A search for patterns in grade retention rates across grades 1-12 to construct simulation research models is described in this paper. Aggregate data from Shepard and Smith (1989) on grade retention rates from 12 American states for the year 1979-80 and from 11 states for 1985-86 were examined for across-grade patterns, supplemented by Dade County School District data for 4 consecutive years, 1982-83 through 1985-86. First, the elementary level pattern is explored. A process to explain the pattern is described and a mathematical function derived from the explanation is proposed to fit the data. Next, a pattern is sought for the highest magnitudes of retention rates, and the elementary grade function is extended to provide a proposed comprehensive fit. Fluctuations in the states' data are accounted for, enhancing the model's credibility. Finally, median summaries of the data by grade are presented and compared to an across-grade model. Findings are as follows: (1) the pattern of the rate magnitude distribution in grades 1-6 suggests a negative exponential function as the target population is depleted across the grades; (2) the retention rate magnitudes show a pronounced tendency to peak at the beginning of each grade level; (3) this finding is supported by shifts in the magnitude of the peaks following the move from a junior high to middle school configuration; and (4) simple equations fit the patterns logically and empirically. A conclusion is that the simulation approach offers an unobtrusive way to investigate patterns of different populations in the retention pool. Five figures are included. (6 references) (LMI)
Comparative Aggregate Patterns of Grade Retention Rates

Don R. Morris
Dade County Public Schools

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Comparative Aggregate Patterns of Grade Retention Rates

Don R. Morris
Dade County Public Schools

In this paper, aggregate data on grade retention rates from 12 American states for the year 1979-80, and 11 for 1985-86, were examined for across-grade patterns. The results were: 1) the pattern of the rate magnitude distribution in the first six grades strongly suggests a negative exponential function as the target population is "depleted" across the grades. 2) the retention rate magnitudes show a pronounced tendency to peak at the beginning of each grade level. 3) shifts in the magnitude of the peaks following the move from a junior high to a middle school configuration decidedly support the preceding finding. 4) both logically and empirically, rather simple equations fit the patterns quite well.

Introduction

Purpose

This paper is about the "progression" of grade retention patterns across grades 1 to 12. The purpose was to describe an exploratory search for patterns in the grade rates general enough to serve for the construction of models suitable for use in simulation research.

Although the literature on grade retention is relatively voluminous, little is said about patterns of rates across grades. For example, there is no mention of the topic by C. T. Holmes, who has authored two meta-analyses of the literature in a decade. Additionally, a sampling of abstracts from his bibliographies uncovered no allusions to such a perspective. In general, patterns per se have not been examined (cf. Shepard and Smith, 1989, who published and discussed the data reproduced for analysis here).

However, it seems to the current author that certain consistencies in grade retention patterns were observable in several different data sets. The availability of published data in a number of American states provided an opportunity to see if the consistencies held up. To this end, data on 11-12 American states for two nonadjacent years were taken from the above mentioned source and analyzed graphically. These data were then supplemented by selected results from a study of four consecutive years of data from the Dade County School District, a large urban district in southern Florida.

The Organization of the Paper

First, the pattern for the elementary level (taken to be the first 6 grades) is explored. A process is described to explain the pattern, and a mathematical function that follows from the explanation is proposed to fit the data.

Next, a pattern is sought for the highest magnitudes of retention rates, and the function applied to the elementary grades is extended to provide a proposed comprehensive "fit".
Known changes in grade configurations in the district data set are explored with the aim of accounting for fluctuations in the states data, enhancing the credibility of the proposed fit of the suggested model to the full range of the of the data.

Finally, median summaries of the data by grade are presented and compared to a model fit across all the grades. These results are then followed by a discussion of their implications.

The Data and Methods

Retention data have recently been published for 14 states and the District of Columbia for two time periods. These data are displayed in graphical form in Figures 1 and 2. In this form, patterns common to the various states stand out. Certain similarities strike the eye: peaks of high and low rates tend to occur at certain grades.

The interest here is not in the absolute rates contained in the data but is rather in the relative highs and lows of the rates from grade to grade. This being the case, the focus is in an ordinal or ranking problem that also permits the skirting of questions about the consistency of methods from one state to the next.

The Kendall coefficient of concordance W was used to measure the extent of association in the grade rankings of the several states. (See Siegal, pp. 229-238.) Of the states for which data were published by Shepard and Smith, twelve in 1979 and eleven in 1985 had data available for all grades 1 through 12. The data for these states were ranked and for 1979-80 Kendall’s W was found to be 0.78. For 1985-86, the W coefficient was 0.70. These coefficients are significant well beyond the 0.001 level, and this is taken as sufficient evidence that the states share common patterns in the relative magnitudes of their reported retention rates by grade.

The best estimate of the ‘true’ ranking of the grades, given that W is significant, is taken to be the order of the various sums of ranks (cf. Siegal, p. 238). Table 1 shows the rankings for the two years derived from the summed ranks for the states. The ranks are identical except that Grade 6 is above 4 and 5 in the latter year. All the elementary grades 2-6 are in order at the bottom for 1979 and also for 1985 except for the movement of 6. The grades with the highest magnitudes for both years are (from rank 1 down) 9, 1, 10, and 7.

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<th>Rank</th>
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Figure 1. Percentages of students retained in each grade 1 - 12 at the conclusion of the 1979-80 school year. The graphs were produced from tabular data published in L. A. Shepard and M. L. Smith, editors, *Flunking Grades: Research and Policies on Retention*, p. 7. Omitted here are kindergarten and total (all grades summary) percentages.
Figure 2. Percentages of students retained in each grade 1 - 12 at the conclusion of the 1985-86 school year. The graphs were produced from tabular data published in L. A. Shepard and M. L. Smith, editors, Flunking Grades: Research and Policies on Retention, p. 8. Omitted here are kindergarten and total (all grades summary) percentages. Blank graph boxes indicate data not available.
Explaining the Positioning of the Lowest Magnitudes

Retaining from a Fixed Pool at a Fixed Rate

The overall pattern at the elementary level is pretty nearly universal in the reported states. The retention rate starts very high in the first grade and drops rapidly to the very lowest rank by the 5th or 6th grade. The rates for the states for the grades 3 through 6 were the lowest in magnitude (ranks 9 through 12) in both years. A look back at Figures 1 and 2 indicate that in the majority of instances the rates for grades 1 - 6 seem to drop rapidly and then level off. One plausible explanation for this is given in the two following paragraphs.

Imagine an entering “at-risk” population at the elementary level, that remains stable in size throughout the duration of the elementary period. Further assume that the individuals making up this at-risk population possess characteristics which, if detected, will result in retention. Assume also that teachers detect these characteristics, albeit imperfectly, and retain the students at the rate of (say) 35 percent a year. The effect is that 35 percent of the whole group is retained the first year, another 35 percent of those that remain unretained are retained the second year, and so on. The number of students in the group who remain unretained will approach zero as the period progresses.

This is not to say that the retention rate is 35 percent. Suppose that the at-risk group from which retentions are being made is a third of the enrollment. Then in the first year the retention rate will be about 12 percent (35 percent of a third), and drop steadily from there, forming a smooth pattern curving downward.

The Resulting Curve

If one retention is sufficient (i.e. if multiple retentions are rare), then the retention rate from grade 1 through grade 6 (assuming all these grades are in the elementary level) should be very close to that of a negative exponential function. Figure 3a shows this “ideal” pattern, which is expressed mathematically as \( r = k + c(1-p)^x \), where \( r \) is the retention rate and \( x \) indicates time in period, in school-years. The k, c, and p are positive constants that represent the minimum rate, the initial rate less the minimum, and the inclination of the curve, respectively, and p never exceeds unity.

![Graphs showing retention rate patterns at the elementary level](image)

Figure 3. Retention rate patterns at the elementary level. Graph 3a on the left displays the proposed negative exponential model displayed at dimensions similar to those covered by the data from the states. Diamond shaped symbols represent the value of the function at each grade. Graphs 3b and 3c show the mean (square symbols) and median (cross symbols) averages by grade for 12 states in 1979-80 and 11 states in 1985-86.
Graphs 3b and 3c show both means and medians by grade from the states discussed above, for each of the two years. The close agreement of mean and median testify to the symmetry and uniformity of distribution. Both graphs resemble the graph on the left quite closely.¹

The Relative Maximum Magnitudes of the Rates

The Highest Ranked Grades

The next topic of interest is the pattern of magnitudes of the "peaks" that prevails across the various states. A review of Table 1 shows that the high retention rates tend to occur at specific grades. These grades are the first, seventh, ninth, and tenth. There appears to be a direct and uncomplicated explanation for this. Each of these grades seems to mark a major change in the students' environment. In a sense, there is a "new" retention-prone population generated at each major change.

This appears to be usually demarcated by a level change. The first grade needs no explanation. The seventh is also clearcut, being the first secondary grade. Depending on the configuration of the levels, either the ninth or the tenth grade can be the first year of the senior high level. One cannot know from the aggregated data, but it is reasonable to assume that the states observed represent a mix of two middle level configurations, the middle school and the junior high. This point is further discussed below.

Extending the Elementary Pattern

It is but a short step to apply the reasoning developed to explain the retention process in the elementary grades, to all levels as a simple repetition of the same curve. It is assumed that the environment must be "relearned", that students prone to difficulties in school in a very basic sense "start over" in learning to cope at each level. This concept is graphed in Figure 4, where exactly the same model shown in Figure 3a is applied to each level in turn, with the single exception that the peak magnitude for the middle level is started at the 2nd year magnitude of the 1st level. This is in fact a reasonable exception, if one accepts the quite plausible notion that the size of the second peak relative to the first is an indicator of the degree of carryover of experience from the first level environment.

Stability of the Pattern Under Conditions of Change

Stability of the Ranks Over Time

As noted earlier, the rank order for the two years 1979-80 and 1985-86 is nearly identical (see Table 1). The Spearman's rho correlation of the two sets of summed ranks is 0.98, and of course, this is a highly significant relationship, statistically.² The fact that this represents very little change over the five year period large enough to change the relative positions of the grade magnitudes suggests that the patterns discussed above are quite stable, at least over short time periods.
Figure 4. Extension of the negative exponential model beyond the elementary level, to include the middle and senior levels, covering the entire grade span. The square symbols represent the value of the function at each grade.

The single discrepancy, visible also in the comparison of Figures 3b and 3c, is the rise in magnitude of the 6th grade, pushing it higher in rank than grades 4 and 5. This is contradictory to the explanations given above, for the rationale of the proposed model. To take this into account, and to deal with the fact that both 9th and 10th grades are very high in 1979-80, some additional and more detailed data are introduced from another source, the Dade County Public School District.

Explaining the Positioning of the Peak Magnitudes

Dade County, over the past decade, has been undergoing a change in the configuration of the levels of its system, from a junior high organization of grades to a middle school arrangement. While both middle school and junior high are variously defined, there as elsewhere, a sizable portion of this growing number of the middle schools have the makeup of grades 6 through 8, while the diminishing junior highs are very largely 7th through 9th.

Specifically, the number of middle schools of this particular makeup increased from 7 percent of the total number of regular schools at the middle/junior high level in 1982-83 to 31 percent in 1988-89. The number of elementary schools made up of grades 1 through 5 increased from 10 percent to 26 percent. The number of senior high schools with a 9th grade went from 29 percent to 58 percent.

Simultaneously, the number of junior high schools (with grades 7-9) decreased from 76 percent to 48 percent of the total middle level regular schools, the number of senior highs without 9th grades decreased from 71 percent to 42 percent, and the number of elementary schools with grades 1-6 decreased from 74 percent to 58 percent.

For purposes of comparison, the data for each of the two groups, taken separately, for the four years 1982-83 through 1985-86 were converted to ranks and subjected to the same analysis as the data for the states. The W coefficient for each group across the time period
period was 0.9 or higher, indicating that there was no significant change in the order of the ranks over time for either group, and permitting the summed ranks to be used for comparisons, rather than dealing with each year separately.

Similar to the states, these two groups showed retention rate peaks at the 7th, 10th, and 1st grades, and - for the middle school group only - the 9th grade, which was rank 3 in that group, and rank 8 in the junior high group. The 6th grade ranked 5 in the middle school group, and dead last in the junior high group. These two grades had the largest differences in rank from one to the other group, of any grade.

The salient point here has to do with the location (level) at which the 6th and 9th grades are placed, and the subsequent effect on the distribution of retention rate magnitudes. It was suggested above that retention rates tended to increase disproportionately at points of major change in the school environment, and that the most obvious place for this was at the change in level. In this middle school configuration the 6th and 9th grades become the first grade of a level, where they had not been in the junior high configuration. Retention rates in both grades were considerably higher in the middle school context. The 6th grade came up, although still behind the 7th, where secondary curriculum starts in earnest. The 9th grade increased to a very high rate, although in the Dade case the 10th grade remained high in both groups. [Other pressures are also known to apply to the 10th grade in that district and in Florida in general.]

No data concerning the configuration of the levels are available specifically on the states under analysis here. However, there are nationally aggregated data publicly available that cover the trend to the middle school over the years in question. (U. S. Department of Education, 1989, p. 98) Those data show that of schools with elementary grades only, the number of schools reported as middle schools (defined as schools with grade spans beginning with 4, 5, or 6 and ending with grade 6, 7, or 8) increased from 3.2 percent of all such schools in 1970-71 to 12.9 percent in 1987-88.

During the same period, of the number of all schools with secondary grades only, those schools reported to be junior high schools (defined as schools with grades 7 and 8 or grades 7 through 9) decreased from 32.9 percent to 23.6 percent.

These definitions leave something to be desired in the description of the middle school and the junior high for the purpose of this discussion. Moreover, the overall decrease in the number of schools nationally inflates the middle school upward trend and lessens the junior high downward trend. Nevertheless, the fact that the middle school configurations are increasing at the expense of the junior high schools is clear.

From 1979-80 to 1985-86, not only did the 6th grade increase enough in the states under study to change the rank order of that grade, but as the grade medians shown below will indicate, the 9th grade rate increased and the 10th grade rate decreased during that period. The information presented in this section suggests that the changes were likely due to increasing reconfiguration in favor of a middle school organization. Therefore, the assumption, built into the model as shown in Figure 4, that higher magnitudes in retention rates occur in the first grade of a level, is strengthened by this supplementary information.
We can now observe the change over the five year period with respect to a version of the model introduced in Figure 4. Figure 5 shows this version fitted to the median rates of all grades in the state data.

![Figure 5](image)

**Figure 5.** Extended model fitted to the retention rate grade medians of the states. The square symbols show the value of the median average of the states at each grade. The line connects the fitted curve of the function r. Model parameters are the same for both years.

The square symbols in the graphs are the grade medians, and the model in both years has the same settings, or "fit", indicated by the continuous line. The settings assume a "repeated retentions effect" of near 0.5 percent (0 in year 1 of each period), a middle grades initial "difficulty level" of 70 percent that of the 1st grade and a senior high "difficulty level" equal to that of the 1st, and a progressively less step drop from the peak (a lower value of p), for each higher level.

The overall fit is good. The fit in the sense of shape is better for the elementary grades in the earlier year, and for the middle and senior grades in the latter year. It is weakest at the middle level, where two grades yield very little information for the observation of trends.

**Discussion**

Shepard and Smith, who in 1989 presented and discussed the data reproduced for analysis here, include another article in their volume, of which Shepard is a coauthor. It contains the following statement: "in virtually all school systems a disproportionate share of retentions occur in the early elementary years, especially in first grade." (Grissom & Shepard, p. 47)

That statement is significant for its omissions. The data presented here in Figures 1 and 2 are presented in that book in tabular form, and the patterns that seem so apparent in the graphs shown above are simply not evident.

**Immediate Implications for Application**

The graphical analysis and visual fit of the model can be helpful in a heuristic way. Descriptively, the findings suggest some practical applications of the model. For one thing, anticipation of the peaks can help administrators concentrate resources on grades where they count. Once the local parameters for the model are established, it can serve as a sort of baseline for estimating effectiveness of at-risk help programs.
For another, in none of the middle school literature of which I am aware is there a warning or indication that the switch to a middle school configuration carries a problem of retention shift with it. The implications of this effect are completely unexplored. It may be, for example, that the raising of retention rates a year earlier for the at-risk who enter senior high influences the drop out rate of these students. A model such as this might alert decision-makers to the possibility, and permit them to take appropriate action to ameliorate the effects when planning structural changes.

Some Research Implications

The major research challenge is to determine what these rather clear aggregate patterns mean. Do they involve the same (or highly overlapping) subpopulations, one characteristic of which is a very slow ability to adapt to structural changes in the social/learning environment? Do they involve essentially different populations (a small overlap from one level to the next in the “retention-prone population”) that encounter problems at one level but not others? Are there for example whole groups of students who do very well with one teacher and small classes, but cannot adjust to many teachers, self-scheduling, and more specialized subjects proposed from multiple points of view?

As one example, it may be the case that retentions are made from more than one distinct sub-population. C. T. Holmes, in his review, found a small subset of studies where retention seemed to “work”. According to him they had the following characteristics in common: “It appears that all of the studies were conducted in settings described as suburban and included few if any black subjects. The populations were...lower middle to upper middle class....These children all tended to have IQ scores at or about 100....Thus...where retention ‘worked’, [the children] were systematically more able than the traditional population of retainees who were more likely slower learners with below average IQ and achievement.” (p. 25)

The general thrust of the recent research presented by the authors in the Shepard and Smith volume seems to be that retention is “all of a piece,” and always harmful. The present point, that there may be more than one population in the “retention pool”, is another that seems not to have been considered, and the simulation approach offers an unobtrusive way of investigating what patterns of different “mixes” might look like. Simulations that give indication of the different retention patterns for different mixes of students may contribute to the identification of situations requiring different policy decisions.

In Conclusion

These first efforts at a grade retention model may seem overly simplistic. It is unlikely that retention rates are constant from year to year, or school to school, or teacher to teacher. But this very simplicity can form the foundation of more powerful models, as this model’s constants are replaced by empirically derived functions.

The fact is that the model fits very well, both conceptually and empirically. Certain exceptions (the wandering 6th and 9th grades) support the point. The negative exponential model appears to capture a basic underlying pattern, particularly at the elementary level. Just such simple assumptions as were made here to generate the model may contribute to research efforts into the differential effects of variables thought to affect retention, for good or ill, and for the simulation of more complicated patterns, by laying down a baseline pattern against which to observe and measure deviations.
Notes

1. Graphs b and c drop off more sharply and flatten more quickly than the model, but there are other forces at work also. In particular, my early simulations indicate that flattening may be characteristic of systems that practice extensive second-time-retentions.

2. The Kendall’s W coefficients associated with the summed rank “best estimates” in Table 1 bear a linear relationship to the average Spearman rank coefficient (Rho avg) taken over all combinations of the k groups, such that:

   \[ \text{Rho avg} = \frac{(kW-1)}{(k-1)} \]
   \[ = 0.75 \quad \text{for} \ W=0.78, \text{States 79} \]
   \[ = 0.67 \quad \text{for} \ W=0.70, \text{States 85} \]

Thus W is directly comparable to the Spearman coefficients without having to compute the correlations of all possible pairs of rankings necessary to compute Rho avg.

3. The following figures have been compiled from the Dade County District and School Profiles, for the years noted. The data were not available prior to 1982-83 in a form that permitted the breakdown given here.

4. The retention rates were computed for each group of schools by summing the enrollment and number reported retained by grade for each year, and dividing the latter number by the former. The total number of schools (elementary, middle, and senior) in the middle school configuration increased from 27 in 1982 to 57 in 1986. The totals for the junior high configuration were 183 schools in 1982 and 155 in 1986.
References


