The first of two documents presented, "Better Mathematics for K-8" (Iris M. Carl), describes the underlying assumptions, purpose, origin, formation, goals, and content of the "Curriculum and Evaluation Standards for School Mathematics" produced by the National Council of Teachers of Mathematics (NCTM). Information on implementing the Standards is given and four planning assumptions are identified. The second document, "SUM: Des Moines' Total Answer" (Kathleen Bullington), describes the origin, purpose, and nature of the Success Understanding Mathematics (SUM) program that uses manipulatives and a concrete-representational-abstract instructional sequence. (CLA)

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Technological advances in the last half-century have dramatically changed mathematics. If students are to become adequately prepared, principals and teachers will have to take the lead by setting appropriate standards for quality and excellence.

Today, when the destiny of our students depends on mathematical literacy, their level of understanding of mathematics is found wanting. Of vital concern is the demand for a quality mathematics education for all students, one that will last their lifetimes. The early support of the NAESP helped the National Council of Teachers of Mathematics (NCTM) move forward to identify our mutual goals of addressing the educational needs of an increasingly diverse population in our elementary schools. The result has been a three-year collaborative effort, culminating in the NCTM’s comprehensive 258-page Curriculum and Evaluation Standards for School Mathematics.

No Barriers to Math

What were the underlying assumptions of the Standards? First, we feel that every student should have access to a full range of mathematics and that neither language proficiency nor past performance should present insurmountable barriers to every student’s potential for progress. The Standards, therefore, is a set of challenging national goals for school mathematics. They also represent an unprecedented degree of consensus in the mathematics community about what should be done to improve the mathematics education of every student.

The writers of the Standards were NCTM members—teachers, administrators, and teacher educators—who constructed a framework of appropriate learning outcomes for productive citizens of the 21st century. Their draft document was examined extensively by segments of the mathematics and general education community as well as by members of the broader society.

The Standards is a coherent vision of school mathematics designed not as a scope-and-sequence guide but rather as a descriptive set of criteria to inform and support local initiatives to change a school’s mathematics curriculum and evaluation.

Call for Change

During the 1980s, a proliferation of reports gave consistently negative findings on schools, schooling, and student achievement. The impetus for the current wave of educational reform was generated by the sustained attention of that period at both the national and local level.

In this context of the drive for educational reform, we have intensified the search for evidence of the failure of existing mathematics programs to educate most students. In the Second International Mathematics Study (SIMS), the comparisons of the United States with other industrialized countries (excluding cultural and historical differences) indicated that the mathematics curriculum in this country is "one of
minimum expectations that resist the changes necessary to keep pace with the demands of preparing students for contemporary life."

The narrow objectives of the elementary school mathematics curriculum, weighted down with the rudimentary skills of the nineteenth century, have hobbled steps toward improvement. The results from the National Assessment of Educational Progress (NAEP) show that, although the majority of students can compute, they lack the knowledge and ability to apply those computing skills to solve problems. An analysis of the data attributes poor student performance to outdated curriculum and to their not being able to make sense of mathematics.

A popular item from NAEP illustrates the gross misdirection. "An army bus holds 36 soldiers. If 1,128 soldiers are being bused to the "training site, how many buses are needed?" Only 23% of the students identified 32 buses as the correct response. Seventy percent of the students computed correctly, but marked 31 or 31-remainder-12 as correct answers.

If we are to prepare productive citizens for the future, people who understand mathematics and its applications, then we must design strategies for curricular equity in what we teach and how we teach every student. Right now, we are perpetuating minimal mathematics for most students, while offering mathematics leading to the study of algebra for just a few students. The absence of problem solving and real-world application in today's school mathematics also limits students' present and future performance not only in school but also in the workplace and the university.

It's an unpleasant fact that students in most of our elementary schools today are taught in a milieu in which the prevailing conditions critical to their success—what they need to know and be able to do—are restrictive and outmoded.

### New Math Opportunities for Students

The Curriculum and Evaluation Standards for School Mathematics was created to build a foundation for all students to gain access to a comprehensive mathematics program throughout their schooling. The Standards focuses on five goals to promote student self-confidence in mathematics by developing mathematical literacy:

1. **Becoming mathematical problem solvers.** Students' experience in applying mathematics to solving problems should extend into all areas of the curriculum. Problem-solving tools, including calculators and computers in individual and group settings with simple and complex tasks, are essential for conceptual development.

2. **Learning to communicate mathematically.** Students' oral and written expression should demonstrate a fluency with the language of mathematics. A knowledge of signs, terms, and symbols is increased through required use in expressing an understanding of mathematics.

3. **Learning to reason mathematically.** Students' attempts to explore and make conjectures should demonstrate logical thought. An ability to support decisions and solutions requires the development of effective approaches to clarifying thinking.

4. **Learning to value mathematics.** Students' natural curiosity should be encouraged and stimulated. An appreciation for mathematics can be developed through its relevance and connection in real-life contexts; through its cultural, historical, and scientific evolution; and through positive teacher attitudes.

5. **Becoming confident in one's own ability.** Students' view of mathematics as making sense should produce success with new and unfamiliar problems.

A variety of experiences "doing" mathematics outside of school as well as in school develops trust in one's own mathematical thinking.

Additionally, the changes presented in the Standards should influence curriculum and instruction. These changes are based on the following underlying assumptions about students, about mathematics, and about students learning mathematics:

- **Mathematics is something a person does.** Students should know mathematics by being able to use it in meaningful ways and should learn mathematics by being engaged in exploration and thinking.

- **Mathematics has broad content encompassing many fields.** Students should be exposed to a wide range of content and experiences, upon which foundation they can build an understanding of the place of mathematics in a technological society.

- **Mathematical power can—and must—be at the command of all students in a technological society.** Students can gain knowledge and confidence in mathematics and develop strategies for continuing to learn and use it in their changing world.

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**The Standards**

The foregoing concepts lay at the heart of the NCTM's work to develop the Standards, which is a holistic vision of K-12 mathematics. It contains 40 curriculum standards divided by grade level—K-4, 5-8, and 9-12—and a comprehensive set of 14 evaluation standards.

Unique to the construction of the Standards is the concept of mathematics as an integrated whole, kindergarten through twelfth grade, rather than mathematics as a collection of individual topics isolated by grade levels. For example, algebraic and geometric concepts are developed even in the earliest grades, where teachers help children...
explore, reason, and infer with patterns and shapes as a way of avoiding the later shock of the formal—and often formidable—secondary school courses.

For each grade-level group—K-4, 5-8, and 9-12—the curriculum content emphasizes opportunities for students to develop an understanding of mathematical models and structures. Within each group, each standard is rooted in certain assumptions, the most prominent of which is that knowing mathematics is "doing" mathematics. The format for all the standards is the same: each one identifies the topic, indicates student activities, develops the rationale, and gives examples, materials, and instructional strategies.

The first three standards cut across all grade levels: Mathematics as Problem Solving, Mathematics as Communication, and Mathematics as Reasoning. Mathematical Connections, the fourth curriculum standard, is the connecting link showing how mathematics is related both across mathematical topics and across other disciplines.

Guided by these four common standards, sustained developmentally throughout the K-12 continuum, students can experience the power, beauty, and usefulness of mathematics. Nine or ten additional topical standards are also given for each grade-level group.

As an evaluation and assessment:

Fourteen evaluation standards and new methods of assessment accompany these curricular changes in context, content, and method. The evaluation standards are divided into three categories:

1. General Assessment strategies for the curriculum.
2. Student Assessment, intended to give feedback to teachers both on the effectiveness of their instruction and on students' mathematical progress.
3. Program Evaluation, provides help in the evaluation of programs and instruction, offers alternative methods of assessment to reflect the diversity, scope, and intent of our vision of school mathematics, and guides and supports change.

In particular, the K-4 Standards

Thirteen areas are covered in the K-4 standards. I spell them out here in order to indicate the specific types and

**SUM:**
Des Moines' Total Answer

Kathleen Bullington

"Send us graduates who can think!"
"Assure us that your students learn the math skills they'll need in the work place!"
"Raise test scores!"

Demands such as these are placed on busy principals who are faced with many other pressing needs.

How, then, does a conscientious principal find time to be a strong instructional leader, helping teachers meet community and NCTM demands to improve in the teaching of mathematics?

One place to look for help is "Educational Programs that Work," the U. S. Department of Education's list of more than 400 programs deemed exemplary, based on their proven effectiveness, cost efficiency, and transferability. Many of the 400 programs focus on the teaching and learning of mathematics. One of these is Success Understanding Mathematics (SUM), a program whose goals match those of the NCTM Standards.

SUM is an acronym for Success Understanding Mathematics. It has two meanings:

The first meaning is "total," and although SUM does not replace textbooks, it does provide a total approach to the teaching of math in the elementary school.

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levels of change the NCTM is recommending. I’m also including a sample of activities and recommended instructional strategies to show how the standards might be implemented. This material comes directly from the Standards document itself.

1. Mathematics as Problem Solving
The class is given the opportunity to plan and participate in an all-school ‘Estimation Day.’ The children, in pairs or threes, are to design estimation activities to be completed by children in other classes. Each group will supply all the necessary materials and monitor the activities. The activities might include guessing children’s heights, the number of candies in a jar, the lengths of various pieces of string, the weight of a bag of potatoes, the length of the room, the number of times they can write their names in a minute, or the length of time required for an ice cube to melt. (From page 25)

2. Mathematics as Communication
Students can write a letter to tell a friend about something they have learned in mathematics class. This type of activity allows the students to consider mathematics for a new purpose. If letters are exchanged, then students learn from the thought processes of their peers. (From page 28)

3. Mathematics as Reasoning
Who am I? I am an even number. I am more than 20 and less than 30. I am not 25. The sum of my digits is 8. (From page 30)

4. Mathematical Connections
(Social Studies) What is the tallest building in Japan? How tall is it? Write a paragraph comparing its height to the height of the Sears Tower in Chicago. (From page 35)

5. Estimation
Children estimate the number of boxes necessary to fill the classroom. A child mentally lines up seven boxes along one edge of the floor and uses them as a unit, or a ‘chunk,’ to estimate the total number of boxes. (From page 37)

6. Number Sense and Numeration
Two children each are given the same number of counters, in this example, 32. One child counts her counters by ones; the other groups his counters by tens and then counts by tens and ones. The children then are asked to compare and discuss their results. (From page 39)

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**Why Was It Developed?**

The second meaning is “answer”; SUM has answered the needs of Des Moines teachers, who developed the program.

SUM methods improve the mathematics achievement of elementary school children by helping them understand the concepts behind mathematical problem solving. They help students become self-confident problem solvers who learn to think and communicate in the language of mathematics.

SUM was developed by teachers in Des Moines, Iowa, who were concerned because student achievement did not match expectations. Their examination of students’ work revealed errors such as these:

\[
\begin{array}{c}
43 \\
-28 \\
25
\end{array}
\]

or this one:

\[
2\frac{1}{2} \times 2\frac{1}{2} = 5/2 \times 5/2 = 25/2 = 12\frac{1}{2}
\]

The errors were not random, accuracy was not the problem, nor was there a lack of knowledge about basic facts. The errors followed patterns. Students were mixing up the steps they had been taught. They were unable to tell that their answers were unreasonable.

They didn’t understand mathematics.

To make things worse, errors led to more errors and the students’ problems became exacerbated from grade to grade.

The teachers had been delivering routine, step-by-step instructions, expecting students to passively follow each step by rote. But when teachers looked closely at the errors, they discovered that their students had not been passive. Rather, the students had been actively constructing their own framework of “understanding,” while attempting to make sense of the procedures.

If their memories failed while they were following the mechanical sequence of steps, they substituted steps from past experience that seemed to fit.

For example, in the subtraction example shown above, the student certainly was not thinking about the meanings of the numbers 43 and 28 but more likely remembered the teacher’s recipe to “subtract the smaller number from the larger one,” then performed the subtraction steps column by column.

In the example that multiplies fractions, the student did not think about the meaning of the numbers but probably remembered the “rule” about common denominators that applies to the addition of fractions—but not to multiplication of fractions.

As teachers in Des Moines became aware of these patterns, they decided to move in and help children make sense of mathematics. They reasoned that if they could match instructional methods with the way children learn, then all the children could learn mathematics.

They knew that children invent strategies and procedures to answer their own questions. This became apparent as the teachers watched young children explain their thinking while joining and separating sets of objects. They realized that the children were inventing ways to solve problems to which they had not yet been introduced.

Research about the ways children learn showed our teachers that children’s reasoning ability develops through stages. They found that elementary school children do not reason abstractly but instead need concrete materials to help them visualize concepts.

During their early years, children learn less by listening and more by doing. Therefore, the teacher’s role is
7. Whole Number Operations
Multiples of a number can be shaded on a hundred chart. Children can then find numbers that are multiples of 2 and multiples of 3 and thus be introduced to the concept of common multiples. Calculators can be useful in exploring multiples of a number through repeated addition. After children become familiar with finding multiples of $3, 6, 9, \ldots$, they can find how many threes make 30 using repeated addition and predict how many threes make 60. The calculator is used to check their predictions. Since work with concepts of operations does not emphasize the computing of answers, calculators are a valuable tool. (From page 43)

8. Whole Number Computation
After clearing the calculator's memory, two children select a target, such as 23, and take turns entering a number from 1 to 5. Each new sum is put into the memory by pressing the M+ key. A player who thinks the target number is in the memory just after his or her turn presses the memory-recall key to check. (From page 46)

9. Geometry and Spatial Sense
When children hold a long loop of yarn so that each hand serves as a vertex, they can explore the effect of changing the size of an angle, or increasing the number of sides while the perimeter is unchanged. (From page 49)

10. Measurement
Given a carton, children can begin to understand an object's many measurable attributes by being asked, "How much does it hold?" (capacity) How tall is it? (length) How large is the front? (area) How heavy is it? (mass or weight) How far around does the border go? (length or perimeter). (From page 51)

11. Statistics and Probability
A class or group project conducted over time enables the students to make predictions and modify them as more data are collected. Suppose, for example, that children are interested in comparing the temperatures in their hometown with the temperatures in two other cities. They can obtain pertinent data from such sources as newspapers and television. They can participate in making decisions about what questions to ask; what data to collect; and how to collect, organize, and display them for others to see and interpret. (From page 55)

Real Solutions to Real Problems

Gradually, as the teachers shared ideas with each other, they developed methods for developing mathematics lessons from real life experiences and the children's interests. Through SUM, teachers present a real question to the students and give them concrete objects, or manipulatives, to guide their learning through questions about the problem.

Mathematics problem-solving abilities are developed as the students move the materials in response to the questions. Through emphasized interaction, the students learn to think and communicate in the language of mathematics.

The physical objects are important, and many kinds may be used. Children also benefit from representing the same idea with a variety of materials.

The primary goals of the SUM program are to develop a sense about numbers and be able to operate with them. Students study mathematics so they can use number concepts for real-life needs, not so they can complete workbook pages of computation exercises. Therefore, time is devoted to helping children make connections between math as they learn it in school and math as it is used throughout life.

They learn, too, to make connections between concrete models that represent numbers, the oral language or those numbers, and the related numerical symbols.

Given the words "twenty-three," for example, students should be able to show a physical model for twenty-three and be able to write the numerical symbol "23." Given the model, they should be able to produce both the oral language and the numerical symbol. Or, given the numerical symbol, they should be able to produce a physical representation and the words.

They learn, for example, that 36 is one more than 35, is between 35 and 37, can be represented as either $30 + 6$ or $20 + 16$, is close to 40, but not close to 400.

Usually, the children work with partners or in small groups. As the children display models and talk with each other, the teachers can both listen and "see" what the children are thinking. By guiding students this way, teachers can help the children clarify their reasoning and prevent patterns of errors.

Manipulatives—Then Books

All children use concrete materials to learn each new concept. By itself, the best textbook cannot provide these experiences, so lessons do not begin with students reading textbook pages.
12. Fractions and Decimals

Children need to use physical materials to explore equivalent fractions and compare fractions. For example, with folded paper strips, children can easily see that \( \frac{1}{2} \) is the same amount as \( \frac{2}{4} \) and that \( \frac{3}{4} \) is smaller than \( \frac{1}{2} \). (From page 58)

13. Patterns and Relationships

Using the constant function on a calculator, children can construct a table of input and output numbers and then express the relationship as an open sentence. (From page 62)

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**Implementing the Standards**

The K-4 standards—and the standards for grades 5-8, also—represent a framework for an elementary and middle school mathematics curriculum that is developed through the collaborative efforts of teachers and administrators.

The standards can be used to revise or create a curriculum, to aid in establishing new basics for a new age, or to serve as touchstones to judge the reform.

The changes envisioned in the Standards for elementary (K-4) and middle (5-8) schools are expected to evolve during the decade of the '90s as a result of careful, long-term, extensive planning. Teams of educators with expertise in the discipline should be given the time and resources needed to develop curricula and to plan and present professional development options that reflect the readiness of the teachers and students involved.

Teachers in some elementary and middle school classrooms are already implementing the Standards; in others they are prepared to start. With encouragement and support from their principals, plus the time and the materials, many more teachers may begin the process of upgrading the math curriculum. Those innovators will adapt the Standards and share the results.

The illustrative examples in the Standards clarify the concepts and approaches and offer possible ways of organizing content and instruction for curricular improvement. The full document is a rich planning resource and has the potential to engender teacher "ownership" of local reform, an essential ingredient of successful efforts.

However, it is important for everyone involved in the crafting and implementation of change to recognize that the mathematics curriculum envisioned in the Standards is neither rule-driven nor

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Manipulatives are used first. After children develop a concept using manipulatives, they are able to understand textbook pictures of the objects. SUM teachers follow manipulative work with representational lessons such as those found in texts. After the visual images are "in their heads," students are able to work with just the numbers at the abstract level.

This is the sequence of lessons:

Concrete \(\rightarrow\) Representational \(\rightarrow\) Abstract

Manipulative models are used to picture an idea at the time the idea is first introduced. Then the same model is expanded and extended as the idea or skill is used in more complex situations later on.

The multiplication concept can be represented in several ways, including a rectangular array. During beginning exploratory lessons for multiplication, children may use three groups of chips with two chips in each group to show "3 times 2." Next, they may organize those same six chips on a graph-paper grid with two chips in each of three rows, with the rows directly underneath each other:

```
[visual representation]
```

The children notice that the graph paper picture for each multiplication example forms a rectangle, and they also learn to superimpose similar pictures for other facts on a blank multiplication fact chart.

Children can independently find the products for all the multiplication facts, counting objects one-by-one, if necessary. They can build a multiplication chart whenever they need to remember a forgotten fact instead of simply memorizing answers from a meaningless list.

To remember a fact like "6 times 8," children can use a variety of thinking strategies:

```
[visual representation]
```

Students may separate their fact picture into two smaller pictures—a picture of "5 times 8" and another picture of "1 times 8":

```
[visual representation]
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Another may "see" "6 times 5" and "6 times 3":

```
[visual representation]
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Visual images from this kind of thinking make it possible for children to estimate appropriate answers to complex computation examples and prepare them to reason with data in the business world.

Before the teachers in the SUM Program began using these arrays, their students did not understand multipli-
The curriculum should be organized around real-world problem-solving situations as an experience that permeates all topics.

4. The curriculum should include mathematics other than arithmetic and its application within mathematics and in other disciplines.

The empowerment of teachers as change agents has major implications for the assessment of student achievement as well as for teacher appraisal. Teachers implementing the curricular standards will be providing exploratory and investigative activities for their students as problem solvers; they’ll be using calculators and computers as well as manipulatives and other materials. Their students will be engaged in cooperative team learning; they’ll be exercising critical thinking skills, as they work through questions that they and their teachers pose; they will identify connections among mathematical topics or between mathematics and other disciplines. This is the kind of excitement that is in store for schools that begin to implement the NCTM’s *Curriculum and Evaluation Standards for School Mathematics*.

As recently as December 1990 in the report of the *Weekly Reader Survey*, researchers have confirmed what elementary educators have always known: i.e., American students like mathematics and want to be successful. Principals and other school administrators now have the NCTM *Standards* to help them develop greater opportunities for students to achieve their full potential in this essential subject.

The NCTM invites the men and women of the NAESP to help make mathematical literacy a reality for every child in America.

An Invitation to Try

With the help of funds from the National Diffusion Network, the SUM program is available to schools whose needs are similar to those for which SUM was originally designed. SUM staff members offer training workshops for teachers, access to program manuals and management materials, and follow-up consultations.

Schools may purchase a set of sample problems students can use when they practice the problem-solving strategies. Other support materials include parent and inservice booklets as well as a set of blackline masters with which teachers may produce inexpensive manipulative activities.

Although the program began as a Title I—now Chapter 1—program, it is appropriate for all students and can be used with equal success by all classroom teachers. In fact, during the first five years that SUM was funded, teachers from 38 states implemented its methods in a wide variety of schools with a broad diversity of students.

We have seen the SUM approach adopted by schools in the Southwest, the Southeast, in a school operated by the Bureau of Indian Affairs, a rural Southern school, the Catholic Archdiocese in two metropolitan areas, in one of the schools sponsored by an oil company in the Middle East, and in a school in an American territory in the South Pacific.

If other schools are interested, the Des Moines teachers would be happy and proud to share the SUM program with them.