving linear equations
A132	Application/problem solving activities related to use of equations (textbook word problems, problems relate to real world situations, recreational problems, challenging problems, etc.)
VD2	
A083	Using arithmetical reasoning (in solving equations): Given $7x + 5 = 40$ what number increased by 5 is 40?
A091	Using rules (in soving equations): collect all constant terms on one side of the equatio and all variable terms on the other, etc.
VD3	
A017	Solving linear equations: $4x - 3 = 19$
A075	Using properties of equality with operations with numbers
A014	Evaluating formulae for given values of the variables
A072	Development by use of physical situations
A084	Using arithmetical reasoning
A095	Presenting formulae and explaining the meaning of the terms in the formula
A096	Having the student inspect graphs and find formulae to express the relationships portrayed by the graph
A097	Providing data from which formulae or equations are developed
A104	Distance-Rate-Time problems
A106	Area-Volume problem
R024	Proportions as equivalent ratios

Table 24. Teacher Clusters Based on Teaching Methods for Volume

Cluster 1 (n=17)		Cluster 2 (n=27)		Cluster 3 (n=5)		Cluster 4 (n=6)	
Teaching Item	Test Item	Teaching Item	Test Item	Teaching Item	Test Item	Teaching Item	Test Item
VV1	136	M039	136*	VV2	072	M031	039
A106		M106		VV3		M113	104
M107				M027			168
				M033			
				M035			
				M137			

* Ranked second among clusters but value is close to the first.

Note: This table lists teaching and test items on which a cluster had the highest mean among all clusters.

Table 24a Teaching Variables for Volume Problems

Variable Numbers	Variable Contents
VV1	
M042	I have my students estimate the size of real world objects
M043	I have my students identify objects whose measurement is as close as possible to a given number of units
VV2	
M037	Graduated cylinders as aids for teaching measurement
M038	Containers as aids for teaching measurement
VV3	
M032	Measuring tape as aids for teaching measurement
M034	Aids represent non-standard units of measurement (papre clips, hand spans, foot lengths, etc.)
M036	Aids represent standard units for measurement (cm squares, cm cubes, or cm rods, etc.)
A106	Area-Volume problem
M031	Rulers as aids for teaching measurement
M033	Trundle wheel as aid for teaching measurement
M035	Geoboards, graph paper, or grids as aids for teaching measurement
M039	Fillable models of geometric solids as aids for teaching measurement
M106	Teaching volume of box by formula V = L x W x H
M107	Teaching volume of box by figure with its faces marked off in square units
M108	Teaching volume of box by providing unit cubes and ask students to build rectangular prisms
M113	I used centimeter cubes and decimeter cubes to establish relationships among units
M137	Activities related to finding the volumes of solids (rectangular solids)

Table 25. Teacher Clusters Basesd on Teaching Methods For Equations and Formulas

Cluster 1 (n=16)		Cluster 2 (n=12)		Cluster 3 (n=4)	
Teaching Item	Test Item	Teaching Item	Test Item	Teaching Item	Test Item
A015*	016	A015	052	VE1	019
		A128	149	A097	
			055		

* Ranked second among clusters but value is close to the first.

Note: This table lists teaching and test items on which a cluster had the highest mean among all clusters.

Table 25a Teaching Variables for Equation and Formula Problems

Variable Numbers	Variable Contents
VE1	
A096	Having the students inspect graphs and find formulae to express the relationships protrayed by the graph
A098	Having the students collect data on related variables and formulate the ralationship between the variables
A099	Having students create new formulae based on known, simpler formulae
A015	Deriving formulae or equations
A097	Providing data from which formulae or equations are developed
A128	Activities related to deriving formulae or equations (where data is derived from experiments or given to students)

Table 26

Partial Correlations between Teaching Methods and Achievement: Proportional Reasoning

Test Item	Numerical Methods (Abstract)	Numerical Methods (Real Context)	Open ended problems (Real Context)	Construction Measurement (Similar Triangles)
Word Problems				
047	-.09	.12	.12	-.01
079	.25+	-.12	.16	-.19
143	-.04	.06	.10	.13
190	-.08	-.08	.04	.25+
026	-.13	.06	.31*	-.10
Table Problems				
142	-.27*	.09	.45**	.32*
152	.09	.28*	.15	.11
Diagram Problems				
156	-.28*	.00	.01	.03
197	-.09	-.11	-.13	.11

Note: Partial correlations control for pretest achievement.
Unit of analysis = class mean.

+ $p<.06$
* $p<.05$
** $p<.01$

OTL = 5

Table 27

Partial Correlations between Teaching Methods and Achievement: Distance-Rate-Time

Test Item	Solving Equations (Algebraic)	Open ended Algebra problems (Real Context)	Graphs Tables (No real Context)
Word Problems			
078	.02	.10	.22
141	.24+	-.12	-.18
192	-.26+	.15	.26+
Table Problem			
152	.15	.30*	.08
Graph Problems			
066	.14	.26+	-.15
160	.16	-.07	-.23
161	.25+	.30*	-.15
Algebraic Problems			
017	.03	.20	-.29*

Note: Partial correlations control for pretest achievement
Unit of analysis = class mean.

+ $p<.06$
* $p<.05$

CTL=5

Table 28

Partial Correlations between Teaching Methods and Achievement: Volume

Test Item	Manipulable Measuring Aids	Containers	Open-ended problems (Manipulable)	Formula	Finding Volume Using Unit Cubes
Word Problems					
039	-.09	-.22+	.09	-.01	.25*
104	-.28*	-.12	.13	-.05	.25*
136	-.11	-.05	.01	.19	-.21+
168	.05	-.06	.16	.01	.02
Diagram					
072	-.20	.16	-.03	.11	-.03

Note: Partial correlations control for pretest achievement
Unit of analysis = class mean.

+ $p<.06$
* $p<.05$

OTL = 5

Table 29

Partial Correlations between Teaching Methods and Achievement: Generating Formulas or Equations

Test Items	Data in Tables, Graphs (No Context)	Combining Variables (Real Context) Formulas
Word Problem		
016	-.16	-.24
052	.41*	.31*
149	.11	.27
Table Problems		
019	.04	.16
055	-.21	-.04

Note: Partial correlations control for pretest achievement
Unit of analysis = class mean.

* $p<.05$

OTL = 5

FIGURE 1. PROPORTIONAL REASONING: WORD PROBLEMS

ID=047 Rotated Form

If the ratio of 2 to 5 equals the ratio of n to 100, then n is equal to

A 10

B 20

C 40

D 150

E 250

ID=143 Rotated Form

If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?

A 90

B 100

C 900

D 1000

E 9000

ID=079 Core Form

A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the color he wants. If he has 28 liters of green paint, how many liters of yellow paint should be added?

A 11

B 16

C 28

D 49

E 196

ID=190 Core Form

Cloth is sold by the square meter. If 6 square meters of cloth cost \$4.80, the cost of 16 square meters will be

A \$12.80

B \$14.40

C \$28.80

D \$52.80

E \$128.00

ID=026 Rotated Form

On level ground, a boy 5 units tall casts a shadow 3 units long. At the same time a nearby telephone pole 45 units high casts a shadow the length of which, in the same units, is

A 24

B 27

C 30

D 60

E 75

FIGURE 2. PROPORTIONAL REASONING: TABLE PROBLEMS

ID:142 Rotated Form Form A Item 16

x	3	6	P
y	7	Q	35

The table above shows the values of x and y, where x is proportional to y. What are the values of P and Q?

A P = 14 and Q = 31

B P = 10 and Q = 14

C P = 10 and Q = 31

D P = 14 and Q = 15

E P = 15 and Q = 14

ID:152 Rotated Form Form C Item 26

A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds is given by $d = 4t$. In the table below, x is equal to

t	d
0	0
1	4
2	8
3	x
4	16

A 6

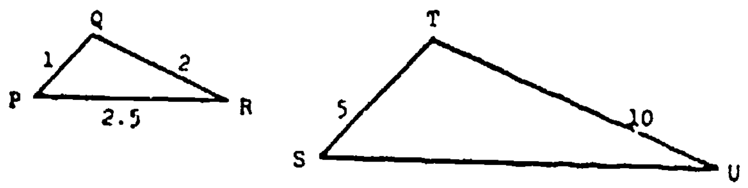
B 10

C 12

D 14

FIGURE 3. PROPORTIONAL REASONING: DIAGRAM PROBLEMS

ID=156 Core Form Form Core Item 40



Triangles PQR and STU are similar. How long is $\overline{SU}$?

A 5

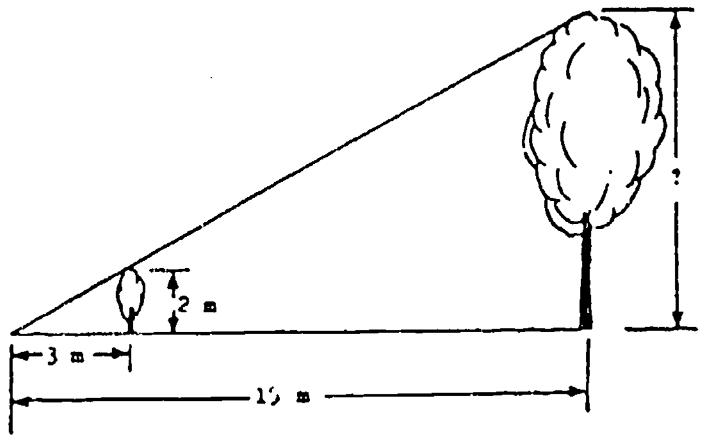
B 10

C 12.5

D 15

E 25

ID=197 Rotated Form Form B Item 32



The picture above shows how Pedro used a short tree to find the height of the tall tree. What answer should Pedro get?

A 10 meters

B 12 meters

C 14 meters

D 17 meters

E 20 meters

FIGURE 5. DISTANCE - RATE - TIME

WORD PROBLEMS

ID=078 Rotated Form

A runner ran 3,000 meters in exactly 8 minutes. What was his average speed in meters per second?

A 3.75

B 6.25

C 16.0

D 37.5

E 62.5

ID=141 Rotated Form

The speed of sound is 340 meters per second. How long will it take before the sound of a car horn reaches your ears if the car is 714 meters away?

A 0.21 seconds

B 2.1 seconds

C 21 seconds

D 210 seconds

E None of these

ID=192 Rotated Form

A car takes 15 minutes to travel 10 kilometers. What is the speed of the car?

A 30 kilometers per hour

B 40 kilometers per hour

C 60 kilometers per hour

D 90 kilometers per hour

E 150 kilometers per hour

TABLE PROBLEM

ID=152 Rotated Form

A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds is given by $d = 4t$. In the table below, x is equal to

t	d
0	0
1	4
2	8
3	x
4	16

A 6

B 10

C 12

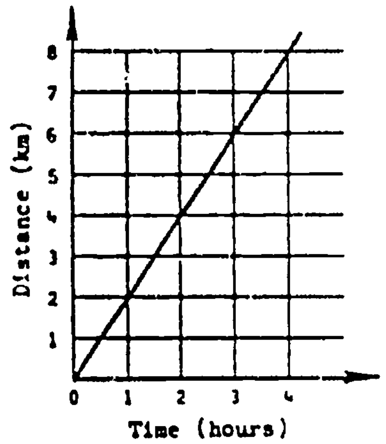
D 14

E None of these

FIGURE 6. DISTANCE - RATE - TIME

GRAPH PROBLEMS

ID=066 Rotated Form



The graph shows the distance traveled by a tractor during a period of 4 hours. How fast is the tractor moving?

A 1 kilometer per hour

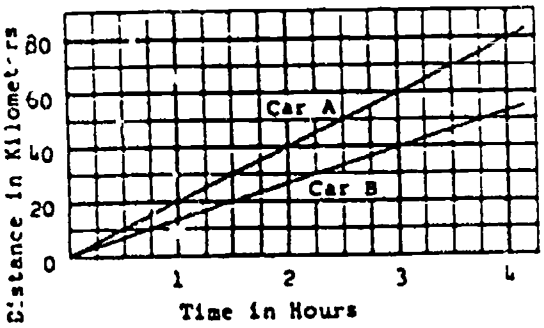
B 2 kilometers per hour

C 4 kilometers per hour

D 8 kilometers per hour

E There is not enough information

ID=160 Rotated Form Form C Item 10



Three hours after starting, car A is how many kilometers ahead of car B?

A 2

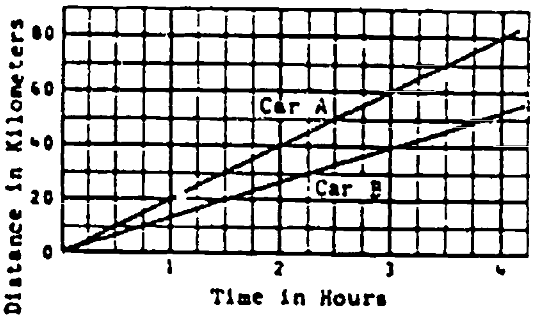
B 10

C 15

D 20

E 25

ID=161 Rotated Form Form B Item 19



How much longer does it take for car B to go 50 kilometers than it does for car A to go 50 kilometers?

A 1 hour 15 minutes

B 1 hour 30 minutes

C 2 hours

D 2 hours 30 minutes

E 2 hours 35 minutes

ALGORITHM

017 Core Form

If P = LW and if P = 12 and L = 3, then W is equal to

A $\frac{3}{4}$

B 3

C 4

D 12

E 36

FIGURE 8. VOLUME

WORD PROBLEMS

ID=039 Rotated Form

What is the volume of a rectangular box with interior dimensions 10 cm long, 10 cm wide, and 7 cm high?

A 27 cm^3

B 70 cm^3

C 140 cm^3

D 280 cm^3

E 700 cm^3

ID=104 Rotated Form

Michael has a large number of wooden blocks which are cubical in shape with each edge 1 centimeter long. What is the maximum number of these blocks that can be used to fill a rectangular box with interior dimensions 10 centimeters long, 10 centimeters wide and 7 centimeters high?

A 27

B 70

C 140

D 280

E 700

ID=136 Rotated Form

What is the capacity of a cubic container 10 cm by 10 cm by 10 cm?

A 1 liter

B 10 liters

C 100 liters

D 1000 liters

E 1000 centimeters

ID=168 Core Form

A solid plastic cube with edges 1 centimeter long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimeters long?

A 8 grams

B 4 grams

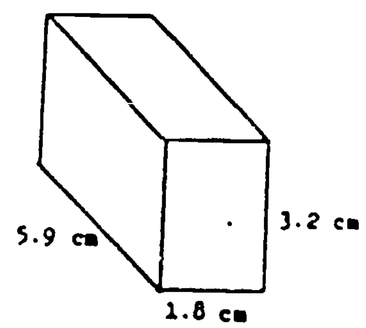
C 3 grams

D 2 grams

E 1 gram

DIAGRAM PROBLEM

ID=072 Rotated Form



The figure above shows a rectangular box. Which of the following is closest to the volume of this box?

A 16 cm^3

B 18 cm^3

C 28 cm^3

D 36 cm^3

E 48 cm^3

FIGURE 10. GENERATING FORMULAS OR EQUATIONS

WORD PROBLEMS

ID=016 Rotated Form

Soda costs a cents for each bottle, including the deposit, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A $ax + by$ cents

B $ax - by$ cents

C $(a - b)x$ cents

D $(a + x) - (b + y)$ cents

E None of these

ID=052 Rotated Form

The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of printing n cards?

A cost = $(100 + 6n)$ cents

B cost = $(106 + n)$ cents

C cost = $(6 + 100n)$ cents

D cost = $(106n)$ cents

E cost = $(600n)$ cents

CID=149 Core Form

A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing $2y$ kg. What weight of tea does he now have?

A $x - 15 - 2y$

B $x + 15 + 2y$

C $x - 15 + 2y$

D $x + 15 - 2y$

E None of these

TABLE PROBLEMS

ID=019 Core Form

The table below compares the height from which a ball is dropped (d) and the height to which it bounces (b).

d	50	80	100	150
b	25	40	50	75

Which formula describes this relationship?

A $b = d^2$

B $b = 2d$

C $b = \frac{d}{2}$

D $b = d + 25$

E $b = d - 25$

ID=055 Rotated Form

m	-1	1	2	4
n	-1	3	5	9

For the table shown, a formula that could relate m and n is

A $n = m$

B $n = 3m$

C $n = -m^2 + 1$

D $n = m^2 + 1$

E $n = 2m + 1$

FIGURE 12. STATISTICS PROBLEMS

WORDS

ID:067 Rotated Form

==

Joe had three test scores of 78, 76 and 74, while Mary had scores of 72, 82 and 74. How did Joe's average compare with Mary's?

A Joe's was 1 point higher.

B Joe's was 1 point lower.

C Both averages were the same.

D Joe's was 2 points higher.

E Joe's was 2 points lower.

ID:132 Rotated Form

==

A team scored an average of 3 points per game over 5 games. How many points altogether were scored in the 5 games?

A $\frac{3}{5}$

B $\frac{5}{3}$

C 3

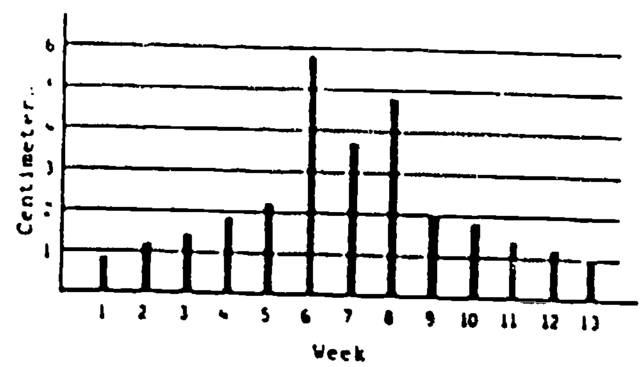
D 5

E 15

BAR GRAPH

ID:099 Rotated Form For

==



In the graph, rainfall in centimeters is plotted for 13 weeks. The average weekly rainfall during the period is approximately

A 1 centimeter

B 2 centimeters

C 3 centimeters

D 4 centimeters

E 5 centimeters

NUMERICAL

ID:035 Rotated Form

==

The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to

A 2.40

B 2.55

C 3.75

D 7.65

E None of these

FIGURE 13.
COORDINATE SYSTEM/GRAPH PROBLEMS

WORDS

ID:029 Rotated Form

One of the following points can be joined to the point (-3,4) by a line segment which cuts NEITHER the x NOR the y axis. Which one?

A (-2,3)

B (2,-3)

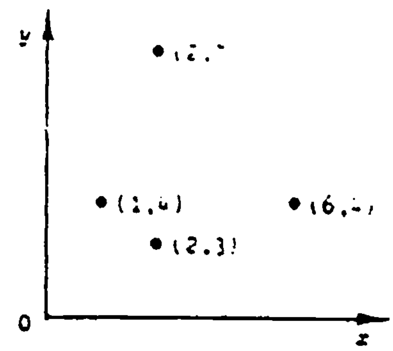
C (2,3)

D (-2,-3)

E (4,-3)

GRAPH + WORDS

ID:126 Rotated Form



The straight line joining the points (2,3) and (2,7) cuts the straight line joining the points (1,4) and (6,4) at the point

A (4,2)

B (1,4)

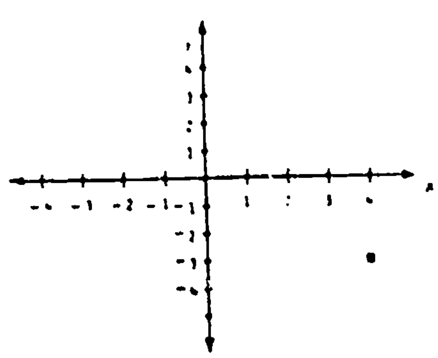
C (1,3)

D (2,3)

E (2,4)

GRAPH

ID:028 Core Form For



What are the coordinates of point P?

A (-3,4)

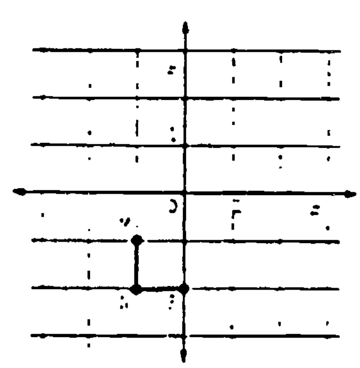
B (-4,-3)

C (3,4)

D (4,-3)

E [illegible]

ID:169 Rotated Form - 8th Grade (Population A) Form



Suppose you start at point M(-1,-1), move a distance of one unit to N(-1,-2), then turn left and move one unit to the point P(0,-2). If you again turn left and move one unit, you will now be at the point with coordinates

A [illegible]

B [illegible]

C (0,-[illegible]

D (-1,-[illegible]

E None of these

FIGURE 14. (PAGE 1 OF 2) TABLE PROBLEMS REQUIRING RESPONSES WITH DIFFERENT SYMBOLIC FORMS

FORMULA

ID=019 Core Form

The table below compares the height from which a ball is dropped (d) and the height to which it bounces (b).

d	50	80	100	150
b	25	40	50	75

Which formula describes this relationship?

A $b = d^2$

B $b = 2d$

C $b = \frac{d}{2}$

D $b = d + 25$

E $b = d - 25$

ID=055 Rotated Form

m	-1	1	2	4
n	-1	3	5	9

For the table shown, a formula that could relate m and n is

A $n = m$

B $n = 3m$

C $n = -m^2 + 1$

D $n = m^2 + 1$

E $n = 2m + 1$

NUMBER

ID=142 Rotated Form

x	3	t	P
y	7	Q	35

The table above shows the values of x and y, where x is proportional to y. What are the values of P and Q?

A P = 14 and Q = 31

B P = 10 and Q = 14

C P = 10 and Q = 31

D P = 14 and Q = 15

E P = 15 and Q = 14

ID=152 Rotated Form

A bowling ball travels 4 meters per second. The distance in meters traveled in t seconds is given by $d = 4t$. In the table below, x is equal to

t	d
0	0
1	4
2	8
3	x
4	16

A 6

B 10

C 12

D 14

E None of these

FIGURE 14. (PAGE 2 OF 2) TABLE PROBLEMS REQUIRING RESPONSES WITH DIFFERENT SYMBOLIC FORMS

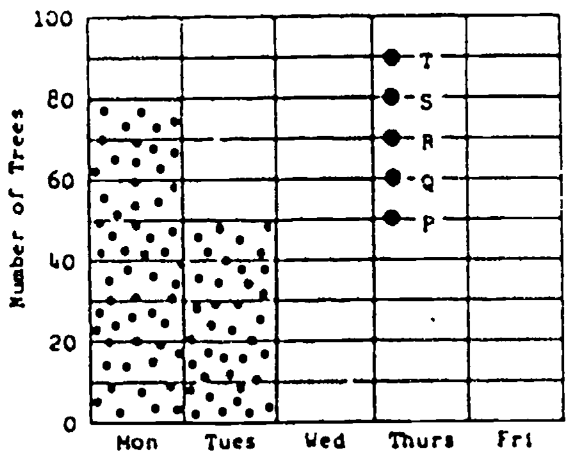
BARCHART

ID=032 Rotated Form Form D Item 34

Here is a table that shows the number of trees planted along a highway in a week.

Days of the Week	Mon	Tues	Wed	Thurs	Fri
Number of Trees Planted	80	50	60	90	75

On the diagram below, the graph for the first two days' planting has been drawn.



If the graph were completed, which point would indicate the top of the bar on Thursday?

A point P

B point Q

C point R

D point S

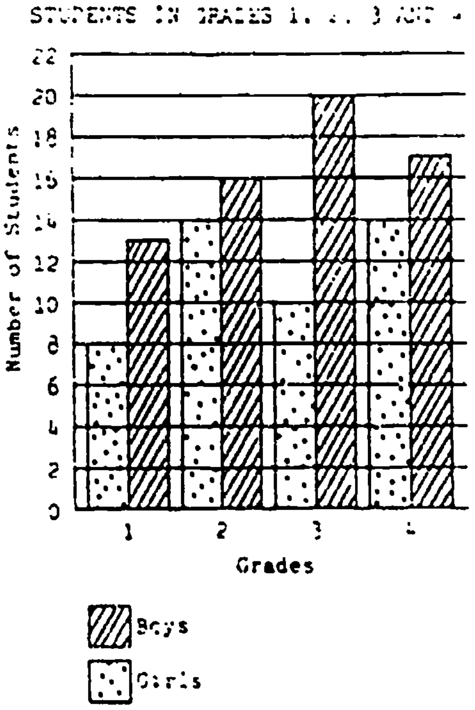
E point T

FIGURE 15. (PAGE 1 OF 2) BARCHART PROBLEMS REQUIRING RESPONSES WITH DIFFERENT SYMBOLIC FORMS

WORDS

ID=034 Rotated Form Form D Iter

==



Which of these is a TRUE statement about the information shown on the graph?

A Grade 2 is the smallest class

B Grades 2 and 4 have the same number of students

C Grade 3 has twice as many boys as girls

D Grade 4 has more girls than boys

E Grade 1 has as many boys as there are girls in grade 4

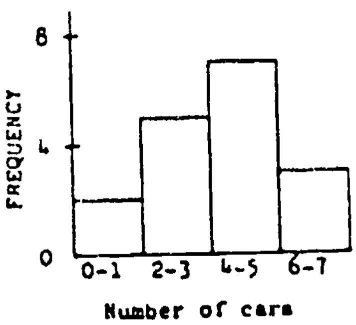
BARCHART + TABLE → NUMBER

ID=098 Rotated Form Form D Item 8

==

Here are a table of data and a graph of the same data. What is *x*?

Number of Cars	Frequency
0 or 1	2
2 or 3	*x*
4 or 5	7
6 or 7	3



A 2

B 3

C 4

D 5

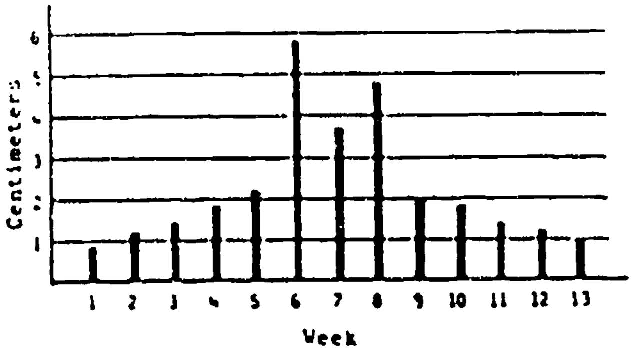
E 6

BEST COPY AVAILABLE

FIGURE 15. (PAGE 2 OF 2) BARCHART PROBLEMS REQUIRING RESPONSES WITH DIFFERENT SYMBOLIC FORMS

NUMBER

ID:099 Rotated Form



In the graph, rainfall in centimeters is plotted for 13 weeks. The average weekly rainfall during the period is approximately

A 1 centimeter

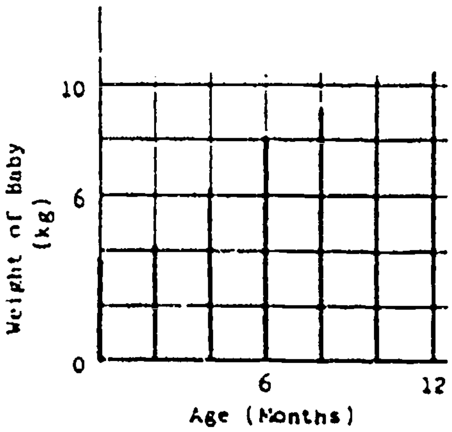
B 2 centimeters

C 3 centimeters

D 4 centimeters

E 5 centimeters

ID:130 Rotated Form



The weight gain from 6 to 10 months was

A 1 kg

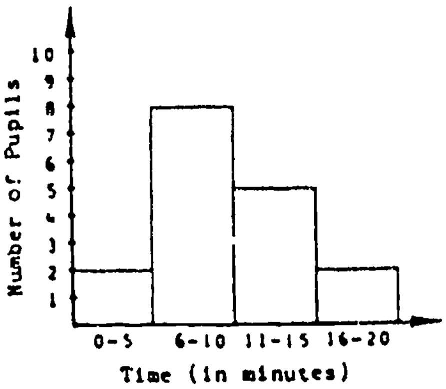
B 2 kg

C 4 kg

D 6 kg

E 8 kg

ID:162 Rotated Form Form C



The graph shows the time of travel by pupils from home to school. How many pupils must travel for MORE than 10 minutes?

A 2

B 5

C 7

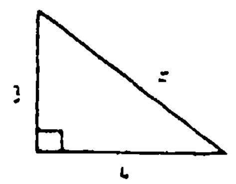
D 8

E 15

FIGURE 16.
DIAGRAM PROBLEMS REQUIRING RESPONSES WITH DIFFERENT SYMBOLIC FORMS

FORMULA

ID=093 Rotated Form



Which of these is a correct statement for this triangle?

A $x^2 = 3^2 + 4^2$

B $x^2 + 3^2 = 4^2$

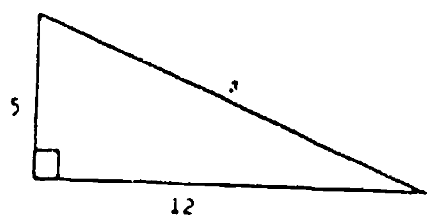
C $x = 4^2 - 3^2$

D $x^2 = 4^2 - 3^2$

E $x = 4 + 3$

NUMBER

ID=027 Rotated Form Form C Item 1



What is the value of a ?

A 7

B 13

C 15

D 17

E None of these