A conceptual framework is provided for generating assessment tasks that provide an instructor with a richer description of student reasoning/thinking than is possible by simply giving students problems to solve. The application of this framework is illustrated with material from introductory statistics courses which focus on: (1) sampling; (2) interval estimation; (3) point estimation; and (4) hypothesis testing. The authors adapt work done in mathematics by R. T. Putnam and others (1990) to the discipline of statistics. Statistical activities are related to a person's understanding of these activities. Examining these relationships provides a framework that helps guide statistics instructors in deciding which assessment tasks to create. The activities can be divided into: statistical problem solving; statistical modeling; and statistical argumentation. Five domains of cognition are defined: (1) understanding as representation; (2) understanding as appropriately integrated and organized knowledge structures; (3) understanding as connections between types of knowledge; (4) understanding as situated cognition; and (5) understanding as the active construction of knowledge. One table provides the framework for generating assessments by crossing activity domains with understanding domains. The use of MicroCAM, a microcomputer assisted measurement tool, is discussed within this framework. Two flowcharts illustrate pretest and posttest knowledge structures for a given student. (SLD)
SOLVING PROBLEMS IS NOT ENOUGH: ASSESSING AND DIAGNOSING THE WAYS IN WHICH STUDENTS ORGANIZE STATISTICAL CONCEPTS

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Solving Problems is Not Enough: Assessing and Diagnosing the Ways in Which Students Organize Statistical Concepts

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Educators are reformulating the goals of both teaching and assessment within content domains. Some of the recurring themes include the need for instruction to promote students as thinkers, problem solvers, and inquirers: the need to conceptualize meaningful understanding as involving active construction; the need for testing for thinking and understanding; and the need for assessment tasks to more closely resemble "real world" learning tasks. According to this view, assessment procedures used in introductory statistics courses should not only assess students' skills in carrying out statistical procedures, but also their understanding and thinking about statistics. Teaching statistical thinking and reasoning would be greatly facilitated, for example, by using instruments capable of assessing the structure and the relationships students impose on their own knowledge of statistical concepts. This paper provides a conceptual framework for generating assessment tasks which provide an instructor with a richer description of students' thinking and reasoning than is possible by just giving students problems to solve. Although the framework is general, its application is illustrated with material from college and beginning graduate level pre-calculus introductory statistics courses and which focuses on the following topics: sampling, interval estimation, point estimation, and hypothesis testing.

A fundamental objective of instruction should be the development of students' connections among concepts and the structuring of their knowledge (Glaser, 1989). A beginning student's knowledge base consists of isolated bits of information reflecting a shallow understanding of concepts and their interrelationships. With instruction and experience, knowledge should become more structured and integrated with prior knowledge.

Research that examines how students process and structure knowledge has direct implications for the design of more valuable assessments (Glaser, Lesgold, & Lajoie, 1987; Lane, 1989; Marshall, 1988; Nitko, 1989). Tests need to assess not only declarative and procedural knowledge but also the relationships that provide the foundation for the organization of students' knowledge structures. There is a need to assess whether a student can obtain the right answer to a problem and also whether the solutions are based on an understanding of underlying principles and concepts, that is, whether they are responding to the deep structure of the problem rather than the problem's surface structure. Diagnosis of learning should focus on how a learner perceives the structure or organization of content and processes information and knowledge to solve problems (Nitko, 1989). Because students' structures of knowledge change over the course of instruction, such assessments should also be sensitive to modifications and reformulations in students' knowledge structures.

In this paper we adapt work done in the mathematics discipline by Putnam, Lampert, and Peterson (1990) to the statistics discipline. We relate statistical activities to various ways of describing a person's understanding or cognition of these activities. The relationship between activities on the one hand and cognitions which underpin these activities on the other, provides a framework that gives statistics instructors guidance concerning the kinds of assessment tasks
to create in order to assess the ways in which their students think about and reason with statistics.

**Statistical Activities**

Statistical activities may be divided into three interrelated domains: problem solving, statistical modeling, and statistical argumentation. Within each of these activity domains instructors can differentiate students in a statistics class. **Statistical problem solving** is the ability to apply statistical concepts to problem situations. Students who are good problem solvers are able to abstract from a given situation the relevant bits of information and their relationships and to perform transformations on the mathematical relationships underlying symbols. Knowledge of conventions, symbols, and systems and their interrelationships underlie successful problem solving activity. Assessment tasks need to capture students' ability to formulate problems, identify principles underlying problems, analyze situations to determine common properties, verify and interpret results, and to generalize solutions.

**Statistical modeling** involves knowing how to develop a model by representing the relationships in a problem graphically and symbolically. Students should also be able to describe the conditions under which a particular model is appropriate. Problem solving and statistical modeling are related by the activity of formulating a problem. Assessment tasks focus on students' ability to identify appropriate models, distinguish among competing models, identify the conditions necessary for application of a certain model, and identify how models are interrelated.

**Statistical argumentation** is the ability to evaluate the truth of propositions and to develop plausible arguments for statistical assertions. Inductive reasoning is required to recognize patterns and to make conjectures. Deductive reasoning is required to verify conclusions, to judge the validity of arguments by considering counter examples, and to construct valid arguments. Assessment tasks focus on students' ability to identify false arguments, to explain similarities and differences among competing arguments, and to articulate the relationships among concepts and procedures.

**Statistical Cognitions or Understandings**

Underpinning a person's performance in each of the statistical activity domains, is a complex array of cognitions which we generally describe as a person's understanding of statistics. We focus on five interrelated ways of describing a person's understanding which we call cognition domains: understanding as representation, understanding as knowledge structures, understanding as connections among types of knowledge, understanding as entailing active construction of knowledge, and understanding as situated cognition (Putnam, Lampert, & Peterson, 1990).

**Understanding as representation** means having internalized ideas, symbols, and systems and being able to move within and between them in ways that allow for successful problem solving activity. When assessing this aspect of understanding statistics, an instructor determines the relationships between an external portrayal of statistical information and the internal cognitive representations of a student. The representational ability of the student is reflected in the degree of organization in a student's knowledge structure (Glaser, 1989). Naive representations of subject matter represent only partial understanding by the student. Assessment tasks focus on student's ability to use statistical vocabulary, notation, and structure to represent ideas, solve
Understanding may be conceptualized as an appropriately integrated and organized knowledge structure. Understanding statistics also means having access to the knowledge and cognitive processes needed to perform various statistical tasks. Problem solving is intimately related to knowledge structures because coherent and organized knowledge structures underlie successful problem solving. Valuable assessments of students' thinking include examining the understandings and models that students construct for themselves during the learning process. Levels of achievement could be viewed as levels of understanding of concepts and their interrelationships that underlie a subject matter domain (Masters & Mislevy, in press). Assessment tasks capable of characterizing knowledge structures underlying problem solving activity provide valuable information about students' statistical understanding.

Characteristics of knowledge structures that can be examined include the number and types of concepts depicted, the number and types of relationships among concepts (including conditional and hierarchical), and the degree of organization. For diagnostic purposes, congruence between a student's knowledge structure and various knowledge structures of experts may be examined. MicroCAM (described later in this paper) is one technique for assessing knowledge structures.

Understanding may be viewed as connections between different types of knowledge such as between conceptual and procedural knowledge and between formal, symbolic knowledge and informal knowledge. From this viewpoint, knowing statistics means understanding the concepts underlying the procedures and being fluent at integrating formal knowledge with informal knowledge developed outside of the class environment. Tasks focus on assessing an ability to create interrelationships across types of knowledge.

Understanding as situated cognition means that understanding statistics involves the interplay between students' cognitive activity and physical and social situations. Learning is the process of constructing understanding out of activities embedded in the social and physical world. Knowledge may be viewed as a product of the activity and situations in which they are produced and used (Brown, Collins, & Duguid, 1988). This involves students' participation in statistical activity or "doing" statistics such as abstracting, convincing, inferring, organizing, representing, inventing, generalizing, explaining, validating, and conjecturing. Ecologically valid instructional and assessment tasks entail being representative of the ways in which knowledge and skills are used in "real world" contexts (Brown, Collins, & Duguid, 1989). This conceptualization of understanding implies the need for assessment tasks to be embedded in activities that have "real world" meaning. In a classroom assessment situation, these activities may be simulated by giving students background context, research questions, and sets of relevant and irrelevant raw data, and asking them to use the data to interpret and answer the research questions.

Understanding as the active construction of knowledge means the process by which knowledge structures have been developed or acquired by students. Learning entails actively reorganizing and integrating new information with existing knowledge. Understanding statistics means having expanded one's own knowledge structures or ways of thinking to incorporate statistical concepts and principles. Assessment procedures need to be sensitive to modifications in students' knowledge structures and requires the assessment of students at various points in time such as before and after instruction in order to trace developing knowledge structures. MicroCAM (described later) is an example of an assessment procedure that may be used to
examine changes in knowledge structures as a result of instruction.

Generating Assessment Procedures and Tasks

The foregoing theoretical conceptualizations of statistical activities and statistical understandings may be used to create a framework which gives guidance to statistics instructors for developing assessment procedures. Assessment tasks can be developed which measure students' levels of problem solving, modeling, and argumentation (reasoning) abilities using as a guide each of the cognition or understanding domains. This will ensure that an instructor obtains a comprehensive assessment of students' thinking that will be especially useful to guide instruction.

It should be noted that the assessment framework we provide requires you to utilize a wide range of assessment techniques; many more techniques, in fact, than are used for assessment in a typical introductory statistics course. Among the assessment techniques needed to adequately utilize our framework are the following question types: essays, short answer, yes-no with student-supplied justifications, concept-oriented multiple-choice items, masterlist, analogical reasoning, graphic inference, concept mapping, and computer guided. Longer term individual and group projects may be required also.

Table 1 provides a framework for generating assessment. Each of the three activity domains is crossed with three understanding domains. The body of the table contains examples of the types of student performances which an instructor may assess. As stated previously, the domains are interrelated, so the table might be better conceptualized as seamless fabric, rather than as discrete cells.

One way to view this table, is to read downward within a statistical activity type and across types of understanding. For example, within problem solving, you can see the wide range of performances a student should display if he or she understands the nature of a statistical problem and its solution. Simply obtaining the correct answer to a problem is insufficient to display deep understanding.

We have illustrations of a variety of assessment tasks that correspond to each cell in the theoretical framework captured by Table 1. These tasks focus on sampling, interval estimation, point estimation, and hypothesis testing, content areas typically taught to majors in education or the social sciences in a college-level pre-calculus introductory statistics course. Because of space limitations, these sample assessment tasks cannot be reproduced here. They may be obtained from the authors.

Assessments in the cognition domains of active construction of knowledge and as situated cognition do not readily fit into the framework of Table 1, but these domains have been shown to underpin a students' understanding of problem solving, modeling, and argumentation (Lave, Smith, & Butler, 1988; Marshall, 1988; Schoenfeld, 1988). A variety of techniques have been developed for assessing knowledge structures and their construction (for reviews see Nitko, 1989; Shavelson & Stanton, 1975). Here we discuss one technique, MicroCAM which uses a Macintosh computer.
MicroCAM

MicroCAM (Ju, 1989) is a microcomputer-assisted measurement tool that assesses students' knowledge structure and interrelationships among different types of knowledge in various subject matter domains. The program was developed in HyperTalk for the Macintosh microcomputer. Using MicroCAM, students provide a spatial display on the computer screen of their knowledge structures, including the concepts, connections among concepts, and the relationships underlying the connections. Figure 1 shows one student's representation of concepts related to basic statistics. To obtain this figure, the student used a mouse to move the concepts in the boxes around the screen to depict how the student associated them. The student then connected the concepts by lines to show the linkages the student perceives. The student then entered words on the lines to describe the nature of the linkages or relationships.

MicroCAM can be used to evaluate students' initial understanding prior to instruction as well as changes in understanding resulting from instruction. Understanding of students' initial organization of knowledge and the relationships underlying their knowledge structures allows the teacher to tailor instruction to promote individual student learning. Students' explicit representations of their knowledge structures prior to and after instruction provides a means to knowing students' misconceptions, weaknesses, or problems and this information may be used as a basis for remediation. Moreover, comparisons of students' knowledge structures before and after instruction can be used to determine the effectiveness of instruction.

Teachers or researchers can develop their own assessment instruments using MicroCAM. the following types of assessments can be developed: (a) the teacher specifies concepts and presents a taxonomy of concept relations to the students, (b) the teacher specifies concepts, but does not present a taxonomy of relations to the student, (c) the teacher does not specify concepts, but does present a taxonomy of relations to the student, and (d) the teacher does not specify concepts, and does not present a taxonomy of relations to the students. The taxonomy of relations that was built into MicroCAM was derived from three sources (Armbruster & Anderson, 1984; Ballstaedt & Mandl, 1985; Fisher, 1988). Ju investigated MicroCAM's stability reliability, concurrent validity, and sensitivity to change during statistics instruction. Results were positive in each of these areas and lend support to the reliability and the validity of using MicroCAM to obtain useful information about students' knowledge structures.

Implications for Integrating Assessment and Instruction

The assessment framework presented in this paper can be a basis for providing valuable information regarding students' learning difficulties. Assessments developed using the ideas in Table 1 are capable of identifying misconceptions and missing concepts and links in students' knowledge provide information to guide the instruction. As another example, MicroCAM can be used prior to instruction to determine the statistical knowledge that students bring to a course including their misconceptions concepts and their interrelationships. Instruction can be aimed at building and organizing students' knowledge structures to facilitate statistical problem solving and reasoning. MicroCAM can also be used after instruction to assess the degree to which students have organized and integrated new information with existing knowledge.
References


## Table I: Framework for the Generation of the Assessment Tasks

<table>
<thead>
<tr>
<th>Domain of Understanding Statistics</th>
<th>Domain of Statistical Activity</th>
<th>Statistical Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Representation</strong></td>
<td><strong>Problem Solving</strong></td>
<td><strong>Statistical Modeling</strong></td>
</tr>
<tr>
<td>a. Formulate problems using statistical concepts and ideas</td>
<td>a. Use geometric, algebraic, and statistical models to represent data, problems and situations</td>
<td>a. Recognize false arguments</td>
</tr>
<tr>
<td>b. Express solutions in terms of appropriate concepts, symbols and systems</td>
<td>b. Describe how practical problems can be restated to fit one or more statistical models</td>
<td>b. Use inductive reasoning to make and validate conjectures</td>
</tr>
<tr>
<td>c. Translate words into statistical symbols and relationships and vice versa</td>
<td>c. Identify data needed to support the fit of a statistical model</td>
<td>c. Critically evaluate conclusions and claims made using data obtained from samples and surveys</td>
</tr>
<tr>
<td>d. Use statistical vocabulary, notation, and structure to represent statistical problems, ideas, and situations</td>
<td></td>
<td>d. Justify selection and use of specific statistical indices in particular research and/or decision contexts</td>
</tr>
<tr>
<td><strong>Knowledge Structure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Identify principles that underlie problems</td>
<td>a. Show how models are derived and interrelated</td>
<td>a. Explain similarities, differences, advantages, and disadvantages among competing statistical arguments</td>
</tr>
<tr>
<td>b. Organize and classify problems</td>
<td>b. Describe conditions under which statistical models are upheld</td>
<td>b. Explain how statistical concepts are linked to specific statistical arguments</td>
</tr>
<tr>
<td><strong>Connections Among Types of Knowledge</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Explain how statistical concepts are used to solve a problem</td>
<td>a. Articulate systems or models, linking several models into a unitary framework</td>
<td>a. Articulate the relationships among concepts, indices, and procedures</td>
</tr>
<tr>
<td>b. Use statistical vocabulary, notation, and structure to describe relationships</td>
<td>b. Articulate how statistical models may be used to answer research and practical questions</td>
<td>b. Articulate relationships among formal and informal statistical knowledge</td>
</tr>
<tr>
<td>c. Analyze situations to determine common properties, structures, and patterns</td>
<td>c. Articulate the relationship of statistical modeling to other forms of knowledge acquisition</td>
<td>c. Articulate how statistical arguments link to other epistemological frameworks</td>
</tr>
</tbody>
</table>
A. Student G., Pretest

B. Student G., Posttest

Figure 1. Knowledge structures of Student G., pre- and post-instruction.