Teaching is complex and layered. The premise of this paper is that teaching is worthy of being an object of systematic inquiry, and that such inquiry will help to clarify and potentially improve the character of teaching. It has also been assumed that a useful approach is to differentiate within teaching those aspects which can and usually are routinized from those which must be uniquely constructed from new arrangements of knowledge. It is acknowledged that the task of teaching is quite different when it is carried out under different models. The differences in tasks are reflected in the routine aspects and in the nature of the knowledge base required for action. Systematic inquiry about the nature of teaching is valuable but should not replace the continued efforts of educational reform based on the understanding of the nature of learning. Reform should be guided by an understanding of both the nature of learning and the nature of teaching, but efforts at reform should not be confused with efforts of investigation and efforts of investigation should not be confused with efforts at reform. Examples of classroom dialogue are included in the document. (LL)
On Teaching

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On Teaching

TEACHING

Teaching is a complex, dynamic, ill-structured process. In this process known and valued information is either newly built, jointly rebuilt, or passed from one source to another. There are multiple approaches to teaching and learning that emphasize different aspects and roles in this process. These approaches stretch along a continuum of who (the teacher, text, or learner) has what type of responsibility for which aspects of presentation and acquisition of knowledge. Conceptions of the learner vary from the rediscoverer and reinventor of human knowledge (Papert, 1980) to an apprentice in a socially situated system (Brown, Collins, & Duguid, 1989, Scribner, 1984a, b) to that of an acquirer of well-designed stacks of information (Gagné & Brown, 1961). Conceptions of the role of the teacher in teaching also vary from seeing the teacher as a relatively passive presence in the discovery model, to one that is more collaborative (with the teacher as a problem poser and arranger of conditions for learning), to one that is primarily didactically directive and/or programmed. There is a trade-off, as the role of the teacher is conceptualized as being more passive, the role of the learner is seen to be more active. Similarly, the fundamental nature of the teacher’s task and the knowledge base that s/he must have are different, depending on exactly how the role and activity is conceptualized. Arguments in support of a particular position are traditionally bolstered by a rich psychological conceptualization of what is

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involved in learning. Comparably rich psychological explorations of teaching are somewhat rarer. Almost never are the two seen in tandem. The systematic study of teaching has a long and rich tradition; however, considering the teacher as a complex, rational planner and organizer and presenting data to support that view is comparably a newer enterprise (Shavelson & Stern, 1981; Yinger, 1980; Yinger & Dillard, 1987).

In the discovery model of learning, teaching is seen as the construction of a situation in which the learner has available all of the necessary tools to discover; personal interest both motivates and structures the task of learning (Dewey, 1963; Montessori, 1965; Neill, 1960; Piaget, 1954). The student selects the topics of inquiry, the path for finding out about the topic, and decides the end point. Unless a student knows to ask for it, the fundamental structure and epistemology of a discipline will remain masked. A student thus has to discover, for example, that keeping note books and records of inquiry in some systematically but neutrally retrievable way is helpful in building up knowledge that goes beyond one episode and in discovery patterns (Siegler & Liebert, 1975). The psychological support for the learner in this role is twofold: first, the student will build up the new knowledge from his or her own existing intuitions and schemas and so the new knowledge will be remembered; second, the student will select topics of inherent interest and value and will work from their own motivations. Psychological criticism is

2 Unfortunately (but not unpredictably) education philosophies of teaching and learning are tangled up in political philosophies and are held as statements about the comparative virtue of the individual holding the particular perspective. The difficulty with the entanglement is that it leads to the usual round of name-calling and intellectual narrowness rather than a cooperative engagement in the task of steady improvement. A significant role, but one which will not be followed up in this chapter, is that the disciplines of inquiry that surround teaching hold predictable positions. Namely psychologists who study the learner hold positions that focus the most active role on the learner, while curriculum and subject matter specialists tend to emphasize the significance of the organizations of the text or subject matter.
pragmatic: first, the student may never discover the "right" thing (Ausubel & Schiff, 1964; Joshua & Dupin, 1987); second, using conventional language and formations aids in recalling the correct piece and in linking it to shared knowledge in a wider community (Leinhardt & Ohlsson, in press). Under a strict view of discovery, teaching is considered good in those cases in which it facilitates but does not interfere with the students' complete construction of all aspects of the self-selected target of knowledge. Teachers are seen as librarians or repositories of information who can be tapped by students. In a discovery model, the teacher must be astute with respect to students' psychological development, insightful observation, and the global pattern of a discipline.

The models which conceive of the teacher as the arranger or collaborative facilitator of conditions are well articulated by Montessori (1965) and Dewey (1963). In these more collaborative models the student constructs knowledge systematically under guided social conditions. Corrections for errors are made through public and private inspection of results and effects (Brown & Palinscar, 1984). In these models teachers are careful observers of students and of the world of knowledge. They are taught to watch and anticipate the thinking and reasoning of a child. They are also taught to observe carefully how a particular pedagogical device is interpreted. Teachers pose questions and offer problems; they facilitate searches of knowledge repositories such as libraries, museums, and natural experiments; and finally they focus attention on particular portions of the enterprise. They are guides and planners. To be effective within the discipline they must have deep disciplinary knowledge. To be effective as teachers they must have deep pedagogical knowledge. This particular tradition has two different identifiable branches both of which treat the
role of the teacher in a similar fashion. The specific knowledge base needed by the teacher is, however, quite different.

One tradition assumes knowledge is most naturally acquired around tasks and projects. This tradition has its modern manifestation in the activity-based learning proposed by followers of Vygotsky (1978); Brown and Reeve (1987); Bereiter and Scardamalia (in press); Scardamalia, Bereiter, and Steinbach, (1984). The contextualization of the problem is a relatively natural "life task" or project. In this setting the teacher's task is to help students draw out the subject matter knowledge content from tasks which themselves require such knowledge. So, for example, a class plans an investigation or writes a book about a specific topic. Roles are decided upon. Disciplinary knowledge is gained from multiple sources and pooled: math, from planning the costs, and from estimating and projecting rates of change and growth; science, from structuring the task and from charting results; writing, from writing about many different aspects of the enterprise (Bereiter & Scardamalia, in press; Scardamalia, Bereiter & Steinbach, 1984). The knowledge is learned in a situated context where the situation itself carries the roots of both the problem and the solution (Schliemann & Acioly, 1989; Carraher, Carraher, & Schliemann, 1983). The task for teachers is to orchestrate, manage, and respond supportively.

In another tradition the project-based contextualization of the problem in life task is reduced and the abstraction increased by contextualizing the problem in the discipline's own task space (see Nesher, in press; White, in this volume). One example is Montessori's didactic material. The binomial cube, the brown stair, and pattern tracing are all concrete representations of abstract mathematical ideas \([(a+b)^2\text{, relative size, linear 2 dimensional portraits of solid 3 dimensional shapes}] and the
teacher's role is to guide the student to see these summarized abstractions. The knowledge is still acquired in a situated context; however, the situation is determined by the discipline more than by the exigencies of life experiences. The tool for the teacher is to be deeply aware of the context of their discipline and to nudge students towards insights that will have disciplinary payoff.

In a recent description of the philosophy of teaching espoused by one group of Japanese educators this latter approach was exactly the model (Becker, Silver, Kantowski, Travers, & Wilson, in press; Nohda, 1987). A problem was posed; the students thought about various solutions; several alternative solutions were posted publicly; and then, in the second phase, the most efficient and general solution was searched for and summarized. Nesher (1989) in presenting a Learning Systems Approach also makes this distinction. Students are introduced to an exemplification (a partial abstraction), taught the language of exemplification and mathematics, and taught applications. Another example which centers student dialogue around specific problems and connects aspects of these to specific mathematics ideas is presented in the descriptions of mathematics instruction provided by Lampert (1986, 1989). Both of these problem centered traditions value the contextualization of problems but they do so in different contexts. One branch contextualizes in the events of daily life, motivating learning through involving projects (Cole, Hood, & McDermott, 1978). The other branch contextualizes in the framework of the underlying discipline, such as mathematics, history, or literature. These two approaches might well be integrated by considering issues of age and maturation. The role of the teacher in both is to guide, challenge and focus,
and often to construct situations in which information is directly explained.

A third approach sees the teacher as the didactic leader and center of knowledge. The students have the job of coming to understand both the content of what is presented and the meaning of why it is presented in that way. Teachers are seen as transmitters of knowledge which derives from university-based pedagogical models of instruction or of text-based models. Thus, one of the reforms of the late sixties and seventies, which came from a vigorous application of behavioral learning theory to schooling, dictated both a very important and very controlled role for teachers, and an equally controlled and responsive posture from students. The fundamental problem with this approach was that it had no room for, and therefore did not deal with, the way in which either the teacher or student might develop meaning and structure of the material being learned. Other reforms of this same period placed heavy emphasis on the meaning and structure of content, and stressed the role of the learner almost to the exclusion of the teacher. But the flaws of these approaches should not be taken as grounds for a sweeping condemnation of all didactic, teacher-led models or of content-based, text-led models. A well constructed model of teaching, with the teacher as didactic leader of the enterprise of learning, has a democratic and egalitarian aspect. Valued topics are taught to all children, not just those who are economically or intellectually privileged. Transparency, focus, and predictability in teaching have some decided advantages and need not be equated with triviality, rigidity, and dogmatism. This caveat is placed here because we are likely to lose valuable knowledge from our own teachers corps as well as knowledge gained from other successful teaching efforts such as those in the Orient, if we dismiss all didactic models of
teaching out of hand. In a strong didactic model, knowledge is still constructed by the student but in response to teacher and text. The control of exploration and the merging of multiple intuitive meanings, however, is not attended to systematically under these models.

Regardless of the model of education selected, the act of teaching is complex both in the intricacy of the actions that are seen and in the complexity of the cognitive activity that generates them. The more the teacher is in control of subject matter, questions and definitions, sequence and timing, the less complex the role is. Acknowledging that teaching is a complex cognitive activity has been made before, but the source and explication of that complexity has not been completely specified (Leinhardt & Greeno, 1986). Teaching is complex, first, because of the tensions of multiple simultaneous goals which can only be met in a particular temporal arrangement; second, because of the overwhelming information processing demands that the environment produces; and third, because the strategic action knowledge that must be coordinated with the content semantic knowledge is often misaligned (Leinhardt & Fienberg, 1989).

This complexity is reduced by choosing to ignore the potential informational input from students or contexts and/or by attending to only one array of goals--those produced by the text or formal presentation of the content. Indeed, as the intricacy from one source increases, teachers seem to trade off interpreting information from other sources. For example, as the subject matter becomes more complex teachers tend to reduce their attention to the individual learner and his or her needs and responses. By the time one is being taught calculus or physics (or most high school courses) there is almost no attention to pedagogical or personal learner issues. (See Borko and Livingston, 1989.)
situation where the demands of the learners themselves are massive, as with young children or handicapped learners, teachers tend to suppress or ignore information about the subject matter in that they tend to choose to hold simpler less subtle conceptualization of subject matter.

All teaching requires that some level of selection and limitation be made by both silencing specific classes of goals and by ignoring some of the information coming in. This selection is a simplification process (Jackson, 1968). Severe reduction of the complexity of teaching does not necessarily produce bad or ineffective teaching. In the extreme it may produce slightly rigid or mechanical teaching. This is a kind of teaching that could be totally captured by a film--one in which all the subtlety occurs within the teacher's script, not because of the dynamics of the teacher with a particular unique class of students. (For an example, see Polya's film, [MAA Individual Lecture Series / Poly 1, 1965].)

When teaching is seen as complex in this way, poor teaching is a consequence of failure to deal with some aspect of the complexity effectively. Failure may include one or more of the following: attempts to achieve multiple goals all at the same time, that is failing to trade off goals, which results in a jumpy, non-fluid lesson; attending too readily to information in the system, for example being too responsive to student behaviors, which may result at one extreme in an overly behavioral or managerial lesson or, at the other extreme, one which degenerates into the solo tutoring of a single child; having the wrong set (or none) of actions available for teaching, such as not knowing how to set up routines for moving through the space of the class, or not knowing how to set up a salient example and discuss its critical features; not having sufficient knowledge to set or produce reasonable goals, not knowing that in order to
have a sensible class discussion the goals of having an intellectually safe environment must be met or not knowing that developing meaning and understanding are not the same thing as developing fluid performance. Teaching is vulnerable, then, when teachers either are missing critical pieces of knowledge, or are failing to restrict or order other knowledge.

Teaching is layered and segmented. Teaching is layered in that there is one body of thought and action that is automatic and routinized and another body of knowledge that permits flexibility and responsiveness to unique situations. It is segmented in that there are different parts to lessons (establishing meaning, posing problems, examining performances, etc.) and different types of lesson combinations (introductions, conceptual explorations, reviews, etc.). This layering and segmentation is part of a response to the inherent complexity of teaching.

Teaching in this chapter will be described as a web of knowledge that works to resolve the tension between automaticity and routines, and unique, conceptually analyzed events. The approach to teaching that will be assumed will be one of collaborative inquiry or didactic leadership. A sense of what makes teaching difficult will be provided by examining in some depth what types of knowledge bases need to be accessed and woven in order to teach successfully. While the definition of what it means to teach has been drawn from a long history of that issue, the discussion of how teaching is done by skillful practitioners will be derived from a much smaller set of discussions--one that has really been on going only for the last fifteen years or so.

Teaching is layered in part because of the tension between the need to be flexible, responsive, and creative and the need to place part of the task under predictable control. Teaching is also layered because there are
multiple types of knowledge that are required in order to teach. These
types of knowledge include knowledge of the subject matter, its content,
and its form or structure. The teacher needs to know the stuff of math and
history but s/he also needs to appreciate how something enters or is
refuted in the discipline. (Schwab, 1978; Shulman, 1987). But unlike the
practitioner of a particular discipline, the teacher also needs to know how
to organize and run a teaching event again in a content based way (what
goes into an explanation and a practice) and in a structural way (how to
order examples, how to keep attention, how much to leave to the student,
and how much to do with them). Knowledge about the structure of pedagogy
includes the skills of classroom management so that students are engaged
and not destructive. Finally, the teacher needs a knowledge of how students
learn; how to tell when or whether they are learning and what to do about
it. In educating teachers we tend to separate these knowledge bases into
courses on methods, subject, child development and learning, et cetera. But
the task of teaching requires that they be accessible at all times
simultaneously. This access needs to be to a cohesive set of knowledge
situated in the context of teaching. The information which is packaged
cohesively by one standard is fragmented and dispersed in the face of the
act of teaching.

The aspect of teaching that makes it complex is the very aspect that
makes it rich in the sense that the knowledge available to inform any action
in teaching comes from multiple sources: experience, example,
developmental knowledge of child learning, or systematic disciplinary
knowledge. This complexity is also a source of another attribute of
teaching which is that it is fundamentally ill-structured. In this chapter
the term ill-structured is defined in a relatively narrow (structure,) way.
namely that it refers to a class of problems whose optimal answer is not known nor obviously deductible, but for which there exist several plausibly best paths to solutions. Ill-structured problems give rise to strategic and tactical sets of solutions or to solutions which are basically inductive and experiential as opposed to deeply principled and deductive. This inductive search, however, is not without constraints.

The knowledge base in any domain can be coherent for many reasons, or, more properly, can achieve coherence in multiple ways. For example, mathematics and theoretical physics achieve a type of coherence from their deductive nature. The basic definitions plus deductive constraints provide a parsimonious way of operating that is internally self-referential and tight (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). Random pieces of knowledge do not float about and collide with one another. Literature can cohere because of its narrative flavor; music coheres because of melody or tonality. Intellectual activity that by its nature must be interactive and responsive and which must reflect multiple and sometimes competing sources of knowledge can achieve coherence in other ways. Familiarity of scene (location, mood, linguistic, problem), or the construction of a personal strategic heuristic for accessing parts of the knowledge or building very specialized local knowledge will all help an apparently disparate set of knowledge components to cohere around a situation. Activities located in the common place of practice (Schwab, 1978) have the flavor of cumulated episodic knowledge which in turn makes them appear highly situated and thus coherent with respect to situation. In sum, teachers can achieve a sense of coherence by staying in the subject matter space entirely or by constructing their own sense of coherence of the situation of teaching.
APPROACHING INQUIRY ABOUT TEACHING AND TEACHERS

Given multiple perspectives on what teaching ought to be, and given the inherent complexity of what teaching is no matter which perspective on teaching is taken, teaching and teachers are a most worthwhile object of inquiry. The tradition of studying teachers and their teaching follows many courses. At various times it has been important to understand how to choose among candidates, so predictive aspects of teachers personality were examined; at other times it has been important to judge good and bad teachers, so various evaluative investigations have taken place; at still other times it has been important to understand which teachers would be likely to be most successful at implementing a reform; at other times concern for the values and beliefs of teachers has held sway. Currently we find ourselves in the midst of calls for educational reform, expectations of vast teacher shortages, and rather minimal resources in our schools of education. We also find ourselves with a relative wealth in terms of insights about how learners learn, and a growing body of knowledge about how teachers learn.

Cognitive psychology has provided a powerful array of methodological and theoretical tools for examining the components of teaching and the nature of the task demands of teaching (Greeno & Simon, 1988; Eisenhart & Borko, 1990). Anthropology has provided us with a stance towards the context, tools for examination, and a frame for debate that is very powerful (Lave, 1985, 1989; Lave, Murtaugh, & de la Rocha, 1984; Scribner, 1984a,b). Many researchers in education have borrowed heavily from the field of cognitive psychology to help structure the task of investigating teaching. One paradigm that has been very useful has been the contrast of novice and
expert performers, borrowed directly from Chase and Simon (1973) and deGroot (1965). This paradigm has been used both to study novices and expert teachers and also to study the more general issue of how more subtle variations in the level of knowledge affect teaching behaviors (Byra, 1989; Shulman, 1986a,b, 1987). Another tool has been the extensive use of protocols and small subject designs. But how to make the nature of education a tractable problem has been the purview of educators themselves.

In the past fifteen years there have been major developments and an evolution in research on teaching. These developments have resulted from the paradigm shift that occurred as researchers turned from the process-product approach to studying teaching to an entire array of approaches that ranged from experimental to ethnographic. Process-product research, which had its intellectual roots in econometrics and experimental psychology, portrayed student learning as an outcome which resulted from a variety of inputs and conditions (Gage, 1978). The "problem" was to define lists of inputs that affected learning outputs. These inputs were quantifiable and atomistic in structure. A major effort was devoted to making certain that the input list was in some way manipulable or changeable for policy purposes (Cooley & Lohnes, 1976; Leinhardt & Putnam, 1987). These two characteristics: manipulability and atomicity meant that studying the problem was fairly well described. The resulting data gave researchers one type of information about what was effective and important in teaching, so that variables like "wait time", "higher order questions", and positive reinforcement were stable and significant, and teachers were told to go "do them". But a cohesive conceptualization of what a teacher needed to know, think, and consider was absent. There were
few useful "inputs" to the teacher nodes. This tradition of considering educational variables as independent inputs continues in the policy arena but these variables are now seen more properly as indicators rather than arguments of teaching.

Lessons in mathematics, or for that matter history, are not all the same. Two things are clearly different even within one teacher; first, that there are different kinds of classes within a lesson topic and second, that there are different segments within classes. When a process-product view of classroom instruction was held, these subdivisions and typologies were deliberately masked; control for the differences in segments or lesson types was achieved by sampling (Cooley & Mao, 1981; Leinhardt, Zigmond, & Cooley, 1981). But when understanding the nature of the shared task and activity was the objective, as in the Bossert (1978), Doyle, (1986), and Vygotskian (Vygotsky, 1978) sense, delineation of these boundaries became necessary. This shift had several implications for research. Under the process-product paradigm the best classroom visiting strategy was to sample randomly from the middle seven months of school. The best new strategy was to sample a single, connected episode, which could run from five to thirty days. In that way one would get an instance of concept introduction, exploratory activity, early presentation of procedure, rehearsal of procedure, and assessment. Within each lesson one would also see delineation of different activity structures. These within-lesson activity structures depended both on the particular theory of teaching the teacher held and where one was in the overall sequence of topic development.

The 1986 Handbook of Research on Teaching (Wittrock, 1986) clearly demonstrated that while quantitative approaches to the study of
classrooms were alive and well, a tremendous amount of work was also
going on in a very different vein (Brophy & Good, 1986; Good & Brophy, 1986;
Shavelson, Webb, & Burstein, 1986). Only a few of the branches will be
reviewed to give the flavor of the "new" activity. One branch tends to
emphasize the connections between generic psychological studies of
expertise, and teaching. This branch has looked at the nature of teaching
from a generic and classical psychological viewpoint. It uses the
novice/expert paradigm to inspect the psychological properties of teachers
with different levels of skill in general teaching (across subject matter).
Not unexpectedly it has been found that experts in teaching resembled other
experts in other fields. The methodology has been primarily experimental
with the use of inventive tasks. From this work a theory of learning to
teach has developed that cracks the complexity problem by assuming a
hierarchy of knowledge. The hierarchy starts by considering general
managerial knowledge as a basis and moves to more subject matter based
knowledge. The research captured by this program has focussed
heavily on
the managerial knowledge base and has not dealt with the role of subject
matter to any major extent (Berliner, 1986, 1988; Berliner & Carter 1986;
Berliner, Stein, Sabers, & Brown, 1987).

A second and very different branch has examined the effects of both
widely and subtly different levels of subject matter knowledge on the
teaching practices of individuals within and across subject matter domains.
This body of work showed that, at the high school level, teachers'
flexibility and thoughtfulness is directly related to their knowledge and
depth of understanding in a field. The methodology used for these studies is
decidedly qualitative and tends to emphasize small, deep case studies.
These studies often compared a teacher teaching two aspects of a single
subject, one aspect in which they were extremely knowledgeable with one in which they were less so. The basis for this research while psychological in origin is also connected with traditions of educational philosophy. It argues, following Schwab, that expertise reflects both knowledge of the content of a domain and knowledge about the epistemology of that domain. In this theory of teaching, teachers develop the necessary capability of transforming subject into teachable content only when they know how the discipline is structured beyond the immediate focus of the material being taught. Lampert's work makes transparent what this means (Grossman, in press; Grossman, Wilson, & Shulman, L., 1989; Gudmundsdottir, in press; Gudmundsdottir & Shulman, 1986; Lampert, 1990; Shulman, 1986a,b, 1987; Wilson & Wineburg, 1988; Wineburg, 1990; Wineburg & Wilson, 1988, in press).

A third branch has focussed on the quantitative and qualitative changes that occur during the process of becoming a teacher at both the elementary and secondary levels. (Feiman-Nemser & Buchmann, 1986, 1987). This research has used several paradigms but tends in general to focus on how teachers think about their teaching as they develop competence or recognize their lack of it. One unique branch has been work that uses introspection about one's own teaching as the fundamental database from which to discuss and describe the phenomena of teaching. This work has multiple themes. Two of the more important themes are 1) a conceptualization of teaching as a process of facing dilemmas in the sense that teachers see their teaching as a web of subject matter knowledge in which multiple paths for next steps are available at all times, and selection of a path always involves a trade-off; 2) the description of teaching as a task in which one comes to understand the real meaning of a
student's utterances rather than one of establishing or clarifying the correctness of the utterance in terms of mathematics. This does three things: first, it makes the classroom a less judgmental place; second, it shifts both power and responsibility for sensibleness onto the shoulders of the child; and third, it makes the course of instruction and learning less predictable for both teacher and child. (Ball 1988; Lampert, 1985, 1986; Morine-Dershimer, 1985; Wilson, 1990)

Work that parallels these branches focuses on the evolution of teachers' planning and the relationship of that planning to in-class action. This combines traditions of research on teachers' thinking and teachers' behavior. It has reflected most clearly the tension between the automatic, routine, or pre-planned and the unique or innovative. The term used for this has been improvisation or planful constrained departures from plans. In findings that are reminiscent of Shulman's, it has been shown that teachers who are knowledgeable develop variations on plans that are fairly successful but that they are less likely to depart from plans if they are not as knowledgeable. (Yinger, 1987; Yinger & Dillard, 1987)

In related work new teachers have been followed as they become more experienced in their first few years of teaching. They are contrasted with experienced teachers who are effective, skillful, and in good control of their subject matter but who are still less than flexible in their use of the necessary pedagogical tools. In supporting the view of hierarchy this work suggests that improvisation is something that develops after a rather more inflexible, sticking-to-plan process is mastered. These plans are both pedagogical and subject matter ones (Borko & Livingston, 1989).

Having sketched a backdrop for research on teaching of a particular kind, the rest of this chapter will trace my own view of the teacher and
teaching. This view has been richly shaped by the currents of research of which it is a part. Since 1980 I have been studying teachers of mathematics, and more recently, history. I have used both the novice/expert contrast combined with an anthropological field-based methodology. The attempt has been to combine concerns of subject matter with concerns of task. This work has focused on understanding the nature of the task of teaching: what makes it difficult, complex, successful, or hard to learn. The strategy has been to take one strand of teaching at a time and investigate it fairly carefully and then to go on to another one, until there were enough strands to weave together. The approach has been neutral with respect to judging the "right" way to teach; however, what has been sought out is teachers who consistently, using multiple indicators, are successful. A successful teacher is simply one whose students consistently learn a lot more than average, where learning is socially defined by the community and the school. Often this means test performance; sometimes it means demonstrative understanding; sometimes it means further learning capabilities. Expertise is always defined by at least one and usually several external criteria. In what follows, the basic divisions and slant, as well as examples, are my own and that of my research team; but the results and perspectives have grown from a shared community of scholarship, only a few of whose members were cited above.

In all of the recent research on teaching there is a stated or tacit awareness of the necessity and dangers of partitioning teaching in order to study it. If only the subject matter is the focus, features are ignored that can cripple a classroom. If the generic aspects of teaching are studied, then the substance and purpose of teaching is lost sight of. If management behavior are studied alone then the conditions for acquiring knowledge are
excluded. If only the social context or political meaning is studied then the
task and rationale for teaching is lost sight of. All research has had to
come to grips with how to partition and focus on the domain of teaching.
This is done by recognizing that any partitioning is arbitrary and to some
extent misleading, but necessary if we are to understand the problem
better. This same issue of partitioning arises again when we consider how
teachers learn to become teachers and how to assess teachers. The
knowledge needed for teaching must be cohesive and well integrated.
However, recognizing these dangers it is still true that teaching has
strands that are detachable and can be studied somewhat separately from
each other; each strand is in turn, layered and woven in with others. These
strands are extracted for purposes of inspection; they are not necessarily
useful divisions for teaching teachers or for testing teachers. In general
each of these strands of teaching tends to have different goal structures
and memory demands for the teacher so they make useful analytic divisions.
In the next part of this chapter these strands are described in some depth.

STRANDS OF TEACHING

In this section those strands of teaching that have been studied and,
for the most part, that show differences between novices and experts are
reviewed in detail. These are not exhaustive; the belief system of teachers
is not discussed nor is the cultural contextual anchoring dealt with, both of
which are important aspects of teaching. Each strand will be discussed
from an exemplary, definitional, pragmatic, and analytic or routine
perspective. That is, each component will be presented as an example, then
examined with respect to its meaning and its relation to the overall task of
teaching. The practical aspects of what happens when this component is
present or absent in a lesson will be examined. Finally, the sense of how the particular strand relates to the tension between routine aspects of teaching and unique innovative aspects will be discussed. The following sections are devoted to methodology, strands of teaching, and teacher education. The strands chosen for discussion include routines for teaching, agendas as plan traces, explanations, representations, and instructional dialogues.

METHODOLOGY

Before turning to specific examples of the strands or components, mention should be made of the methodology and subject matter bases for these analyses. The set of studies that form the core for this next discussion has been carried out over the last ten years. These studies have shared subject matter, grade range, and methodology. The subject matter has been elementary mathematics with an emphasis on subtraction, fractions, and graphing, although most recently history and writing have been included. All of the analysis of lessons and their effects have been built around a commitment to understanding and analyzing the particular content which was being taught. While some components are more easily described than others in relationship to the subject matter all of the conceptual work has focussed on the particular (Schwab, 1978). Grade level is also important. All but eight of the twenty teachers who have been studied have taught in totally self-contained, elementary-school classrooms; none of the teachers taught more than three sections of the same subject matter. All teachers were aware of the age and developing nature of the students with whom they were dealing. In the few studies of high school teachers which are described, many of the specifics of the
components described below are quite different. A somewhat different conceptualization may be more appropriate for high school teachers. In part, this is because high school teachers see well over 150 students a week and "cover" much more content. They do not have access to nor can they make much use of the individual, personal data of the student in the way most elementary teachers can.

Methodologically the work has evolved but the basic approach has been fairly stable. It takes approximately 18 months to two years to find and develop a relationship with a teacher so that they will agree to work with me. Teachers generally work with me for three to ten years. During that time we identify a component block of subject matter to be the core of what will be studied (subtraction, fractions, graphing, the Constitution) and that the teacher normally teaches and agrees is important in the curriculum. In the first year of actual data collection with a teacher, two to four weeks of classroom observation are carried out, sometimes audio taping, at other times taking field notes. All available text books or other teaching materials used by the teacher directly or indirectly are obtained and studied, all of the class handouts or tests are obtained. The purpose of this phase is to learn the context of the teaching, the organizational and hierarchical structure of the school. It is also to learn the language of the classroom (seat work or bell work or brain teasers? dismissal, or line up, or file?) and the social make-up and tempo of the room. During the next data gathering phase each lesson is preceded by a short interview which asks "What do you intend to do today?" and is followed by a short interview asking "How did it go?" All the lessons in a particular block are video and audio-taped (from eight to thirty days). The teacher views the video tapes almost every day and discusses them. This interview is called a stimulated
recall but it is more accurately a process of annotation. All discussions with the teachers are audio taped. In the second and third years of study, after reasonable sketches have been obtained of how the lesson blocks are likely to proceed, students are included in the data gathering. Students are given pre- and post-unit interviews and they are interviewed in class or immediately after class about their work. Samples of the students' work is gathered and photocopied. Annually, all of the data is transcribed and verified (twelve hours of transcription for every hour of videotape). The data is then indexed and roughed out. This involves tabulating what occurred in each lesson, checking whether or not all of the data is available for the lesson (including interviews with the students and the teacher), and identifying major lesson segments. Rarely is the entire episode for a single year reported. More commonly a single aspect of the data base is inspected carefully and several teachers compared. Novice teachers are studied less frequently and for shorter times. Not every teacher has been studied in this way, but all of the research reviewed next has been verified by such studies. In the next section, the strands or components of teaching are presented, starting with the routines of teaching. The selection of the routines of teaching is intentional because it represents one end of the continuum in the tension between the automatic and the analytic.

ROUTINES

What does that mean? Jason?

Try..

Ok now, you said two things: how much in weight and how many of them. What does anybody else think about what Jason said?

Psyche?...
Is she right Jason?
Did you mean how many kilograms?
Or how many fish?
How many kilograms in weight?
John what do you think?
Okay, Karl what do you think it means?

M. LAMPERT INTRODUCING A PROBLEM IN ESTIMATION, AVERAGE, AND DIVISION (Lampert, 5/16/86, I. 16-38).

Who can tell me what fractional name for one are you going to use?
Bingo. 3/3
Why 3/3 Kal?
John?
Jesus?
You can divide 3 by 3 and 9 by 3. Why can't I use 2/2?

KONRAD INTRODUCING CONSTRAINTS FOR REDUCING FRACTIONS (Konrad, 3/9/82, I. 403-415).

The quotations above are from two teachers with extraordinarily different teaching styles, circumstances, and philosophies. The first is from Magdalene Lampert, an educator at both the university and elementary school level; the second is from Konrad (pseudonym), an elementary teacher who has served on many boards for mathematics teaching in her city. Both teachers should be considered experts, but using different criteria. Lampert teaches one class period daily in an elementary school which serves an ethnically and racially diverse population drawn mainly from the children of graduate students in East Lansing. Konrad teaches full time in
an all-black, inner-city classroom. Because of her excellent management capabilities, Konrad is frequently given students with the most difficult discipline problems in the school. Both teachers have very high levels of mathematical knowledge, far exceeding that found for most elementary teachers. Both believe in students developing deep conceptual understanding of their mathematics. Lampert's teaching supports the development of a shared mathematical dialogue that guides inquiry. Konrad teaches from a behaviorally based, individualized program.

Both teachers use routines. Routines are small, socially shared, scripted pieces of behavior. They are so small as to be overlooked by most teachers when describing how or why they teach. But trying to teach without routines is almost impossible. (Bromme, 1983; Leinhardt, 1983; Leinhardt, Weidman, & Hammond, 1987; Yinger, 1979, 1980). Routines help facilitate management in the classroom, such as the people/product-moving events of lining up, pencil sharpening, et cetera; they facilitate or support actual instruction by establishing ways to display or share information (look at the poster, look at page X, when you finish this do that); and they help facilitate exchanges of knowledge, understanding, and evaluation (for example, routines of hand raising, choral response, question posing).

In the two sets of quotes above we see two remarkably similar call-on routines which are of the exchange type. In both, several children were asked to respond to a single question; in each case the response to the questions were similar in that the named child answered in a way that related to the question. The sum total of the routine and general import

4 Interesting questions concerning how routines get established, how diverse they are across schools and settings, how aware students and teachers are of them can all be asked. Routines are most noticeable when they are absent because it is then that things break down.
were different, however. In the Lampert exchange this is a part of the larger routine of "establish and share meaning and terms;" the continuation from one child to the next is done to generate several ideas about the topic that will be then discussed. The Konrad exchange is part of the "cycle to correct" routine, which is used to get a single correct idea out. It stops as soon as the first correct answer is given. (In a close variant, rehearse-to-correct cycling continues until a specified set of students all answer correctly.) The basic structure of the two routines is similar. Lampert uses a few more words--and because we know that she intends these to be generative behaviors we can add import to the fact that she integrates one comment with another. Other teachers use the same language of connection, but with the cycle-to-correct intent.

Routines are vital. They reduce the cognitive processing load for both the student and the teacher; they are very easy to teach because by second grade students have a schema of "learn the routine for x"--they expect them. Routines can be considered efficient when they call up an action with a minimum of fuss and bother, or confusion. They are efficient when actions that could take a long time are done quickly. Effective teachers have management, support, and exchange routines in place by the end of the second day in a school year. They retain 90 percent of these same routines at mid year (Leinhardt, et al., 1987). But routines are also subtle. Because they are used so often they set the tone of a class. In Lampert's classes there are dozens of routines surrounding exchanges and development of ideas (much like the proverbial multiple words for snow among those who live in snow-covered environments). In Lampert's class there are routines for generating small-and large-group answers, for recording reasoning behaviors in personal journals, for indexing pages in journals (by date and
page number), for revising ideas publicly and privately, for deciding the
dimensions of discussions, and so forth. In Konrad's class there are
several exchange routines but they all focus on giving or getting
information from or to the teacher; there are no routines for managing
personal knowledge change or for reaching group consensus. The effect is
to inform students indirectly of what is expected and valued.

When teachers learn to become teachers and after some time wish to
modify their teaching behaviors they often adopt the large pieces of a new
reform (small group, cooperative teams, etc.) but they keep the old routines
for producing and sharing knowledge. That leads to two consequences: first
that the new system does not work and that they have "management
problems;" second, that the class of students receives very mixed
messages.

Figure 1 shows a planning net for establishing definitions in a math
classroom. The planning net was designed to describe the Konrad style of
"calling on." This net is presented to show how one routine, that of calling
on a student, is used in context. In both of the quotations above, students
were called on in response to a question asked by the teacher when a
student had their hand raised. In both cases the end of the episode occurred
when the teacher restated the final meaning. In Lampert's case this was
done when she increased the volume of her voice, stating almost verbatim
the student's final definition of what the question meant. In Konrad's case
it was a flatter restatement of the "looked for" correct answer. In
Lampert's case it would be more reasonable to say that meaning had been
assigned to the terms and that the goal (see Figure 1) was not so much to keep students interested as much as to keep students generating the math work of the day. But both teachers were, as they conducted the lesson, aware of time constraints and the role of other students. The test (see Figure 1) "is it correct" was also done differently in the two lessons. In the Konrad lesson the test was a zero/one, correct or incorrect format. In the Lampert case it was more a question of what are the ambiguities or clarifications inherent in the given answer. Lampert also would appear to have another goal or test operating in this mini exchange--that of discovering how widely shared the views were and how diverse. If she had received a complete and well articulated answer initially she might have "cycled to consensus" (another routine) rather than cycled to correct.

If we consider the issue of how routines relate to the tension between what is unique and what is automatic in the layered task of teaching, routines are, as their name suggests, routine. Routines are often picked up or developed by teachers in an ad hoc, unintentional way. However, both Konrad and Lampert in interviews were aware of a deliberateness about their routine behaviors. They were at one time analytic about behavior that would become automatic. Routines in addition to clarifying and simplifying the teaching task help to give a class its atmosphere and more subtle or tacit messages about teaching and learning. Lampert is quite consistent in requiring students to record certain things in their notebooks and to date and number pages; it is a definite support routine. But she is equally consistent in displaying the non-arbitrary nature of decisions and ideas in mathematics and in social communications. There are routines for revision of ideas and routines for challenging ideas and for soliciting meaning. In Konrad's class there are routines for
answering and routines for probing and questioning. But while the routines for answering are public, the ones for probing are private and are not shared with other members of the class. Thus, the climate of public presentation is that it should be rapid and correct while the attitude about reflection and uncertainty is that it is private and personal. An interesting aspect of routines is that because they are so automatic most expert teachers are not able to document how they do them or design them—they are fairly unpackable pieces of procedural knowledge. As such, routines do not figure prominently in the annotation or recall of actions or in the plan for actions.

Routines then, can be easily missed by researchers, teachers, teacher educators, and educational reformers. But their existence and role is both documented and clearly significant. Routines are the stable answers to the habitual questions of classroom life. Much as early anthropologists looked to see how cultures handled the common problems of food gathering and distribution, the classroom researcher needs to care about these little common solutions to recurring common problems. A second strand in the fabric of teaching is the agenda. The agenda is the short, mental working plan that the teacher carries into each lesson.

AGENDAS

Well, I think when they get back from lunch I'm going to water them down and let them use the restroom to calm them, and then we're going to do uhhhh, a little activity with erasers, paper clips, string, and books. And we're going to talk about pendulums and clocks. We're going to make a pendulum and then they're going to chart the differences between the amounts of
swings and the length of the string. {NOVICE (TWAIN) (END OF
STUDENT TEACHING) PRE LESSON (Twain, 11/23/82, l. 4-13)}

Em, I don't know what happened. Well, I think that I didn't
spend--I see now that that should have taken two lessons to do.
We should have spent one lesson in bar graphs or charts--that
kind of thing--graphs. And uh graphs and counting pendulum
swings--the length of the pendulum making a difference.
Ahmmmm, counting seconds, recording the information that was
one entire lesson. To put the thing into one lesson was just too
much information for them to record and process accurately.
{NOVICE (TWAIN) POST LESSON EVALUATION (Twain, 11/23/82,
lines 3-15)}

I'm going to (student interrupts with question) excuse me Mark.
We're hopefully going to work on finding the fractions of a set.
Given eight objects--eight separate objects to find one--one
half of them, one fourth of them, as opposed to what we were
doing last week, which was finding a fraction of one object.
{EXPERT (KONRAD) PRE LESSON (Konrad, 1/25/83, lines 5-14)}

Mm, they understood the fraction of a set faster than I thought
they would. Of course I'll know (laughs) -I'll know definitely
when I check the Basic Worksheet books. Mm, to make sure they,
ah--as I walked around the room, everyone seemed to mm--know
what they were doing. {EXPERT (KONRAD) POST LESSON
EVALUATION (Konrad, 1/25/83, l. 2-8)}
I'm gonna have kids practice, um the convention for writing ordered pairs, by writing the x value first. But I've decided not to emphasize the x's and the y's but to use it in my speech. But, but I, but not really explicitly teach about it. What I want to teach about today is that there are two places where ordered pairs have been appearing in our work; one is on graphs and the other is on function charts. And after we practice, um making sure we all know that we go over first, and then up, using that alphabet thing, then I'm gonna say, suppose we give letters of the alphabet to these ordered pairs on the function chart that we made yesterday and see where they appear on the graph. And what we, what patterns we can observe and why we think those patterns are there. That's it. I expect that to take at least 45 minutes. LAMPERT (EXPERT) PRE LESSON (Lampert, 10/26/88, I. 101-123)

These three sets of quotations represent the first response of one novice and two experts to the questions of "What are you planning to do today?" and (for the first two teachers) "How did you think it went today?" The response to the first question is the first segment (probe questions elicit more information) of what I have referred to as an agenda (Leinhardt, 1989). An agenda is an operational plan which is concise, focussed, and is the general set of goals and actions that the teacher intends to engage in for the next 40 to 50 minutes. Planning is an intimate and critical part of teaching. It has been researched and described elegantly by several researchers (Clark & Yinger, 1977; Jackson, 1968; Yinger, 1987; Yinger &
Dillard, 1987). What a teacher takes from a plan for use in the teaching is captured to some extent by the verbal trace of the agenda. Learning how to construct a plan and to formulate an agenda becomes routinized, but the specific plan is almost always uniquely constructed for the given lesson based on how prior lessons have gone.

The agenda is the teacher's local mental note pad of the more formal plan. Plans and agendas are both devices which help to store in long term memory strategies for approaching an interactive on-line situation. This storage of strategies reduces the burden of information processing in the midst of teaching. The agenda is expressed largely in terms of non-routine elements of the lesson. That is, routine components which repeat are not mentioned but are assumed. Often an agenda includes major action schemas, such as seat work or demonstrations; but those, if present, will keep markers or flags for places in which the teacher needs to look for student data. We have done a great deal of work which compares the agendas of experts and novices, which will be discussed below. But first to get at some of the concepts consider the two segments above.

The novice's agenda picks up the students where they will be, namely, coming back from lunch. This is salient to her because she knows this will present some discipline problems. She identifies the point of first physical contact, not first intellectual contact. Her solution of "watering them down" is interesting. Students always go to the bathroom after breaks, whether recess or lunch. It is a school-based routine—to violate it would be absurd. But that routine in itself will not in any way help to calm the students; rather it is a subprocedure, which will pose other management difficulties. Also the language is herd-like—one "waters down" cattle.
This probably indicates the sense the student-teacher has of control at risk rather than the more obvious interpretation that the children are animals.

Next, the novice says that she is going to do activities with objects which she lists: erasers, paper clips, string, and books. This is math class; the characteristics of the objects are irrelevant except in her mind because she must make certain they are available. What she fails to mention is how or why the objects are being used. After the object list, there is the pendulum and mention of a chart, again without any mention of the over-riding intellectual goal. Any experienced teacher would recognize several flaws. First, for most classes, there are no immediately accessible, first-order routines to manage the distribution, counting, paper set-up, and charting of all of these different things. That means considerable lesson time will have to be given over to filling in the specifics of the primary schema for "how do we do this activity--or instruction for games, field trips and other unusual happenings." That is, students know to expect the unexpected and have good skills for handling them; but it is a harder action to manage than those things that are direct routine call-ups, such as board work or teams, et cetera. A second flaw in the agenda is the fact that there are clearly two large and long activities planned (making a pendulum and using it). The third flaw is that the teacher is not thinking clearly about the basic educational goal. She has no overarching pedagogical or instructional goals to guide her in making short-term, small corrections. She is losing her center. As the interview proceeds she talks about clocks and the fact that this is the penultimate section before review, but she never mentions the content or topic of the lesson, nor does she connect it to prior work.
In her evaluation of the lesson the novice recognizes that the lesson did not go well. She identifies one of the three flaws in her agenda: it was too much stuff. If she corrects that in the next teaching round she will do well, but she will still have a long way to go before she can construct for herself a meaningful mini plan that will work and will have in it the critical lesson information. In both her actions and in her plans for actions the novice is aware of problems but she does not seem to have any information about how to make the job simpler let alone more effective.

In the first expert’s agenda the first thing noted is the topic: finding fractions of a set. For this particular teacher the actions to support that instruction are already known: a demonstration of work at the board, student dialogue about the connections, and independent work. The only salient feature is the goal. Everything else is in place; all of the routines for presenting new material, for understanding how the students are doing with respect to acquiring the new information, and any activity. It is important to note that this would be a useless agenda for a novice because it is not nearly detailed enough. However, the expert’s brevity is a kind of code which permits more detailed routines to be called up. The goal and the connection to the prior lesson’s work are plenty for the expert to work with.

In Konrad’s review of how the lesson worked out there are two additional features of importance: first, that it went more quickly than she expected; and second, the place to look to see whether it went well is the students’ work. This teacher is ready and willing to consider student knowledge as an acid test of whether the teaching was successful.

In the Lampert agenda there is even more detail about the content of the lesson in terms of subject matter and the role of the students.
are two main tasks: first, to bring out the ordered pair notion explicitly; and second, to tie this specifically to the two locations in the students' experiences in which ordered pairs have been present. Joining together two disparately located but identical concepts is an unusual and important goal for a lesson. She also distinguishes between what she hopes the students will pick up on (the letters x,y) and what she will handle explicitly (ordered pairs). In the actual lesson it became clear that many students had two different meanings for "origin." One meaning, which had been derived from their social studies lessons, used origin as center of a space not just the 0,0 coordinates. The other meaning was the ordered pair 0,0. Lampert decided to change the course of the lesson entirely and to deal with the various meanings of origin (and meridian, etc.) rather than to teach the lesson that she had planned. While this flexibility in departing from plan is the mark of a skilled improvisor (Yinger, 1987) it is beyond this discussion for now.

In general, the agendas of experts look quite different from the agendas of novices. Expert agendas contain: a) some list of actions both of the teachers and of the students; b) some sense of predicted student behavior; c) a set of tests or check points that will help decide how to proceed; d) a connection or placement of this lesson in the wider spectrum of lessons; e) overarching pedagogical rules such as moving from the concrete to the abstract, or overarching, content-driven rules, such as this idea is useful in understanding this next idea (Leinhardt, 1989). It was this latter sense that permitted Lampert to change her lesson on the spot, a sense of what was conceptually important and why; and the lack of it that got the novice into some difficulty. The lesson agenda also clearly identifies the subject matter topic being taught. This is true whether or
not the topic is subtraction, fractions, graphing, or the Civil War. In a more structural vein, most agendas of experts are longer and contain more detail. In the examples above the first expert's agenda is shorter and sparser than usual but in the lesson that followed and in the evaluation it is clearly sufficient.

Usually experts report many more instructional moves and more specific topics of content than do the novices (Leinhardt, 1988a, 1989). Further, experts anticipate problems with either the tactics of approaching the task or with concepts inherent in the material. In addition to mentioning student actions and using student performance to help gauge the success of a particular lesson, experts seem capable of thinking of the lesson along two tracks at once. One track is the actions and thoughts that the teacher must have; the other is the student. That is, the expert sees the status of the student changing throughout the course of the lesson. Novices show no signs of this capability. They do not report overtly that they will monitor the students' behavior, nor do they generally report that they are envisioning the mental progress of the students throughout the course of the lesson. The presence or absence of this parallel sense of what is happening is an important aspect of forming an agenda. It shows a linkage between the content being taught and the kinds of learning behaviors that are likely to lead to acquiring the knowledge. Its absence leads to the frequently noted complaint of new teachers, "But I taught them that."

An extension of this concern is lack of it for the student is the way in which tests or checks are often built into an agenda. Novices almost always describe the lesson as non-interruptible; this is their plan and they will stick to it (Byra, 1989). Experts anticipate the need to be able to alter the initial plan; and the reasons for such alteration will most probably be
that the students are experiencing some difficulty or displaying a lack of facility that had not been anticipated. This, of course, suggests that experts feel that they can alter a plan because they have a store house of solution components that lets them do something else. This planning to check students' understanding by questioning or reading facial expressions allows the expert to manage the natural uncertainty of the teaching task. The novice, by failing to consider the student, will not change plans unless and until a crisis occurs.

Finally, experts often provide a sense of the logical flow or at a minimum a clear goal. This explicit logical flow helps again to handle the little interruptions or even lapses of memory that tend to occur. If the specific action or sequence has to be stopped or altered the expert can manage to refer back to the basic logic of the lesson and substitute comparable moves, while the novice is unable to do this. The novice's knowledge is unintentionally tied to the specifics of the actions.

For both novices and experts the capacity and strategies for constructing a plan and abstracting an agenda must become routinized. But the specific act of forming an agenda is in itself a unique analytic action. It seems that teachers modify an agenda in the course of teaching and in the minutes after a teaching episode. When the lesson is used again (whether a year later or in the next period) subtle changes are brought forward in the form of things to worry about, watch for, or use again. But deeper alterations, such as how to approach a problem, affect the more complex lesson design. (See Putnam, 1987; Putnam & Leinhardt, 1986.) In terms of the tensions described at the beginning of this chapter agendas are strategically automatic but substantively analytic.
Agendas are most often descriptions of activity segments that lead up to and include presentations. Agendas do not record the non-presentational portions of a lesson, such as checking homework, having a game or drill, or small group problem solving. However, not all lessons contain presentations by the teacher or by students. Some lessons are continuations of prior activities or tasks, others consist of working in small group teams with no central new informational exploration, still others are planning or review sessions. If a class does have a presentation then this is the time when a teacher presents, or has the class explore relatively new ideas, materials, procedures, and notations. Presentations are rarely more than a third of the class time and they do not occur every day. During review times the review is the presentation and it takes on a very different character from presentations that occur when a topic is introduced, when concepts, terminology, and notation are being developed or intuitive understandings are being shared. So another strand of the teaching fabric is presentation.

Presentations, in turn, contain explanations and specific rules for generating examples or the examples themselves. Presentations also contain a tacit list of what is important to learn and where the pitfalls may be located—what to watch out for as the students work through the material. This sense of important elements guides the construction of an explanation. Just as all lessons do not necessarily contain presentations, not all presentations contain instructional explanations. An instructional explanation is the system of goals and activities that are involved in the direct communication of subject matter content. In the next section the explanatory strand of teaching is explored.
EXPLANATIONS

T: So I give you a problem to guess. It will be really a problem of solid geometry, what you know as solid geometry, how much is there to know? For instance, everybody knows what is a plane. (writes on B: "plane") A plane is very flat. The top of this desk - this is part of a plane. (puts hand firmly palm down on T desk) Or approximately. The better it's made, the smoother it is, the flatter it is, the better it resembles the ideal plane of mathematics. But the ideal plane of mathematics goes on in all directions. It is infinite. So you know what is a plane. It is flat and infinite. Now my problem is about planes, -- several of them--and to tell the whole story, about 5 planes. (turns to B, writes "s" on end of "planes") So, you imagine 5 planes. (writes "5" in front of "planes") So, if you cut so that is one plane, two planes, three, four, five planes. Now these 5 planes cut this space in many parts or divisions or compartments, or whatever you call it. And that's just the question. How many parts? (picks up yardstick; cuts through the air horizontally, vertically, and diagonally with it) This is my question, or almost. There is something to be added, but I'll wait 'til you find it out by yourself. But you understand it? You imagine it a big piece of cheese, some cheese. . . (writes on B: "how many parts") There are lots of many pieces. And you have to guess how many. . . Guessing that's the important beginning of solving any problem. And the real problem is difficult. A real problem you cannot do it right away, otherwise it wouldn't be a problem. It belongs to the idea of a problem that there is a certain difficulty. So if
you cannot do a problem, what to do? Just wait for an idea? No. The right thing is try to imagine some easier problem which could prepare you for this problem. Some easier problem which could help... You must, you should be suspicious in life, you see. So if I ask you five planes, then you should have asked yourself, why does he ask just five? Why not four? Why not six? So what would you ask? Yes.

S: I guess you ah mean ah, how many ah, planes do ah, say three planes ah,...

T: Three. Good, or what do would you say?

S: Well, the simplest model would be two planes, I would guess.

T: Two? Is that the simplest? One. One plane, of yes. [...] so much trouble to find the simplest. One. Yes. That is the s_, you see, but it is so, in mathematics often the simplest is the best. So here is, here is for you one plane. Oh, you tell me that there's just one line of the blackboard. Yes, it's true. But I mean it in the following way, you see. This line on the blackboard is the intersection of the blackboard with the plane, you see. By this plane I am showing you it's a horizontal plane. And this horizontal plane could be the surface of quiet water, of a reflecting pond. There is nothing else in the world just this surface and over it, air; under it, water. So how many parts?

Ss: Two. Two...

T: So I have just one dividing plane. It is too long to write down dividing, I just write down the end, dividing plane. And there is just one dividing plane; then the number of parts is exactly two. (Pause) Good. (writes on B, between list of numbers and
statement of the task, two abbreviated headings & one set of entries: ...ding parts

This was one plane. So what is the next case? After one, what comes after one?
Ss: Two. Two...
(uses yardstick to draw diagonal line through horizontal [see Fig. 1a])

T: Oh you say, that is not a plane, just a line. Yes, but I mean, it's so. You see, this line is the intersection of the plane of the blackboard - with, you see, with such a plane. . . .

And how many parts do you see?
S: Four. Four...

S: Could I ask you a question? If you...
T: Yes, please, do ask me.

S: ...if the planes were parallel would it still divide them into four parts?
T: Very good question. That's a good question. I have waited just for your question. That's a very good question. If all the planes are parallel, one, two, three, four, five, then it is no problem. If all 5 planes are parallel, little imagination, like that. There there are how many parts?
S: Five/Six.
T: Six. Well then the whole problem would be over. This cannot be the question. Very good. But I wanted you to bring it out.

Yes, my question was incompletely stated. And that is, was as, and was so intentional you see. Because problems in life, real,
even in science, they are often incompletely stated. You have to
find out what the real question is...

Good. It was a very good question. Now, look here. So, one
plane, 2 planes. How many parts?
Ss: Four...
T: Good. Oh we did it already. Now, 3 planes. (writes "3" in
"---ding" column) It would be a bad idea to put just right. That
would be a bad idea... (places yardstick at opposite diagonal
through existing intersection) Oh I know. The blackboard.
(smACKs blackboard with palm of hand) That is the third plane.
Good? Good enough for a third plane. Then look here. So they
are - the 2 planes indicated by the lines, and the blackboard.
How many such?
Ss: Four.
T: Good. Some other parts are outside the room, behind the
blackboard. How many such?
Ss: Four.
T: So altogether, how many?
Ss: Eight...
(writes "8" below "4" in "parts" column. now on B:)

...ding parts
1  2
2  4
3  8
T: I wish to draw your attention to the most important
point in reasonable guessing. If you had more time I would
introduce each much slower, but we have little time. So I take,
tell you right away one interest-, important point in reasonable
guessing to think of extreme cases. . . [POLYA LESSON (Polya,
1965, lines 149-447)]

In this lesson Polya presented two concepts: the constrained use of
guessing in mathematical problem solving and a guided inquiry on how many
parts a space is divided into when it is cut by 5 planes. The first concept
was presented through an annotated demonstration, while the second
concept was handled through a guided inquiry which produced an
explanation. The two constructs were woven together, guessing and planes
cutting space. However, the planes cutting space makes use of
representations (desk cheese, top of a lake, blackboard), a vocabulary and
terminology check (intersections, dimensions, planes), a concrete
demonstration (waving the meter stick in the air and drawing), and
constraints. Instructional explanations in mathematics usually contain
these and other critical features in their goals and actions.

Figure 2 shows a generic planning net for explanations in math
classes. Explanations in classrooms are an intuitively important part of
the lessons; however, relatively little attention has been given to the
properties of explanations that are successful.

Insert Figure 2 Here

In a series of studies (Leinhardt, 1987, 1989; Leinhardt & Greeno,
1986), the properties of successful explanations in mathematics classes
have been explored, and the conceptual model shown above developed and
tested. In these studies explanations are seen as being developed during the presentation of material, as in the Polya example. Explanations are the processes by which new material (concepts, procedures, connections) are put forward in a way that connects it to prior knowledge, locates it within the semantics of the particular discipline, and constrains the meaning and applications of the new concepts or actions. Most teachers have routines for raising a problem which needs to be explained, selecting the sequence of examples, and re-situating or attaching the new knowledge in its logical place.

Explanations are complex clusters of actions which may occur as a single episode or may extend over several lessons. Explanations are given in response to several goals: to develop understanding, to clarify misconceptions, to introduce a new idea, or to review an idea in a different way. In the Polya lesson above it appears that the goals are to develop understanding and to introduce a new idea. In mathematics lessons explanations usually include most of the critical features in the model above. In this model explanation is shown as an action with multiple subgoal states and respective subaction schemas.

In Figure 2, representations known and subskills available refer to the idea that explanations are built on existing knowledge and often use representations of procedures or concepts. Sometimes the representations are physical analogies, such as Dienes blocks or computer micro-worlds; at other times they are internally self-referential. For example, explaining question-formation in the past tense in French may be built up in part from an analysis of English (Kasunic, 1989), discussions of taxation in the 18th century, and an understanding of the historical context of the 18th century.

5 In a second series of studies on explanations in history a different model is being developed, one which we hope will eventually be merged with the model for mathematics (Leinhardt, 1990a,b; Leinhardt & Odoroff, 1990).
century may be related to current debates on taxes. Both goals refer to the fact that the knowledge of the subskills and representations must be in place before an explanation that builds off of them can be successful. In the Polya lesson it is clear the students understand the analogy to desk, plane, meter stick, slicing air, and pond as well as the vocabulary of intersections, dimensions, points, lines, and planes. It is clear because they generate responses using these terms and representations. If they are not in place, then an explanation is using one unknown to elucidate another. In one study of explanations, explanations by experts met these two goals 96 percent of the time while novices did so 25 percent of the time (Leinhardt, 1989).

Explanations may also contain goals for providing *numerical*, *concrete*, and *verbal demonstrations* of material. In one study experts completed their demonstrations in those three areas 100 percent of the time while novices did so only 35 percent of the time (Leinhardt, 1989). This was not because novices ran out of time but rather that they lost their way in the explanation. Novices are, in general, less successful in routines and in their ability to reduce information-processing demands, so they appear to be more easily drawn off the more complex focus of giving an explanation. In the Polya example, the dialogue does answer the mathematical question posed and the representations—-in this case the diagrams—-are completed along with the verbal explanation.

Figure 2 also shows a set of goals that are never achieved by novices and achieved only around fifty percent of the time by experts. These are the *identification of the (disciplinary) problem*, the *conditions of use or limitations*, and the *nature of the principles* which support the particular solution—-the "permissions" if you will. In the Polya explanation both
problems are clearly stated and the constraints on guessing as a tactic are presented continuously. Wild guessing is not encouraged, being suspicious is, and the relationship between guesses and predictions is explored. Some simpler explanations begin with a notion of a problem—a barrier to business as usual; for example, why do we need to regroup with subtraction in some cases, why are there fractions or negative numbers? The problem in the first case is that the procedures for subtraction have to be slightly modified. One modification involves transferring the symbols while retaining the value. The problem in the other two cases is that division and subtraction are not closed over the counting numbers. The problems in the Polya example are how to use guessing and how many parts are created when a space is cut by 5 planes. Often, as in the Polya case, students have a strong intuitive sense of the problem and part of the answer and their intuitions can be incorporated into the explanation or used to help the student develop the other necessary components for self explanation. The constraints and permissions are the part of an explanation that ties it to the discipline. They represent the principles that allow for transformations of numbers or changes of the number system provided other important aspects such as equivalence or consistency are not violated. In the Polya example, going to simpler cases and then working forward with suspicion is the permissible change.

In some explanations new notational systems are introduced. Here we change examples. Figure 3 displays the connections between concepts that were presented in order to explain a graph as a coordinate system grid. The basic idea to be gotten across in the explanation was that the grid had certain properties: lines and spaces, an origin, and a directionality. The concept of ordered pairs was also salient. The teacher, Mr Gene, was trying
to connect the new notion of a graph both to the earlier graphing experiences with bar graphs and to the new text material that related graphs to a city grid. From the point of view of an explanation, Mr. Gene presented a coherent and well connected set of ideas. His representation was useful in covering most of the major ideas in the lesson on properties of positive quadrant graphs (Leinhardt, Stein, & Baxter, 1988). In earlier work it had been shown that expert teachers usually present explanations that have this coherent quality to them, whereas novices give explanations that are island-like in terms of topics. Each topic is presented alone with little or no connection to the main point or to other sub-topics (Leinhardt, 1989; Putnam & Leinhardt, 1986).

Insert Figure 3 Here

The fragmented character inherent in novice's explanations is related to the lack of a cohesive agenda. Novice agendas are list-like and sparse. Expert agendas contain both the instructional moves and, more importantly the guiding concepts such as principles of pedagogy (going from concrete to abstract) or principles of subject (guessing in mathematics, the idea of ordered pairs across at a specific level of formalization). Without these overarching principles a novice's agenda cannot help to signal the explanation, when it occurs, in ways that will both remind the teacher to make interconnections between ideas and show the student these connections.

Constructing an explanation requires an interesting meld of routinized knowledge and constructed, unique, analytic actions. Teachers and texts
together tend to decide on larger issues in an explanation such as scope and
sequence (how much will be covered and in what order) but the specific
words will be unique and occur on the spot. If the explanation occurs from
a text that uses a particular representation then that aspect of the
explanation will be thought out in advance and may become routinized; but
specific numerical examples are more often generated spontaneously,
within certain constraints. Thus, from the point of view of routine versus
analytic tensions, explanations tend to be more analytic than routine but
they are richly supported by a context of routinized activities. (See
discussion below on instructional dialogues).

As has been mentioned, many explanations, especially in mathematics,
are built on some type of representation of the material. In the next
section the teaching strand related to the construction of representations
is described.

REPRESENTATION

Remember Graph City way back a long time ago? But when we
visited Graph City a long time ago it was just kind of a tiny
little town just beginning. Take a look at it now. It's done some
changing. How has Graph City changed? They added the
directions north, south, east, and west. {MR. GENE INTRODUCING
FOUR QUADRANT GRAPHS (Gene, 1/29/87, l. 401-407)}

Although it's dangerous to use analogies because times and
events and people are totally very different. One always gets
into trouble doing that sort of thing. {MS. STERLING EXPLAINING
TO RESEARCHERS WHY SHE AVOIDS THE USE OF ANALOGIES IN EXPLAINING HISTORY. (Sterling, 3/16/89, l. 3-6))

Okay, now I heard somebody say that they had ah, grilled cheese for dinner last night and they did not know if it were a fraction or not. (MS. RIVERS INTRODUCING EQUIVALENT FRACTIONS (Rivers, 1/5/82, l. 79-82))

A major component of many but not all explanations is the use of some type of secondary representation of the targeted material. Representations in instruction do not refer to the learner's internal mental construction but rather to a public pedagogical device for making concepts or processes clearer. Representations may have the property that they retain in some analogical way primary relationships among objects (see Gentner & Gentner, 1983) or they may have the property that they are manipulable objects that are concretely and deeply familiar to the learner. Representations may be of a more abstract, intermediate nature, bridging the symbolic formalisms that are new, or they may be background tags (White, 1989). Representations play a bridging role, but while they are powerful pedagogical tools, they are not cost free. In elaborate representations the student must understand and keep straight both the referential and target material and the connections or rules for mapping between them (Gentner, 1983; Gick & Holyoak, 1980). A representation may precede an explanation of the abstract target material, or it may follow it; it may be presented in parallel or it may form the background context for an explanation.
In the first quotation above the teacher, using the text, builds a model of the Cartesian graphing system by presenting a map of a grid-like city or town. After several lessons he expanded the town into four separate sections corresponding to the four quadrants of a graph. There is a tremendous amount of visual overlap. In this example the representational discussion precedes the abstract one and then is used in concert with it. That is, Mr. Gene moved back and forth between the two 'worlds' to make his points about the coordinate system of four quadrant graphing and graphs. Many key issues such as the significance of ordered pairs are well described by such an analogy. But the core referent of a map line, namely a street, has different properties than a graph. For most cities, the interpretation of a point inside a box (2.5, 5.5) is conceptually different (it's probably inside a house or building) than one that is on the line (or street). In the graphing world there is no such difference; the lines are merely referential.

Another problem is presented, in the third quotation, in which the teacher uses a grilled cheese sandwich as a referent. It shows what happens when the representation chosen is useless at best, inappropriate at worst, and is confusing and plain wrong. Fractionality is not an attribute of a type of food, it is an attribute of a measurement or division of food. Throughout the lesson the referent changed from food types: peas, cheese, and hamburger, to rooms and rows of chairs. The unit was constantly changing in mid discussion. Just as "manipulatives" in and of themselves offer no particular help for the students, representations offer no clarification or support in an explanation unless they are related to the content and carry few misleading entailments. Texts are often quite careless about the specific features of a representation and include aspects
that are likely to be misleading or confusing, such as switching carelessly between ratio and ordered pair referents or fractional part-whole referents. Teachers who are not knowledgeable about the specific subject they are teaching are not always able to analyze what features of a representation are good or not so good. A representation is "good" if it contains elements that make salient the key aspects of target material without large amounts of negative entailments. They are "bad" if they are wrong or confusing.

In the second quotation a teacher's discussion reflects a fairly widely held belief of historians, namely that jumping time frames is a risky business. But as Wilson (private communication) has pointed out the objectives of pedagogy and content may occasionally be at odds with each other. In an interesting exchange Nesher (in press) and Ohlsson (1987) have each described the roles of micro worlds or representations differently. Nesher considers representations to be exemplifications. These exemplifications carry their own language: vocabulary and grammar. They have some familiarity from the real world but they are abstractions. As abstractions they are built to clarify specific aspects of the targeted material and to bridge carefully from the real world ambiguity to the mathematical precision in meaning. Ohlsson (1987) in his interpretation also stresses the linguistic embodiments. Ohlsson is more concerned with the correctness of the mathematical mapping and less with the pedagogical power than is Nesher. Ohlsson presents his definition as a type of test for intermediate representations of mathematical ideas. Representations then are double edged. If they have properties that clarify but do not mislead, and if they can be conveniently used by the group then they play powerful roles in teaching. Representations, in having their own operational rules,
impose a second burden on the teacher and learner. One must know both systems and often their correspondences as well.

Considering representations as an issue in the tensions of teaching between automatic and analytic, representations are both spontaneously generated and carefully drawn from texts or prior experience. When representations are used automatically, there is considerable risk. The experienced teacher who is expert in subject matter and teaching tends to think carefully before selecting and introducing a particular representation. The novice is more likely attracted to the surface features—what makes something catchy or motivating. The analogical power that can be gained from the appropriate use of representations should not be overlooked.

Teachers, of course, are not the only constructors of representations; children often develop or borrow them from other sources, as can be seen in the next section. In the next section the teaching strand that is focussed on is instructional dialogues. In this particular case the dialogue also forms an explanation of fractions. However, dialogues are a form of teaching, a repertoire, not a portion of the lesson or an aspect of curricular presentation.

INSTRUCTIONAL DIALOGUES

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<tr>
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<tr>
<td>111</td>
<td>334</td>
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<tr>
<td>2</td>
<td>7</td>
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<td>10</td>
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<td>113</td>
<td>340</td>
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<tr>
<td>195</td>
<td>586</td>
</tr>
<tr>
<td>1/4</td>
<td>?</td>
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(The above chart was on the blackboard during a fifth grade lesson on functions.)

T: Suppose my input number is one fourth?
S: [chatter]
T: What do you think? What would be the output number?
Philip?
T: I want you to be paying attention please. Um Asa? That's not something you're supposed to be doing. Put that in your desk. Soochow?
S: One and three fourths.
T: How would you explain it please?
S: BECAUSE ONE FOURTH TIMES THREE IS THREE FOURTHS AND THEN YOU JUST ADD A ONE.
T: Okay, so first you times by three and then you add one.
T: Who can explain why one fourth times three is three fourths? Nusoo?
S: ONE FOURTH, LIKE ONE FOURTH OF A PIE AND THEN SOMEBODY BRINGS TWO MORE AND ONE TIMES THREE IS THREE--THREE PIECES OF PIE THAT CAME OUT OF FOUR PIECES OF PIE?
T: Okay, are they all the same size? Those three pieces of pie? Ava?
S: Yes
T: How do you know?
S: Because if you’re adding one fourth times three you’re going to equal parts
T: Okay. Cause I'm, I'm taking three things that are all the same size. They're all the size of one fourth. Ali?
S: IT COULL BE ONE FOURTH [-] COULD BE A WHOLE ONE.
T: Can you explain what you mean?
S: Can I come to the board?
T: Yes, here take this, [chalk] it's easier to see.
S: Here's like a big pie [draws circle and divides it into
fourths]
T: Um-hum.
S: And then you could divide it into fourths, four pieces. AND
THEN ONE FOURTH COULD BE ONE [POINTS TO ONE SEGMENT OF
CIRCLE] AND THEN WOULD BE LIKE THIS ONE [POINTS TO THE
ONE ON THE INPUT SIDE OF THE CHART]
T: I don't understand what you mean. Does anybody else
understand what Ali means? Bridgette?
S: Me_, he means that if you ha_, if you have one fourth and
you make say you color in three of the four pieces [-] equal
one whole
T: Is that what you meant?
S: Yeah.
T: Okay, what do you think about that? Ali is saying three
times one fourth is one fourth [sic]. Add one fourth and
you'd get four so it would be just like here [points to the 4
beside the 1 in the function chart]. But the input number
here was one [writes faint 1 in input column beside the 4]
and now the input number here is one fourth [points to the
1/4 in new chart]. What do you think Sun Wu?
S: HE THINKS THE UM, THE ONE IS LIKE ONE FOURTH. BUT ITS
REALLY ONE, ANOTHER, FOUR.
T: What do you think about that Ali? [draws another circle].

   How many fourths are there in one whole?
S: Four fourths [T draws new circle into fourths].
T: Four fourths? So if I was going to put a number in here I could put one and a fourth [sic] [writes in column]
T: Is there anything I could put in there besides one and a fourth? Elsie?
S: Wouldn't it be one and three fourths?
T: Oh, I'm sorry. It should be one and three fourths like that anyway [changes chart]. Is that what you meant?
S: Yeah
T: Ali? If I count these and this how much do I have? [points to x's in three of the quadrants of the second circle]
S: Three fourths.
T: Three fourths plus one whole -four fourths. [points to other circle]. How many fourths all together?
S: One...there's four fourths altogether.
T: In this one [points to bottom circle] one two three four--but if I add three fourths plus four fourths [writes 3/4 besides the top circle and 4/4 besides the bottom and puts a plus sign]. How could I write the answer besides one and three fourths. Ali?
S: Seven fourths
T: Okay, I could write seven fourths. Could you explain why?

{LAMPERT IN EARLY LESSONS ON FUNCTIONS AND GRAPHING
(Lampert, 10/24/88, I. 261-370)}
The exchange above is an example of an instructional dialogue. Instructional dialogues are forms of explanations in which fundamental, discipline-based ideas are brought forth for public and explicit discussion. They most often occur during presentational segments and offer a different form of explanation. The path is not predictable (notice the students developed the slightly problematic representational system) and the specific form of knowledge acquired is not always evenly shared by all participants. (Of course, in a more didactic presentation, it is not evenly shared either, but the clarity of the explanation is somewhat more even in that the presenter is more sophisticated in taking into account the audience [See Lampert, 1985].) This particular exchange, which will be analyzed below in some detail, took place on the first day of a series of lessons that involved graphing and functions. The teacher is Magdalene Lampert; the students are a fifth grade class.

The context of the episode is important. The agenda for the lesson was to begin to relate the function charts to graphs. This portion is, to some extent, a review of identifying the function rule and testing it. The segment was a usual opening for Lampert, in which she poses some type of problem for the students to consider and discuss. The routines in use were exchange ones; primarily she used call-on routines until the target meaning was clear and shared.

The task of focus was establishing the rules of correspondence in the function table (the rule was $3x + 1$). The instructions were to discover the rule, use it to complete the partial chart with their group (tables of four students each), and then to describe the rule without giving the rule away. After working through several items, Lampert posed the question of an input number of $1/4$. This was an interesting choice for several reasons.
First, it was a classically "illegal" problem. Texts and teachers tend to stay within certain boundaries when doing problems. When students are given problems that are way outside these boundaries, the problem similarities often disappear for the solver and both teacher and students have trouble seeing connections (see Leinhardt and Smith, 1985). By choosing a fraction as an input number, Lampert was also integrating information about fractions that had been brought up in lessons shortly prior to these lessons. But there was an additional twist. The other numbers in the input list got much larger through the multiplicative process, and the additive part--plus 1--increased the value of the output number only slightly more; the answer was in the multiplication. This was especially true in the number pair that preceded the 1/4 input (195/586). In changing the 1/4, however, adding 1 to the base does more than multiplying by 3. This meant that the observed difference between 1/4 and 1 and 3/4 was due primarily to the adding of 1 not the multiplying by 3. This is counterintuitive, and may be, in part, a source of the confusion that Ali had.

In the instructional dialogue the first segment that is shown in all caps indicates the "correct" answer that was given by Soochow, quite quickly; but in searching for the views held by other students Lampert uncovers an interesting hypothesis that Ali holds (second use of all caps). Ali seems to wander close to several very useful mathematical ideas which are not followed up. He starts with a clear effort at proportional reasoning: If 1 goes to 4 why can't we consider the 1/4 of a pie as one piece of pie? Then we would have four pieces of pie, or one whole. Then he seems to be saying, with Elsie's help, 1/4 is 3/4 less than one whole (still using the 1 to 4 rules) so the answer should be 3/4 less than 4/4 or 1 and 1/4. But in this he also is uncertain of how to operate with fractions to confirm
or disconfirm his answer. Lampert focuses on getting him and the other students to see why 3 times 1/4 is 3/4 and then on two different ways to think about adding the one whole or 4/4 to the 3/4.

Using an instructional dialogue for teaching is much more than getting students to talk in class. While Lampert accepts all students' ideas, values them, and directly teaches that students should value each other, she does not let just any answer stand. The posture is not "I'm ok, you're ok." It links quite tightly to a sense of the mathematical agenda. This means that not all individual lines of thinking are explored. In an instructional dialogue, because so much of the thinking is turned over to the child, the path is unpredictable. But there are still many points of choice. Both insight and pragmatics guide the choice.

If Lampert had followed Ali down the proportional reasoning path she would have had to make explicit the two-step process of the function rule and the distinction between an additive and multiplicative relationship or mapping, as the chosen rule contains two different steps. Ali, in focussing on only one step (one is three different from four), missed the point of the twostaged process (although he could do it, he just didn't know that that made a difference). Untangling that misconception directly might have been confusing to the rest of the students. It also would have taken a great deal of time. So the teacher, in conducting an instructional dialogue, must not only construct a setting in which the student can be heard and will give detailed enough dialogues so they can be analyzed, but must also have a finely tuned sense of the mathematical import of different paths and a sense of which is best to go down first.

Another point to be raised from the dialogue with Ali is that he had chosen the representation of a pie. This representation supports and may be
responsible for his misconception. In our language we distinguish between one pie and one piece of pie. We can count pieces of pies as whole objects or we can speak about them as fractions of the whole pie, in which case we drop the word piece (i.e. we refer then to one fourth of a pie; if we say one fourth of a piece we mean something different). As with all real and concrete referents for fractions, the base unit is what is slippery. In Ali’s misconception if one were referring to pieces of pie as the whole countable object then there would indeed be four pieces which would incidentally be equal to one whole pie. (Look at the protocol when Ali says “And then you could divide it into fourths FOUR PIECES) What is "incorrect" about Ali’s reasoning is that he assumes that the unit to be added to the product will be the same unit as the units being multiplied, just as they had been in all the other cases; but in this case they are not. Lampert’s dialogues characteristically do not directly correct a student but allow the student to self revise. However, since the unit base problem was not in the frame of reference for this student he would have had a hard time inventing it. It is also possible that Lampert, as she says in the dialogue, simply did not understand Ali’s reasoning on the fly and did not see a path ou.

It is important to note that these dialogues are very different from whole class tutorials which would result from an analysis of one-on-one Socratic exchanges (Collins & Stevens, 1982). Lampert in this exchange must balance both Ali’s intellectual needs with those of the entire class and the total picture of meaning that is on the table. Instructional dialogue is a more risky enterprise because the misconception, rather than the insight, may be the thing that sticks with the entire class. The outcome is an intellectual product that is both shared and completely personal. Part of the work space is shared but the knowledge gained is personal.
Instructional dialogues are routine only in their form; the substance is entirely analytic and unique.

**SPECULATIONS ON THE TEACHING OF TEACHING**

Learning to teach, unlike, for example, learning to ski or learning to project a vector in \( N \) dimensional space, is peculiarly difficult because that natural demarcation between what is known and what is not known is blurred. In skiing, if one cannot make a kick turn it is knowable on the flat and on the steep. (For the non-skier, a kick turn is a simple maneuver learned on the flat but whose main utility is in very tight and steep slopes --knowing whether one knows it is obvious). In contrast, if one cannot establish a safe learning environment for classroom discussion in which errors can be made and corrected, it may never be known because the goal for discussion will never be set. If one does not know how to manage an opening routine, the start of a lesson may seem lengthy, and the core problem not identified. ("These students are so immature; it takes them forever to get started.") Finally, as is often noted, one can fall back on what is perceived to be the routines used by one's own first grade teacher.

One reason then why it is difficult to learn to teach is because it is particularly hard to disentangle what is known from what is not known. Another reason that it is difficult to learn to teach is that multiple pieces of the skill must be accessed simultaneously and coherently, at some elementary level. The entire interwoven collection of knowledge salient to teaching must be broken apart into more manageable pieces while not losing coherence. But how should the task be approached?

This is comparable to the discussion at the beginning of the chapter on how something should be studied when that something is totally
integrated such that partitioning the task changes its nature in a profound way. One approach to dividing up the teaching task is to consider the common academic subdivisions of learning theory: subject-matter content, methods, and management. Another common separation is between generic teaching knowledge and subject-matter content knowledge, or between management and subject matter. Berliner (1988) has suggested that teachers should learn to manage before they learn to teach. But at the extremes, learning to manage smoothly with nothing to say is useless and learning to create brilliant, exciting learning activities is hopeless in a chaotic classroom. Or one might want to convince novices to behave like experts. The problem with this approach is twofold: knowing how an expert tends to behave does not help in getting someone to that point and, more importantly, in a context such as teaching, it will lead to a conservation and lack of appropriate innovation.

Still another approach would try to make use of the visible and naturally occurring boundaries of teaching: segments of lessons, types of lesson, or routines and the analysis of them. Such a division would build on the natural subdivisions in the tasks, and would have the advantage of recognizable context when the new teacher begins to teach (see Leinhardt, 1988b). Micro-teaching was of course just such an attempt; teaching teachers to handle smaller-sized (usually 6 to 8 students), more manageable classes. Micro-teaching at some level should most probably be included in any teacher education program. However, micro-teaching tends to mask the level of management demands on a teacher and falsely inflates the availability of time for analysis, reflection, and tutorial support. Regardless of the way the task is cut up (there may be an optimal slicing but as of yet the fact that each way of slicing has its own problems for
reassembly of the total task has not been addressed), the education of teachers needs to include some way of reconstructing the whole as it appears in reality.

Given a context of subject matter teaching, it might be useful to consider teaching new teachers analytic and teaching skills simultaneously. What follows is a speculation on how teaching teachers might proceed. It is based on a notion of starting with a master teacher and a real, intact classroom. Consider a five-staged process for learning to teach: observation, prediction, criticism, generation, and theoretical analysis (see Figure 4). This means student teachers would learn to observe classrooms (teachers, children, curriculum); they would learn to predict actions, given a sequence; they would learn to critique the components, identifying what works and does not in a particular circumstance; they would learn to generate alternative scenarios for portions of lessons; and finally, they would learn to analyze the parts of lessons from a variety of theoretical perspectives--annotating lessons with theory. One might teach student teachers to identify and select segments or aspects of lessons for focus and work through all stages (moving vertically down a column). For any one segment the teacher-to-be should be connected to the surrounding teaching activities by a master teacher until that part of the teaching is learned sufficiently for another piece to be taken on. This is in distinction to letting a student teacher work on an entire lesson separately from the teacher. To give an example, consider three segments of lessons in mathematics that might be chosen: review of prior class material or opening a lesson, supporting questions or guiding discussions (latter middle lesson), setting up homework or review (end). The image is one in which the student teacher, in coordination with a master teacher starts with one
part of a lesson in conjunction with a master and gradually takes over lesson parts and lesson types under the guidance of a university (or other external analyst) educator and the teacher. Figure 4 shows a matrix of student teacher actions crossed with examples of lesson components. The central unit of work for the student teachers then is the lesson.

The proposal is that student teachers become steeped in the context of the classroom, but not permitted to drown. Many aspects of analysis can be taken on simultaneously, essentially working both vertically and horizontally through the system. A student teacher could learn to observe multiple components of the master teacher's performance or could develop one component all the way through from the observation stage to generation. Ideally the student teachers could be exposed to several master lessons and several theoretical positions on the same topic so they would have the opportunity to contrast multiple gross and subtle details.

The pedagogical progression that seems to grow fairly well from the kind of analysis of the cognitive skills of teaching would introduce a student teacher to the entire classroom as a systematic observer. Other researchers have suggested this as well as suggesting that student teachers not simply look at teachers but have their attention focused on particular aspects of the teaching. Thus, in the framework of constructs that we have discussed, the student teachers would observe routines, agendas (plans), presentations (and other segments), explanations, representations, and dialogues about a specific piece of subject matter, as
they themselves were working on a particular aspect of a lesson. Actual transcriptions of lessons would be used as devices for focusing attention and inspecting the detail. The student teachers should come to understand the intricacy and trade-off of many of the moves of several teachers, and learn to borrow existing solutions until they are ready to work on those aspects themselves. After the student teacher is completely competent in observing/describing many of the elements in considerable detail, they might move to predictions.

Predictions refer to the student teacher's ability to accurately predict what will happen next. This would be easiest with strands such as routines. Video cameras could be stopped before low-level and simple routines are used and the student teacher could predict a known master teacher's next set of actions. Immediately, the prediction could be compared to what the known teacher actually does, by resuming the videotape. This prediction capability is analogous to predictions students should make in reading and math. Just as with reading, prediction of events teaches attention to the flow, rhythm and course of classroom activity. As routines for management, intellectual support, exchange, and learning get catalogued and known by the student teachers, the student teacher is taught to discuss. That is, the student teacher becomes a critic by describing alternatives or trying to improve what might be done. If the student teacher is actually going to go into a particular class after observing the regular teacher this careful analysis of routines alone will be very helpful, because s/he can build on what is there or modify it—but modify it knowingly, not in an accidental way. As with routines, the student teacher can use various forms of plans to predict how a teacher will actually execute them. As the student teacher moves closer to the time in which
s/he is ready to take over a class s/he should work collaboratively with a master teacher designing plans and executing them. The student teacher needs to do more than analyze and mimic a master teacher; generating their own components is central.

The following is a hypothetical example. The lesson is on graphing. Suppose the component that the student teacher selects to take on first is homework assignment and correction (column 3). This is a common activity and it requires close attention to what is supposed to be taught on a given day, so modifications must be planned for, and it is controllable outside of class. After learning how to observe all aspects of the primary teacher's teaching and to predict various actions, the student teacher begins to focus on when and how homework is assigned (routines of homework) and what homework consists of (analysis). S/he does all of the work assigned to students by the master teacher and notes what is problematic and powerful for the students as they do their work (critiquing homework). S/he may discover, for example, that the prototypical graphing homework assignment--find an example in the newspaper or on television--sounds better than it is. S/he may do some protocol work with one or two students as they work through their homework assignments out of class. S/he learns all of the routines surrounding homework--its assignment, discussion, and correction. Then s/he designs some assignments for the class, introduces them and corrects them. Perhaps s/he has students do one text-based problem and design an exercise of their own. The student teacher carefully modifies the tasks until they are well synchronized in spirit and difficulty with those of the lesson still being given by the teacher. After the homework is assigned, the student teacher should work through the assignment with a student and analyze it. Learning how to assign, correct,
and use homework, the student teacher is in a position to handle a common segment of most classes—assigning problems or seat work to be done either individually or in teams in class and supporting those activities. As this phase is reached the student teachers must return to analysis both of their own performance and of their performance in contrast to the prediction of what the cooperating teacher would have done. In analyzing the homework activity the student teacher would be pressed to examine routine, automatic behavior and analytic, reflective behavior.

The point of going so slowly and of moving from this particular part of the lesson would be to quickly focus the student teacher on the subject matter, the learner, and managing the task in creative but consistent ways. The example intentionally used parts of the teaching task that permit the clock to be slowed down a bit. In actual class presentations the clock flies, especially if there are management problems. As the student teacher takes on more and more of the space and ideas of the classroom there can and should be an increase in both flexibility and personal style. As the number of segments are increased and layered, the student teacher should return to analysis with another task, that of annotating lessons with theory. Activities that are goal driven and successful, such as explaining subtraction of fractions, should become sites for exploration of theory.

What does this lesson look like with respect to constructivism (Cobb, Yackel, & Wood, 1988) or direction instruction (Gersten, Carnine, & White, 1984)? This would attach theory to real and well understood practice and would permit a language of comparison and analysis of teaching to be developed. The objective is to empower the student teacher with a systematic way of talking and thinking about lessons while at the same time requiring them to stay connected with the totality of teaching.
students particular subject matter content. It takes seriously the role of a student teacher as a student but it also recognizes that because these students must work with children, an unchecked, bumbling, discovery mode of learning is not prudent. It attempts to teach a student teacher to account for both the actions and the intent of the actions of another teacher and then to apply that analytic capability to themselves. This supportive stance towards a student teacher would encourage rather than discourage the use of textbooks. The textbook would be seen as a natural and useful scaffold. Dependence on the text can be reduced first by learning to analyze it, then by critiquing it, then perhaps by using multiple texts, and finally, but not necessarily, by working without a text at all.

CONCLUSIONS

Teaching is complex and layered. The approach of studying or critiquing teaching from one aspect or another in isolation will not help us understand the phenomenon clearly nor will it help us improve teaching. However, such complexity cannot be handled all at once, so there is a dilemma. How to keep the totality in view while providing detail on one aspect is the challenge. One cut is to separate the grey, repetitive, continuous, and often overlooked, routine aspects from the colorful, unique, constructed, and innovative aspects. The challenge then is not to underestimate the role or significance of the routine, nor to overestimate the significance of the unique. The approach I have tried to take in my research is not to declare what should be, but to come to understand first what is and then why it is so.

The premise of this paper has been that teaching is worthy to be an object of systematic inquiry—that such inquiry will help to clarify and
potentially improve the character of teaching. It has also been assumed that a useful approach is to differentiate within teaching those aspects which can and usually are routinized from those which must be uniquely constructed from new arrangements of knowledge. It is acknowledged that the task of teaching is quite different when it is carried out under different models of teaching. The differences in tasks are reflected in the routine aspects, in the uniquely constructed aspects, and in the nature of the knowledge base required for action.

The study of teaching may be informative to those sciences of human thought and action as well. It represents an unusual location for displaying both the static organized knowledge bases and the action systems of productions and it displays them under conditions of change and response. In contrast, the physician diagnosing a patient or the aircraft mechanic inspecting a part does not have an instantaneous change to multiple patients or airplanes as a consequence of each move. Production systems for medicine and mechanics can be built that inspect knowledge systems slowly and systematically—productions for teaching are forged on line in more dynamic circumstances. Psychology as a discipline is struggling with the role of knowledge and understanding in use; teaching is an interesting case. I would also argue that the systematic inquiry about the nature of teaching is valuable and that it should not replace or be confounded with the continued efforts of educational reform based on the understandings of the nature of learning, as important as that is. Reform should be guided by an understanding of both the nature of learning and the nature of teaching; but efforts at reform should not be confused with efforts of investigation and efforts of investigation should not be confused with efforts at reform.
References


presented at the National Council for Teachers of Mathematics' Effective Mathematics Teaching Research Agenda Conference, Columbia, MO.


Kasunic, M.C. (1989). *Questions formation in the passé composé: How it is explained and how the explanation is received*. Unpublished manuscript, University of Pittsburgh, Pittsburgh, PA.


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List of Figures

Figure 1. Planning net for definitions.

Figure 2. Model of an explanation.

Figure 3. Semantic net of explanation given by Mr. Gene, Lesson 8: Graphing and Graph City.

Figure 4. Framework for teacher education.
PRESENTATION

Terms have been defined

Student states definition

Teacher states definition

Student is responder

Keep lesson moving

Call on student

Keep student interest

Is it correct?

Is time OK?

Is there another knowledgeable student?

If yes: goal is met

If yes: if yes

If not: if not

If yes: if not

If not: if not

Figure 1
Figure 2
Figure 3
## Lesson Components

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*Figure 4*