Two longitudinal studies of expert-level cognitive skills and their acquisition are reported. The first study focused on the skill of an otherwise normal subject who increased his digit-span to over 100 digits with 4.5 years of practice using the mnemonic system developed by W. G. Chase and K. A. Ericsson. The second study followed two college undergraduates who became mental calculation experts by practicing for 3 to 4 years with strategies used by previously studied experts. The studies relied on the tenets of skilled memory theory. Observations of subjects' learning and performance suggest that acquired knowledge, rather than exceptional native talents or general aptitudes, constitutes the foundation of expertise, and that the knowledge structures and processing that support expertise can be analyzed. Theoretical analyses reveal that rapid and reliable storage and retrieval of information in long-term memory (LTM) appears to be a key characteristic of experts' information processing. Acquired memory skills enable experts to use LTM in order to reduce the constraints that a limited working memory capacity imposes on performance in complex tasks. The studies also document the effects of efficient information-processing strategies on cognitive skill acquisition and expert performance. The report distribution list is appended. A 5-item summary of publications, a 92-item list of references, 11 figures, and 9 tables are included. (TJH)

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Final Report

Ski. . Skilled Memory

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This report summarizes the findings of two longitudinal studies of expert-level cognitive skills and their acquisition. Their empirical, theoretical, and methodological contributions are described and some practical implications of this work are discussed.

Comprehensive analyses of subjects' learning and performance demonstrate that acquired knowledge, rather than exceptional native talents or general aptitudes, constitutes the foundation of expertise in the skills studied. They show that the knowledge structures and processing supporting expertise can be analyzed at a fine grain. Theoretically-motivated analyses reveal (a) that rapid and reliable storage and retrieval of information in LTM appears to be a key general characteristic of experts' information-processing, and (b) that acquired memory skills enable experts to use LTM efficiently to reduce the constraints that limited working memory capacity imposes upon performance in complex tasks. The studies also document the effects of efficient information-processing strategies on cognitive skill acquisition and expert performance.
1. Introduction

This is the final report of a project investigating the nature and development of expert-level cognitive skills with the support of ONR Contract N00014-85-K-0524. It summarizes the findings of two longitudinal studies of cognitive skill acquisition, in which the skills of the participating individuals were subjected to detailed cognitive analyses.

The accomplishments of this project fit into three broad categories. First, it has produced two rich databases on human expertise that trace the transition from the novice level to some of the highest levels of skilled performance systematically observed. Second, the analyses of the experts' performance have added depth and breadth to our understanding of the knowledge and memory dynamics supporting exceptional levels of human performance. Third, it has demonstrated the theoretical and practical value of the general approach to "knowledge engineering" employed by this project.

This report first reviews the general issues motivating this research project and its objectives. Subsequent sections summarize its main empirical, theoretical, and methodological contributions. The final section discusses some practical implications of this work.

2. Issues and Objectives: Expertise, Knowledge, and Skilled Memory

The studies described here were motivated by several related issues. The objectives of this project are reviewed in the context of these issues.

The most general goal was to refine our knowledge of what enables experts to perform demanding cognitive tasks with such extraordinary proficiency that most people believe they possess talents not found in the "normal" population. From an information-processing perspective, understanding such exceptional performance involves asking questions such as: How can we characterize experts' information-processing capabilities? What qualitative and/or quantitative differences account for the typically large disparity between the performance of experts and novices? The general phrasing of these questions assumes that certain invariants characterize expertise and its cognitive substrate across the variety of tasks or domains in which it is demonstrated. Considering the fundamentally adaptive nature of human expertise, however, it is important to question this assumption and ask if expertise has any general, context-free characteristics. If so, what methods might be most suited to detect them? This project addressed these issues by analyzing the cognitive structures and processes, particularly those related to
memory, that support expert-level cognitive skills and their development in two domains, mnemonic skill and mental arithmetic.

The second general goal was to analyze and describe the knowledge that supports expert level performance in the skills studied. Three distinct lines of prior research direct investigation to expert knowledge, its representation, its use, and its acquisition. First, there is the skilled memory effect, the superior memory experts typically exhibit for material found in their realm of expertise. The very restricted nature of their memory advantage relative to novice controls links this effect to their prior experience in a particular domain. The interpretation that a highly-organized, domain-specific body of acquired knowledge accounts for this effect has received substantial empirical support (cf. Chase, 1986; Simon, 1979; Tech. Rept. 89-2).

Second, the idea that knowledge is the foundation of expertise has been supported by the capabilities of computer programs known as expert systems. Essentially, these programs codify and use knowledge extracted from human experts in domains such as medical diagnosis, chemical analysis, and computer system configuration to perform problem-solving and decision-making tasks at levels at or near those of human experts (Duda & Shortliffe, 1983). Although these systems may not represent, store, and operate upon this knowledge in ways that human experts do, their capabilities show that the knowledge at their core is sufficient to produce expert performance (Feigenbaum, 1989). Third, Chase & Ericsson’s (1981, 1982; Ericsson, C. se, & Faloon, 1980) longitudinal study of the acquisition of mnemonic expertise directly linked acquired knowledge to unquestionably exceptional performance. Collectively, studies of the skilled memory effect, expert systems, and the acquisition of cognitive skill converge upon the conclusion that knowledge, rather than general aptitudes or talents is the foundation of expertise.

This conclusion leads to a theoretical problem and another objective: to identify and describe memory structures and processes that mediate experts’ application of knowledge. This problem is called the "paradox of expertise" (Barsalou & Bower, 1984; Smith, Adams, & Schorr, 1978; Posner, 1988). The issue is to reconcile experts’ effective and efficient use of a large knowledge base with the well-documented limitations that components of the human memory system impose upon information processing capacity (Miller, 1956; Shiffrin, 1976; Simon, 1976). This issue lies at the heart of Simon & Chase’s theory of expert skill (Chase. 1986; Chase & Simon, 1973a, 1973b; Simon & Chase, 1973; Simon & Gilmartin, 1973; Simon, 1979) and its offspring, the framework of theoretical principles that constitute skilled memory theory (SMT)
(Chase & Ericsson, 1982; Technical Report 89-1). Sharing several key assumptions of its precursor, SMT seeks to explain aspects of expert performance related to memory dynamics for which the Chase-Simon theory could not account. Key objectives of this project were to evaluate SMT's explanation of the paradox of expertise, particularly its generality, and better understand the cognitive structures and processes that implement skilled memory in specific expert skills.

2.1. Principles of Skilled Memory

SMT postulates that expert-level performance depends upon experts' efficient use of a vast, domain-specific knowledge base. This implies that an expert's knowledge base contains more than just content knowledge; becoming an expert also involves developing memory management skills. SMT asserts that through extensive practice in a particular domain, experts acquire knowledge structures and procedures for efficiently encoding and retrieving task-relevant information in long-term memory (LTM). Three general principles describe how experts use memory efficiently to excel in their particular domains.

The Mnemonic Storage Principle states that experts use abstract, semantic memory structures developed through extensive experience to quickly recognize and encode familiar patterns of information and maintain that information for later use. This principle essentially states that chunking is a mechanism experts use to process large amounts of information in a limited capacity STM. Studies of expert mnemonists (Chase & Ericsson, 1981, 1982; Ericsson, Chase, & Faloon, 1980) and experts from domains such as chess (Chase & Simon, 1973a, 1973b), bridge (Charness, 1979), the games of go (Reitman, 1976) and gomuku (Eisenstadt & Kareev, 1975), electronics (Egan & Schwartz, 1979), architecture (Akin, 1982), and computer programming (McKeithen, Reitman, Rueter, & Hirtle, 1981) support this generalization. All suggest that experts use knowledge acquired through years of practice in a particular domain and stored as organized units in LTM to encode large amounts of information economically.

The Retrieval Structure Principle states that experts use their acquired understanding of the material and tasks of a particular domain to create mechanisms for indexing chunks of information in LTM in a way that facilitates their orderly recovery. Retrieval structures are memory mechanisms that govern organized storage and retrieval of content information by "addressing" information at the time of storage to provide a systematic means of later retrieving it. Their function is analogous to that served by cataloging systems used in libraries or filing
systems used in offices. Retrieval structures represent a solution to the problems of explaining the large number of chunks (sometimes in excess of the storage capacity of STM) that experts can quickly store and easily retrieve in a prescribed order.

Finally, the Speed-Up Principle states that the speed and reliability of both memory encoding (i.e., chunking and LTM indexing) and retrieval processes increases with practice. Assuming that LTM storage and retrieval times decrease continuously with practice (Pirolli & Anderson, 1985), this principle implies that with sufficient practice experts can store and access virtually unlimited amounts of information in LTM with the speed and reliability normally associated with STM storage and retrieval.

How does the development of highly efficient LTM encoding and retrieval processes relate to skilled performance? Theoretically, efficient and reliable storage and retrieval processes enable experts to circumvent basic information-processing limitations, particularly limited STM capacity and relatively slow LTM encoding processes (Simon, 1976), that severely constrain novice performance on most complex tasks. In effect, the development of skilled memory enables experts to increase their working memory capacity for familiar materials. Increasing the amount of information available for processing and the speed at which it can be accessed should increase the overall processing capacity of a system per unit of time. To the extent that complex cognitive skills require the appropriate sequencing of elementary cognitive operations, skilled memory theory, through its postulated retrieval structure mechanism, suggests a means by which efficient procedural control is achieved. The enhanced processing capacity predicted by SMT is consistent with the speed and accuracy that typifies expertise in complex cognitive skills.

3. The Training Studies

Two longitudinal studies of the acquisition of cognitive skills have been completed under this contract. The first focused on the skill of an otherwise normal subject (DD) who increased his digit-span to over 100 digit with 4.5 years of practice using the mnemonic system invented by Chase & Ericsson's SF. His level of performance is the highest ever observed on this task, exceeding by more than an order of magnitude the performance of normal subjects. Because prior analyses of DD's skill had established that his performance was consistent with the principles of skilled memory, investigation focused on analyzing the mechanisms underlying his skill, their properties, and their interaction.
The second study followed two college undergraduates who became mental calculation experts by practicing for three to four years with strategies preferred by previously studied experts. This study represented an important step in generalizing SMT; its aim was to test how well predictions derived from SMT could account for expertise in a task where superior information retention is not the principal goal, but is necessary for fast and accurate performance. Analyses of their skills have added depth and breadth to our understanding of expertise, its cognitive foundations, and, especially, skilled memory and its role in expert performance.

The main empirical findings of these studies are summarized following a brief description of the training procedures used.

3.1. Practice Regimens

3.1.1. Digit-Span Training Study

DD's practice regimen involved practicing the standard forward digit-span task under laboratory observation 3 to 5 days per week. He began training with as many as 26 trials per 45 minute session, however the number of trials per session gradually decreased as his span and the duration of each trial increased. For more than 60% of his 1079 practice sessions, he received 3 trials per session.

The "up-down" procedure was used to measure DD's span (Ericsson, Chase, & Faloon, 1980; Watkins, 1977). Under this procedure, span is measured as the longest list that can be recalled on 50% of the trials. Each trial begins with an experimenter giving DD the length of the forthcoming list. Once DD indicates he is ready (after a brief period of preparation), the list is read to him at a rate of one digit per second. Following a interval of silent list rehearsal, he begins his serial recall. If his serial recall of a given list is correct, one digit is added to the length of the list presented on his next trial. The next list is reduced by one digit if DD's serial recall is not perfect.

In most practice sessions, DD provided a verbal report of how he encoded the items in each list immediately following his serial recall. After the last digit-span trial, each practice session ended with a free recall task. Here, DD was asked to recall as much material as he could from the previously presented digit lists in any order. All sessions were recorded on audio tape.
Throughout DD's training, experimental sessions were frequently inserted in place of practice sessions. These sessions were used either to conduct exploratory and more formal hypothesis-testing experiments or collect concurrent verbal protocols.

3.1.2. Mental Calculation Training Study

Two undergraduate volunteers, GG and JA, routinely practiced mental multiplication under laboratory observation for approximately 1/2 hour per day, 3 to 5 days per week. JA attended 268 sessions for approximately 175 hours of practice over a 3-year period. Over 4 years GG accumulated 618 sessions resulting in about 300 hours of practice. Practice consisted of solving multiplication problems using an unconventional, general computational strategy employed by the majority of expert calculators whose methods for multiplication have been documented. The trainees' practice sessions were frequently augmented or else replaced by sessions in which either verbal protocols were taken or experiments were conducted.

In practice sessions two manipulations were used to systematically vary the memory demands of the multiplication problems given. These manipulations involved independently varying problem size and presentation conditions. To vary problem size, the trainees regularly practiced on problems whose multipliers were either one- or two-digit numbers and whose multiplicands ranged from one to five digits in magnitude. These manipulations produced the nine problem-size categories shown in Table 1. All problems were randomly generated and presented in blocks containing one problem from each size category.

Two modes of problem presentation were employed, oral and visual. In the oral condition, problems were read to the subjects. After receiving a subject's ready signal, an experimenter (E) would read first the larger of the operands of the current problem, pause for approximately two seconds, and then give the word "times" followed by the second operand. The operands of visually presented problems were typewritten in the center of 3 x 5 cards, according to the display convention shown in Table 1. The E would simply display the printed card face to present a problem to the subject in the visual-presentation condition.

An additional procedural difference distinguished the two presentation conditions. Visually presented problems remained displayed until the subject either gave an answer or gave up. Thus, problem operands were available throughout the course of computation in the visual condition, whereas oral presentation required subjects to maintain both operands of a problem in memory to
solve the problem successfully. The extra memory load imposed by oral problem presentation was expected, therefore, to increase problem difficulty.

One further manipulation was employed as an instructional intervention. It involved only one of the trainees (GG) and occurred after he had practiced for 500 sessions. Problem-by-problem analyses of the procedures used by the trainees showed that the general algorithms they used to perform 2x multiplication differed both in computational complexity and in the memory demands they imposed. Process models of the trainees' procedures (discussed below) confirmed these findings; JA had devised and used a general algorithm for two-place (2x) multiplication that was more efficient than GG's. In an instructional session between GG's 500th and 501st practice sessions, JA's strategy was explained to GG and he was told to apply this strategy to all 2x problems he received in subsequent practice sessions. This experiment provided an opportunity to study the flexibility of a heavily-practiced skill and test the validity of the model on which this intervention was based.

3.2. Expert Performance: Some Empirical Generalizations

Figure 1 shows the digit-spans of DD and Chase & Ericsson's SF as a function of practice. Notice the spans of both individuals at the beginning of practice; both fall within range expected for normal adults of 7 plus-or-minus 2 digits (Miller, 1956). Over the course of practice, DD increased his span to 106 digits. To put his performance in perspective, DD's span exceeds the average normal span by approximately 50 standard deviations. DD's span exceeds SF's span of 84 digits by roughly 25%. There is no evidence that DD's span represents an absolute ceiling on his performance. In sessions testing DD's supraspan serial recall capabilities, he has recalled lists as long as 110 digits perfectly.

DD's mnemonic skills have been compared to other mnemonists on another benchmark memory task. The Luria Matrix (Luria, 1968) is a memory task that has been used to assess the skills of several exceptional mnemonists (Ericsson & Chase, 1982; Hurt & Love, 1972). The matrix is a symmetric display of 50 random digits arranged as shown at the top of Figure 2. On a test trial subjects are given the matrix to study for a series of serial recall trials that follow. These trials involve recalling either all of the matrix elements or a specified subset of elements in a prescribed serial order. The various recall tasks are shown in the lower half of Figure 2. Subjects' instructions are to minimize study time on the matrix without sacrificing accuracy in the test phase.
After over four years of practice on the digit-span task DD was tested on the Luria Matrix over 12 consecutive sessions. He received four matrices per session. After study of each matrix, upon his signal the matrix was removed and he received six consecutive recall trials of its contents. The first and last were always "entire matrix" trials. The intervening trials contained the remaining partial recall trials, their order being counterbalanced across all matrix presentations and unknown to DD for any given matrix.

DD's performance in the testing sessions was impressive. His recall performance averaged 98.5% correct for his first recall of the entire matrix across all sessions. His mean accuracy over the remaining recall tasks (second column down, third column down, third column up, zig-zag, 2nd entire matrix recall) was 99.5%. His mean study time along with his mean recall times for each of the recall tasks are given in Table 2 along with similar measures of performance taken from expert and novice subjects. His superiority on all measures indicates that his skill is not limited to the digit-span task, and shows some flexibility. In general, the data indicate that DD's mnemonic capabilities represent the peaks of human memory performance.

Turning to the results of the mental calculation training study, Figure 3 and Table 3, respectively, show the improvements in the trainees' speed and accuracy for problems differing in size and mode of presentation. Figure 4 compares their final solution times as a function of problem size and presentation mode against the performance of a mental calculation expert (AB) independently regarded as one of the world's best. In short, both GG and JA have achieved levels of performance that qualify them as experts.

The achievements of DD, GG, and IA are important in several respects. First, they provide information about the relative contribution of different variables to the acquisition of expert-level cognitive skills. Second, they show that expertise can be acquired with less than a decade of practice. Third, they demonstrate that interventions that teach expert strategies can facilitate the development of expertise. Fourth, the databases describing their performance and learning represent benchmarks for testing recently-proposed global theories of human cognition.

Neither DD, GG, or IA began practice with any identifiable exceptional intellectual

1The time listed for entire matrix recall reflects his first trial. His second recall of the entire matrix averages 10 seconds faster.

2All three show comparably low overall error rates.
abilities, talents, or cognitive capacities. Their achievements, along with other recent studies of expert performance (Ceci & Liker, 1986; Schneider, Korkel, & Weinert, 1989) indicate that such predispositions, particularly high levels of general intelligence, are not essential for acquiring expertise. Rather, the changes observed in performance as a function of practice support the claim that practice is the key variable, as Simon & Chase (1973) argued nearly 20 years ago. This conclusion does not deny the importance of other variables. Individual differences in aptitudes and previous experience in related domains may well differentially affect rates of skill acquisition or determine peak performance levels. This is an issue that calls for systematic study. Motivation is an important variable (Charness, 1989); subjects who devote years of practice to any demanding task are obviously very highly motivated.

By the standards of most psychological studies, the duration of these training studies is long, however the amount of practice needed by the trainees to become experts is short by standards found in the literature on expertise. For instance, Van Lehn (1989) states that the label expert is usually reserved for individuals with several thousand hours of experience. Hayes (1985) has argued that a minimum of a decade of practice is necessary to become a world-class expert. Differences among tasks, subject differences, and differential training conditions caution against making broad generalizations about the amount of practice needed to become an expert, however the results of the digit-span and mental calculation training studies reveal that the investment of time, effort, and resources required to achieve expert-level skills is not as great as prior studies suggest.

The training studies show that the processing strategies subject practice are important determinants of their learning and performance. This holds true whether the strategies are discovered by the subjects or explicitly taught to them. In both the digit-span and mental calculation studies, DD, GG, and JA were instructed to use strategies known to be used effectively by previously studied experts. Additional subjects also originally involved in these studies were observed as they practiced under identical conditions. They were instructed to use different strategies, however. While all subjects showed improvement, none using the strategies predicted to be less effective progressed much, either in terms of practice or performance, relative to the those given the expert strategies.

For instance, at the same time DD began practice, Chase & Ericsson (1982) also gave another subject extended practice on the digit-span task. She was given a strategy that promoted
the encoding of digits as meaningful chunks, but did not lead to the development of a retrieval structure. Her span improved at a rate comparable to SF’s and DD’s, but she hit an asymptote at 18 digits, grew frustrated, and quit practicing.

The mental calculation study originally began with 6 subjects (Staszewski & Chase, 1984). Using random subject assignment, three were instructed to use the traditional right-to-left procedure, whereas GG, JA, and another were assigned the left-to-right strategy. Although large individual differences were evident in subjects’ performance, as a group, subjects in the right-to-left condition were slower and less accurate than their cohorts and the marginal advantage for the left-to-right group increased as demands related to problem size and presentation mode increased. Subjects in the right-to-left group also expressed more frustration than those in the left-to-right condition. Two of the right-to-left trainees quit before completing 12 sessions and the third completed only 22. Although the low number of subjects involved and lack of control of various subject factors make it difficult to unambiguously attribute the achievements of DD, GG, and JA to the strategies they were given, the pattern of subject performance, attrition, and strategies observed in these studies suggests they played a very important role.

The clearest evidence that strategy-based interventions facilitate the development of expertise is the improvement observed in GG’s performance following his introduction to and adoption of JA’s 2x computational strategy. A significant feature of this intervention is that it was applied at a relatively advanced stage in GG’s training. Its success demonstrated that a heavily practiced performer can learn and benefit from new and effective strategies, without suffering the heavy negative transfer that some studies of skill acquisition have found (Shiffrin & Schneider, 1977).

One further accomplishment of this project is that it has contributed two large and rich data sets to the empirical database on human expertise and its acquisition. Studies of human expertise have had a tremendous impact on cognitive theory and research over the past twenty years, however the database on which our understanding of expertise rests is disproportionately small. The scarcity of expert subjects and the time and resources required to thoroughly analyze their skills explains this situation. The datasets from these studies add significantly to the expertise database, containing quantitative and qualitative measures of the trainees’ practice performance (including chronometric, error, and verbal report data) covering thousands of practice trials. These databases not only describe human performance at some of the highest levels ever systematically observed, they also describe the learning that led to these levels.
The availability of these databases is particularly relevant to the emergence of "cognitive architectures" in cognitive science. Exemplars include ACT* (Anderson, 1983), SOAR (Newell, Rosenbloom, & Laird, 1989), ICARUS (Langely, 1989), and connectionist architectures (Rumelhart & McClelland, 1986). Implemented as simulation systems, these architectures represent general theories of human cognition designed to account for human performance and learning over a wide range of tasks. Their validity as general theories of cognition rests largely on the range of cognitive performance which they can accurately model. Because the databases built up in this project describe extremes of human learning and performance, modeling its findings represents an acid test of current and future architectures.

3.3. Cognitive Foundations of Skilled Memory

3.3.1. Expertise, Knowledge, and LTM

The fundamental assumption underlying the concept of skilled memory is that improvements in performance as a result of practice are due, at least in part, to subjects' increasingly efficient use of LTM to encode, maintain, and retrieve information critical for successful task performance. SMT asserts that experts expand working memory capacity by learning to use circumscribed portions of LTM to maintain information needed for future processing in a readily accessible state. Organized knowledge acquired through practice in specific tasks or domains mediates experts' LTM encoding and retrieval processes. Evidence supporting these assertions is described in the following sections.

Support for the LTM Storage Hypothesis

Evidence that LTM, rather than STM, is the locus of DD's list storage comes from several sources. One is his performance on the final free recall task that concludes each of his practice session. He recall for the contents of the lists presented in these sessions is impressive (Mean = 93%, SD = 4%) This level of retention is clearly inconsistent with data on STM forgetting (Brown, 1958; Murdock, 1961; Peterson & Peterson, 1959) and quite consistent with the skilled memory effect. Enduring knowledge representations are predicted, if information is encoded in LTM.

Further evidence for LTM storage comes from an experiment using a variant of the Brown-Peterson distractor paradigm. In this experiment DD was given digit-span trials in which lists
either 25, 50, or 75 digits\(^3\) long were read to him at the rate of one per sec. Unlike usual practice procedures which allowed DD to initiate serial recall when he was ready, a distractor task involving visual search was interpolated between list presentation and DD’s serial recall. The distractor interval lasted either 1, 2, or 4 min. Results showed no effect of the distractor upon the accuracy of DD’s serial recall, regardless of its duration. His accuracy averaged 99, 98, and 95 percent correct for 25-, 50-, and 75-digit lists, respectively.

Finally, some compelling evidence for DD’s storage of lists in LTM comes from a study testing his memory for a 100 digit-list after a 24-hour interval. Following two consecutive days without practice (to minimize interference from previous days’ lists), DD was presented with a 100-digit list in what he assumed was a normal practice session. After his perfect serial recall of the list, the experimenter feigned an equipment problem to end the session. The next day DD arrived for practice at the same time as he had the previous day. To his surprise, he was asked for a serial recall of the list he had received the previous day. He recalled 99 of the digits in their appropriate locations. When told his recall was incorrect, he spontaneously identified the incorrect digit and its location, and gave the correct one. He then reported that during retrieval, he had narrowed search to two candidates, but not sure which was the correct digit, he guessed.

Variants of the traditional memory span testing paradigm have produced results that rule out the counterargument that DD does not store information in LTM, but, rather, has somehow expanded STM capacity. If this argument were true, his span for all materials that he encoded in STM should be enhanced. However, when either alphabetic symbols or words are read to DD at the customary one per sec rate, his span\(^4\) for these materials resembles that of unpracticed subjects (Crannel & Parrish, 1957). It is also known that STM retention is minimally affected by changes in item presentation rate relative to the large adverse effects that speeding up item presentation has on LTM encoding (Glanzer & Cunitz, 1966; Murdock, 1962). When DD’s span was tested using digits presented at a rate of four per second instead of the customary rate, his span measured at 10, an order-of-magnitude decrease from the span exhibited under normal practice conditions. These sizeable reductions in his memory span with changes in either materials or presentation rates indicate a high degree of specificity to the conditions of practice.

\(^3\)DD’s span at the time of this experiment was in excess of 100 digits.

\(^4\)The up-down procedure was also employed for all experiments measuring DD’s memory span.
Consistent with the principles of skilled memory, results from free recall and recognition tasks show that expert mental calculators also use LTM to store vital information during their computations.

One of the procedures for testing GG’s and JA’s LTM retention was similar to that used in testing DD’s final free recall. After solving the final problem of a practice session, the trainees were asked to recall as many of the problems presented in the session as accurately as possible. For most sessions, advance notice of the recall task was given prior to the start of calculation practice. On a number of occasions, the trainees received no advance warning. A problem was scored as correctly recalled if the number of digits in both operands was correct and 50% or more of the digits were given in their proper places.

The trainees’ recall performance was consistent with the predictions of SMT. Recall testing began after GG had accumulated 92 practice sessions and JA 136. In the initial testing sessions, GG recalled an average of 31% of the problems presented in the course of a practice session and JA recalled 37%. With further calculation practice, JA’s recall gradually rose to 46% near the end of his training and GG’s reached 61%. Consistent with studies of experts’ incidental memory (Lane and Robertson’s (1979); DD’s 24-hour delayed recall cited above), the trainees’ level of recall was unaffected by whether or not they knew they would be tested for recall on a given day.

Although their ability to recall problems supported the LTM coding hypothesis, it was bothersome that the amount of material GG and JA could recall was small relative to the amounts recalled by SF (Chase & Ericsson, 1982) and DD in post-practice free recall. A plausible explanation was that the problems presented to the trainees in the course of a practice session (and the memory representations generated in solving them) interfered with their recall of information from previously presented problems. This account was supported by an analysis that showed that the probability of recalling a problem increased as a function of its presentation order within a session for both trainees. In light of this interference, a recognition task was used as a more sensitive means of testing the calculators’ reliance on LTM storage. Following each of two practice sessions, GG, JA, and AB5 were presented with a set of 108 problems and asked to differentiate between those presented in the immediately preceding practice session and those drawn from a randomly-generated set of distractors. Results showed that the subjects could easily distinguish old problems from new on the majority of trials (GG 92%, JA 87%, AB 82%).

5The world-class calculator whose calculation skill served as a basis for comparing the trainees’ performance.
Consistent with the knowledge-based coding hypothesis, evidence for practice-specific memory coding skills is also shown in expert mental calculation. Although GG solves 2x4 and 2x5 problems faster and more accurately than AB, his advantage does not extend to larger problem sizes. In other words, GG’s expertise is limited by his experience; his performance falls when he is presented with problems larger than those regularly received in practice sessions.

This specificity is shown by GG’s performance on randomly generated 3x3s and 4x4s, problem sizes which AB practiced regularly, but GG had never practiced. Tested on 3x3s, GG’s average solution time for 12 problems (Mean = 41.23 sec, SD = 25.78) was almost double that of AB (Mean = 21.69, SD = 17.43), while showing comparable accuracy. Presented with eight 4x4 problems, GG experienced extreme difficulty and quit computation before reaching an answer on all but the last two. He correctly solved only the last one. His computation times exceeded five minutes on all of the 4 x 4s, and his overt behavior on all but the last trial resembled that of a novice struggling to solve a 2x5. On all but the last two trials, GG cited memory overload as the reason for failure, stating specifically that he “didn’t have the right schemes for finding and manipulating the numbers.” Significantly, after his attempt on the next-to-last 4x4, the first one for which he reached an answer, he reported discovering an effective representational scheme. His successful solution of the final 4x4 confirms his report.

In general, the evidence for LTM storage and the specificity of superior retention skills observed in these studies supports key assumptions of SMT. These findings, along with the generality of the skilled memory effect, suggest that experts in a wide variety of other domains strategically use knowledge to represent and maintain task-relevant information in LTM.

3.3.2. Mechanisms of Skilled Memory and their Implementations in Expert Skills

SMT asserts that two types of memory mechanisms mediate experts’ LTM encoding and retrieval: semantic memory representations that recognize and encode information as meaningful chunks and retrieval structures that index chunks in LTM in a way that facilitates their later retrieval in the appropriate serial order. It asserts that these mechanisms develop with practice, and, once they are established, further practice increases the efficiency with which they are used. The following sections review the evidence that skilled memory supports the remarkable performance of DD, GG, and JA and, in the process, describe how its component mechanisms are implemented by these subjects.
Exceptional Memory Skill: Structure and Process

Substantial progress has been made in developing an information-processing theory of DD's skill. A variety of studies were performed to analyze the encoding operations he employs during digit-span trials, the representations he creates, and the processes he uses to retrieve information from LTM. Their findings have led to the identification of knowledge structures that mediate DD's encoding and retrieval operations as well as a fairly detailed description of the functional organization of these components of his skill and the nature of their interaction.

In theory, the key to DD's exceptional memory is his ability to quickly create elaborate, organized, information-rich LTM codes for the short 3- or 4-digit sequences into which he systematically parses digit lists and (b) and to access these representations for orderly retrieval. These encoding and retrieval operations are inter-related and mediated by three types of knowledge structures in his knowledge base. These mechanisms are identified as (a) his semantic coding system, (b) his retrieval structure, and (c) his contextual coding system.

The contents of Tables 4 and 5 provide a concrete context for describing these mechanisms and their role in DD's performance. These tables contain verbatim transcriptions of verbal reports given by DD after digit-span trials in which he perfect recalled a 75- and a 50-digit list. For sake of exposition, the left-hand column of numbers in each table displays the contents of each list organized according to their grouping in each report. In these reports, DD describes how he encoded the elements of each list.

Retrieval Structures

DD's reports reflect both how he organizes his encoding and serial recall of digit lists as well as the mechanism that imposes a common structure on both of these processes. As his reports show, DD's basic unit of encoding is a digit group, either three or four digits in size. These protocols reveal regularities in his formation of digit groups that generalize far beyond these particular lists.

A salient feature of Tables 4 and 5 is the redundancy in DD's list parsing. Examination of the arrangement of digit groups in these protocols reveals an abstract hierarchical scheme of organization. As these reports show, DD's digit groups are arranged in sequences of uniformly-sized groups, sometimes four, but mostly three groups in length. These sequences form higher-
order units labeled "supergroups." Still higher-order units, called "supergroup clusters" pair supergroups made of 3-digit groups and 4-digit groups. The scheme for organizing these units is clearly illustrated in Table 4. This protocol shows that DD organized the first 16 items of this 75-digit list as a single supergroup composed of four consecutive 4-digit groups. The next 21 digits represent his first supergroup cluster. It consists of a supergroup of three 3-digit groups followed by a supergroup of three 4-digit groups. The supergroup cluster pattern is repeated again, and once again, albeit incompletely in this list. Table 5 shows that the same organizational scheme is applied to a 50-digit list for as far as its digits extend.

The common structure that these lists share is neither coincidental nor idiosyncratic to these particular lists. Analyses of DD's protocols, analyses of his errors on digit-span trials, and chronometric analyses of his list encoding, serial recall, and memory search (Tech. Rept., in preparation-a) show that the organization of the lists shown in Tables 4 and 5 reflects a general organizing strategy based on an abstract, hierarchical structure. A variety of studies presenting DD with 50- and 75-digit lists showed that he uniformly applied the organizational schemes instantiated in these protocols. Throughout his training he also applied this general scheme to lists larger and smaller, truncating or adding abstract organizational units as the length of the particular list dictated. The uniformity of this scheme across different list lengths can be seen in Figure 5, which graphically represents his organizing schemes for lists 50, 75, and 100 digits long.

The mechanism underlying DD's organization of lists is called a retrieval structure. Essentially, this mechanism operates as a memory indexing system. It is used by DD to store semantically encoded digit groups in LTM in a way that (a) maintains the ordinal relations between the groups and (b) supports a systematic retrieval strategy. In general, DD's retrieval structure anticipates the problem of retrieving a large number of coded digit groups from LTM in proper order and encodes these items so that processes governed by this mechanism can access them.

To make the concept of an indexing system more concrete, consider some common examples: postal systems that organize mailing addresses, library systems (e.g., the Dewey Decimal System) that specify the locations of books in a library, and address systems used to locate a particular piece of information in the memory of a digital computer. All of these systems define locations within an abstract relational scheme where a particular content can be
"delivered," stored, and later accessed. The key to their effectiveness is that some "content" is stored and addressed according to a stable system, so that knowledge of the addressing system later supports organized search and efficient retrieval.

The mnemonic system known as the method of loci represents a specific memory indexing system that has several of the basic properties that characterize DD’s retrieval structure. Individuals using this method memorize lists of items by first forming an image of each to-be-remembered content item and then associating each image with a predetermined physical location found in a well-known physical environment. Retrieving the stored items involves mentally traversing the path connecting the locations and "picking up" the content associated with each location. In theory, recalling each location provides a set of cues for recalling the associated items. The key parallel between this system and DD’s is that a familiar, systematically organized body of knowledge, whether it be of a physical environment or an abstract set of relations, coordinates the storage and retrieval of information.

In essence, retrieval structures are mechanisms that organize and coordinate encoding and retrieval processes. They combine a well-structured knowledge base with processes that operate upon this knowledge to systematically generate "addresses" for storing information. These "addresses" consist of abstract features associated with to-be-remembered content at the time of storage. At the time of recall, the retrieval structure is again invoked to systematically regenerate the same addresses, which then serve as retrieval cues for accessing the stored information. Using the method of loci example, this translates to using a familiar path through a familiar region to generate the imaginal locations to which to-be-remembered items are associated. To retrieve the stored items, one systematically regenerates the sequence of locations to access their associated content.

Evidence for these claims about retrieval structures and their role in DD’s skill come from investigations of his performance on several tasks (Tech. Rept. 89-1, in preparation). For example, protocol data like those in Tables 4 and 5 were used to generate models of retrieval structure organization and which were then validated using chronometric analyses of DD’s serial recall. This work showed that DD’s serial recall for lists of varied length was organized in the manner predicted by the structural models. Application of these same models to temporal patterns obtained from studies of self-paced list encoding showed that the same structures were used to organize DD’s list encoding. The isomorphic relation between DD’s list encoding
processes and his retrieval processes for serial recall was also shown by substantial correlations between the pauses in DD’s list encoding and the pauses in his serial recall of individual digits at identical list locations. The additional finding that the temporal patterns that characterize both list encoding and serial recall are highly intercorrelated over the overlapping portions of different-sized lists supports the claim that a single, general retrieval structure is used consistently.

The idea that digit groups are addressed in LTM with a unique set of features that identify their relative location within DD’s retrieval structure has been tested in several ways. In general, this view implies that encoded digit groups are relatively independent of one another in memory and leads to the following prediction: given the location or address of a digit group within a list of specified length, DD should be able to both encode and retrieve the digits associated with that location flexibly. Two experiments were performed that tested this prediction.

In the first, lists 25 and 50 digits long were presented to DD under self-paced conditions. One digit was presented at a time on a CRT and remained displayed until a response from DD displayed the next digit. The digits were not presented in the order in which they were later to be recalled, however. Rather, a graphic representation of his retrieval structure was displayed and a pointer indicated where a particular sequence of three or four digits belonged within this structure. For each location, the sequence of digits was given in the order in which they were to be recalled. The order in which the locations were presented for study on each trial varied randomly. Following the “mixed” presentation of digit groups of each list, DD was asked to recall the list as he usually would, first with the initial four groups of four digits, the next three groups of three digits, and so forth. Despite the novelty of these list display procedures, DD’s serial recall for 25- and 50-digit lists averaged 95% correct. This result is consistent with the hypothesized independence of retrieval structure locations and demonstrates this independence in list encoding.

A memory search experiment modelled on Sternberg’s (1967) memory scanning paradigm demonstrates similar flexibility in retrieving digit groups (Tech. Rept. in preparation-a). In this study DD received 50-digit lists read at a rate of one digit per sec. Following list presentation, he was given a series of cued recall trials. These trials presented digit groups from randomly selected retrieval structure locations as cues. Depending upon the search condition specified prior to list presentation, DD’s task was to give the entire digit group that either preceded or followed the cue in the list. Results showed that DD retrieved the correct digit group on 94.4% of trials.
This level of accuracy compares with the accuracy with which novices retrieve information from STM (Sternberg, 1967). Analysis of both response latencies and posttrial protocols showed (a) that the pattern of DD’s latencies was consistent with a model of retrieval that assumes retrieval structure mediation and (b) that DD could use the cued information to directly access information specifying cue locations.

To summarize, evidence from a variety of tasks and measures provides support hypotheses about the form and function of retrieval structures.

**Semantic Encoding: A Chunking System**

Another salient feature of DD’s list encoding that is evident in Tables 4 and 5 is his categorical labelling of digit groups. Essentially, his representation of each group in terms of either a running time, an age, date, or miscellaneous pattern reflects his imposition of subjective meaning upon otherwise random information. The advantage of making meaningless information meaningful has been known since the time of Ebbinghaus (1964, originally published 1885) and is well-established experimentally (Crowder, 1976). Not surprisingly, this general strategy has been identified as one commonly used by exceptional mnemonists whose memory skills have been studied under laboratory conditions (Ericsson, 1985). The implication is that these memory experts can quickly relate new information to existing knowledge to exploit information in LTM as a mnemonic aid.

One of the noteworthy accomplishments of this project is the detail in which it has analyzed DD’s semantic coding processes and their underlying knowledge. We have found that DD’s ability to create a meaningful memory representation for any random digit sequence he encounters is supported by an elaborate, semantically-rich, hierarchically organized knowledge base which also supports multiple retrieval strategies.

How can DD’s memory representations be characterized in terms of structure and content? His protocols reveal that he uses a small set of abstract coding structures to encode digit groups. These structures are presented in Table 6. The content these structures take is presented in Table 7, which shows the semantic categories that he uses to give coherence and meaning to otherwise meaningless digit sequences.
The protocols in Tables 4 and 5 clearly indicate that DD’s encoding of digit groups in terms of their semantic content involves more than simply assigning a category label to a digit group. Rather, they show that he frequently assigns a number of meaningful features to create a well-elaborated representation for a digit group. For instance, he distinguishes the sequence 420 not just as a 1-mile time but as "a good high school mile time." He encodes both 6938 and 5802 as 10-mile times, but differentiates between them by noting that the former is "a really slow 10-mile" whereas the latter is "a good pace 10-mile." The sequence 142 is represented as a 1/2-mile time, "right around the world record." The sequence 063 is encoded not just as an age, but as an age "right around retirement time."

The beneficial effects of meaningful elaboration upon recall are well established (Bobrow & Bower, 1969; Craik & Lockhart, 1972; Hyde & Jenkins, 1969; Stein & Bransford, 1979). In theory, there are several advantages. First, elaborating a representation enhances the probability of recall by increasing the number of potential retrieval cues (Tulving & Thomson, 1973) or paths (Anderson, 1983; Anderson & Reder, 1979) that can be used to retrieve a stored representation. When several representations share features that raise the threat of interference, the presence of distinguishing features reduces this threat. It appears, however, that there is another feature of DD’s semantic encoding that contributes to his superior recall performance. This is the elaborate organization of his semantic knowledge base.

Several sources of evidence suggest that DD’s semantic knowledge base is organized along the lines of the semantic network pictured in Figure 6. One source of support for this representational hypothesis comes from his protocols. The elaborations that frequently qualify a general category label suggest that DD uses a multileveled hierarchy of conceptual categories to encode digit groups according to their membership in a set of stable, well-defined classes.

Supporting evidence comes from the organization of DD’s recall on the free recall task that concludes digit-span practice sessions. Analysis of his recall protocols shows that his recall is organized by his semantic encoding categories. The initial digit groups he reports are those encoded as 1/4-mile times. When he can recall no more 1/4-miles, he then turns to recalling 1/2-mile times, again reporting as many of the digit groups coded with this label as he can. He then proceeds to recall 3/4-mile times, 1-mile times, 3-kilometer times, proceeding through the coding categories in the order in which they are listed in Table 7. Analysis of his recall from a 10-session sample shows that 94% of all digit groups recalled are clustered within categories. His recall of items within these categories is ordered according to the magnitude of the coded values.
Further support for this representational hypothesis comes from recently constructed simulation programs developed to model DD's semantic encoding of digit groups and the organization exhibited in his final free recall. The encoding model assumes that a semantic network organized along the lines of the network in Figure 6 governs DD's categorical encoding of digit groups. Parsing digit lists into uncategorized, digit groups as DD would with his retrieval structure, the program performs a set of tests designed to mirror the decision-making procedures DD reports using to determine semantic codes. The results of these tests lead to a category assignment for each digit group. Comparison of the program's categorization performance with DD's on identical lists shows that it matches DD's category assignments on 98% of the digit groups it receives.

The free-recall model assumes that retrieval of digit groups is governed by the same knowledge representation used for encoding. This model assumes a process equivalent to a depth-first top-to-bottom activation of the nodes of his semantic network and that activation of nodes and their associated labels represents activation of cues used to retrieve digit groups whose representations contain the same semantic features. The model then performs a systematic within-category search as this process has been inferred from analyzing concurrent verbal protocols of his final free recall (Example: "OK, quarter-miles in the forties... four-seven-six... quarter-miles in the fifties... five-eight-one, five-nine-oh, five-nine-nine"). Evidence for the validity of the model is its ability to predict the order in which digit groups are recalled by DD in free recall. The average rank-order correlation between the model's predictions and DD's performance on a sample of items recalled in 10 practice sessions is .92 (SD = .05).

Further evidence for the postulated representation comes from Chase and Ericsson's (1982) investigation of the internal structure of DD's 1-mile-time category. They presented DD with 3-digit sequences that he always coded as 1-mile times printed on cards. His task was to examine the items and sort them into groups based on his perception of their similarity. Chase and Ericsson found that DD (like SF) sorted these items into a variety of categories that suggested a hierarchical knowledge structure containing several levels of mutually-exclusive subcategories.

I have used a similar approach to replicate the Chase and Ericsson findings for DD's 1-mile category and examine whether his other semantic categories were similarly structured. Procedures differed from those of Chase and Ericsson (1982) in the following respect: after DD would sort cards into groups, he was asked to label all groups created and then combine these
groups to form larger groups and, again label the new set of more inclusive groups. This process continued until DD had produced a single group representing one of the categories listed in Table 7.

Using this procedure and the same materials Chase and Ericsson used to analyze DD’s 1-mile category, results nearly identical to theirs were obtained. Presented with items that fall into his 2-mile category, DD’s sorting indicated that this category exhibited a similarly detailed hierarchical internal structure. The semantic network pictured in Figure 7 reflects the representation inferred from his sorting of items from the 2-mile category. The structure revealed for these categories suggests a powerful mechanism for chunking random digits into meaningful units and also elaborating their representation in a way that both associates semantically-similar chunks with higher-order category labels and differentiates them at lower levels.

As Figure 6 suggests, however, DD’s semantic categories differ quite a bit in the amount of internal structure they exhibit. DD sorts items from his 1/4-mile and 3-kilometer categories into relatively few meaningful subcategories. He reports having no subcategories that he consistently uses to differentiate digit groups categorized as 3-mile times, dates, or miscellaneous patterns. A hypothesis currently being tested is that the degree of structure within a coding category is inversely related to the proportion of all possible digit sequences to which that category can be applied. The rationale is that the more items that fall into a category or subcategory, the greater the need to encode items with features that discriminate them to ward off interference. This raises the issue of how DD deals with the threat of interference when multiple items fall within his less-differentiated semantic coding categories or subcategories. The issue is addressed in the forthcoming discussion of contextual encoding.

In general, the simulations of DD’s categorization and free recall, his verbal protocols, and his performance in the sorting tasks provide converging support for the theoretical description of the organization and content of DD’s semantic knowledge base. With this picture of his knowledge base, attention turns to how its organization supports DD’s encoding and retrieval of list items in the context of digit-span trials.

How does the kind of knowledge organization shown in these studies contribute to DD’s performance? As indicated earlier, well-differentiated coding categories mediate his encoding of elaborate, meaningful, and relatively unique memory representations for digit groups. The
literature shows that such characteristics promote retention. But there is a more fundamental way in which the organization of DD's knowledge base supports such encoding. The organization shown in DD's semantic memory reduces the amount of processing related to his recognition and encoding of meaningful patterns. Of course, the organization of his retrieval structure and the manner in which it reliably guides his systematic parsing of lists into codable groups serves a similar role and it is evident that the operations of these two mechanisms are well integrated. Using the knowledge represented by these mechanisms, information about the size of a given digit group and its first digit or two sharply constrains search for the appropriate set of coding features to a relatively small semantic space.

Although DL's semantic coding system represents a powerful mechanism for recoding digits as meaningful chunks of information, encoding is not its only function. There is also evidence that relates the organization of his semantic knowledge to his serial recall. Thus, like his retrieval structure, his semantic coding system mediates both encoding and retrieval of information in the digit-span task, although the temporal structure of his serial recall indicates that his retrieval structure is the primary access mechanism.

The role his semantic knowledge base plays in serial recall is clearest in situations in which DD experiences difficulty in retrieving a digit group at a particular list location. Most lists in the range of 100 digits often include such a group or two. They are easily distinguished because their retrieval times are measured either in tens of seconds or sometimes minutes. Periods of silence this size stand out clearly in DD's typically fast and fluent serial recall.

Both concurrent and retrospective protocol data collected on such occasions reveal three knowledge-based strategies for retrieving the missing digits. When DD has a few semantic features about a hard-to-recall group, he restricts his search for its contents to a relatively small range of candidates, probing memory using category labels subordinate to those he holds. Together, the retrieval cues he holds and his implicit knowledge of the organization of his knowledge base provide the constraints that narrow his search for additional cues. In situations when DD cannot recall the semantic category used to code a group, his protocols show that he resorts to using his semantic coding categories in an orderly generate-and-test strategy for retrieval. He searches through his coding categories as he does in his final free recall, naming each to himself to see if he recognizes the semantic code for the group in question. If he can establish a category with reasonable confidence, he then uses its internal structure to guide further
search. In the infrequent instances in which his recovery of a complete semantic code does not produce the missing digits, he generates a sequence of digit-groups testing each for recognition provided that the number of candidates is small. The interesting feature of these "back-up" retrieval strategies is that they reveal DD’s ability to intentionally exploit the organization of a particular data structure in his semantic memory.

On the Relations between DD’s Knowledge Components

Up to now, description of DD’s retrieval structure and system for semantic coding has implicitly emphasized their independence. Two points are in order to clarify current assumptions about their interaction and similarity.

First, there are noteworthy structural and functional similarities between DD’s retrieval structure and his semantic coding system. The knowledge representations employed by these systems are both multilevel hierarchies with multiple branches at each abstract level. Functionally, both are used to organize and encode to-be-remembered material in LTM and later mediate its orderly retrieval. Recall how retrieval structures guide DD’s serial recall in the digit-span task and his semantic knowledge structure guides both his free recall and reconstructive retrieval of hard-to-recall digit groups in serial recall. The content information that these mechanisms generate may be different, but their structural and functional similarity is striking. Further, the similarities between these mechanisms and the mechanism known as a discrimination net in Feigenbaum and Simon’s (1962, 1984; Richman & Simon, 1989) EPAM model of memory suggest that DD’s semantic coding system and his retrieval structure represent very sophisticated implementations of a general EPAM-like memory mechanism adapted to handle the demands of the digit-span task.

Second, it is important to realize that the operation of these mechanisms has to be extremely well coordinated, particularly during list encoding, considering the complexity of DD’s coding processes and the presentation rate used in the digit-span task. Independent theoretical arguments about the real-time retrieval capabilities of discrimination nets (Feigenbaum & Simon, 1984) make such precise temporal meshing plausible, lending further credibility to current assumptions about the interaction of these knowledge structures in DD’s list encoding and retrieval.
Although the evidence reviewed in the preceding sections indicates that DD's semantic coding system and retrieval structure are critical components of his skill, the operation of these mechanisms alone does not explain all aspects of his performance on the digit-span task and related experiments. For instance, protocols taken when DD's span was in the 80-100 digit range (like those in Tables 4 and 5) showed that DD's list encoding involves more than imposing meaningful interpretations on systematically parsed list segments. Also, the development of his semantic coding system and retrieval structure cannot account for improvements in DD's digit-span that occurred after these mechanisms were well established. After 3 years of practice (when his span had not yet surpassed SF's) verbal protocols taken in regular practice sessions suggested that these mechanisms were intact and operational. Subsequent monitoring of DD's protocols, studies designed to examine the structure and function of these mechanisms, and replications of these studies at later points in DD's practice also indicated that his coding and retrieval processes, as they relate to these mechanisms, remained quite stable. This implies that an additional mechanism or additional mechanisms were developed and/or refined to lift his span to its peak.

Similar reasoning by Chase and Ericsson (1981) in their analysis of SF's skill led them to propose that practice-related speed-up of memory processes was an essential part of the development of skilled memory. They argued that after SF had established his semantic coding system and retrieval structure, increased efficiency in the operation as a result of practice accounted for subsequent improvements in SF's digit-span. Consistent with this argument were data showing that the time SF needed to encode digit groups decreased monotonically with practice.

This finding generalizes to DD. Using a self-paced list presentation procedure to measure DD's encoding speed at yearly intervals, steady decreases in his encoding times (per digit group) were seen over his third, fourth, and fifth years of practice. But another important and logically-related development accompanied improvements in his coding efficiency.

Over this period, the emergence of a new coding mechanism was revealed in both the temporal character of DD's serial recall and his verbal reports. With increasing frequency, pairs of digit groups and triplets that composed supergroup units were recalled with unusually short intergroup intervals. In his retrospective protocols, DD regularly reported coding more than just
the meaning and location of these sequences. Almost invariably, he reported explicitly coding relations among the symbolic elements he held in memory. Together, these two related phenomena suggested that increased encoding efficiency had spawned a new mechanism for encoding higher-order patterns of information.

These patterns, and the labels DD uses to represent them, belong to the broad class of encodings called contextual codes. Contextual coding refers to DD's representation of a wide variety of relations that he discovers as he receives a list on a digit-span trial. What differentiates his creation of contextual codes from his encoding of retrieval structure locations and semantic encoding of digit groups is the irregular and variable nature of contextual encoding. Whereas DD invariably encodes the relative location and semantic content of each digit group in a list, the contextual codes he creates, if any, depend upon contextual variables such as the contents of a particular trial list, the contents of any preceding lists, and his representation of list contents. It is important to emphasize here that the content information available to DD during list presentation is not equivalent to that available to a novice, due to DD's knowledge-based pattern recognition capabilities.

What do contextual codes consist of and what accounts for their creation? Beneath the superficial diversity that characterizes DD's contextual coding from trial to trial, orderly "deep structures" exist. Analysis of DD's retrospective verbal reports from over 100 digit-span trials has revealed several abstract categories of contextual codes. Typically, these categories are identified in his protocols by distinctive verbal labels. Tables 4 and 5 contain a few such as "back-to-back," "add-em up," "faster than," and "other [semantic code]."

In these protocols DD reports coding relations between semantic codes whose creation can be separated by either a little or a lot of time and processing activity. Such relations link (a) semantic codes created for digit groups presented contiguously within the same digit-span trial, (b) codes in the same list whose creation is separated both by time and DD's coding of intervening digit groups, and c) codes from different lists presented in the same session. More concretely, in Table 4, DD recognizes that the third and fourth digit groups are both miles in the 6-minute range. He relates these items by coding the second sequence as a faster time than the first. In a similar fashion, he reports that two contiguous sequences (6938 and 5802) occurring later in the same list were both encoded as 10-mile times and differentiated on the basis of their assignment to subcategories within the 10-mile category. Likewise, at several points in Table 4, DD reports having noticed that codes for contiguous digit groups have redundant elements.
Redundancy is not the only basis for relating contiguous codes, however. DD regularly recognizes and encodes sequences of semantic codes that occur in ascending (e.g., "half-mile, three-quarter, one-mile") or descending (e.g., "three-quarter-mile, half, quarter") order within his semantic coding system. He also notices and encodes alternating sequences of codes such as "age, mile, age," or "two-mile, ten-mile, two-mile." Interestingly, his coding of such relations between triplets of semantic codes is restricted to situations in which redundant (e.g., "mile, mile, mile"); ascending, descending, or alternating codes occur within a supergroup unit of his retrieval structure. This constraint suggests that abstract location information and semantic codes serve as building material for contextual codes.

Contiguity is not essential for DD to notice semantically similar codes and relate them. In Table 4 that he reports noticing that 9396 is nearly identical to the sequence 9393 that occurred in a previous trial. The scope of such discovered relations sometimes extends across sessions. On one occasion, DD noticed that a particular digit sequence he was encoding had occurred in an identical position in a list in the previous day's session. A check of the lists presented on the previous day verified DD's observation. This anecdote, and, in general, DD's ability to discover redundancies of the type described here reflects two salient characteristics of his skill. The first is a remarkable retention of information over intervals of considerable length, during which a wealth of potentially interfering information is encoded. The second is his ability to recognize the threat of interference that redundant coding of digit groups creates and to encode relational information that links and differentiates the redundant codes simultaneously. The result is a unique memory code which, in theory, resists interference.

DD's encoding of different relational patterns of information is not restricted in content to semantic codes. He also reports noticing a variety of relations among digits. For example, Tables 4 and 5 show several instances in which he notices the repetition of contiguous digits, both within and between digit group boundaries. The phrase "back-to-back" typically identifies contiguous and redundant symbols, be they semantic codes or digits. He also frequently reports coding symmetric relations between individual digits and pairs of digits which he codes with the label "frontwards/backwards" (cf. DD's coding of the sequence 8558 in Table 4). His use of this latter category label for a variety of different contents (e.g., 191, 8558, age-mile-age) illustrates the abstract nature of his contextual coding patterns.

His protocols show that arithmetic relations among digits and digit groups form the basis for
another class of contextual codes. There are frequent occasions on which DD reports coding pairs of digits within 4-digit groups in terms of the difference between the quantities represented by each pair. For example, in Table 4, DD reports coding the groups 4346 and 4548 in terms of an attribute denoting a subtractive relation (e.g., "apart") and the value "three." Additive relations are also noted for several digit groups and identified by the label "add-em-ups." This label is applied when some subset of digits within a group sum to either another digit or combination of digits within that group. The protocols related to the digit groups 352, 642, 716, and 4272 in Table 4 reveal several instances in which this general relation is encoded. In the case of the contiguous sequences 352 and 642, DD explicitly mentions noticing a double redundancy in his coding of these items; the pair are coded as being both "back-to-back" 1-mile times and "add-em-ups."

Evidence suggests that DD uses contextual coding to reduce memory interference on the long trials he received at advanced stages of practice. The longer lists DD gets as a result of improving his digit-span increase the amount of potentially interfering information with which he must deal. This idea is consistent with the view that interference is the principal threat to recall of information stored in LTM (Anderson, 1985; Crowder, 1976). Because DD's creation of contextual codes is a form of elaborative encoding, this activity should enhance retention, provided it does not hinder other "regular" coding operations.

Several sources of evidence show that interference is a very real threat to success on extended digit-span trials. Chase and Ericsson (1982) have shown for both SF and DD that accuracy of serial recall diminishes as a function of trial order within a practice session and that list rehearsal time increases. In addition, the probability of correctly recalling a digit group in postsession free recall increases as a function of trial order. Subsequent work with DD has replicated these findings, although the magnitude of the effects has diminished with practice, when list length is held constant. Further evidence for interference has come from studies using error analysis and protocol analysis. These studies relate serial recall errors to confusion of new information with information encoded earlier within a list or in previous trials. Regularities in these errors suggest that they are due to confusion of specific types of information created by DD's different coding mechanisms. For example, errors involving the transposition of entire

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6Rehearsal time is defined as the time between presentation of the last digit in a trial list and the point at which DD begins his serial recall.
digit groups from different supergroup clusters implies confusion of retrieval structure locations within a list. Other errors can be attributed to the semantic similarity of incorrectly recalled digit groups to previously presented digit groups.

Experimental evidence supports the idea that contextual coding enhances DD's list retention by reducing interference between his memory codes during serial recall. In a series of experimental sessions DD was presented with 50-digit lists whose contents were not randomly generated, but carefully constructed to manipulate the opportunities for contextual coding they presented. "Enriched" lists provided many potential opportunities for contextual coding, whereas "depleted" lists were designed to minimize the number of such opportunities. It is important to note that manipulation did not in any way alter DD's normal list parsing, semantic encoding of digit-groups, or retrieval structure indexing.

Table 8 presents measures of DD's serial recall performance as a function of list type. DD's serial recall was near-perfect and did not differ as a function of condition. Consistent with expectations, indices of retrieval speed show that the contents of enriched lists were recalled much more quickly than those of the depleted lists. Analysis of recall times as a function of list type and trial order within a session supported the hypothesis that contextual coding reduces LTM interference. On the first trial of each session, where interference should be minimal, recall times were nearly equivalent for both list types. Recall times rose sharply for depleted lists on subsequent trials within a session, where increased interference is expected. Recall times for enriched lists did not increase appreciably until the fifth and sixth trials, and then only modestly relative to increases observed for depleted lists.

This work provides converging evidence for DD's encoding of contextual relations and suggests that this activity plays an important, but not indispensable role in enabling him to achieve nearly perfect serial recall of rapidly presented 50-digit lists. It seems likely that contextual coding plays a much more important role in achieving perfect serial recall as the length of trial lists increases. Consistent with the view of Chase and Ericsson (1982) and in support of the theory that motivated this study, contextual coding appears to be a mechanism employed to reduce the interference that remains, even after well-elaborated memory traces have been created using mechanisms that semantically encode short random sequences and code their ordinal relations.
An important theoretical point is that DD’s contextual coding represents an emergent mechanism used to achieve exceptional memory performance. This mechanism can be understood in terms of the development of skilled memory. The coding strategies and structures related to contextual coding are based on information created by DD’s use of a retrieval structure and a similarly organized body of semantic knowledge. Practice with these mechanisms increased the efficiency of their operation. The consequence is that large amounts of task-relevant information are encoded, accessed, and recalled both reliably and efficiently. This efficiency is important in that it provides DD with the resources to strategically process information available to him in a working memory whose capacity is expanded by efficient LTM coding and retrieval mechanisms.

Some General Conclusions About DD’s Skill

Collectively, studies of DD’s skill support the hypothesis that DD creates richly elaborated LTM representations for the materials presented on digit-span trials. Various sources of evidence described in this report suggest that the composition of these representations is consistent with the abstract tripartite structure originally proposed by Chase and Ericsson (1982) and shown schematically in Figure 8. In addition, these studies show that the same mechanisms play important roles in retrieving stored information.

Recently, developers of computer models of human learning and skilled performance have discovered the advantages of combining information from different knowledge modules within a system to produce intelligent behavior (Newell, 1989, 1990). Consistent with these findings, this analysis shows that DD’s skill can be decomposed into separate knowledge components whose coordinated interaction produces extraordinarily high levels of skilled performance.

To sum up, DD’s exceptional memory skill results from the interaction of practice, knowledge, and strategies. At the start of his training DD exhibited no unusual general aptitudes or memory abilities. He was selected, however, for his unfamiliarity with a particular body of knowledge that he could bring to his training and given an effective strategy for applying that knowledge to extend his digit-span.

Through practice DD developed new resources for handling the memory demands of longer and longer lists. Practice with the strategies he was given (and the memory mechanisms he developed to implement these strategies) improved his performance in the digit-span task until his
progress brought him to the limitations of these mechanisms. Their use, however, provided him
with the resources to fashion and implement new strategies for dealing with the demands that
larger lists posed. His exploitation of the resources available to him — which include a large and
very well organized knowledge base, a wide repertoire of encoding strategies, and enhanced
information processing efficiency — enabled him to increase his information-processing capacity
and extend the limits of human performance on the digit-span task.

I emphasize what I see as the most novel and important contribution this research makes to
understanding human expertise. In the literature, there is nearly universal agreement that a
cornerstone of expertise is experts’ ability to rapidly encode global patterns of familiar, task-
relevant information (Chase, 1986; Chase and Simon, 1973a, 1973b; Chase & Ericsson, 1982;
Fricsson & Staszewski, 1989; Glaser & Chi, 1988; Newell, 1990; Olson & Rcuter, 1987; Posner,
1988; Tech. Rept. 88-1, 89-1, 89-2). With the exceptions of studies carried out in the context of
SMT, few studies address how these patterns or chunks are represented, retrieved, and used.

Studies of DD’s skill go further. They catalogue the variations in content, structure, and
complexity of the patterns an expert strategically creates to achieve exceptional performance. They
also dissect the knowledge base that supports high-level pattern recognition capabilities,
identifying specific mechanisms used to create one level of pattern information and whose
efficient and interactive operations generate the material and resources for the creation of higher-
order patterns. They also show the same mechanisms are used to retrieve the encoded patterns.
In describing how practice, knowledge, and strategies relate to expert performance, these studies
describe how specific component mechanisms of DD’s skill interact with each other and with
practice to support the development of new and adaptive knowledge structures and memory
coding processes.

Expert Mental Calculation: Structure and Process

Several sources of evidence show that the mental calculation trainees’ memory management
strategies are consistent with the principles of skilled memory. The three following sections
outline the support for this claim. Detailed analyses of the trainees’ computational procedures
also show that efficient computational strategies can reduce the memory demands of mental
calculation and that their adoption can improve performance.
Evidence for Skilled Memory

Retrieval structures in expert mental calculation

One source of evidence for retrieval structures are concurrent verbal protocols collected from GG and JA. Approximately 25-30 hours of verbalized solutions have been collected from each. These data were obtained by asking the trainees to "think aloud" (Ericsson & Simon, 1980; Newell & Simon, 1972) as they solved problems just like those presented in practice sessions in terms of size and mode of presentation.

Clear evidence for retrieval structures comes from the manner in which both trainees structure problem multiplicands, intermediate results, and products in their protocols. For instance, when GG encodes a five-digit multiplicand upon presentation, the pause between his enunciation of the second and third digit is noticeably longer than those separating other consecutive pair of digits (which are approximately equivalent). The same temporal pattern characterizes his subsequent references to this operand within a solution protocol. The finding that this pattern is consistently observed for the vast majority of five-digit multiplicands in the 1x5's and 2x5's indicates that this is an abstract representational format. The pattern of pauses suggests that this format is hierarchically structured, consisting of two intermediate level abstract units, the first containing the first two digits and the second containing the remaining three. The temporal structure of his four-digit multiplicands suggests a abstract hierarchical structure containing two groups of two digits. Similar regularities are evident for his structuring of intermediate results and final products, although it appears that the organizational format used to represent any number that appears in his computations depends upon variables such as its magnitude, the size of the problem, and the function it serves in computation (problem operand, intermediate result, or final product). JA’s protocols reveal similar structural regularities in his representation of numbers, however the specific formats used by JA differ from GG’s.

Chronometric data have been used to confirm these observations. Because listening to the multiplicands recalled by the trainees in post-practice problem recall sessions suggested the same sort of temporal organization seen in their concurrent protocols, the final free recall protocols provided an opportunity to test notions of retrieval structure organization based on the concurrent protocols. Therefore, predictions were made about the structure of multiplicands for different sized problems for each of the trainees and then tested by measuring the pauses between the individual digits of the multiplicands recalled in final free recall.
For example, the tree-structures drawn in the right-hand corner of each of the plots in Figure 9 represent the retrieval structure organizations predicted for GG's 3-, 4-, and 5-digit multiplicands. Note that predicting a single structure for all problems with a common multiplicand size, disregarding the variables of multiplier size (1x or 2x), problem presentation mode (oral or visual), and multiplier value, implies that a common abstract structure is used to code many varied instances.

Making the same assumption used test models of DD's retrieval structure organization (i.e., that a tree traversal process is used to access digits represented by the terminal nodes of a hierarchical associative memory structure), longer pauses are predicted between pairs of digits that span retrieval structure unit boundaries. The data is plotted as a function of intervals between consecutive pairs of digits and problem presentation mode, represent means for a minimum of 50 observations. The important finding is that statistically reliable increases in pause times occur at the predicted inter-digit intervals, confirming that retrieval structures are used to store and access task-critical information in LTM.

Further evidence validating the trainee's use of retrieval structures comes from the successful prediction of the trainees' solution times achieved by process models discussed in a later section of this report. A more detailed description of the retrieval structures employed by the trainees is found in Technical Report 88 i.

Evidence for chunking

A characteristic of expert mental calculators repeatedly cited in the literature is an extensive knowledge base of interrelated and easily accessible number facts (Ball, 1892; Bidder, 1856; Bryan, Lindley, & Harter, 1941; Hunter, 1962, 1977; Jakobsson, 1944; Mitchell, 1907; Mueller, 1911; Sandor, 1932; Smith, 1983). For example, Hunter (1977) reported that Aitken could "automatically" report whether any number up to 1500 is a prime or not and, if not, immediately give its factors. Bryan, Lindley, & Harter (1941) reported that another expert, AG, knew "by heart" the multiplication tables up to 130 x 130, the squares of all numbers up to 130, the cubes of numbers up to 100, fourth powers up to 20, and more. Consistent with these reports, AB exhibits a similarly elaborate knowledge base of number facts and relations, which he reports to have developed not through intentional memorization but, naturally, through years of practice at squaring and multiplying.
In spite of the limited amount of practice GG and JA have had relative to the experts cited above, several kinds of evidence indicate that each has acquired a store of declarative knowledge which resembles the knowledge acquired by lifelong experts in use, if not in volume.

The first signs of such development occurred for both trainees at a remarkably similar point in their training. Around their 200th session, both showed an increasing incidence of unusually fast solution times on certain 1 x 2s. The times for these problems were closer to those normally observed for identity problems (whose multiplier is 1) and "decade" problems (whose multiplicands are multiples of 10) than to the times for problems whose products presumably had to be computed rather than retrieved from memory. When questioned about these instances, both GG and JA invariably reported recognizing a familiar problem and consequently deviating from their usual solution procedures. In many of these instances, they reported immediately "knowing" an answer upon receipt of a problem. In addition, both reported occasionally noticing such familiar problems embedded in larger problems and altering their computational plans accordingly. On the basis of these reports, post-trial retrospective protocols were taken from GG and JA during practice sessions on a regular basis to determine the frequency and circumstances under which such events occurred. In addition, the frequency with which concurrent protocols were taken was increased.

The retrospective and concurrent protocol data both showed evidence consistent with the mnemonic encoding principle of skilled memory. The retrospective protocols showed that the frequency with which the trainees' report noticing familiar subproblems with practice problems increased fairly steadily with practice. Between sessions 226-235, both reported noticing such subproblems on about 30% of the practice trials on which such pattern recognition can occur. These percentages stood at 89% for GG after 450 session and 64% for IA at the end of his practice.

The concurrent protocols show these numeric patterns are treated in two qualitatively different ways in the course of computation. First, these patterns are expressed as quantities rather than as concatenations of single digits. For instance, with the problem 25x4, the multiplicand would be expressed as "twenty-five," rather than as a ordered pair of digits (i.e., "two, five"). Second, such familiar patterns are distinguished by the ways their products are produced. Typically, there is no record of intermediate computation intervening between attention being paid to a familiar subproblems operands and the generation of its product, which
occurs almost instantly. This is consistent with the very fast solution times observed for such 1x2 problems in practice.

In what sense are these patterns meaningful? The answer is best revealed by the trainees' protocols. Concurrent protocols show that their recognition of particular patterns leads to the selection of one of a variety of fairly local computational strategies that each has available. These strategies are local in two senses. First, the patterns on which they are based are small and consist of two elements, a multiplier and multiplicand. GG's patterns, even at the conclusion of his training, were rarely larger than 1 x 2. This was generally true for JA, although occasionally he reported encoding patterns as large as 1 x 4. The limited size of these patterns, which falls within estimates of STM capacity, suggests the constraint that STM imposes on coding processes. Second, these strategies are local in the sense that they are implemented within the larger stereotypic control structures that represent the general left-to-right algorithms that JA and GG use for one- (1x) and two-place (2x) multiplication. Essentially, the patterns that GG and JA recognize and encode represent familiar subproblems which can be solved efficiently with specialized strategies.

Both the patterns and strategies to which particular patterns relate differ for the trainees. In general, JA has a larger repertoire of strategies and a greater variety of pattern classes to which they are related. Like GG, JA has "expanded his multiplication tables" so that there are a variety of 1 x 2s whose products he can retrieve and report in a second or less. His ability to identify quickly the factors of particular numbers enables him to combine factoring and retrieval as a means of solving certain problems and subproblems. In addition, in the course of practice JA discovered an abstract pattern of results related to an abstract class of problems that led him to devise a computational strategy whose basic procedures resemble those taught in the Trachtenberg system of speeded mathematical computation (Cutler & McShane, 1960). Basically, this strategy is a rule-based computational system applicable to problems or subproblems whose multiplier is 9 and whose multiplicand is a sequence of digits that are either identical (e.g., 9 x 444) or else ascend or descend in units of either 1 or 2 (e.g., 9 x 876, 9 x 579, 9 x 234, etc.). Exploiting the redundancies in the products of such problems, JA's strategy enables him to eliminate addition operations from his computations, thus saving him time.

Although GG's strategies are less varied and original than JA's, his strategies illustrate how rapid pattern recognition and efficient strategy use can improve calculation speed. Table 9
presents the principal results of a study that investigated the relation between his strategy use and performance.

In this study the efficiency of GG's strategies was examined by testing him on the entire population of 1 x 2 multiplication problems (excluding problems that use 0 as a multiplier). In the oral condition, each of a set of 810 problems was read just as in oral practice blocks. In the visual condition, problems were presented visually on a CRT. Problems in each condition were presented in random order, and approximately 100 trials were presented in each of 8 sessions conducted on consecutive days. On each trial GG's task was to report the product of the presented problem as quickly as possible and afterward report the strategy he used to solve the problem.

Four basic strategies previously observed in GG's protocols were reported. His Identity strategy, applied to problems with a multiplier of one, is intuitively obvious; GG would simply report the multiplicand that had been presented. He described his second strategy, labelled Retrieval, as one in which he would report the product that he "immediately knew" upon problem presentation. GG's other two 1 x 2 strategies involved sequential arithmetic operations, in contrast to the two already mentioned. The strategy labelled Calculation involved solving 1 x 2s in the way that GG originally solved them at the beginning of his training, just as novices would, using two operations to generate simple products and a third operation to add them. His remaining strategy, labelled Grouping, represents an abbreviated version of full computation. According to GG, this procedure involves two consciously controlled steps. The first operation, he reports, is his immediate and simultaneous retrieval of two simple products upon receipt of a problem. The second operation is their addition. Note that with the exception of problems on which the "full" computation strategy is used, GG's concurrent verbal protocols indicate that he represents multiplicands as single quantities or chunks rather than as discrete symbols. This suggests that the patterns driving strategy selection are specific pairs of quantities.

Table 9 shows the proportion of trials on which each strategy was employed for both presentation conditions, and GG's mean reaction times aggregated as a function of reported solution strategy. Because detailed presentation of these results exceeds the scope of the present discussion, the data will be used to make three general points. First, the "full" strategy is used in only about 5% of the situations where GG used it as a novice. Second, the "retrieval" and "grouping" strategies produce solutions much more quickly than full calculation. It is also the
case that these strategies produce fewer intermediate results, an important consideration is that such results represent potential sources of interference when it is time to retrieve a product for output. Third, GG’s average response latency (the interval between problem presentation and response initiation) for all visually presented 1 x 2s (Mean = 692 msec, SD = 136) approximates solution times that unpracticed adults produce in solving visually presented simple (1 x 1) multiplication problems (Aiken & Williams, 1973; Campbell, 1987). In general, these findings illustrate how knowledge acquired through practice can produce dramatic improvements in calculation speed. The knowledge referred to here consists of meaningful patterns of information associated with specialized computational strategies. These patterns are meaningful in the sense that they are explicitly encoded and used to enable JA and GG to achieve their principle goal, to solve multiplication problems as quickly and as accurately as possible.

The general point here is that AB, GG, and JA all exhibit a form of mnemonic coding that resembles the pattern recognition capabilities of experts from other domains (Chase & Simon, 1973a, 1973b; Eisenstadt & Kareev, 1975; Reitman, 1976). Through extensive practice with a wide variety of problems, these experts have learned to recognize multi-digit patterns that randomly occur both in isolation and embedded in larger problems. These patterns are meaningful in the sense that they are linked to specific computational strategies that reduce calculation times. In their discussion of the role of pattern recognition in the play of chessmasters, Simon & Chase (1973; Chase & Simon, 1973b) suggested that the patterns experts hold in memory are linked to plausible “good” moves. As a result, their ability to rapidly recognize familiar patterns enables them to select moves more efficiently than less skilled players. The current work shows that similar knowledge-based pattern recognition capabilities enable expert mental calculators to employ computational algorithms that decrease solution times. Thus, this work explicitly links complex pattern recognition with strategy selection and high-level performance.

It is theoretically significant that Siegler’s studies of children’s arithmetic (Siegler & Jenkins, 1989; Siegler & Shrager, 1984) show that children’s skills parallel those of GG and JA in several respects. For instance, children discover and employ a variety of computational strategies for solving simple arithmetic problems. In addition, there is good evidence that their strategy selection is apparently determined by their recognition of specific familiar combinations of problem cards. Finally, as their skills improve with practice, memory retrieval replaces multi-
step computation as the preferred solution strategy for an increasing number of problems. These parallels suggest a fundamental continuity in the skill acquisition process across age levels, practice levels, and tasks.

To sum up, it appears that expert mental calculators use semantic memory in three principal ways to achieve fast and accurate performance. First, consistent with Skilled Memory Theory's mnemonic encoding principle, they use an elaborately interrelated knowledge base to recognize and encode meaningful patterns of numbers that occur either as problems or embedded subproblems, thus promoting their retention. Second, much like chessmasters apparently use their pattern recognition capabilities to efficiently select effective chess moves, calculation experts use their unique pattern recognition capabilities to select efficient computational strategies on a problem-by-problem basis. Finally, experts use their knowledge to replace computation with retrieval as a means of generating products and intermediate results, thereby decreasing solution times. Just as SF and DD became expert mnemonists by learning to use semantic memory to encode meaningful patterns of digits, this work shows that GO and JA have developed knowledge bases which they use in a similar fashion to become experts in the domain of mental calculation.

Evidence for speed-up

The speed-up seen in the trainees' solution times implies an underlying increase in the speed with which meaningful patterns are recognized, encoded with retrieval structures, and later retrieved. These data do not provide conclusive support for SMT's speed-up principle, however. Improvements in solution speed can result also from the discovery and use of efficient computational algorithms that decrease both the processing and memory demands related to computing solutions. Clearer support for the speed-up principle comes from data that relate speed-up more directly to memory encoding and retrieval processes.

Figure 10 plots GG's learning curves combining curves for orally and visually presented problems of corresponding problem size in each plot. The shaded area in each plot depicts the visual advantage, that is, the amount of time by which mean solution time for orally presented problems exceeds the mean for visually presented problems. This measure is obtained by subtracting the mean solution time for visually presented problems from the mean for orally presented problems for each block of 5 practice sessions. Unshaded areas between the two
functions indicate blocks in which problems presented orally were solved more quickly than visual problems.

The most salient common feature of these plots is the reduction in shaded area as a function of practice. This reflects a gradual convergence of solution times for the oral and visual presentation. Statistical comparison of oral and visual solution times from sessions 476-500 reveals that a reliable difference between means occurs only for 1 x 1s; here, orally presented problems are solved more quickly. In addition, a consistent but non-significant oral advantage is observed for 1 x 2s and 1 x 3s. Considering GG’s ability to use direct retrieval to solve a good proportion of 1 x 2 problems, presentation of the first problem operand in the oral condition may prime associated patterns in semantic memory and lead to faster retrieval times than those found in the visual condition. For all other problems sizes, the means still reflect a slight visual advantage that is swamped by the variability in solution times. The main point, however, is that the visual advantage evident in the early stages of practice diminishes in all cases with practice. The interpretation of this trend is that GG’s skills at storing and retrieving problem information in LTM have improved with practice to a point where he encodes and operates upon internal representations nearly as quickly as he processes external representations.

The same general pattern, that of a reduction of the visual advantage as a function of practice for all problem sizes larger than 1x3’s, is also observed for TA. Consistent with this practice related trend, further evidence for the speed-up principle is seen in AB’s performance. His solution times show no reliable differences as a function of presentation mode over the same range of problem sizes.

The relevant difference between presentation conditions lies in the demands they place upon memory. Recall that, in the visual condition, problem operands are constantly available for inspection while the solution process proceeds, whereas in the oral condition getting the correct answer to a problem depends on perfect retention of problem operands. Thus, it seems reasonable to assume that the visual advantage stems from the extra time used to encode and retrieve problem operands (and their constituent elements) in the oral condition.

The key empirical finding is that the visual advantage evident in the early stages of practice and logically related to memory load diminishes with practice. The interpretation of this trend is that the trainees’ skills in storing and retrieving problem information in LTM have improved with
practice to a point where they encode and operate upon internal representations nearly as quickly as they process external representations of the same information. This development is consistent with the speed-up principle of skilled memory.

Proces... Strategies, Capacities, and Performance

Although the evidence obtained indicates that skilled memory represents one way experts can increase information-processing efficiency, it is not the only means. Automatizing processing operations represents another (Schneider, Dunais, & Shiffrin, 1984) is another. The use of efficient strategies is still another (Chase & Ericsson, 1982; Cheng, 1985; Hunter, 1977; Simon, 1975; Siegler & Jenkins, 1989). Several findings from the mental calculation training study support this latter claim.

First, the training study originally contrasted the effect of practicing the left-to-right and conventional right-to-left methods of multiplication on the assumption that lightning mental calculators use the most efficient strategies (Chase & Ericsson, 1982; Hunter, 1977). The pattern of early practice results obtained was consistent with this assumption.

Second, analyses of concurrent verbal reports taken from JA and GG as they solved problems revealed a significant difference in the way in which they implemented left-to-right strategy. Although their procedures are more similar than different, sharing many common features including the use of chunking and retrieval structures, the general strategies they applied on 2x problems clearly differed in terms of the number of processing steps involved. JA's strategy was clearly more efficient than GG's, particularly in the number of operations that were devoted to maintaining intermediate results in memory.

Third, model-based analyses also showed differences in processing complexity between JA's and GG's 2x strategies. In fact, it was the protocol evidence that prompted construction of process-models of the trainees' procedures. The purpose was to represent their thought processes in a way that the relative efficiency of their computational strategies could be measured objectively and precisely.

Briefly, these models were designed with control structures that produced the sequence of operations that characterized each trainee's solution procedures across different problems of varying size. A theoretically important feature of the models was their assumption that both GG
and JA used retrieval structures to encode and retrieve problem operands and intermediate results in working memory during their calculations. Because the protocols indicated that most of their arithmetic operations involved only elements of operands and intermediate results, both models assumed that (a) all multiplication and addition operations are carried out on pairs of symbols representing single digits\(^7\), and (b) retrieving individual symbols, nested within hierarchically organized knowledge representations, carries with it the overhead of traversing the abstract architecture used to organize and access these symbols. Thus, basic memory search and retrieval operations that access the inputs for arithmetic operations constitute a major portion of the processing needed to execute each simple arithmetic operation involved in solving a problem. A measure of task complexity related to solving a particular problem is obtained by summing the number of elementary memory operations needed to execute the arithmetic, rehearsal, or reformatting operations that a solution algorithm dictates for a specific problem.

Once the trainees' procedures were captured in running programs, the different programs were given a test set of identical 2x problems. Comparison of the models' performance showed that JA's method was more efficient than GG's in terms of the number of operations it required to produce solutions. A more detailed examination of the models' performance as a function of problem size showed that the relative advantage of JA's strategy varied proportionally with problem size; JA's strategy was only marginally more efficient than GG's in solving 2 x 2s, but its margin of superiority increased monotonically with each increase in multiplicand size.

These findings were consistent with the pattern of results shown repeatedly when JA's and GG's solution times were compared at equivalent levels of practice using samples taken between sessions 100 and 300. GG's average solution times were faster than JA's for all levels of 1x problems, roughly equivalent to JA's times for 2 x 2s, and slower by an increasing margin as problem size increased to 2 x 5s.

To test the validity of these models more directly, the same problems that the trainees received in practice were fed into the simulated programs. Their runs produced estimates of task complexity for each problem of each problem category. These measures were used to

\(^7\)These first-approximation models make no provisions for either trainee's ability to retrieve solutions to familiar 1 x 2s or encode and operate upon their double-digit operands as unitary quantities. Therefore, despite the encouraging results obtained using these models as analytic tools, they face further development and testing before they can be confidently regarded as complete, psychologically valid models of the trainees' skills.
predict the trainees’ solution times. The results were both encouraging and informative. First, across several samples both GG’s and JA’s models could consistently account for about 80% of the variance in their respective solution times. The parameter estimates produced by these analyses implied that JA took roughly 300 msec per assumed processing operation and GG about 250 msec, values corresponding to independent estimates of the durations of goal-directed cognitive operations (Newell, 1990; Simon, 1979).

From this finding, it follows that GG’s solution times should be faster than JA’s on problems whose solutions required approximately the same number of operations. Moreover, because the models show that GG’s solutions for 2 x 3s, 2 x 4s, and 2 x 5s require increasingly more operations than JA’s solutions, the models predict that his solution times should fall farther and farther behind JA’s as the size of 2x’s increases. These predictions fitted the pattern of results shown repeatedly when JA’s and GG’s solution times were compared. Thus, the model-based analyses offered an explanation for the differences observed in trainees’ performance. GG’s relative advantage over JA in processing speed is reflected in his faster solution times on 1x’s, but this advantage is negated on the larger 2x problems by the additional processing operations that his 2x algorithm requires.

To directly test the hypothesis that the differences in the efficiency of JA’s and GG’s 2x computational algorithms might account for this pattern of performance, the following experiment was performed. At the beginning of GG’s 501st practice session, JA’s method for 2x calculation was described to him and he was instructed to use this new method in all subsequent practice sessions. In order to compare GG’s performance using the two strategies in as controlled a fashion as possible, the problems presented in sessions 401-500 were re-presented in sessions 501-600 in the same order and under their original presentation conditions.

Figure 11 plots GG’s average solution times for sessions 476-500, sessions in which he was still using his original 2x algorithm, and sessions 576-600, sessions in which he was fairly well practiced in using JA’s 2x algorithm. Comparison of the functions shows an improvement in solution times for all problems sizes with practice. The average improvement for 1x problems is approximately 7% and represents a baseline against which improvements due to the experimental manipulation can be measured. Focusing on the times for 2x’s, the effects related to switching strategies are interesting in several respects.
First, the general pattern of improvement was predicted in advance by the simulation models' estimates of task complexity; solution times for 2 x 2s, 2 x 3s, 2 x 4s, and 2 x 5s showed decreases of 9%, 25%, 31%, and 40%, respectively. Note that the greatest decreases occur for the problem sizes on which GG's solution times are faster than AB's (2 x 3's, 2 x 4's, and 2 x 5's). The implication is that GG, left to his own devices, would have required considerably more practice to achieve the level of performance that instructional intervention has produced. This result demonstrates how information obtained by analyzing expert performance can be used to "engineer" human expertise in cognitive skills as well as perceptual skills (Biederman & Shiffrin, 1987).

Second, GG was able to adapt to the new algorithm with surprisingly little difficulty. Quantitatively, close inspection of GG's 2x learning curves showed a relatively small and temporary increase in GG's solution times immediately after switching strategies. A small (2%) and temporary increase in his 2x error rates also occurred at the same point. Qualitatively, concurrent verbal protocols taken during the first few days of the strategy switch also showed a corresponding decrease in the fluency of GG's sequence of operations. In retrospective reports, he mentioned having to pay "a little closer attention" to sequencing his operations on each 2x trial, which he believed slowed him down. Significantly, both types of protocols revealed that GG could encode problem operands and intermediate results via his retrieval structures and execute his pattern-driven computational strategies within the new algorithm without any apparent difficulty.

Theories of skill acquisition (Fitts & Posner, 1967; Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977) that emphasize the development of automaticity predict that high levels of practice under stable conditions produce relatively inflexible skills. While there is some evidence for the negative transfer predicted by such theories in the current experiment, the performance decrement related to GG's switch from a familiar strategy to a novel one is trivial compared to that shown when the task environment in which subjects had automatized their skills was altered radically (Experiment 1, Shiffrin & Schneider, 1977). The relative absence of negative transfer is not entirely surprising, because the high variability built into the trainees' practice environment is not conducive to the development of automaticity (Schneider, Dumais, & Shiffrin, 1984).

Although it seems likely that automatization of relatively low-level processes (pattern recognition, memory retrieval) makes an important contribution to G's impressive performance,
the ease with which he adapted to using a new algorithm indicates a considerable degree of
control and flexibility in his skill. This finding supports Bartlett's (1932) views on expertise and
flexibility. This experiment also shows that the acquisition of expert-level skill is a complex
process involving more than the automatization of mental operations (Cheng, 1985) and the
development of skilled memory. Strategy discovery and use play important roles.

4. Methodology: Implications for Theory

The approach this project uses to analyze expertise and its development departs from the
traditional methods used by experimental psychologists and computer scientists in several
respects. The distinguishing features are described followed by a brief discussion of its advantages
and disadvantages.

First, it analyzes complex, goal-directed behavior that stretches often over several tens of
seconds. The activities studied are complex in the sense that they require selection and
coordination, particularly serial organization, of a variety of cognitive processes. The intent is to
catalogue as completely as possible the key structures and processes that contribute to exceptional
human performance as well as the control structures that orchestrate their operation.

Analysis of subjects’ learning and performance is fine-grained. Individuals rather than
groups are studied, and their performance is analyzed on a trial-by-trial basis as they adapt their
activities to the demands of a particular task environment over long periods of practice. The
danger of blindly averaging over subjects and trials is that subtle regularities in behavior that
represent precise and flexible adaptation can be hidden.

The analyses are comprehensive. Once again, the intent is to catalogue as completely as
possible the key mechanisms that contribute to exceptional human performance and the control
structure(s) that govern their interaction. Therefore, multiple tasks are used to study subjects’
performance (i.e., variants of the digit-span task, letter-span, word-span, the Luria Matrix, free
recall, probed recall, etc.) often with multiple methods (verbal protocol analysis, experimental
hypothesis testing, simulation), using multiple measures (chronometric measures, accuracy,
verbal reports). Such a strategy is designed to identify specialized subsystems mediating complex
behavior and to test the range of their application. Also, keeping in mind that the phenomena
under investigation, knowledge and cognitive processes, can only be studied indirectly, the use of
multiple methods is extremely important to insure construct validity. This multitask,
multimethod approach assumes that the strategy of converging operations is the best route to the soundest scientific conclusions.

The obvious disadvantage of the idiographic approach is the difficulty of producing findings that generalize across subjects. However, in spite of the relatively few subjects studied under the rubric of the Skilled Memory Project (by Chase, Ericsson, and Staszewski), common characteristics consistent with the tenets of SMT have been abstracted from different expert subjects from different skill domains. Of course, comprehensive analysis of even a single subject, let alone several, using multiple methods and measures is inherently expensive in terms of time and resources.

Hopefully, the foregoing pages show the reader that the advantages of this approach to studying expertise outweighs the costs. In general, the advantage of this approach is that it can yield relatively detailed, coherent local theories of expertise and its development from which more general theoretical principles can be abstracted. In contrast to much of the previous research on expertise, this approach follows the methodological advice Newell (1973) urged upon cognitive psychology for the sake of sustained theoretical progress. It aims for a comprehensive understanding of the structures, processes, knowledge that experts employ and the way in which these elements are organized as integrated, adaptive information-processing systems that produce goal-directed behavior. Newell (1990) cites the theoretical value of this approach to understanding the complex information-processing that characterizes intelligent thought.

The longitudinal character of this approach is important because it reveals how intelligent systems adapt to the demands of particular tasks with experience. This approach has shown how practice-related changes in knowledge representations and processes produce quantitative and qualitative changes in behavior. Whereas other approaches to the study of expertise have led to the inference that acquired knowledge is its foundations, this approach has succeeded in demonstrating the validity of this claim. Siegler and Jenkins (1989) note how this approach to the study of learning is particularly useful for understanding the relation between cognitive strategies and performance, an important but relatively neglected topic. Most importantly, from the perspective of this project's objectives, this approach has produced new empirical and theoretical insights into how normal individuals overcome innate impediments to learning and performing complex cognitive tasks to achieve levels of performance once considered beyond their capabilities.
5. Practical Implications

Because this project falls into the category of basic research, its main goal was to produce a body of theory and data that would add to our scientific understanding of human expertise and its development. Fundamentally, it has shown that expert knowledge can be analyzed at a relatively fine grain and has demonstrated the value of such analyses for modeling expert performance and designing interventions that facilitate the development of expertise. These accomplishments hold practical implications for the enterprise of "knowledge engineering."

In the context in which it was introduced (Feigenbaum, 1977), the phase "knowledge engineering" referred to the development of expert systems by computer scientists studying artificial intelligence. Researchers and practitioners generally agree that extracting knowledge from experts so that it can be represented in functional programs is perhaps the most crucial and difficult aspect of building such systems (Olson & Reuter, 1987; Waterman, 1986; Winston, 1984). The direct knowledge extraction methods typically used and taught account for this problem to no small degree. They are only suited to identifying knowledge that an expert can consciously access and communicate accurately. Much psychological evidence on the flaws of self-report measures (Nisbett & Wilson, 1977) and the nature and penetrability of expert knowledge indicates that inherent limitations hamper conventional methods.

This project's successful modeling of aspects of its experts' performance suggests that its comprehensive approach to knowledge extraction, combining a complementary variety of direct and indirect methods, offers a viable and potentially valuable alternative to the conventional approach. In addition, the theoretical accomplishments of this project offer knowledge engineers a body of knowledge that can be used to guide their efforts in knowledge extraction. To the extent that this project offers a body of theoretical and methodological principles that computer scientists can apply, it can help make knowledge engineering less an art (Feigenbaum, 1977) and more the scientific activity that the term engineering implies.

The findings of this project's training studies also suggest that "knowledge engineering" extends beyond the field of artificial intelligence to that of instructional design. They show that cognitive science has the methods to discover and explicitly describe expert processing strategies and that interventions based on such research can be used to facilitate the development of expertise. Its description of knowledge structures supporting expertise suggests that organized semantic networks and retrieval structures can be used by educators as targets for instruction and learning (Glaser, 1989; Glaser & Bassok, 1990).
6. Summary of Publications


7. References

Aiken, L. R., & Williams, E. N. (1973). Response times in adding and multiplying single-digit numbers. Perceptual and Motor Skills, 37, 3-13.


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Figure 2

LURIA’S MATRIX

6 6 8 0
5 4 3 2
1 6 8 4
7 9 3 5
4 2 3 7
3 8 9 1
1 0 0 2
3 4 5 1
2 7 6 8
1 9 2 6
2 9 6 7
5 5 2 0
X 0 1 X

TYPES OF RECALL INSTRUCTIONS

<table>
<thead>
<tr>
<th>ENTIRE MATRIX</th>
<th>THIRD COLUMN</th>
<th>SECOND COLUMN</th>
<th>SECOND COLUMN UP</th>
<th>ZIG-ZAG DIAGONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Trainees' Initial and Final Solution Times

**Legend**
- Initial Oral
- Initial Visual
- Final Oral
- Final Visual

**Figure 3**

**GG**

**JA**
Figure 4

Comparison of Trainees and AB

Legend
- o GG Oral
- o GG Visual
- o JA Oral
- o JA Visual
- o AB Oral
- o AB Visual

Solution Time (Sec.) vs Problem Size
Figure 5

50-Digit Lists

75-Digit Lists

100-Digit Lists
### Figure 6

**Semantic Coding Categories**

#### Running Times

- **1/4m**
  - Below 50: 045-049
  - Good College: 120-140
  - World Class: 141-149

- **1/2m**
  - Below 20: 200-240
  - Good College: 241-259

- **3/4m**
  - Below 30: 300-349
  - Good College: 350-370

- **1m**
  - Below 40: 400-449
  - Good College: 450-499

- **2m**
  - Below 50: 500-559
  - Good College: 560-599

- **3k**
  - Below 60: 600-649
  - Good College: 650-699

- **3m**
  - Below 70: 700-749
  - Good College: 750-799

- **10k**
  - Below 100: 1000-1159
  - Good College: 1160-1219

- **10m**
  - Below 100: 1300-1359
  - Good College: 1400-1449

**Non-running Categories**

- **Dates**
  - Near 100: 1050-1059
  - Under 100: 090-099

- **Ages**
  - Over 50: 500-549
  - Under 50: 045-049

**Misc**

*Additional subcategories not pictured*
Figure 7

Running Time

3K

2 Mile

Slow

Race
Times

1100-1159

1000-1049

High School
Races

Under
10 Min
Barrier

Under
9 Min
Barrier

Just
Under 10
953-959

10 Sec Under
959-951

DD's 1st
952

Other
H.S. Runners
940-949

State Meet
Winner
611-920

Just Over
Barrier
900-910

Under 9 Min
Barrier
850-859

1 Sec
Under PR
921

PR
922

State Meet
923

Best Times

College Races

2nd or 3rd
Place
811-820

Go
College Times
820-847

Record
Times
800-810

World
Class
Figure 9

Pause Times in GG's Problem Recall

3-Digit Multiplicands

4-Digit Multiplicands

5-Digit Multiplicands

Inter-Digit Interval
Figure 10

Speed-up in Memory Access

Solution Time (Sec.)

1x5

Solution Time (Sec.)

1x4

Solution Time (Sec.)

1x3

2x5

Solution Time (Sec.)

2x4

Solution Time (Sec.)

2x3

Practice (5-Session Blocks)

Practice (5-Session Blocks)

Practice (5-Session Blocks)
Figure 11

Effects of 2x Strategy Change

Legend
- Sessions # 476–500
- Sessions # 576–600
<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 1$</td>
<td>$x \ 6$</td>
</tr>
<tr>
<td>$1 \times 2$</td>
<td>$x \ 4$</td>
</tr>
<tr>
<td>$1 \times 3$</td>
<td>$x \ 9$</td>
</tr>
<tr>
<td>$1 \times 4$</td>
<td>$x \ 8$</td>
</tr>
<tr>
<td>$1 \times 5$</td>
<td>$x \ 7$</td>
</tr>
<tr>
<td>$2 \times 2$</td>
<td>$x \ 38$</td>
</tr>
<tr>
<td>$2 \times 3$</td>
<td>$x \ 52$</td>
</tr>
<tr>
<td>$2 \times 4$</td>
<td>$x \ 76$</td>
</tr>
<tr>
<td>$2 \times 5$</td>
<td>$x \ 69$</td>
</tr>
</tbody>
</table>
### Table 3

Trainees' Errors Rates (%)

#### First Ten Sessions

<table>
<thead>
<tr>
<th>Presentation</th>
<th>Problem Size</th>
<th>1x1</th>
<th>1x2</th>
<th>1x3</th>
<th>1x4</th>
<th>1x5</th>
<th>2x2</th>
<th>2x3</th>
<th>2x4</th>
<th>2x5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GG</strong></td>
<td>Oral</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>55</td>
<td>70</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>Visual</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>25</td>
<td>20</td>
<td>35</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td><strong>JA</strong></td>
<td>Oral</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>15</td>
<td>25</td>
<td>15</td>
<td>40</td>
<td>41</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Visual</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>15</td>
<td>35</td>
</tr>
</tbody>
</table>

#### Final Thirty Sessions

<table>
<thead>
<tr>
<th>Presentation</th>
<th>Problem Size</th>
<th>1x1</th>
<th>1x2</th>
<th>1x3</th>
<th>1x4</th>
<th>1x5</th>
<th>2x2</th>
<th>2x3</th>
<th>2x4</th>
<th>2x5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GG</strong></td>
<td>Oral</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>13</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Visual</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>10</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td><strong>JA</strong></td>
<td>Oral</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>11</td>
<td>9</td>
<td>13</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Visual</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>
Table 4

Verbal Protocol: DD's Encoding of a 75-digit List

<table>
<thead>
<tr>
<th>Group</th>
<th>DD's Report:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0204</td>
<td>OK, first group was a half mile, oh, two, oh, four. I said oh, two, oh, four, half mile.</td>
</tr>
<tr>
<td>4927</td>
<td>And then, ah, I had back-to-back four, that's forty-nine twenty-seven, a ten mile, and I said that two and seven add up to that nine and had that forty-nine.</td>
</tr>
<tr>
<td>5832</td>
<td>Then, ah, five, eight, three, two was a ten mile and I just said I got back-to-back ten miles and then the three and two add up to that five. I said OK, five's the first digit and these add up to it.</td>
</tr>
<tr>
<td>1800</td>
<td>And, um, then the eighteen hundred I just said was a date.</td>
</tr>
<tr>
<td>352</td>
<td>And then, um, seven, no, three, five, two was a mile time. I said it's a real fast mile time and it's an add-'em-up.</td>
</tr>
<tr>
<td>642</td>
<td>And six, four, two was a mile time, was an add-'em-up and I just said, OK, they're both add-'em-ups, but they're like totally different. I mean one is so much faster than the other one, but they were both back-to-back miles, add-'em-ups.</td>
</tr>
<tr>
<td>928</td>
<td>And then nine two eight was a two mile.</td>
</tr>
<tr>
<td>4658</td>
<td>Then forty-six fifty-eight was a ten mile, and I just said that was twelve apart between the forty-six and fifty-eight.</td>
</tr>
<tr>
<td>4753</td>
<td>And then forty-seven fifty-three was a ten mile and it was six apart, and I said, OK, I got back-to-back ten miles.</td>
</tr>
<tr>
<td>4346</td>
<td>And then ah, four, three, four, six was a mile time, three apart between forty-three and forty six.</td>
</tr>
<tr>
<td>716</td>
<td>Then ah, seven, one, six was a three thousand meter add-'em-up.</td>
</tr>
<tr>
<td>284</td>
<td>Then ah, two eight four was an age.</td>
</tr>
<tr>
<td>444</td>
<td>I had back-to-back fours, it was just four, four, four, was a mile time.</td>
</tr>
<tr>
<td>9025</td>
<td>Then nine, oh, two, five was a two mile</td>
</tr>
<tr>
<td>9390</td>
<td>and nine three nine oh was a two mile. I said, OK, I had nine, three, nine, three before [in a previously presented list], this is just nine, three, nine, oh. That was back-to-back two miles.</td>
</tr>
<tr>
<td>8578</td>
<td>Then the third one was a two mile, so I got three two miles in a row here. It was eight, five, five, eight and it was forward and backwards.</td>
</tr>
<tr>
<td>225</td>
<td>Then, ah, two, eight, five was an age. I said OK, I had, I just had two, eight, four. This is two, eight, five, one tenth of a year older.</td>
</tr>
<tr>
<td>762</td>
<td>Then, ah, seven, six, two was an age. I didn't really do anything with that.</td>
</tr>
<tr>
<td>869</td>
<td>And then eight, six, nine was an age. I said OK, it's almost eighty-seven. I just said OK, I got two back-to-back ages that I really wasn't crazy about. I just wanted to rehearse and get back to them as fast as possible.</td>
</tr>
<tr>
<td>4393</td>
<td>And then the four, three, nine, three was a mile, I said fifty apart.</td>
</tr>
<tr>
<td>4548</td>
<td>And then four, five, four, eight was a mile and I said there was three apart between those.</td>
</tr>
</tbody>
</table>
Table 5

Verbal Protocol: DD's Encoding of a 50-digit List

<table>
<thead>
<tr>
<th>Group</th>
<th>DD's Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>3785</td>
<td>First group, um, the whole first four groups of four, it just went two ages, mile, mile, two ages, and the miles were similar and the two ages were similar, so I just was set on that. I mean I was in great shape. So it was two ages, they were two apart, first group.</td>
</tr>
<tr>
<td>6307</td>
<td>Then the mile, just a little over six and a half minutes I said.</td>
</tr>
<tr>
<td>6261</td>
<td>And the next one, six, two, six, one. I said, OK, it's faster and it's sixty-two sixty-one, and it's one apart between the sixty-two and sixty-one.</td>
</tr>
<tr>
<td>6871</td>
<td>And then the last one was eighty-eight, seventy-one. It's two ages and I just, I didn't figure any age difference, but I knew that the first age was in the eighties and my first group was ages in the eighties, so I was OK with all of that.</td>
</tr>
<tr>
<td>420</td>
<td>Then, ah, four-twenty was a mile. I just said four twenty flat, that was easy enough, a good high school mile.</td>
</tr>
<tr>
<td>799</td>
<td>Seven, nine, nine was an age. I said it was almost eighty years old.</td>
</tr>
<tr>
<td>810</td>
<td>And eight ten was a two mile. I just said it's a really fast two mile.</td>
</tr>
<tr>
<td>6938</td>
<td>Then sixty-nine, thirty-eight was a ten mile, and I just said it was up there, it was like a really slow ten mile.</td>
</tr>
<tr>
<td>5602</td>
<td>And then, ch, fifty-eight oh two was another ten mile and I said, OK, it's almost fifty-eight minutes, it's, it's a good pace ten mile.</td>
</tr>
<tr>
<td>3798</td>
<td>Then uh, thirty-seven ninety-eight was a 10K. It wasn't a legitimate 10K, but I just remember saying OK, it's almost thirty-eight minutes, if you think about it like that.</td>
</tr>
<tr>
<td>663</td>
<td>And then ah... oh, six, three was an age. I said it was, ah, like right around retirement age.</td>
</tr>
<tr>
<td>142</td>
<td>And one forty-two was a half mile. I said it right around world record half mile.</td>
</tr>
<tr>
<td>886</td>
<td>And then eight, eight, six was an age. I didn't really do much with that, because then all of a sudden I had back-to-back sixes, so I linked those two up,</td>
</tr>
<tr>
<td>6933</td>
<td>and it was sixty-nine, thirty-three, another ten mile.</td>
</tr>
</tbody>
</table>
### DD’s Semantic Coding Structures

<table>
<thead>
<tr>
<th>Coding Structure</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Three-digit groups</strong></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>3:52</td>
</tr>
<tr>
<td>Time + Decimal</td>
<td>56.4</td>
</tr>
<tr>
<td>Age + Decimal</td>
<td>79.9</td>
</tr>
<tr>
<td>&quot;0&quot; + Time</td>
<td>049</td>
</tr>
<tr>
<td>&quot;0&quot; + Age</td>
<td>063</td>
</tr>
<tr>
<td>Misc Pattern</td>
<td>111</td>
</tr>
<tr>
<td><strong>Four-digit groups</strong></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>49:27</td>
</tr>
<tr>
<td>Time + Decimal</td>
<td>9:02.5</td>
</tr>
<tr>
<td>Age + Age</td>
<td>8785</td>
</tr>
<tr>
<td>&quot;0&quot; + Three-digit code</td>
<td>02:04</td>
</tr>
<tr>
<td>Date</td>
<td>1955</td>
</tr>
<tr>
<td>Misc Pattern</td>
<td>9876</td>
</tr>
<tr>
<td>Misc Pattern + Decimal</td>
<td>963.2</td>
</tr>
</tbody>
</table>
Table 7

**DD’s Semantic Coding Categories**

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4m</td>
<td>497</td>
</tr>
<tr>
<td>1/2m</td>
<td>142</td>
</tr>
<tr>
<td>3/4m</td>
<td>315</td>
</tr>
<tr>
<td>1m</td>
<td>420</td>
</tr>
<tr>
<td>3k</td>
<td>716</td>
</tr>
<tr>
<td>2m</td>
<td>928</td>
</tr>
<tr>
<td>3m</td>
<td>1430</td>
</tr>
<tr>
<td>10k</td>
<td>2904</td>
</tr>
<tr>
<td>10m</td>
<td>4753</td>
</tr>
<tr>
<td>Date</td>
<td>1800</td>
</tr>
<tr>
<td>Age</td>
<td>284</td>
</tr>
<tr>
<td>Misc.</td>
<td>987</td>
</tr>
</tbody>
</table>
Table 8

DD’s Serial Recall Performance as a Function of List Type

<table>
<thead>
<tr>
<th></th>
<th>Enriched</th>
<th>Depleted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>List Type</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>% Correct</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>99.8</td>
<td>99.8</td>
</tr>
<tr>
<td>SD</td>
<td>0.9</td>
<td>4.4</td>
</tr>
<tr>
<td>t</td>
<td>1.098n.s.</td>
<td></td>
</tr>
<tr>
<td><strong>Rehearsal Time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>29.1</td>
<td>49.9</td>
</tr>
<tr>
<td>SD</td>
<td>13.5</td>
<td>19.4</td>
</tr>
<tr>
<td>Median</td>
<td>25.0</td>
<td>48.0</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td>4.037***</td>
</tr>
<tr>
<td><strong>Recall Time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>43.3</td>
<td>66.7</td>
</tr>
<tr>
<td>SD</td>
<td>27.5</td>
<td>51.5</td>
</tr>
<tr>
<td>Median</td>
<td>34.5</td>
<td>54.5</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td>1.972</td>
</tr>
<tr>
<td><strong>Total Recall Time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>72.4</td>
<td>115.6</td>
</tr>
<tr>
<td>SD</td>
<td>37.4</td>
<td>63.1</td>
</tr>
<tr>
<td>Median</td>
<td>57.8</td>
<td>101.5</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td>2.577**</td>
</tr>
</tbody>
</table>

Note. All tests one-tailed, df = 46. *p < .05; **p < .01; ***p < .001. Times reported in seconds.
Table 9

GG's Strategies for 1 x 2s

<table>
<thead>
<tr>
<th>Condition</th>
<th>Strategies</th>
<th>Identity</th>
<th>Retrieval</th>
<th>Grouping</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oral</td>
<td>Mean RT</td>
<td>230</td>
<td>303</td>
<td>430</td>
<td>839</td>
</tr>
<tr>
<td></td>
<td>Proportion of Trials</td>
<td>11.1</td>
<td>23.2</td>
<td>58.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Visual</td>
<td>Mean RT</td>
<td>527</td>
<td>631</td>
<td>728</td>
<td>1014</td>
</tr>
<tr>
<td></td>
<td>Proportion of Trials</td>
<td>11.1</td>
<td>24.3</td>
<td>61.0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Note: Times given in milliseconds.
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