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ABSTRACT

Calls for the unification of mathematics education research have been made from many pedagogic perspectives, particularly with respect to the need for raising dynamic models that bridge the learning-teaching gap and that portray the ways that the meanings of specific mathematical procedures and concepts are constructed within individual classrooms. Such a model is presented in an attempt to examine a unifying theoretical representation of the intellectual processing that occurs between the identification of a task within a classroom milieu and the commencement of actions planned as a resolution of that task by the teacher and students. There exist three important assumptions made with respect to this model: (1) instruction as it occurs in the classroom is a social-cultural/anthropological process involving "negotiated tasks" that are attributed "community meaning" that is acquired through workings external to individual or idiosyncratic constructions of meaning and are derived from shared teacher-student and student-student discourse; (2) the teacher and the students come to that discourse with internally constructed "situated tasks" that are the result of dynamic interactions among individual attending, individual frameworks of knowledge, and individual systems of perception; and (3) aspects and interactions of both cognition and affect are significant to the establishment of situated tasks. (47 references) (JJK)

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THE MAKING OF MEANING: A MODEL FOR INTERPRETING MATHEMATICS EDUCATION RESEARCH

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INTRODUCTION

The body of research on the learning and teaching of mathematics has grown exponentially in the past twenty years. A majority of the theories and philosophies that have been developed to guide and sustain that research and to provide logical epistemologies for the analysis and synthesis of the results that research have been specific to well-defined domains within mathematics. In a relatively new area, as modern mathematics education research is, the reliance on domain specific theories, philosophies and models has been an efficacious and intellectual necessity. Now, however, we are at a point where the body of knowledge related to those domain specific areas has reached a critical mass that lends itself to an examination (and in some cases, a re-examination) of more global theories, philosophies and models that had to be tabled while the domain specific ones were developed and refined.

The model presented in this paper is one attempt at examining a unifying theoretical representation of the intellectual processing that occurs between the identification of a task within a classroom setting and the commencement of actions planned as a resolution of the task by the teacher and students. The model depicts the interactions among attending, encoding, decoding and negotiating, which are viewed as the primary processes used to "give meaning" to an identified task in a mathematics classroom.

The model is designed from an information processing interpretation of attending, encoding and decoding. However, the information processing approach taken is a "weak" one, using Cobb's (1990) definition of "weak," in that the claim made about the value of the computer metaphor for understanding mathematical cognition is a moderate or weak one. The philosophy underlying the development of the model is more "social" in nature, reflecting, as Cobb (1990) recommends, that students are actively engaged with teachers in a classroom community that includes negotiation. Both the teacher and the student are assumed to be engaged in constructing cognitive knowledge and affective perceptions through the use of verbal, figural, kinesthetic and other representations. There are several important assumptions made in relationship to the model: 1) Instruction as it occurs in the classroom is a social-cultural/anthropological process involving "negotiated tasks" that are attributed "community meaning" that is acquired through a process external to individual or idiosyncratic constructions of meaning and are derived from shared teacher-student and student-student discourse. 2) The teacher and the students come to that discourse with internally constructed "situated tasks" that are the result of dynamic interactions among individual attending, individual frameworks of knowledge, and individual systems of perceptions. 3) Aspects and interactions of cognition and what has commonly been identified as "affect" both are crucial to the formation of situated tasks.

LINKING TEACHING AND LEARNING

A call for the unification of different areas of mathematics education research has been made on many fronts. Romberg and Carpenter (1986) called for a rethinking of the school mathematics program in light of implications in two areas of research: inquiry on how students learn mathematics and inquiry on teaching. After careful reviews of the current state of mathematics instruction, recent developments on student learning and recent developments on teaching, Romberg and Carpenter (1986) identify several areas where researchers should concentrate their efforts in the next two decades. One of those areas is the construction of dynamic models that bridge the learning-teaching gap. Carpenter and Romberg make the case that these models should reflect the roles of change, learning, and growth and should depict the way meaning of specific mathematical tasks is constructed in classrooms.

The work on integrating research on the teaching and learning of mathematics has been one focus of the National Center for Research in Mathematical Sciences Education (Fennema, Carpenter, & Lamon, 1988). In a recent book from that group, Fennema and Carpenter (1988) present a general model for research and curriculum development for integrating cognitive and instructional science (see Figure 1). They describe the model as one that provides a promising new paradigm for the study of teaching and learning, and that:

> ...assigns a central role to teachers' and students' thinking. Classroom instruction is based on teachers' decisions and the effects of instruction on students' behaviors and learning are mediated by students' cognitions. ...teachers' decisions are presumed to be based on their knowledge and beliefs as well as their assessment of students' knowledge through their observation of students' behaviors. (pp. 8-9)

Thus, any global model related to research in mathematics education would profit from building on the initial work done in linking teaching and learning. However, other links must also be considered, particularly those that extend the definitions of cognition and that integrate information processing theory with affective and cultural theories.

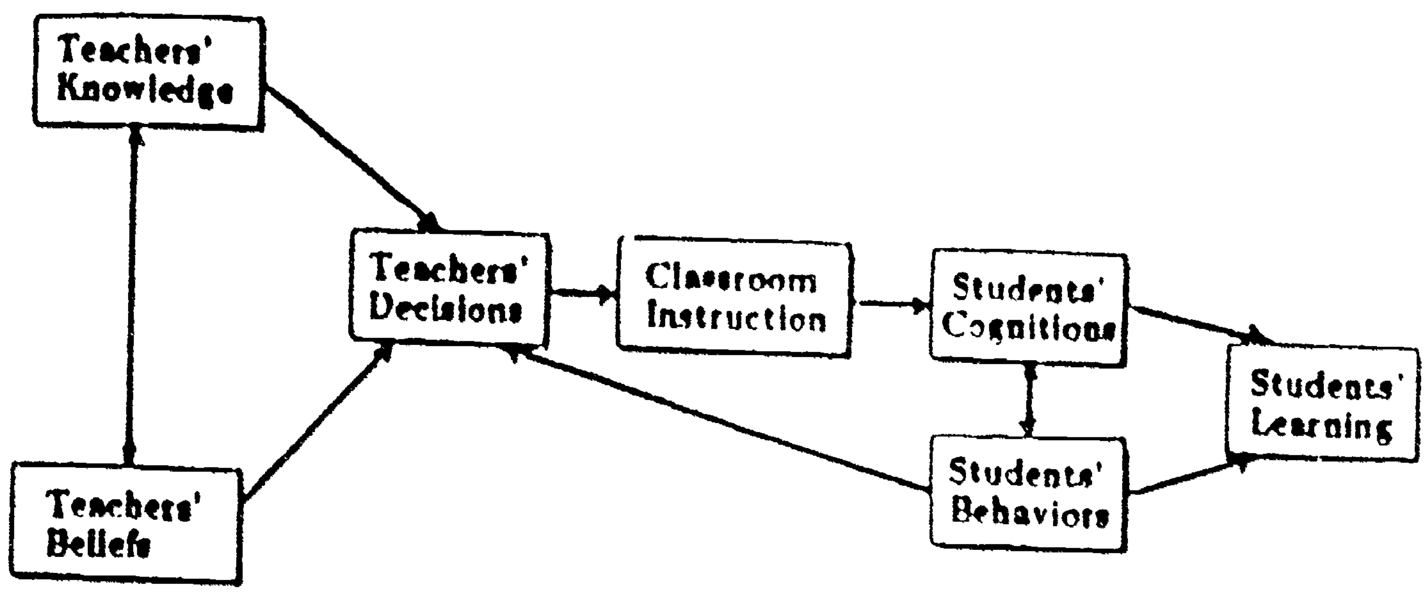

Figure 1. Model for research and curriculum development from Fennema, E. & Carpenter, T.P. (1988), p. 9.

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LINKING COGNITION, AFFECT AND CULTURE

While acknowledging the central role that information processing theories have played in enhancing the research on mathematics education, Greer and Verschaffel (1990) summarize the major criticisms directed at information processing theories: 1) emphasis on cognition that de-emphasizes affect, context, culture and history; 2) emphasis on symbol manipulation that de-emphasizes meaning; and 3) emphasis on precision of description that devalues knowledge or thinking that cannot be symbolically or precisely represented.

McLeod (1990) and McLeod and Adams (1989) provide thorough reviews of the need for integrating affect and culture with cognition in mathematics education research. McLeod (1988, 1990) suggests that Mandler's application of an information-processing linked theory of affect to mathematics education (Mandler, 1984, 1989) provides an essential basis for describing children's emotional reactions to mathematical problem solving. Mandler's (1989) application centers on the effect of an interruption or discrepancy in the completion of a schema-activated action sequence. McLeod (1990) demonstrates how effective this theory is in describing children's actions on a "chickens and pigs" problem (If there are x chickens and pigs altogether and y legs, how many of the animals are pigs?).

However, much of this description and much of Mandler's theory focuses on actions that occur after the children have "understood" or assigned meaning to a situation. Systematic analysis of the interaction of affect, culture and cognition during the formation of meaning prior to the initiation of an action sequence may advance the application of Mandler's theory.

Brown, Collins and Duguid (1989) make a strong case for viewing learning and cognition as fundamentally situated in the activity and context in which learning takes place. Lave (1988), Saxe and Gearhart (1988), and others also make a strong case for the role that culture and situation play in the application of mathematics as well as the learning of mathematics. Thus meanings for concepts, procedures and situations are partly the result of the activity, context and culture in which the learning takes place. Greer and Verschaffel (1990) support the need to make context and culture a major part of explaining learning; however, they encourage keeping the information processing theory as well. They state, "maximising the realizable value of cognitive research in educational terms depends crucially on multi-disciplinary partnership." (p. 8) It seems, then, that the development and verification of models linking aspects of information processing theories with aspects of situated cognition theories could benefit researchers by providing the means for a combined application of successful but competing viewpoints for understanding how children learn mathematics.

MAKING OF MEANING MODEL

The proposed "making of meaning" model for interpreting mathematics education research, see Figure 2, was constructed using a template designed to account for links between teaching and learning and for links among cognition, affect and culture. The model reflects the idea that the chief protagonists in linking the teaching and learning processes are the teacher and the student. Thus the models contains the external and internal

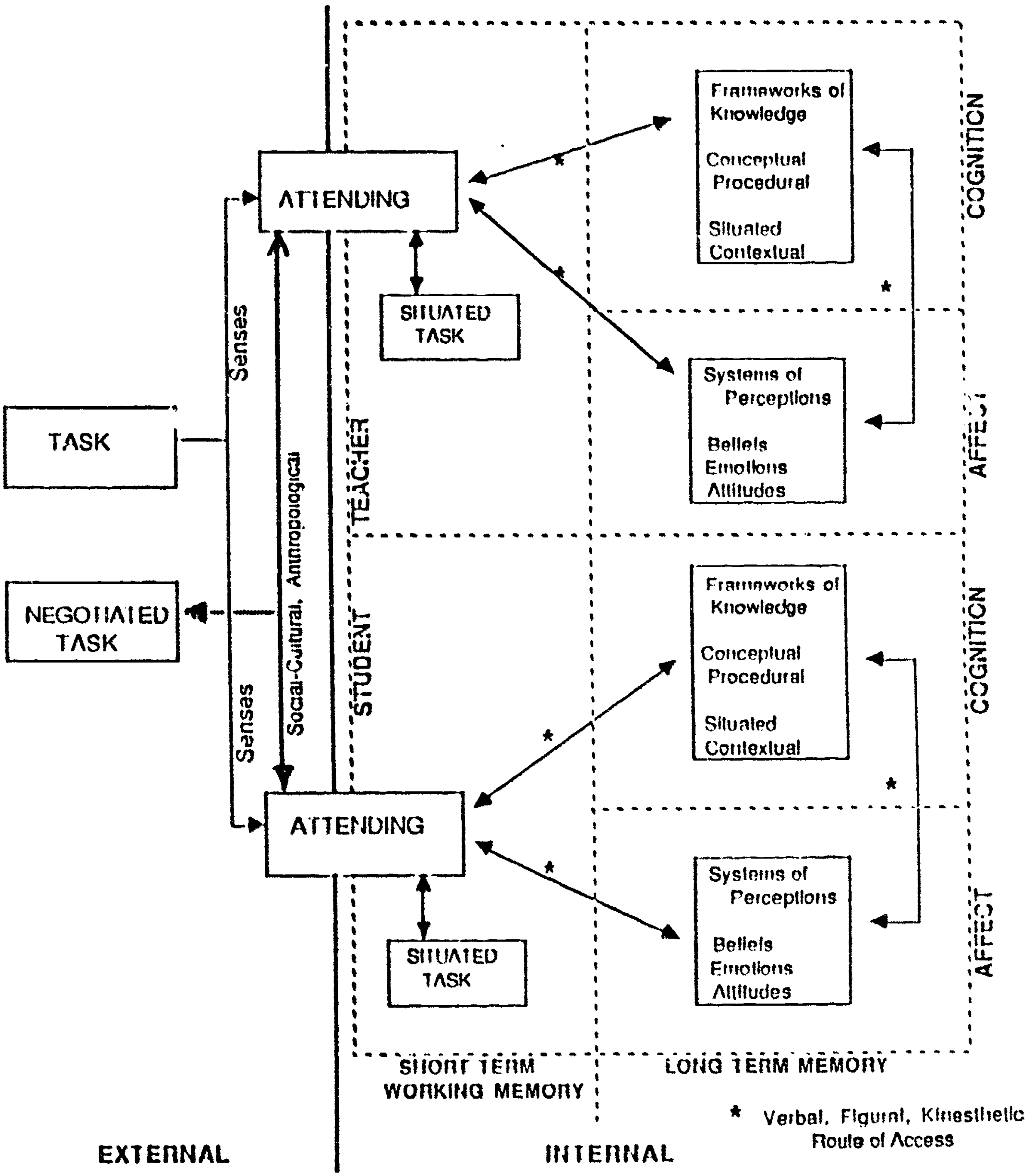

Figure 2. Making of meaning model for interpretation of mathematics education research.

processing arenas for both the teacher and the students. The student processing arenas are represented as a single student, but should be thought of as a stack, one arena for each member of the class. Thus, attending to the TASK, constructing an internal individual SITUATED TASK, and constructing a NEGOTIATED TASK occur not just between two protagonists, but many. The gateway between the internal and external arenas is the attending process. The external arena is the milieux of the classroom and the internal arena is working memory space and long-term memory.

The notion of the internal and external arenas and of an internal SITUATED TASK is similar to that suggested by Sowder (1985) in his discussion of what psychology has to offer with respect to encoding mathematical tasks. His diagrams (see Figures 3 and 4) for solution sequences for routine and non-routine problems depict external and internal processing, as well as external and internal versions of tasks.

TASK VARIABLES

The entry point into the model is the TASK node in the external arena. Within a classroom a task is considered, whether presented by the teacher, a text or a student. The TASK has associated with it a set of variables. The number and forms of these variables are not fixed, but are open to interpretation by the human players in the classroom through their physical and cognitive processes of attending, decoding and encoding. Although these variables are not fixed, they may be classified according to conventional systems.

In his hierarchy of task variables, Kulm (1984) links content, context and syntax variables with the students' surface and semantic analyses performed during the understanding-the-problem stage of problem solving,

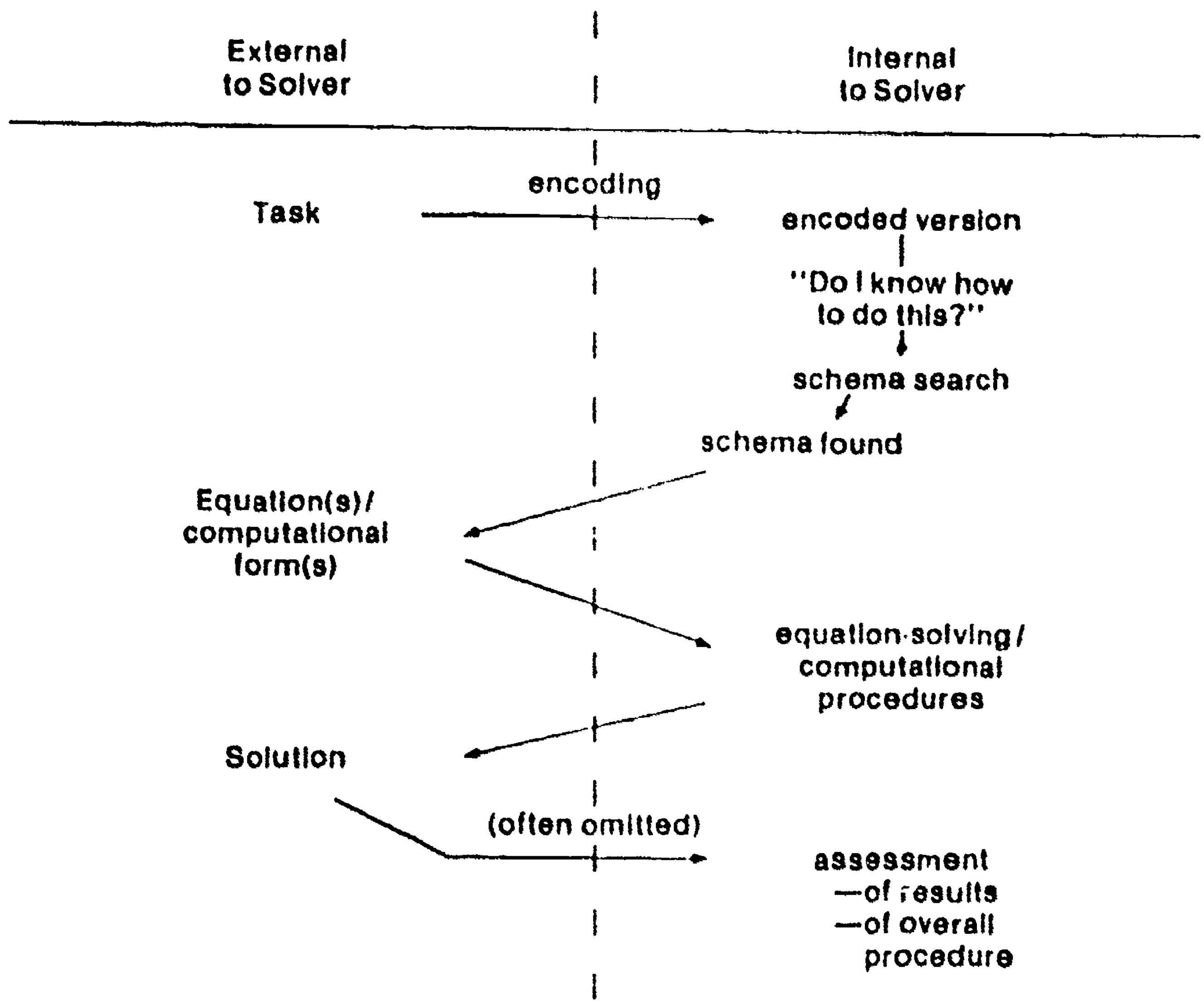

Figure 3. Simplified solution sequence for routine problems or exercises from Sowder (1985), p. 141.

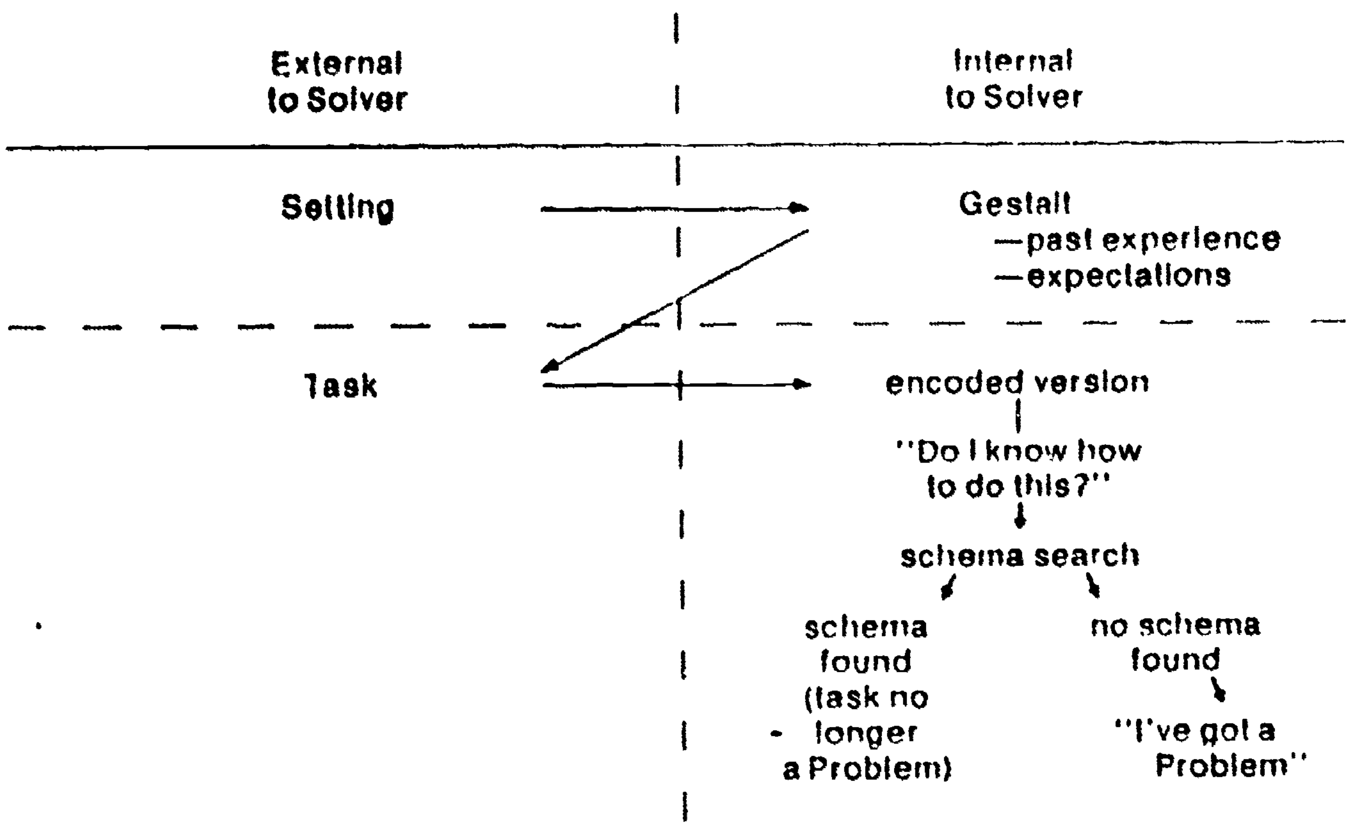

Figure 4. Sequence illustrating the distinction between a routine problem and a genuine problem from Sowder (1985), p. 142.

which may be characterized as the stage in which students begin to make a task or situation meaningful. Syntax variables are defined as "those variables describing the arrangement of and relationships among words and symbols in a problem" (Kulm, 1984, p. 16). For the purposes of this paper, the words and symbols themselves, separate from any connotation or denotation they may have when examined together, are considered to be syntax variables. Other syntax variables include those listed by Barnett (1984): length variables, grammatical complexity variables, numeral and symbol forms, question sentence position and form, and sequence variables.

Kulm's definitions of content and context variables also hold for this paper, with a few major additions. Content variables are those that refer to the mathematical meanings of a situation such as content area (or domain), type of operation involved and the mathematical complexity of the problem. Context variables are the non-mathematical or incidental meanings, described by Webb (1984) as those that refer to the form of the problem (problem embodiment or representation, verbal context or setting, and information format). However, as has been shown in studies of students' beliefs about the nature of mathematics (Kouba & McDonald, 1991), the class of context variables must also include where and when the concepts or processes depicted in a situation were learned, practiced and used. Thus, tasks have both surface and semantic cultural variables. Also, another set of context variables that accompany the task variables through the ATTENDING gateway are the environmental variables that have been shown to affect the attending process (such as noise level, temperature of the room, time of day, distractions outside the windows, comfort of the chair, arrangement of the chairs, number and identity of people in the room, etc.)

ATTENDING AND THE INTERNAL ARENA

The TASK is "sensed" by the teacher and the students. That is, the teacher and the student selectively attend to the TASK variables. The actual ATTENDING process works as described in most information processing literature. Information is attended to, taken in, decoded, rearranged and encoded, and stored in some form in the short-term portion of working memory. All of these processes are regulated by some type of decision-making and/or valuing mechanism, which in turn is influenced by the existing nature of the person's long-term memory. The model depicts only the underlying influencing interactions: the ATTENDING process is mediated by existing FRAMEWORKS OF KNOWLEDGE and existing SYSTEMS OF PERCEPTION.

To reiterate, FRAMEWORKS OF KNOWLEDGE and SYSTEMS OF PERCEPTION influence and indirectly control what the teacher and student attend to in a TASK; how the variables related to the TASK are represented and given meaning during the decoding, rearranging and encoding; and how the TASK gets stored as a SITUATED TASK. Thus, the attending, encoding and decoding processes as influenced by frameworks of knowledge and systems of perception are the means by which TASKS become SITUATED TASKS.

Frameworks of Knowledge. Although both the teacher and the student have the same general categories of frameworks of knowledge and systems of perceptions, there are differences. Teachers have FRAMEWORKS OF KNOWLEDGE that consist of formal and informal knowledge about:

a) the content of mathematics,

b) learning,

c) teaching (pedagogy),

d) other content areas (which often serve as contexts for mathematical situations).

e) beliefs about the nature of mathematics, and

f) schooling as a social endeavor.

Students have formal knowledge about the content of mathematics and about other content areas. They have informal knowledge about beliefs about the nature of mathematics and about schooling as a social endeavor. If students have knowledge about teaching and learning, it, too, is at an informal level.

Formal knowledge is that which has been constructed as part of systematic study, usually in schooling situations. Formal knowledge closely approximates a social consensus of the structure of that particular area of study. For example, formal conceptual and procedural knowledge about mathematics approximates the structure presented in texts and the structures envisioned by mathematicians.

As an aside, one could engage in a philosophical discussion over which of the task variables constitute the formal mathematical "truth" or "reality" for a given situation or a given set of concepts and procedures. However, the position taken in this paper is that there is no external "truth" that constitutes the content of mathematics or the content of a situation. Mathematics has negotiated conventions and negotiated base postulates that then are processed by the prevailing logic system (again, a negotiated system). Thus, formal knowledge is knowledge that approximates the negotiated consensus. For example, the areas of additive, multiplicative and algebra structures are not "external truth," but are negotiated systems drawn from the internally situated knowledge of those who engage in the negotiation process. For the most part, "additive structures" have a stable negotiated delineation. "Multiplicative structures,", on the other hand, are still being negotiated. Several groups of researchers have differing viewpoints about the nature of multiplicative structures. Negotiation is taking place in the mathematics education literature as each group explains the internal processing and logic that supports their interpretation. As addressed in a later section of this paper, this making of meaning model may prove useful in examining and furthering that negotiating process by providing a means to juxtapose and integrate the differing perspectives.

Informal knowledge has been acquired through individual experiences. It may be systematic in nature, but it has not been acquired through an organized program of study.

Knowledge about the content of mathematics has been examined by Hiebert and Lefevre (1986), who have provided definitions and a careful analysis of CONCEPTUAL and PROCEDURAL knowledge in mathematics. Briefly, they define conceptual knowledge as a connected web of knowledge rich in relationships, "a network in which the linking relationships are as prominent as the discrete pieces of information" (Hiebert & Lefevre, 1986, pp. 3-4). They identify two levels (a continuum, really) of conceptual knowledge: primary, which is tied to context and therefore not very abstract; and reflective, which is less tied to specific contexts and more abstract than the primary level. Procedural knowledge they define as consisting of two parts:

> One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists

of the algorithms, or rules, for completing mathematical tasks. (Hiebert & Lefevre, 1986, p. 6)

Although these definitions were designed for conceptual and procedural knowledge in mathematics, I believe they are reflective enough in nature to generalize to other areas of formal knowledge that teachers possess. Conceptual and procedural knowledge about learning, teaching, and other content areas can be identified. For example, researchers looking at the interaction of learning and teaching in the mathematics classroom (Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C. & Loef, M., 1989; Cobb, P., Yackel, E., & Wood, T., 1988) suggest that teachers should possess the following knowledge about student learning: knowledge of content domain, knowledge of problem difficulty, knowledge of distinctions among problems that result in different processes of solution, and understanding of the stages students pass through in acquiring concepts and processes in a domain. These areas of knowledge may be classified as conceptual knowledge. Likewise, the following, which were also suggested as good requisite knowledge for teachers, may be classified as primarily procedural knowledge (although all of them also have components of conceptual knowledge): knowledge of ways to assess students' knowledge in the domain, knowledge of the processes used to solve different problems at each stage of acquisition of concepts, understanding of the nature of the knowledge that underlies these processes, knowledge of the classroom discourse procedures that promote the development of different solution processes, and knowledge of the social procedures and interactions of classroom teaching.

CONCEPTUAL and PROCEDURAL knowledge also may serve as adequate initial categories for knowledge about other subject areas such as science, art, and social studies, at least in terms of how those subjects are integrated with mathematics. However, teachers and students must also possess knowledge about those areas as contexts for the application of mathematics. More will be said about this in a moment.

Items (e) and (f), beliefs about the nature of mathematics and schooling as a social endeavor, differ from the other areas of teacher and student knowledge considered thus far. While the areas have conceptual and procedural aspects, they are, in essence, the context in which all of the other areas of knowledge are considered. Furthermore, beliefs about the roles of the teacher and the students and about the process of schooling are also part of knowledge about schooling as a social endeavor. Thus, teachers and students must possess CONTEXTUAL knowledge about beliefs, schooling and other subject areas.

As with CONCEPTUAL knowledge, it is useful to distinguish between two levels of CONTEXTUAL knowledge, a primary level and a reflective level. The primary level of conceptual knowledge (that tied to contexts) may be considered a sub-category of the primary level of CONTEXTUAL knowledge, because the primary level of conceptual knowledge is limited to mathematics topics as contexts. Hiebert and Lefevre (1986) use the primary relationship of linking two pieces of information about decimal numbers as an example. The primary level of CONTEXTUAL knowledge includes those links as well as links between a mathematical concept and an application of that concept or a belief about that concept.

The reflective level of CONTEXTUAL knowledge is the abstract recognition of patterns within contexts. It is the realization that mathematics may be decontextualized or generalized. The ability to operate at the reflective level of CONTEXTUAL knowledge may be a prerequisite for developing all reflective conceptual knowledge, most primary conceptual knowledge and any meaningful procedural knowledge. To elaborate, Hiebert and Lefevre (1986) seem to treat conceptual and procedural knowledge as knowledge decontextualized from situations and culture. However, if Brown, Collins and Duguid (1989) are correct, the majority of the content knowledge held by some teachers and most students is embedded in the learning contexts, the belief systems and the cultures of the person. Thus, much of the conceptual and procedural knowledge is constructed and stored as SITUATED KNOWLEDGE. SITUATED KNOWLEDGE may be defined, then, as conceptual and procedural knowledge that has not been decontextualized, but remains integrally a part of the situation, beliefs and culture in which it was learned.

A person who has a well-developed reflective level of contextual knowledge knows how, why and to what extent conceptual and procedural knowledge are embedded in situated knowledge. He or she knows about the role of context and culture in learning and knows what aspects of meaning are lost or altered when knowledge is generalized to the abstract levels of conceptual and procedural knowledge. This person knows, too, the kind and extent of the transformations that may take place in meaning when the abstract conceptual or procedural knowledge is reapplied in new situations, that is, "recontextualized."

Perhaps it is reflective contextual knowledge that Kaput (1987a) is calling for in his recent remarks about the need for students to possess knowledge about representations. Reflective contextual knowledge about representations may be the key in addressing the failure of students to cross between symbol and referent, a failure that Kaput (1987a) points out leads to failure to estimate or to have a sense of the magnitude of a calculation. Kaput laments that mathematics educators and researchers have given little attention to teaching how various mathematical and nonmathematical representation systems work and how they relate to one another. Perhaps mathematics educators and researchers have not attended to those issues because there existed no theoretical framework for helping to make sense of the situation. Kaput (1987a, 1987b) lays the groundwork for such a theory, which when developed may fit into a larger theory for "making meaning" as an aspect of reflective contextual knowledge.

As a summary for this section on FRAMEWORKS OF KNOWLEDGE, a more detailed diagram for the portion of the model is offered (see Figure 5). ATTENDING links directly to SITUATED KNOWLEDGE, which may be transformed through the use of CONTEXTUAL KNOWLEDGE into CONCEPTUAL and PROCEDURAL KNOWLEDGE, and vice versa.

Systems of Perception. In humans, affect, in its broadest definition, is a mediation process on cognition. Cognition does not occur without affect (Mandler, 1989; McLeod, 1989, 1990). McLeod (1990) and Hart (1989), in syntheses of the affect domain, identify beliefs, attitudes and emotions as constructions (with "cold" to "hot" valuing or arousal properties) that influence the construction of knowledge and are part of that knowledge. Hart (1989) provides useful definitions for beliefs, attitudes, emotions and affect:

> Note that instead of referring to affective variables, affect, or attitudes towards mathematics, I am now using the terms beliefs, attitudes, and emotions, which, for me, lessens the confusion associated with the meanings of affective variables, affect, and attitudes toward mathematics. Here, belief is used . . . to reflect certain types of judgments about sets of concepts. . . .

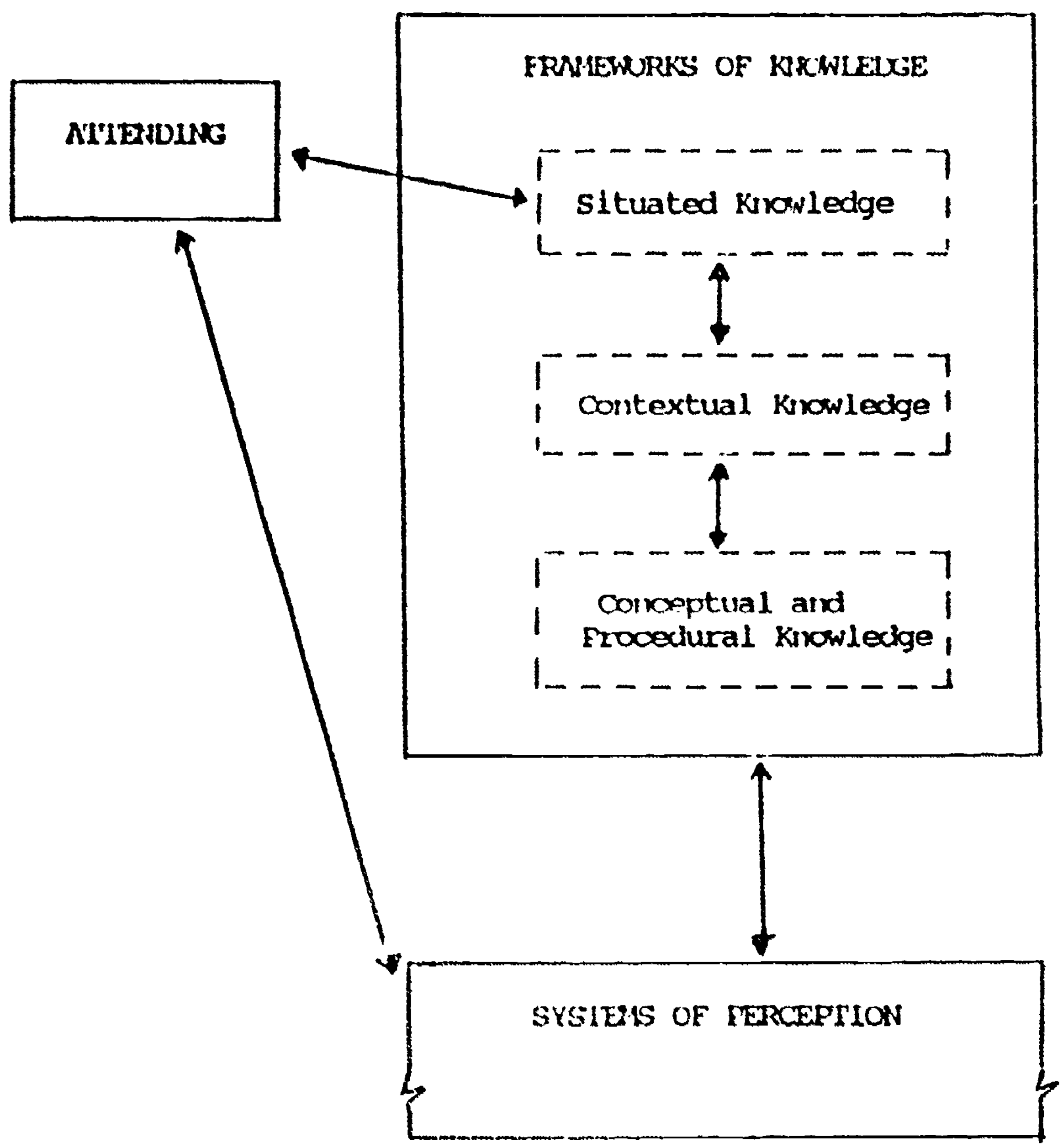

Figure 5. Detailed diagram of FRAMEWORKS OF KNOWLEDGE portion of the Making of Meaning Model.

> I use attitude toward an object to refer to emotional reactions to the object, behavior towards the object, and beliefs about the object. Emotion is used here . . . to represent a hot gut-level reaction. Affect is used only as a synonym of emotion; many educators use the term in a more general sense. (p. 44)

McLeod (1990) makes some further distinctions among beliefs, attitudes and emotions, "So, the terms beliefs, attitudes, and emotions are listed in order of increasing affective involvement, decreasing cognitive involvement, increasing intensity, and decreasing stability." (p. 17)

In the model presented in this paper, SYSTEMS OF PERCEPTION has been deliberately chosen as the label for this area of concern. Perceptions are defined in most dictionaries as cognition, as products of the process of perceiving, and as recognitions of moral or aesthetic qualities. Systems of perception are constructed in conjunction with and in symbiotic exchange with frameworks of knowledge. In theory and for the ease of discussion, we can separate the two, but in actual human functioning they cannot be separated.

The systems of beliefs that are included in SYSTEMS OF PERCEPTION differ from the beliefs about the nature of mathematics mentioned in the section on frameworks of knowledge in the extent of valuing, the type of judgment made, and the object of the belief. BELIEF in SYSTEMS OF PERCEPTION refers to the beliefs one holds about oneself and beliefs that carry a "good" or "bad" connotation. A belief about the nature of mathematics that has little valuing judgment attached is that mathematics is only concerned with numbers and operations. A belief that has a valuing judgment and that therefore would be part of SYSTEMS OF PERCEPTION is that mathematics is for "nerds" who are social outcasts.

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McLeod and Adams', (1989) book on affective issues and problem solving is an excellent summary of research and construction of SYSTEMS OF PERCEPTION. It may be helpful, however, to highlight some of the major areas within SYSTEMS OF PERCEPTION that influence the formation of meaning. Two areas of BELIEFS that have been studied in depth are confidence, especially in relationship to gender differences (Meyer & Fennema, 1988; Reyes, 1984), and causal attribution theory (Dweck, 1986; Heckhausen, 1987), which explores the reasons children have for explaining their success and failure. Areas related to attitude that have received recent attention are motivation and the setting of goals. And, an area within emotion that has traditionally received heavy attention in mathematics education is anxiety. Specific examples of the interaction of these areas will be considered in the next section.

Situated Tasks. Situated tasks are the products of the interactions among attending, frameworks of knowledge and systems of perceptions as both the teacher and the students assign meaning to the initial TASK. The assignment of meaning to the initial task is akin to "making sense," which von Glasersfeld (1987) reminds us, in a constructivist epistemology, relies on the use of material (in this case, frameworks of knowledge and systems of perception) that a person already has.

To clarify, re-examine Sowder's (1985) diagrams for solution sequences (Figures 3 and 4). In the internal portion of his diagrams an encoded version of the task is followed by questions that elicit schema searches. Prior to this kind of search, however, a schema search to assign meaning must occur as a means for encoding a version of the task. Thus, during and after the intake of task variables, the internal processing proceeds as though the following kinds of questions were asked: Do I know anything about these variables? Do I recognize these variables? What do these things mean? What do I know, believe or feel that can help me make sense of this and put it all together? Schema that can provide answers to these questions are sought.

Because this model of making meaning is built on constructivist assumptions (von Glasersfeld, 1987), this making sense involves finding a way of fitting available schema into a pattern that is circumscribed by the specific constraints of the task variables, the environmental variables, and the rest of the frameworks of knowledge and systems of perception residing in long-term memory. A key question at this point is how the seeking is done. What is the means of communication? In the model the routes of access among attending, frameworks of knowledge and systems of perceptions are verbal, figural, kinesthetic and other means of representation. An important point to remember in understanding the nature of the process of situating a task is that although the teacher and the students theoretically have access to the same TASK variables, the representations chosen to internally represent those variables need not be an exact match to any external definitions or connotations. To be adequate representations, they need only be compatible and not clash with the sense of the TASK or with the other classroom protagonists' expectations. (Inadequate representations, misrepresentations, or misconceptions are not addressed in this paper, but could be addressed by the model, and may need to be addressed in future applications and modifications of the model.)

Some examples may help to clarify how the teacher and students use the interactions among attending, frameworks of knowledge, and systems of perception to form situated tasks. The first example is drawn from individual interviews with first through third graders who were asked to solve multiplicative word problems (Kouba, 1990). For a problem involving how 6 horses could fairly share 24 apples, three children were adamant that each horse should get just one apple. These children recognized the problem as one involving grouping, but accepted that there would be "a lot" of apples left over. They also were able to solve correctly a similarly structured problem about people sharing tomatoes. All the evidence showed that they had attended to a division structure and had some schema for making sense of division problems that involved sharing. However, further probes during the interviews revealed that they had attended to the context of horses, which had activated a schema related to their experience with horses. The children believed that it would be unhealthy for horses to eat more than one apple, thus each horse should get just one apple. These children had situated the task in their experience (all were from a rural home setting) and the answer they gave was correct for the SITUATED TASK they were responding to. The children's meaning for the task did not conflict with the task. Nor did it appear to conflict with the expectations of the other protagonist in the situation, for the interviewer accepted all answers without indicating that the children should think of a problem only in terms of the numbers and mathematical operations. The children probably were not cued by the setting into applying "mathematics classroom" actions or schema because the interview was conducted in a separate classroom.

The second example is from a similar study of young children (Kouba, 1989), in which a third grader was asked to respond to a task similar to the apples-and-horses task (finding how many elements in group when given the total and the number of groups). The child responded, "Oh, that's making groups and that's division." She evidently used existing knowledge frameworks constructed from prior experience with formal instruction to give meaning to the situation. However, she also used some systems of perception in the form of beliefs when she went on to say, "We haven't learned division yet, so I can't do this." She evidently held the belief that she could do only what had been formally taught. She would not attempt the problem, even though she had physical materials available for a trial-and-error approach and even though she had solved correctly other division problems that involved finding how many groups when given the total and the number of elements in a group. She did not refer to these latter problems as "division," and thus, appeared to have situated them differently.

Many other examples are present in the research literature, especially in children's problem solving and in examination of students' and teachers' beliefs and attitudes (Kouba & McDonald, in press; Nesher, 1988; Ohlsson, 1988; Schoenfeld, 1988; Thompson, 1988).

NEGOTIATED TASK AND THE EXTERNAL ARENA

The external arena is the classroom, for this model. Yackel, Cobb, Wood, Wheatley and Merkel (1990) in their chapter on the importance of social interaction, describe classrooms in which student-teacher and student-student interactions promote the development of personally meaningful solutions to tasks. For this to take place, however, some negotiation or discourse must occur so that the teachers and students have a community understanding about the nature of the tasks and the nature of the classroom as a place for interaction.

One of the teacher's roles in the negotiation is to work towards a generalized NEGOTIATED TASK that allows for the possibility that a range of individually situated tasks will "fit" the negotiated task.

Classroom negotiation, which should result in a NEGOTIATED TASK, is also the time and place for both the students and teachers to check on the viability of their situated tasks. In this sense, the teacher and the students are communicators in a common context and function as von Glasersfeld (1987) describes. That is to say, for satisfactory communication to occur, the communicators' individual SITUATED TASKS need be compatible enough that they do not clash with the other communicators' expectations. If clashes are apparent, then negotiation should occur. It should be noted that negotiation means that the teacher is as willing as the students to reexamine his or her meaning or situating of a task. The assumption is made that SITUATED TASKS are not immutable. Refocusing or re-attending to a different set of task variables would result in an altered SITUATED TASK that would still be constructed by the individual.

Also, because the teacher is the only adult protagonist involved, it is the teacher's responsibility to know enough about the internal process of constructing situated tasks, enough about the underlying structures of the mathematics involved, and enough about communication and the social context of the classroom to help all protagonists (self included) re-situate a task internally and check that there is a consistent organization for that situating, which, according to von Glasersfeld (1987) would lead to "understanding."

Not enough research has been done to provide a full range of descriptions for negotiation or discourse in the mathematics classroom. However, initial studies on the use of discourse, especially in small group work are providing some direction (Cobb, P., Yackel, E., & Wood, T., 1988; Lampert, 1986; Lemoyne & Tremblay, 1986; Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C. & Loef, M., 1989; Resnick & Ford, 1981; Yackel, Cobb, Wood, Wheatley and Merkel, 1990). As summarized in Kouba and Franklin (in press) some common features for the negotiating discourse include discourse that is: centered on child-generated actions and ideas with the children as main participants; facilitated by the teacher so that all ideas, even incorrect ones are critically examined by all participants; linked to previous experiences with physical and pictorial models as consensus is negotiated so that no one type of representation mode dominates; and comprehensive in the formal and informal language used so that the consensus is not tied to any one verbal representation. Much still needs to be done in this area.

RESEARCH ON SIMPLE MULTIPLICATIVE STRUCTURES

The model presented in this paper may be used as a means for analyzing aspects of the major theoretical approaches in the research on children's processing of multiplicative situations.

Nesher (1988) three ~~four~~ major theoretical approaches in the analysis of simple multiplicative structures. Two emphasize mathematical constructs and are exemplified by the work of Vergnaud (1983, 1988), Schwartz (1979, 1988). Vergnaud's and Schwartz's approaches are also classified as "dimensional analyses" because they are concerned with the relationship between the number and the nature of a quantity (a number and its label or unit). The third major approach is a textual analysis focused on the propositional structure of word problems. Nesher's work (Nesher, 1988; Peled & Nesher, 1988) exemplify this approach.

All three approaches depict children attending to number and word task variables in a multiplicative situation (usually a word problem). In this respect, all three approaches have a component that links CONCEPTUAL and PROCEDURAL FRAMEWORKS OF KNOWLEDGE to the meaning that children assign multiplicative situations. The nature of and the extent of the emphasis on the conceptual and procedural knowledge differ, however.

Vergnaud's (1983, 1988) analysis, diagramed as four quantities related in both a scalar and functional direction between and across two measure spaces, assumes that children have a CONTEXTUAL BELIEF about the nature of mathematics that appears to value the abstract above the context. This valuing of the abstract is demonstrated by the penchant for representing relationships within and subcategories of multiplicative structures as numbers in pure symbol notation (scalar and function) freed from the constraints of the units attached to the numbers. In essence, the nature of the quantities is controlled. The dimensional analysis is a means for moving numbers as efficiently as possible to the most abstract, generalized spaces of CONCEPTUAL AND PROCEDURAL KNOWLEDGE. This is done with little attention to the issues about understanding that have been raised previously in the discussion of the reflective level of CONTEXTUAL KNOWLEDGE.

Schwartz's (1979, 1988) analysis also is centered in CONCEPTUAL and PROCEDURAL FRAMEWORKS OF KNOWLEDGE, but has more CONTEXTUAL BELIEFS components that may be attributed to those valued in science, and thus is

SITUATED in science. The nature of the quantities, rather than being controlled, is used as the focus for categories. The labels or units for quantities remain linked to the quantities and rate relationships between quantities are treated as entities that may be perceived differently. For example miles-per-hour, although of the same category as candies-per-bag, may be perceived differently because it can be thought of as a single entity, "speed." In this sense, Schwartz's analysis is less abstract than Vergnaud's and addresses some of the representational understandings that are a part of reflective CONTEXTUAL knowledge. Schwartz's analysis may be viewed as one means for describing the effect of situating mathematical structures (whose most abstract natures may be aptly portrayed by Vergnaud's analysis) within the social conventions of science.

Nesher's (Nesher, 1988; Peled & Nesher, 1988) analysis moves farther away from abstraction and describes a modification phase that is more situated and contextual that Schwartz's, but also accounts for the influence of language in more detail than a dimensional-analysis-of-units level. In Nesher's analysis, the textual propositions, which are language dependent situational relationships among the quantities and quantity/unit pairs, must be transformed at the reflective level of contextual knowledge into conceptual and procedural knowledge.

Thus it seems that the making-of-meaning model provides a means for viewing theories about multiplicative structures as successive or possible alterations along or within a complex level-of-abstraction scaffold. The model may also help to incorporate other research on multiplicative structures. In particular, the work of Fischbein, Deri, Nello, and Marino (1985), on hypothesized primitive, intuitive models and the related work of

Bell, Greer, Grimson and Mangan (1989) on a competing claims theory for choices in giving meaning to and solving multiplicative problems should be examined.

As research helps to clarify the relationships among attending and systems of perception, the multiplicative theories should be re-examined and expanded to compensate for some of their current drawbacks. The existing theories assume consistency of affect over time, consistency of goal and motivation, and consistency of self-confidence. Without these considerations, the theories will hold for generalized result but will not be useful for looking at small classroom groups or individuals.

Likewise, this model for the making of meaning should be re-examined, modified, and refined in light of continued dialogue about how people make meaning. In particular, the negotiating processes and the interactions along the social-cultural/ anthropological routes of access between tasks and attending need to be expanded and clarified. This area could benefit from further consideration of discourse theories, the social construction of meaning and knowledge, and further research on teacher decision-making.

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