This study, involving 65 undergraduates at the Georgia Institute of Technology (Atlanta), explores a scheme for representing problem-solving knowledge and predicting transfer as a function of problem-solving subgoals acquired from examples. A subgoal is an unknown entity (numerical or conceptual) that needs to be found in order to achieve a higher level goal of a problem. Subjects studied three algebraic word problems dealing with work. Twenty-three subjects studied examples in which the representations for each worker's rate and time were provided, as well as the actual solution; 21 subjects studied examples in which only an explicit representation for each worker's rate and the solution were provided; and 21 subjects studied examples in which only an explicit representation for each worker's time and the solution were given. Results indicate that learners who had read examples illustrating certain subgoals did a better job of achieving those subgoals in novel test problems requiring new representations compared to subjects who had not been exposed to those subgoals in their examples. It is contended that if researchers can determine subgoals for solving problems in a domain, particularly from a novice's viewpoint, they can develop examples that are more likely to teach those subgoals. Four sample word problems are included. (TJH)
THE EFFECTS OF LABELS ON LEARNING SUBGOALS FOR SOLVING PROBLEMS

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Title: The Effects of Labels on Learning Subgoals for Solving Problems

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Objectives and Framework

Problem solving research has shown that, when people study worked-out example problems in domains such as algebra and probability, they often can not solve novel problems requiring a solution procedure different from the ones illustrated in the examples (Catrambone & Holyoak, in press). Essentially, learners acquire a superficial, cookbook-like approach to solving problems without necessarily gaining any understanding of the underlying structure to the problems. Learners often rely on a strategy of preserving object correspondences. That is, they often try to put similar numbers and objects from an example into similar positions in a new problem (Reed, Dempster, & Ettinger, 1985; Ross, 1987, 1989). If the roles for these numbers or objects are different in the new problem, then learners often either do not know what to do or they commit errors based on this faulty transfer of example information to novel problems. For example, when subjects studied probability problems involving mechanics picking cars to repair, they tended to give the same mathematical roles to humans and inanimate objects in a test problem even if the test problem actually involved inanimate objects picking humans (such as secretaries being assigned to certain chairs) (Ross, 1989).

The current paper explores a scheme for representing problem solving knowledge and predicting transfer as a function of subgoals acquired from examples (see also Catrambone & Holyoak, in press). A subgoal is an unknown entity (numerical or conceptual) that needs to be found in order to achieve a higher-level (sub)goal of a problem. Examples can be represented in terms of the subgoals that they convey to learners. Depending on how an example or series of examples are constructed, a learner could acquire or fail to acquire particular subgoals. Test problems can also be represented in terms of the subgoals needed to solve them. Examples can be constructed in order to vary the subgoals they teach to students and thus affect their success on non-isomorphic test problems.

Consider the algebra example in Table 1a in which one has to determine how long it would take someone to do a job given that certain information about their work rate and another person’s work rate is given. This problem involves using an equation for determining work that requires representing each worker’s work rate, time, and task completed:

\[(\text{Rate}_1 \times \text{Time}_1) + (\text{Rate}_2 \times \text{Time}_2) = \text{Tasks Completed}\]

Learners are good at memorizing how to solve problems isomorphic to the one in Table 1a. In this problem, both workers’ rates are represented as constants. The time for one worker is represented as a variable and the other worker’s time is represented as a function of that variable. However, learners may not encode the example solution in terms of determining a representation for each rate and time, but rather have a more superficial understanding of the solution procedure that involves plucking numbers from the problem statement and inserting them into the equation. As a result, if a new problem requires a different representation of the rates and times, these learners may be unable to solve the problem. That is, if learners may not have learned that certain subgoals exist, the subgoals of representing each worker’s rate and time, and that these subgoals might be achieved by different methods (i.e., different ways of representing rate and time depending on the given in the problem).

For instance, the problem in Table 1b requires that one of the worker’s rates be represented as a variable. In addition, instead of having the workers’ times be represented as a variable and a function of that variable, the times are now represented as a constant and a function of that constant. Nevertheless, the new representations can be inserted into the same equation as the one used for the example.

It is hypothesized that if learners acquire a particular subgoal during training then they will be more likely to correctly solve a novel problem requiring that subgoal to be used in a new way compared to a learner who did not acquire that particular subgoal. It is assumed that a learner is more likely to acquire a subgoal from an example if that subgoal is labeled in the solution.
Table 1

Example Work Problems

a) Mary can rebuild a carburetor in 3 hours and Mike can rebuild one in 4 hours. How long would it take Mary to rebuild a carburetor if she and Mike work together, but Mike works for 1/2 hour more than Mary?

Solution

\[
\frac{1}{3} = \text{Mary's rate}, \\
\frac{1}{4} = \text{Mike's rate}, \\
\frac{1}{2} = \text{time Mike spent rebuilding carburetor}, \\
\frac{1}{2} = \text{time Mary spent rebuilding carburetor}, \\
\frac{1}{2} = \text{time Mike works for more than Mary.}
\]

\[
\frac{1}{3} + \frac{1}{4} = \frac{7}{12} = \text{Mary and Mike's combined rate}, \\
\frac{7}{12} = \text{time they work together}, \\
\frac{7}{12} = \text{total time Mary spent rebuilding carburetor.}
\]

b) Mr. Jones can refinish a dresser in 5 hours. After working for 2 hours he is joined by his wife. Together they finish the job in 1 hour. How much of the job could his wife do in 1 hour when working alone?

Solution (not seen by subjects)

\[
\frac{1}{5} \times (2+1)) + (w \times 1) = 1 \\
\frac{3}{5} + w = 1 \\
w = \frac{2}{5} = \text{wife's rate}
\]

so, in 1 hour wife could do \( \frac{2}{5} \) of job

c) Barbara and Connie can finish a job in 6 hours when they work together. Barbara works twice as fast as Connie. How much of the job could Connie do in 1 hour when working alone?

Solution (not seen by subjects)

\[
(2c \times 6) + (c \times 6) = 1 \\
12c + 6c = 1 \\
18c = 1 \\
c = \frac{1}{18} = \text{Connie's rate} \\
\text{so, in 1 hour Connie could do} \frac{1}{18} \text{of job}
\]

d) Joe can stack a shelf of groceries in 3 hours. Sheila can stack a shelf of groceries twice as fast as Joe. If Joe works for 1 hour alone stacking a shelf and then Sheila starts to help him, how long will Sheila be working with Joe until the shelf is stacked?

Solution (not seen by subjects)

\[
\frac{1}{3} (t+1) + \frac{2}{3} (t) = 1 \\
\frac{4}{3} t + \frac{1}{3} = 1 \\
t + \frac{1}{3} = 1 \\
t = \frac{2}{3} \\
\text{so, Sheila will work} \frac{2}{3} \text{of an hour}
\]

Method

Subjects. 65 students from introductory psychology classes at the Georgia Institute of Technology participated in the experiment for course credit.

Materials and Procedure. Subjects studied three isomorphic example word problems dealing with work (see Table 1a for an example). Some of the subjects (n=23) studied examples in which the representations for each worker's rate and time was provided (lines 1-4 in the solution to the example in Table 1a) as well as the actual solution. These subjects will be referred to as the RTL subjects. Some of the subjects (n=21) studied examples in which only an explicit representation for each worker's rate was given along with the solution (i.e., lines 2 & 4 from the example were deleted). These subjects will be called the RL subjects. Some of the subjects (n=21) studied examples in which only an explicit representation for each worker's time was given along with the solution (i.e., lines 1 & 3 from the example were deleted). These subjects will be called the TL subjects.
After studying the examples subjects received three problems to solve. One was isomorphic to the training examples while the other three involved new and old ways of representing rate and/or time (see Tables 1b, c, d).

On test problems requiring a rate to be represented in a way other than as a constant, it was assumed that the RTL and RT subjects would be more successful than TL subjects since the former groups were hypothesized to be more likely to possess the subgoal of representing a worker's rate. The TL subjects, on the other hand, would be more likely to have learned something akin to taking numbers (that happen to represent the workers' rates) and plugging them into the equation without necessarily understanding what they represented. Conversely, the RTL and TL groups were expected to be more successful than the RL group at representing a time in a way other than a variable or function of a variable since the subgoal of representing time was hypothesized to be learned by the former groups during training.

Results

Subjects' performance on the isomorphic test problem was 96%, 95%, and 95% for the RTL, RL, and TL groups, respectively. Performance on Problem 2 was 74%, 57%, and 43% for the groups. This was a marginally significant difference, $\chi^2(2) = 4.47, p = .11$. Performance on Problem 3 was 70%, 52%, and 43% for the groups. This difference did not approach significance, $\chi^2(2) = 3.34, p = .19$.

For the second and third test problems the measures of most interest are how successfully subjects represented workers' rates and times in situations where the representation was not the same as in the training examples. The prediction is that the RTL and RL groups will be more successful than the TL group in representing rates when they are unlike those in the training examples and that the RTL and TL groups should be more successful than the RL group in representing workers' times in novel ways. Collapsing across the second and third test problems, the RTL and RL groups' success in representing workers' rates correctly for situations requiring a representation other than a constant was 88% while for the TL group the success rate was 62%. This is a significant difference, $\chi^2(1) = 16.66, p < .001$. The RTL and TL groups' success in representing workers' times correctly for situations requiring a representation other than a variable or function of a variable was 65% while for the RL group the success rate was 62%. This is not a significant difference, $\chi^2(1) = 0.2$.

Discussion

It was predicted that learners who had labels for certain subgoals in training examples would better achieve those subgoals in novel test problems requiring new representations compared to learners who did not have the labels. This prediction was confirmed for one subgoal (representing rates) but not for another (representing times). The failure to find support for time representation may be due to several factors. One could be that representing rates is somehow easier than representing times and thus, may have benefitted from the relatively minimal manipulation of simply labeling them in the training examples. A second explanation is that the labeling manipulation is not very powerful and thus, is not always likely to help learners form subgoals. Rather, a more detailed elaboration may be necessary. This elaboration might take the form of providing an explanation for why the rate and time in a certain example is represented the way that it is and perhaps why that representation may change in other problems. This level of explanation may be more likely to help learners acquire the subgoals (cf. Reder, Charney, & Morgan, 1986).

Perhaps the most important issue raised in this experiment is determining what are the useful or important subgoals for solving problems in a domain. If researchers and teachers can determine these subgoals, particularly from a novice's point of view, then they can develop examples that are more likely to teach those subgoals. Prior work has suggested that subgoals can be determined in a relatively straightforward way (e.g., Catrambone & Holyoak, in press). Thus, helping teachers to determine these subgoals would be an important step in improving instruction in math and science. Training examples could then be designed by teachers and textbook writers to help students to acquire the necessary subgoals. Problems requiring variations on the methods for achieving subgoals could be left as exercises or test problems for students. This approach would help ensure that the test problems are more diagnostic since the instructor would have a sense of the "distance" from the training examples to the test problems and thus, be in a position to determine whether certain problems are needlessly difficult or easy. Additional research can focus on factors that make learners more likely to learn the previously identified subgoals.
References


