This manual contains materials for a numeracy course for adult industrial workers. In addition to assessment tests, seven units are provided. Unit topics are whole numbers; fractions; decimals; percents, median, and range; measurement and signed numbers; ratio/proportion and introduction to algebra; and computer literacy using algebra software. Materials within each unit include objectives, dictionary (words and definitions), some explanation, drill sheets (exercises), and a unit review. Unit tests are included. A final section of the manual contains suggested instructional strategies for each unit and some general suggestions for using the manual. (YLB)
Math/Measurement for Upgrading Skills of Industrial Hourly Workers

Math Manual

by

Joan McMahon

Gearing Up for the Future

Project #98-0044
Pennsylvania Department of Education

Northampton Community College
Math/Measurement Literacy for Upgrading Skills of Industrial Hourly Workers

MATH MANUAL

by

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Curriculum Developer

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Disclaimer

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## Table of Contents

<table>
<thead>
<tr>
<th>Assessment Tests</th>
<th>.................................................</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1: Whole Numbers</td>
<td>..................................................................</td>
<td>25</td>
</tr>
<tr>
<td>Unit 2: Fractions</td>
<td>..................................................................</td>
<td>50</td>
</tr>
<tr>
<td>Unit 3: Decimals</td>
<td>..................................................................</td>
<td>112</td>
</tr>
<tr>
<td>Unit 4: Percents, Median and Range</td>
<td>..................................................</td>
<td>139</td>
</tr>
<tr>
<td>Unit 5: Measurement and Signed Numbers</td>
<td>..............................................</td>
<td>159</td>
</tr>
<tr>
<td>Unit 6: Ratio/Proportion and Introduction to Algebra</td>
<td>..........................................</td>
<td>207</td>
</tr>
<tr>
<td>Unit 7: Computer Literacy Using Algebra Software</td>
<td>...........................................</td>
<td>232</td>
</tr>
<tr>
<td>Unit Examinations</td>
<td>..................................................................</td>
<td>234</td>
</tr>
<tr>
<td>Suggested Instructional Strategies/General Suggestions for Using Manual</td>
<td>........................................</td>
<td>252</td>
</tr>
<tr>
<td>Appendix</td>
<td>..................................................................</td>
<td>264</td>
</tr>
</tbody>
</table>
ASSESSMENT TESTS
Write the number.
1. twenty-three ______
2. nine thousand, two hundred, sixty-eight ______

Write the word.
3. 47 ________________
4. 1,091 ________________

How many thousands, hundreds, tens, and ones?
5. 4,679 _____ thousands, _____ hundreds, _____ tens, _____ ones
6. 9,067 _____ thousands, _____ hundreds, _____ tens, _____ ones

Circle the largest number.
7. 1,020 1,021 1,121 8. 999 9,099 9,909

Put these numbers in order. Write the largest first.
9. 5,089 _____ 10. 1,010 _____
   5,090 _____
   5,900 _____
   5,908 _____

Add or subtract.
11.  50
    + 30
    _____
12.  78
    - 42
    _____
13.  113
    + 245
    _____
14.  746
    - 301
    _____
15. There were 34 yes votes and 62 no votes. How many voted at all? 

16. A dealer had 529 cars in stock. 302 cars were sold. How many were left? 

Subtract.

17. 3,146  18. 560  19. 7,005
- 1,908  - 98  - 418

Add.

20. 414 + 36 + 389 + 20 = 

Write the numbers.

21. Fifty thousand, four hundred ninety-eight

22. Eight hundred million, seven thousand, four hundred seven

Find the Answer.

23. 90, 855
- 2, 411

24. 58 + (13 + 32 - 9) - 17 =

25. Find the average of 77, 68, 93, 55, and 82.
26. Write the proper protractor reading shown below.

Estimate to the nearest ten thousand. Circle the correct answer.

27. 4,978,040 people voted. 1,561,820 voted for Scott. How many voted for someone else?

6,420,000 3,420,000 2,360,000

Multiply.

28. 4 x 9 = ____

29. 6 x 5 + 4 - 2 x 3 =

Divide.

30. 81 + 9 = ____

31. 14,877 + 1102 =
   (Express remainder two ways - fraction and decimal)

32. 6 + 6 = ____

Solve this problem.

33. Eduardo gave Spanish lessons. He charged $7 a lesson for each student. There were 9 students in his class. How much did he make for each class?

Multiply.

34. 87
   x 9
   ____

35. 345
   x 8
   ____
Divide. Show if any numbers are left over.

36. 3)398

37. 37)8,650

Find the answer.

38. Mr. Gomez worked 236 days last year. He made $11,800. How much did he make each day?

Add or subtract.

39. 8 9
     11
+ 5
    11
_____
 13

40. 3 5 2
     14
- 1 6 11
    14
_____
 6

41. 2 5 5
    8
- 9 3
    4
_____
 5

42. 3 7 2
    8
- 1 9 5
    6
_____
 3

43. Express 227 as a mixed number.

44. Three pieces are cut from the length of angle iron shown in the following sketch. What decimal fraction of the original length of angle iron (36 in.) is the length of angle iron remaining? Include all 1 in. cuts in the computations.

(Note! in. = ""
45. If a carpenter needs nails longer than 1 3/8-inch, should she use 1 3/4-inch or 1 5/16-inch nails?

46. An electrician needs 20 3/4 feet of wiring for one job and 32 3/8 feet for another job. How many feet of wiring are needed for both jobs?

47. Patrick needs 1 3/4 cups of sugar for a cake and 1 3/4 cups for the frosting. If he has 3 cups of sugar, does he have enough? If not, how much more does he need?

Multiply or divide.

48. \( \frac{3 \frac{1}{2}}{1 \frac{3}{4}} \)  

49. \( \frac{1 \frac{1}{5} \times 2 \frac{1}{4}}{1} \)

50. Find the center of the object below.

51. Find the missing dimension.
Add or Subtract
52. Add .9 and .3

53. Find the center of the circle below.

Multiply or Divide.
54. .0 4 1
    x .0 3

55. 34 ) .0 8 1 6

56. .6 ) 5 .4 7 2

57. .0 2 5 ) 2 3 7. 5

Solve.
58. 9 is percent of 45?

59. What is 5% of 900?

60. 80% of what number is 60?
62. A circle is made up of 360° therefore, a semi circle is made up of _______ degrees.

63. In the circle shown below angle A is 85°, therefore, angle B is _______ degrees.
64 - 71. Below is an American (English) tape measure. Fill in the missing dimension below.
72. You are given the materials and a print of the object on the right. Your job is to lay out the center-lines for the 4 up-right objects to 1/16th + of an inch accuracy so that the spaces are the same size.

Method One -- Solve by using all feet.

Step 1. Draw a sketch of the object and dimension.

Note: There are 4 uprights, but there are 5 spaces.

Step 2. Divide by the number of spaces.
(Carry the decimal 3 places if necessary.)

Step 3. Convert the decimal so you can read the dimension on an American (English) tape measure. Multiply by the desired denominator.

Step 4. Make a list of each separate dimension.
73. Find \( \frac{1}{2} \) of 4'9" = ________

74. Find \( \frac{1}{2} \) of 8 lb. 5 oz. ________

75. Find the length of the brace on the right. Convert answers so it can be read on an American (English) tape measure.

76. Read the settings on the English micrometer scales shown below (0.001").

a. ________

b. ________
Please do the work in the space provided. Place your answers on the answer sheet. Unless otherwise requested, express all decimal answers to two (2) decimal places and reduce all fractions to lowest terms.

Subtract
1. \(7,005 - 418\)

Add
2. \(414 + 36 + 389 + 20\)

Find the Answer
3. \(90,855 - 2,411\)

4. \(58 + \frac{13 + 32 - 9}{3} - 17\)

5. Find the average of 77, 68, 93, 55, and 82.

6. Write the proper protractor reading shown below.

\[\text{Reading} \]
Estimate to the nearest ten thousand. 
Circle the correct answer

7. 4,978,040 people voted. 1,561,820 voted for Scott. How many voted for someone else?

| 6,420,000 | 3,420,000 | 2,360,000 |

Multiply

8. 6 x 5 + 4 - 2 x 3 = ____________

Divide

9. 14,877 ÷ 1,102 = ____________
   (Express remainder two ways - fraction and decimal)

Multiply

10. 87 11. 3,405
    x 9    x 37

Divide - Show if any numbers are left over

12. 3 ) 398
    13. 37 ) 8,650

Find the answer

14. Mr. Gomez worked 236 days last year. He made $11,800. How much did he make each day?

Add or Subtract

15. 89 16. 35 3
    + 11 - 16 11
    + 5    + 14
    + 11

19. Express 237 as a mixed number. 33

_________
20. Three pieces are cut from the length of angle iron shown in the following sketch. What decimal fraction of the original length of angle iron (36 in.) is the length of angle iron remaining? Include all 1 in. cuts in the computations. 

(NOTE: in. = "")

21. If a carpenter needs nails longer than 1 3/8-inch, should she use 1 3/4-inch or 1 5/16-inch nails?

22. An electrician needs 20 3/4 feet of wiring for one job and 32 3/8 feet for another job. How many feet of wiring are needed for both jobs?

23. Patrick needs 1 3/4 cups of sugar for a cake and 1 3/4 cups for the frosting. If he has 3 cups of sugar, does he have enough? If not, how much more does he need?

Multiply or divide

24. \( rac{3 \frac{1}{2}}{1 \frac{3}{4}} \)  

25. \( \frac{1 \frac{1}{5}}{2 \frac{1}{4}} \)
26. Find the center of the object below.

![Diagram of a rectangle with dimensions 11' - 7 7/8']

Solve

27. Find the missing dimension.

![Diagram of a rectangle with missing dimension labeled with 2 7/16 and 2 7/16]}

Add or subtract

28. Add .9 and .3

29. Find the center of the circle below.

![Diagram of a circle with a diameter labeled 4 13/16]}

Multiply or Divide

30. \[
\begin{array}{c}
\text{.041} \\
\times \text{.03}
\end{array}
\]

31. \[
\frac{34}{.0816}
\]
32.  \[ \frac{.6}{5.472} \]

33.  \[ \frac{.025}{237.5} \]

Solve.

34.  9 is percent of 45?

35.  What is 5% of 900?

36.  80% of what number is 60?

37.  \[ \frac{22}{4} = \frac{55}{x} \]

38.  A circle is made up of 360 degrees therefore, a semi circle is made up of ______ degrees.
39. In the circle shown below angle $\alpha$ is $85^\circ$, therefore, angle $B$ is $\underline{\hspace{2cm}}$ degrees.

40 - 47. Below is an American (English) tape measure. Fill in the missing dimension below.
48. You are given the materials and a print of the object on the right. Your job is to lay out the center-lines for the 4 upright objects to 1/16th + of an inch accuracy so that the spaces are the same size.

Method One --- Solve by using all feet:

Step 1. Draw a sketch of the object and dimension.
Note: There are 4 uprights, but there are 5 spaces.

Step 2. Divide by the number of spaces.
(Carry the decimal 3 places if necessary.)

Step 3. Convert the decimal so you can read the dimension on an American (English) tape measure. Multiply by the desired denominator.

Step 4. Make a list of each separate dimension.
49. Find 1/2 of 4'9" = ________________

50. Find 1/2 of 8 lb. 5 oz. ________________

51. Find the length of the brace on the right. Convert answers so it can be read on an American (English) tape measure.

52. Read the settings on the English micrometer scales shown below (0.001"").
   a. ________________
   b. ________________

Round these numbers to the nearest hundreds
53. 5,364 54. 4,921

Round these numbers to two decimal places
55. 3.14159 56. 73.84793
57. \(3.64 \times 10 = \) 

58. In an isosceles triangle, two angles are equal. In some isosceles triangle, Figure below, angles A and B are equal. Angle C is 15.8 more than angle A. Find the three angles.

59. A storage room is 2.5 times longer than its wide. See Fig. below. The perimeter is 28 m. Find the length and width.

60. The total cost of an electrical shop and the lot was $84,800. The building costs $830 more than eight times the cost of the lot. Find the cost of the building and of the lot.

61. A conductor 65 ft long is cut into two pieces. One piece is 5 ft longer than one-half the other. How long is each piece?
62. The sum of the three angles in any triangle is always 180, regardless of the size and shape. In a right triangle (one 90° angle), one acute angle is twice the other, Fig. below. What are the two acute angles?

\[ \text{Fig. below} \]

63. A sphere with a diameter of 3.00 in.

64. A rectangle solid with dimensions: 44 cm; 35.35 cm; and 10.0 cm.

65. \(3.5 \text{ ft} = \) \underline{__________} \text{ in.}

66. \(25 \text{ m} = \) \underline{__________} \text{ cm.}

67. \(0.0003 \text{ kg} = \) \underline{__________} \text{ g.}

Found the total surface area of the following objects. Round as necessary.
ANSWER SHEET

1. __________  23. __________  45. __________
2. __________  24. __________  46. __________
3. __________  25. __________  47. __________
4. __________  26. __________  48. __________
5. __________  27. __________  49. __________
6. __________  28. __________  50. __________
7. __________  29. __________  51. __________
8. __________  30. __________  52. __________
9. __________  31. __________  53. __________
10. __________  32. __________  54. __________
11. __________  33. __________  55. __________
12. __________  34. __________  56. __________
13. __________  35. __________  57. __________
14. __________  36. __________  58. __________
15. __________  37. __________  59. __________
16. __________  38. __________  60. __________
17. __________  39. __________  61. __________
18. __________  40. __________  62. __________
19. __________  41. __________  63. __________
20. __________  42. __________  64. __________
21. __________  43. __________  65. __________
22. __________  44. __________  66. __________

24
UNIT 1: WHOLE NUMBERS

OBJECTIVES:

After studying this unit, you will be able to:

- add, subtract, multiply, and divide whole numbers
- understand the meaning of a whole number or unit
- be able to pick out key words in written problems
- understand the order of operations in solving problems
- solve word problems

Dictionary: Look for the following helpful words which will determine the type of problem.

<table>
<thead>
<tr>
<th>Dictionary</th>
<th>Arithmetic Sign or Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. total</td>
<td>+</td>
</tr>
<tr>
<td>2. sum</td>
<td>-</td>
</tr>
<tr>
<td>3. remaining</td>
<td>-</td>
</tr>
<tr>
<td>4. difference</td>
<td>-</td>
</tr>
<tr>
<td>5. increased by</td>
<td>+</td>
</tr>
<tr>
<td>6. decreased by</td>
<td>-</td>
</tr>
<tr>
<td>7. more than</td>
<td>+</td>
</tr>
<tr>
<td>8. less than</td>
<td>-</td>
</tr>
<tr>
<td>9. added to</td>
<td>+</td>
</tr>
<tr>
<td>10. plus</td>
<td>+</td>
</tr>
<tr>
<td>11. minus</td>
<td>-</td>
</tr>
<tr>
<td>12. subtracted from</td>
<td>-</td>
</tr>
<tr>
<td>13. calculate</td>
<td>perform operations indicated by words or signs</td>
</tr>
<tr>
<td>14. solve</td>
<td>calculate</td>
</tr>
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<td>---</td>
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</tr>
<tr>
<td><strong>15.</strong> find</td>
<td>calculate</td>
</tr>
<tr>
<td><strong>16.</strong> answer units</td>
<td>report answer in same measure or name as applied to the numbers in the problems</td>
</tr>
<tr>
<td><strong>17.</strong> measure</td>
<td>inches, pounds, acres, devices, gallons, etc.</td>
</tr>
</tbody>
</table>
Section 1: Using a Calculator (based on available model)

1-A Instruction for addition (+): 13 + 98 =

1-B Examples
1-B.1 25 + 39 =
1-B.2 107 + 1425 =
1-B.3 111 + 17 + 25,739 =

1-C Instruction for subtraction (-): 98 - 13 =

1-D Examples
1-D.1 129 - 35 =
1-D.2 203 - 193 =
1-D.3 1195 - 50 - 165 =
DRILL SHEET

UNIT 1: USING CALCULATOR - ADDITION AND SUBTRACTION

1D-1  3 + 79 =
1D-2  11 + 97 =
1D-3  14 + 35 =
1D-4  33 + 33 =
1D-5  79 - 14 =
1D-6  55 - 52 =
1D-7  83 - 14 =
1D-8  22 - 6 =
1D-9  44 + 33 =
1D-10 25 + 18 =
1D-11 44 + 13 - 11 =
1D-12 58 + 58 - 47 =
1D-13 47 - 3 + 4 =
1D-14 29 + 29 - 23 =
1D-15 33 - 19 - 11 =
1D-16 1,001 + 1,275 =
1D-17 1,975 + 10,345 =
1D-18 11,989 - 8,763 =
1D-19 5,040 - 3,009 =
1D-20 25,876 + 23,999 =
1D-21 33,011 - 17,666 =
1D-22 38,848 + 160 =
1D-23 38,848 - 1,606 =
1D-24 74,929 - 119 =
1D-25 73,919 - 108 =
Section 2: Using a Calculator

2-A Instruction for multiplication (x): 5 x 16 =
1. set decimal indicator at zero
2. enter 5
3. press [x] key
4. enter 16
5. press [=] key; display shows 80

2-B Examples
2-B.1 14 x 11 = _____
2-B.2 179 x 555 = _____
2-B.3 139 x 93 = _____

2-C Instruction for division (÷): 75 - 5 =
1. set decimal indicator at zero
2. enter 75
3. press [÷] key
4. enter 5
5. press [=] key; display shows 15

2-D Examples
2-D.1 555 ÷ 15 = _____
2-D.2 40 ÷ 8 = _____
2-D.3 900 ÷ 50 = _____
<table>
<thead>
<tr>
<th>DRILL SHEET</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNIT 1: USING CALCULATOR - MULTIPLICATION AND DIVISION</strong></td>
</tr>
<tr>
<td><strong>2D-1</strong></td>
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<td><strong>2D-24</strong></td>
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<tr>
<td><strong>2D-25</strong></td>
</tr>
</tbody>
</table>

30
Section 3: Place Value

A. You are a clerk at a retail store. All you have is dimes and pennies. How do you give someone 43 cents change using the fewest number of coins?

   B. 43 = 10 10 10 10 1 1 1

   28 =

   C. 43 =

   + 28 =

OR

D. Decimal System Place Value

   1. One hundred twenty-three
   2. Two thousand thirty-five
   3. Thirty thousand seven
   4. Sixteen thousand five hundred

hundred  ten
millions thousands thousands thousands hundreds tens ones

1. 1 2 3
2. 
3. 
4. 
Whole numbers are to the left of the decimal point, and numbers less than one whole (.) are to the right of the decimal point.

(In written number form, the word and represents the decimal point.)

Using the chart above as a guide, write the following numbers:

1. Thirty thousand seven

2. One hundred twenty thousand one and thirty seven hundredths

3. Two hundred two and five thousandths

4. Seven and forty-five ten thousandths

5. Ten million six hundred thousand forty-one
Section 4: What is a whole?

The circle above could represent a whole grapefruit, or a whole dollar. It could represent anything that was complete, meaning nothing leftover or missing.

We use integers which are the numerals we associate with counting to make up whole numbers. These are 1, 2, 3, 4...

Word Problems - Addition & Subtraction

Earlier in Lessons 1 and 2 we learned how to add, subtract, multiply, and divide using a calculator. We will now learn that there are words that instruct us what to do in arithmetic operations.

Examples

1. Bob’s hours for the past week are listed below:

   a. What is the total number of hours he worked?

   b. If any hours over 40 hours per week are overtime, how many hours of overtime did he work?

   Monday: 7
   Tuesday: 10
   Wednesday: 6
   Thursday: 12
   Friday: 11

2. Sue’s yearly salary at the XYZ Company was $26,700. When the company relocated, Sue accepted a position with the ABC Company for
$1,900 less than she was making at XYZ. How much is she making at ABC?

3. Shown below is a board used to manufacture micro-electronic devices. The letters, A through H, represent dimensions on the board; the dimension for each letter is measured between lines touched by the arrowheads.

In this exercise, 12 different sets of dimensions are given with 2 dimensions missing in each set. Solve for the missing dimensions (in millimeters); place answers in blank spaces: * Helpful if a straight edge is used under each line.

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<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<td>6.</td>
<td>92</td>
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</tr>
<tr>
<td>9.</td>
<td>237</td>
<td>196</td>
<td>41</td>
<td>103</td>
<td>701</td>
<td>172</td>
<td></td>
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<tr>
<td>10.</td>
<td>39</td>
<td>29</td>
<td>47</td>
<td>29</td>
<td>100</td>
<td>101</td>
<td></td>
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<tr>
<td>11.</td>
<td>96</td>
<td>68</td>
<td>47</td>
<td>48</td>
<td>210</td>
<td>190</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>207</td>
<td>109</td>
<td>117</td>
<td>290</td>
<td>400</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

34
DRILL SHEET

UNIT 1: WORD PROBLEMS - ADDITION, SUBTRACTION, MULTIPLICATION & DIVISION

3D-1 A section of a concrete wall and footing for a clean room are shown in the figure below. What is the total height of the wall and footing?

3D-2 During the first week of April, the following number of wafers were cut from silicon rods: 5,570 wafers on Monday; 7,855 wafers on Tuesday; 7,236 wafers on Wednesday; 6,867 wafers on Thursday; and 6,643 wafers on Friday. During the following week, 4,050 more wafers were cut than during the first week. Find the total wafers cut during the first two weeks of April.
3D-3 The figure below shows a graph constructed for the product 12. Construct a graph for the product 24. Will it have the same shape?
A duplicating machine operator at ALPO makes copies of printed material for various departments. A log is kept to record the number of copies made for each department. The monthly log is shown in Fig. 1-2:

<table>
<thead>
<tr>
<th>DEPARTMENT</th>
<th>WEEK 1</th>
<th>WEEK 2</th>
<th>WEEK 3</th>
<th>WEEK 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>853</td>
<td>712</td>
<td>956</td>
<td>1088</td>
</tr>
<tr>
<td>Eng. &amp; Design</td>
<td>1050</td>
<td>936</td>
<td>277</td>
<td>732</td>
</tr>
<tr>
<td>Personnel</td>
<td>2756</td>
<td>1935</td>
<td>2080</td>
<td>993</td>
</tr>
<tr>
<td>Accounting</td>
<td>830</td>
<td>0</td>
<td>344</td>
<td>130</td>
</tr>
<tr>
<td>Sales</td>
<td>1202</td>
<td>555</td>
<td>3859</td>
<td>2444</td>
</tr>
<tr>
<td>Data Processing</td>
<td>85</td>
<td>53</td>
<td>0</td>
<td>187</td>
</tr>
<tr>
<td>Purchasing</td>
<td>1932</td>
<td>1637</td>
<td>767</td>
<td>845</td>
</tr>
<tr>
<td>Inspection</td>
<td>177</td>
<td>286</td>
<td>53</td>
<td>0</td>
</tr>
<tr>
<td>Rec. &amp; Shipping</td>
<td>538</td>
<td>613</td>
<td>423</td>
<td>778</td>
</tr>
</tbody>
</table>

Fig. 1-2

a. Find the total number of copies made in week 1
b. Find the total number of copies made in week 2
c. Find the total number of copies made in week 3
d. Find the total number of copies made in week 4
e. Find the total number of copies made for the month:

ALPO was charged $465 for repairs to the company shuttle bus. The charges for labor are $196. Paint and materials cost $67 and replacement parts cost $110. How much profit is made by the outside contractor?
3D-6 The tool and die department must know dimensions A and B in order to finish grind surfaces F and G of the component for a testing machine shown in the figure below. All dimensions are in millimeters:

![Diagram of component with dimensions A and B indicated.]

a. Solve for A; report answer in millimeters

b. Solve for B; report answer in millimeters

3D-7 The cafeteria baker makes a batch of cookie mix which weighs 48 pounds. Pastry flour and other ingredients are used to make the mix. The weights of the other ingredients are 13 pounds of almond paste, 12 pounds of margarine, 5 pounds of egg whites, and 7 pounds of granulated sugar. How many pounds of pastry flour are used?

3D-8 Sue March went to town and bought a skirt for $22, and a hat for $7. How much money did she receive in change from two $20 bills?

3D-9 During a basketball game, seven players scored the following points: 12, 9, 21, 16, 14, 17, and 35. If these were the only scores for the team, what was the team’s final score?

3D-10 There are three machines available to saw-cut silicon rods. Machine A cuts 13 inches per minute, Machine B cuts 8 inches per minute, and Machine C cuts 11 inches per minute. How many inches of silicon can be cut in 1 minute if all three machines are working?
3D-11 If four containers in a storage area have a capacity of 12 gal., 27 gal., 55 gal., and 21 gal., could 100 gal. of acid be stored in those containers?

3D-12 How many feet (') of floor marking are needed to enclose the inspection area illustrated in the following figure:

```
3D-13 During a one-month period, the maintenance department installed the following numbers of switch outlets: 23, 14, 36, 27, 19, 21, 34, and 28. What is the total number of outlet boxes installed?

3D-14 The credit union clerk received the following amounts from employees: $85, $125, $137, $96, and $109. How much total money was received?

3D-15 The shipping department air freighted the following amounts of micro-electronic components: 7,286 lb., 8,106 lb., 7,832 lb., and 8,215 lb. What was the total weight shipped?

3D-16 The following pounds of solder were purchased by ALPO during a one-month period: 40, 75, 125, 70, 150, 80, 95, 110, and 60. What is the total number of pounds of solder purchased that month?
3D-17 Three lengths of tubing for packaging are needed: 15 in., 18 in., and 10 in. Can these pieces be cut from a length of tubing 42 in. long?

3D-18 An ALPO pipefitter needs 23 pieces of pipe. Each piece must be 443 centimeters (cm) long. What is the length of pipe needed?

3D-19 How many reams of paper are required by a printer who has 12 jobs to do for ALPO, each of which requires 32 reams of paper?

3D-20 If the maintenance contractor has 15 employees who are paid $185 each per week, what is the weekly payroll for the employees?

3D-21 A truck can haul 400 fixtures. How many fixtures can be moved by the truck in 15 trips?

3D-21 Thirty-five rolls of electrical cable were ordered, each contains 1025 ft. What is the total number of feet of cable on the rolls?

3D-22 An ALPO construction estimator earns $2600 per month. How much is earned in 6 months?

3D-23 How many resistors are needed to build 30 TV’s if each TV requires 305 resistors?

3D-24 An assembly line can produce 23,090 parts a day. If the assembly line has no problems, how many parts can be produced in 20 working days?

3D-25 What is the weekly payroll if 48 employees each earn $218 a week?

3D-26 In one ALPO building where 46 outlets are being installed, 1472 ft. of cable are used. What is the average number of feet used per outlet?

3D-27 Twelve water tanks are constructed in a welding shop at a total contract price of $14,940. What is the price per tank?
Fifteen packages of conduit are purchased. The total weight of the shipment is 3120 lb. What is the weight per package?

Now try this!

Arrange numbers in the figure below so that the sum around each circle and along each straight line is the same.

With experience, the distributive property can be used for more difficult mental computations.

Thought Process

\[
35 \times 102 = 35 \times (100 + 2) \\
= (35 \times 100) + (35 \times 2) \\
= 3500 + 70 \\
= 3570
\]

Thought Process

\[
45 \times 98 = 45 \times (100 - 2) \\
= (45 \times 100) - (45 \times 2) \\
= 4500 - 90 \\
= 4410
\]

Can you find these products mentally?

\[
15 \times 103 \quad 101 \times 92 \quad 18 \times 99
\]
Section 5: Order of Operations

1. First, do all operations within grouping symbols. Grouping symbols are parentheses (), brackets [], and braces {}.

2. Next, do multiplication and division operations in order from left to right.

3. Last, do addition and subtraction operations in order from left to right.

* Reminder that * is a sign for multiplication also.

Examples

1. Find the value of

   \[(15 + 6) \times 3 - 28 - 7 = \]
   \[21 \times 3 - 28 - 7 = \]
   \[63 - 28 \div 7 = \]
   \[63 - 4 = \]
   \[= 59\]

2. \[9 + 12 - 5 = \]
   \[21 - 5 = \]
   \[= 16\]

3. \[35 + 30 \div 5 = \]
   \[35 + 6 = \]
   \[= 41\]

4. \[(35 + 30) - 5 = \]
   \[65 - 5 = \]
   \[= 13\]

5. \[(10 \times 8) - (5 \times 4) = \]
   \[80 - 20 = \]
   \[= 4\]

6. \[(240 \div 80) \times 15 - 3 = \]
   \[3 \times 15 - 3 = \]
   \[45 - 3 = \]
   \[= 15\]

42
<table>
<thead>
<tr>
<th>DRILL SHEET</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNIT 1: ORDER OF OPERATIONS</td>
</tr>
<tr>
<td>4D-1</td>
</tr>
<tr>
<td>4D-2</td>
</tr>
<tr>
<td>4D-3</td>
</tr>
<tr>
<td>4D-4</td>
</tr>
<tr>
<td>4D-5</td>
</tr>
<tr>
<td>4D-6</td>
</tr>
<tr>
<td>4D-7</td>
</tr>
<tr>
<td>4D-8</td>
</tr>
<tr>
<td>4D-9</td>
</tr>
<tr>
<td>4D-10</td>
</tr>
<tr>
<td>4D-11</td>
</tr>
<tr>
<td>4D-12</td>
</tr>
<tr>
<td>4D-13</td>
</tr>
<tr>
<td>4D-14</td>
</tr>
<tr>
<td>4D-15</td>
</tr>
<tr>
<td>4D-16</td>
</tr>
<tr>
<td>4D-17</td>
</tr>
<tr>
<td>4D-18</td>
</tr>
<tr>
<td>4D-19</td>
</tr>
<tr>
<td>4D-20</td>
</tr>
</tbody>
</table>

43

48
DRILL SHEET

UNIT 1: PRACTICAL APPLICATIONS REQUIRING ORDER OF OPERATIONS OF ARITHMETIC EXPRESSIONS

5D-1 An engine is used by ALPO’s receiving department to lift heavy crates on the loading dock. The horsepower needed to lift crates can be found as follows:

horsepower = (weight of crate x distance lifted - time) ÷ 550

Find the horsepower needed to lift each of the crates listed in the following table:

<table>
<thead>
<tr>
<th>CRATE</th>
<th>WEIGHT OF CRATE</th>
<th>DISTANCE LIFTED</th>
<th>TIME</th>
<th>HORSEPOWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>660</td>
<td>10</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1100</td>
<td>12</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1650</td>
<td>14</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3300</td>
<td>8</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>2640</td>
<td>10</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

5D-2 The accounting department computes the annual depreciation of each piece of tooling, equipment, and machinery in ALPO. From a detailed itemized list, the accounting department groups all items together that have the same life expectancy (number of years of usefulness) as shown in the following table:

<table>
<thead>
<tr>
<th>GROUP</th>
<th>COST</th>
<th>FINAL VALUE</th>
<th>YEARS OF USEFULNESS</th>
<th>ANNUAL DEPRECIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Tooling</td>
<td>$14,500</td>
<td>$1,200</td>
<td>5 years</td>
<td></td>
</tr>
<tr>
<td>b. Equipment</td>
<td>$28,350</td>
<td>$3,750</td>
<td>6 years</td>
<td></td>
</tr>
<tr>
<td>c. Equipment</td>
<td>$17,900</td>
<td>$2,040</td>
<td>17 years</td>
<td></td>
</tr>
<tr>
<td>d. Machinery</td>
<td>$67,700</td>
<td>$7,940</td>
<td>9 years</td>
<td></td>
</tr>
<tr>
<td>e. Machinery</td>
<td>$80,300</td>
<td>$10,600</td>
<td>10 years</td>
<td></td>
</tr>
</tbody>
</table>

TOTAL ANNUAL DEPRECIATION

Find the annual depreciation for each group and the total depreciation of all tooling, equipment, and machinery, as follows:

Annual Depreciation = (cost - final value) ÷ number of years of usefulness
A general rule is used in the health field to determine a child’s dosage of medicine; it is as follows:

\[
\text{Child’s dosage} = \frac{\text{age}}{\text{age of child}} \times \frac{\text{age}}{\text{of child} + 12} \times \text{average dosage}
\]

What dose (number of milligrams) of morphine sulfate should be given to a 3 year-old child if the adult dose is 10 milligrams?

It should be noted that the weight of a child plays a large role in the dosage of medicine.
UNIT 1: REVIEW

-1R1- \[362 \div 1,491 + 73 + 29,248\]

-1R2- \[4,793 - 404\]

-1R3- \[7,878 \times 403\]

-1R4- \[1755 \div 27\]

-1R5- \[58 + \frac{13 + 32 - 9 - 17}{3}\]

-1R6- A printing shop prints 9,600 ALPO letterheads on Monday, 11,760 on Tuesday, 13,354 on Wednesday, 8,846 on Thursday, and 12,215 on Friday. How many letterheads are printed during the week?

-1R7- Five machines at ALPO produce the same product. Each machine has a counter which records the number of parts produced. Counter readings for the beginning and end of one week's production are shown in the following table:

<table>
<thead>
<tr>
<th>Machine</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counter Reading Beginning of Wk</td>
<td>18,925</td>
<td>14,382</td>
<td>8,408</td>
<td>36,604</td>
<td>903</td>
</tr>
<tr>
<td>Counter Reading End of Week</td>
<td>47,763</td>
<td>41,206</td>
<td>36,441</td>
<td>70,325</td>
<td>28,027</td>
</tr>
<tr>
<td>Number of Parts Produced</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. How many parts does each machine produce during the week?

b. What is the total weekly production?
In order to make the photostat fixture shown below, a tool and die maker must determine dimensions A, B, C, and D.

- Dimension A
- Dimension B
- Dimension C
- Dimension D

ALPO's Pioneer Store orders 18 cartons of hand calculators. Each carton contains 64 calculators. They pay $9.00 per calculator, and then sell each at $7.00 profit. What is the total money collected when all the calculators are sold?

If one piece of shipping tubing is 111 in. long, how much will be left if you cut a 79-in. piece off the tubing?

Joe weighs 236 lb. now. Joe weighed 294 lb. two years ago. How many pounds did Joe lose during the past two years?

The cooling system of a car will hold 24 qt. of water and antifreeze. If the cooling system is full and contains 2 gal. of antifreeze, how many quarts of water are in the cooling system? (1 gal. = 4 qts.)
If your church social group made 4,000 hoagies for a benefit and distributed 2,641 of them, could they fill an order for 1,540 more?

A ream of paper contains 500 sheets. How many sheets are contained in 23 reams of paper?

ALPO has 17 cars with six-cylinder engines that need the spark plugs replaced. If the plugs come in packages of eight, how many packages do they purchase to handle this job?

A part-time computer operator earned $1,755 in 13 weeks. How much did he earn per week?

How long a piece of carpet (in inches) should you buy for a runner for the steps shown in the figure below:

\[ \text{NOTE: There are 5 steps} \]
-1R18- Find the missing measure in the following figure:

-1R19- For her annual salary, a furnace operator received the following monthly pays: $1096; $1117; $1067; $1742; $1896; $2056; $2308; $1483; $1597; $1223; $1074; and $1029. What was the furnace operator's annual salary?

-1R20- If a machine setter earns $23,400 annually, what is the monthly salary?

-1R21- \[21 \div (7 \times 3) + (4 \times 5) + 25\]  

-1R22- \[(720 \div 360) + (180 \times 2) - 1\]
UNIT 2: COMMON FRACTIONS

OBJECTIVES

After completing this unit, you will be able to:

- understand the meaning of a fraction, or part of the whole
- define the terms numerator, denominator, improper fraction, proper fraction, and mixed number
- write equivalent fractions
- express fractions in lowest terms
- understand the meaning of a mixed number
- express fractions as mixed numbers
- express mixed numbers as fractions
- determine lowest common denominator
- add fractions, mixed numbers, and whole numbers
- subtract fractions, mixed numbers, and whole numbers
- multiply fractions, mixed numbers, and whole numbers
- divide fractions, mixed numbers, and whole numbers
- follow order of operations involving fractions
- solve word problems involving fractions

Dictionary:

<table>
<thead>
<tr>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
</tr>
<tr>
<td>Numerator</td>
</tr>
<tr>
<td>Denominator</td>
</tr>
</tbody>
</table>
4. **Terms**
   - numerator and denominator in a common fraction

5. **Common**
   - familiar.

6. **Common Fraction**
   - one whole number divided by another whole number

7. **Proper Fraction**
   - less than a whole or one

8. **Improper Fraction**
   - more than a whole or one

9. **Mixed Number**
   - a whole number with a proper fraction

10. **Equivalent**
    - identical or same
Section 1: Concept of a Fraction

1-A What is a fraction?

Note in the above illustration that a part, slice, or fraction of the whole circle or "pie" is missing.

Now note in the above illustrations that there is a whole circle or "pie" on the left and a part or slice of the same size circle or pie on the right. This slice is a fraction of the whole remaining, or "leftover."

Thus, the subject of fractions is nothing more than the study of parts or portions that make up the whole number.

A whole number, as defined earlier in Unit 1, means "nothing leftover." For example when you divide any number by itself, the result will always equal one.

Examples

\[
\frac{11}{11} = 1; \quad \frac{135}{135} = 1; \quad \frac{63}{63} = 1; \quad \frac{1025}{1025} = 1
\]

1-B When we divide one whole number by another whole number, we create a common fraction.

Every common fraction consists of two parts: a whole number above the division sign, called the numerator; and a whole number below the division sign, called the denominator.

The denominator indicates the fractional parts that make up the whole. When we speak of a steel rule or ruler or
yardstick having an inch with quarter (1/4), eighth (1/8), sixteenth (1/16), or thirty-second (1/32) graduations, we are indicating the number of parts of these that make up the whole: 4; 8; 16; 32.

There are two types of common fractions, proper and improper. In a proper fraction, the numerator (upper number) is always less, i.e., smaller than the denominator (lower number). In an improper fraction, the numerator is always greater, i.e., larger than the denominator.

1-C Examples

<table>
<thead>
<tr>
<th>proper</th>
<th>improper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3, 5, 9</td>
<td>5, 2, 7, 12, 37</td>
</tr>
<tr>
<td>4 8 8 16 32</td>
<td>4 8 6 16 32</td>
</tr>
</tbody>
</table>

As you can see from the preceding examples, proper fractions are less than a whole, and improper fractions are more than a whole, (or greater than 1).

Note: One reason for reducing an improper fraction such as 13, is that measuring-instrument parts (increments) are usually written and located in reduced form.

Example: [It is easier to find 1 5 on a tape measure than to count out thirteen 1 parts.]
DRILL SHEET

UNIT 2: PROPER AND IMPROPER FRACTIONS

Complete the following:

<table>
<thead>
<tr>
<th>Proper Fractions</th>
<th>Improper Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D-1 1/2</td>
<td>1D-16 5/2</td>
</tr>
<tr>
<td>1D-2 1/4</td>
<td>1D-17 1/4</td>
</tr>
<tr>
<td>1D-3 1/8</td>
<td>1D-18 1/8</td>
</tr>
<tr>
<td>1D-4 7/16</td>
<td>1D-19 1/16</td>
</tr>
<tr>
<td>1D-5 7/32</td>
<td>1D-20 7/32</td>
</tr>
<tr>
<td>1D-6 7/6</td>
<td>1D-21 7/6</td>
</tr>
<tr>
<td>1D-7 7/9</td>
<td>1D-22 7/9</td>
</tr>
<tr>
<td>1D-8 7/12</td>
<td>1D-23 7/12</td>
</tr>
<tr>
<td>1D-9 7/15</td>
<td>1D-24 7/15</td>
</tr>
<tr>
<td>1D-10 64/64</td>
<td>1D-25 64/64</td>
</tr>
<tr>
<td>1D-11 50/50</td>
<td>1D-26 50/50</td>
</tr>
<tr>
<td>1D-12 27/27</td>
<td>1D-27 27/27</td>
</tr>
<tr>
<td>1D-13 23/23</td>
<td>1D-28 23/23</td>
</tr>
<tr>
<td>1D-1 25/25</td>
<td>1D-29 25/25</td>
</tr>
<tr>
<td>1D-15 17/17</td>
<td>1D-30 17/17</td>
</tr>
</tbody>
</table>

54
Section 2: Mixed Numbers

When dividing the numerator of an improper fraction by the denominator (i.e., the upper number by the lower number), the result will be some whole number and a fractional part remaining or "leftover".

For example \( \frac{37}{32} = 1 \) whole and a remainder of \( \frac{5}{32} \),

or \( \frac{32}{32} = 1 + \frac{5}{32} \), written as \( 1 \frac{5}{32} \);

this is a mixed number.

2-A Examples

1. \( \frac{47}{16} = 2 \frac{15}{16} \)

   \[ \text{solution} \]
   \[ \frac{16}{16} = 1; \quad \frac{16}{16} = 1; \quad \text{remainder} \frac{15}{16} \]
   \[ \frac{16}{16} \]
   adding:
   \[ 1 + 1 + \frac{15}{16} = 2 \frac{15}{16} \]

2. \( \frac{73}{64} = 1 \frac{9}{64} \)

   \[ \text{solution} \]
   \[ \frac{64}{64} = 1; \quad \text{remainder} \frac{9}{64} \]
   \[ \frac{64}{64} \]
   adding:
   \[ 1 + \frac{9}{64} = 1 \frac{9}{64} \]

3. \( \frac{123}{32} = 3 \frac{27}{32} \)

   \[ \text{solution} \]
   \[ \frac{32}{32} = 1, \quad \frac{32}{32} = 1, \quad \frac{32}{32} = 1; \]
   \[ \frac{32}{32} \]
   remainder \( \frac{27}{32} \)
   adding:
   \[ 1 + 1 + 1 + \frac{27}{32} = 3 \frac{27}{32} \]
**DRILL SHEET**

**UNIT 2: EXPRESSING IMPROPER FRACTIONS AS MIXED NUMBERS**

Convert each improper fraction to a mixed number:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2D-1</td>
<td>67 = 64</td>
<td>2D-16</td>
<td>7 = 3</td>
</tr>
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<td>2D-15</td>
<td>33 = 19</td>
<td>2D-30</td>
<td>14107 = 2439</td>
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Section 3: Equivalent Fractions

In order to solve common fraction problems, it is necessary to be able to express any of these as an equivalent fraction. This means the numbers will change but the value will not.

3-A Examples

1. Express $\frac{23}{7} = \frac{?}{28}$ as an equivalent fraction.

   **Solution:**
   - **Step 1** Set up a multiplier fraction, e.g.,
     $$\frac{23}{7} \times \left(\frac{-}{-}\right) = \frac{28}{28}$$
   - **Step 2** divide 28 by 7 = 4
   - **Step 3** substitute 4 in the numerator and the denominator
     $$\frac{23 \times (4)}{7 \times (4)} = \frac{22}{28}, \text{ Answer}$$
   - **Note:** Since $4 = 1$, and $1 \times \frac{23}{7} = \frac{23}{7}$, therefore
     the value of $\frac{23}{7}$ does not change, although the resultant numbers changed to $\frac{22}{28}$.

2. Express $\frac{15}{32} = \frac{?}{160}$ as an equivalent fraction.

   **Solution:**
   - **Step 1** Set up a multiplier fraction.
   - **Step 2** divide 160 by 32 = 5
   - **Step 3** substitute 5 in the numerator and denominator
     $$\frac{15 \times (5)}{32 \times (5)} = \frac{75}{160}, \text{ Answer}$$
   - **Note:** Since $5 = 1$, and $1 \times \frac{15}{32} = \frac{15}{32}$, the value of $\frac{15}{32}$ does not change, although the resultant numbers changed to $\frac{75}{160}$. 
### DRILL SHEET

#### UNIT 2: EQUIVALENT FRACTIONS

Express each common fraction as an equivalent fraction:

<table>
<thead>
<tr>
<th>3D-1</th>
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<th>3D-13</th>
<th>3/3</th>
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<td>7/71</td>
<td>=</td>
<td>3D-15</td>
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<td>=</td>
<td>16/16</td>
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<td>3D-4</td>
<td>9/18</td>
<td>=</td>
<td>3D-16</td>
<td>3/16</td>
<td>=</td>
<td>64/64</td>
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<td>=</td>
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<td>7/8</td>
<td>=</td>
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<td>=</td>
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<td>9/32</td>
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<td>=</td>
<td>3D-19</td>
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<td>4/15</td>
<td>=</td>
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<td>7/16</td>
<td>=</td>
<td>48/48</td>
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<td>6/11</td>
<td>=</td>
<td>3D-22</td>
<td>14/17</td>
<td>=</td>
<td>51/51</td>
</tr>
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<td>3D-11</td>
<td>23/25</td>
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<td>3D-12</td>
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<td></td>
<td>3D-25</td>
<td>5/32</td>
<td>=</td>
<td>96/96</td>
</tr>
</tbody>
</table>
Section 4: Expressing Fractions In Lowest Terms

Unless specifically instructed not to, common fractions are to be reduced to their lowest terms.

4-A Examples

1. Express \( \frac{148}{36} \) as a fraction in lowest terms.

   solution: Step 1 Determine one common number that can be divided into 148 and 36 so that the results are whole numbers, e.g.,
   
   \( 148 \div 4 = 37, \text{ and } 36 \div 4 = 9, \text{ thus } \frac{148}{36} = \frac{37}{9} \), Answer

   Step 2 Continue this procedure until no further common number can be divided into both the numerator and denominator so that the result is a whole number.

2. \( \frac{3}{6} = 3 \div 3 = 1, \text{ and } 6 \div 3 = 2, \text{ thus } \frac{3}{6} = \frac{1}{2} \), Answer

3. \( \frac{5}{30} = 5 \div 5 = 1, \text{ and } 30 \div 5 = 6, \text{ thus } \frac{5}{30} = \frac{1}{6} \), Answer

4. \( \frac{6}{16} = 6 \div 2 = 3, \text{ and } 16 \div 2 = 8, \text{ thus } \frac{6}{16} = \frac{3}{8} \), Answer

5. \( \frac{180}{15} = 180 \div 15 = 12, \text{ and } 15 \div 15 = 1, \text{ thus } \frac{180}{15} = \frac{12}{1} \) or 12, Answer

6. \( \frac{21}{49} = 21 \div 7 = 3, \text{ and } 49 \div 7 = 7, \text{ thus } \frac{21}{49} = \frac{3}{7} \), Answer

59
UNIT 2: EXPRESSING FRACTIONS IN LOWEST TERMS

Express each common fraction in lowest terms:

4D-1  \[\frac{9}{27} = \frac{1}{3}\]
4D-2  \[\frac{12}{16} = \frac{3}{4}\]
4D-3  \[\frac{24}{32} = \frac{3}{4}\]
4D-4  \[\frac{4}{8} = \frac{1}{2}\]
4D-5  \[\frac{14}{8} = \frac{7}{4}\]
4D-6  \[\frac{21}{9} = \frac{7}{3}\]
4D-7  \[\frac{30}{16} = \frac{15}{8}\]
4D-8  \[\frac{28}{8} = \frac{7}{2}\]
4D-9  \[\frac{14}{12} = \frac{7}{6}\]
4D-10 \[\frac{32}{96} = \frac{1}{3}\]
4D-11 \[\frac{48}{96} = \frac{1}{2}\]
4D-12 \[\frac{32}{64} = \frac{1}{2}\]
4D-13 \[\frac{22}{16} = \frac{11}{8}\]
4D-14 \[\frac{12}{8} = \frac{3}{2}\]
4D-15 \[\frac{16}{8} = \frac{2}{1}\]
4D-16 \[\frac{32}{16} = \frac{2}{1}\]
4D-17 \[\frac{24}{8} = \frac{3}{1}\]
4D-18 \[\frac{8}{24} = \frac{1}{3}\]
4D-19 \[\frac{16}{32} = \frac{1}{2}\]
4D-20 \[\frac{51}{17} = \frac{3}{1}\]
4D-21 \[\frac{17}{51} = \frac{1}{3}\]
4D-22 \[\frac{28}{56} = \frac{1}{2}\]
4D-23 \[\frac{64}{128} = \frac{1}{2}\]
4D-24 \[\frac{26}{4} = \frac{13}{2}\]
4D-25 \[\frac{48}{3} = \frac{16}{1}\]
Section 5: Expressing Mixed Numbers As Fractions

In order to be able to multiply and divide mixed numbers, you must be able to express these as common fractions.

Express each mixed number as a common fraction.

5-A Examples

1. \( \frac{11}{32} = \frac{63}{32} \)
   
   solution:  
   Step 1 multiply the whole number by the denominator, or \( 1 \times 32 = 32 \)
   
   Step 2 add the numerator to the product resulting in Step 1, or \( 32 + 31 = 63 \)
   
   Step 3 place the result obtained in Step 2 as the numerator over the original denominator, or \( \frac{63}{32} \), so that
   
   \( \frac{11}{32} = \frac{63}{32} \), Answer

2. \( \frac{7}{8} = \frac{31}{32} \)
   
   solution:  
   Step 1 multiply the whole number by the denominator, or \( 3 \times 8 = 24 \)
   
   Step 2 add the numerator to the product resulting in Step 1, or \( 7 + 24 = 31 \)
   
   Step 3 place the result obtained in Step 2 over the original denominator, or \( \frac{31}{8} \), so that
   
   \( \frac{7}{8} = \frac{31}{32} \), Answer

3. \( \frac{15}{16} = \frac{253}{16} \)
   
   solution:  
   Step 1 multiply the whole number by the denominator, or \( 15 \times 16 = 240 \)
   
   Step 2 add the numerator to the product resulting in Step 1, or \( 240 + 13 = 253 \)
Step 3  place the result obtained in Step 2 as the numerator over the original denominator, or \( \frac{253}{16} \), so that:

\[
\frac{15}{16} \times \frac{13}{16} = \frac{253}{16} , \text{ Answer}
\]

Note: Two of the most common applications of complex fractions in trade and industry are:

1) Converting a fraction part of a denominator number to a decimal.

Examples: 1 \( \frac{3}{4} \) inches is \( \frac{3}{4} \) parts of a foot or \( \frac{12}{1458} \) feet as decimal. 2 \( \frac{1}{8} \) ounces is \( \frac{1}{8} \) 16th parts of a pound or \( \frac{1}{16} \) a complex fraction.

\[
\frac{1}{12} \times \frac{1}{16} = \frac{1}{192} , \text{ Answer}
\]

2) Changing percents to decimals.

Examples: 1\% is \( \frac{1}{100} \) of 100 parts or \( \frac{1}{8} \), a complex fraction.

\[
\frac{1}{100} \times \frac{1}{8} = \frac{1}{800} , \text{ Answer}
\]

\[
\frac{100}{.125} = \frac{800}{1} , \text{ Answer}
\]

15 \( \frac{1}{2} \)\% is 15 \( \frac{1}{2} \) of a 100 parts or 15 \( \frac{1}{2} \), a complex fraction.

\[
\frac{15}{100} \times \frac{1}{2} = \frac{15}{200} , \text{ Answer}
\]

\[
\frac{100}{15.5} = \frac{800}{127} , \text{ Answer}
\]
Drill Sheet

Unit 2: Expressing Mixed Numbers as Fractions

Express each mixed number as a common fraction:

5D-1 \[ \frac{3}{8} \]

5D-2 \[ \frac{11}{16} \]

5D-3 \[ \frac{13}{8} \]

5D-4 \[ \frac{7}{2} \]

5D-5 \[ \frac{13}{32} \]

5D-6 \[ \frac{14}{8} \]

5D-7 \[ \frac{7}{13} \]

5D-8 \[ \frac{7}{11} \]

5D-9 \[ \frac{7}{7} \]

5D-10 \[ \frac{97}{9} \]

5D-11 \[ \frac{105}{5} \]

5D-12 \[ \frac{22}{9} \]

5D-13 \[ \frac{33}{32} \]

5D-14 \[ \frac{19}{16} \]

5D-15 \[ \frac{11}{3} \]

5D-16 \[ \frac{24}{13} \]

5D-17 \[ \frac{7}{11} \]

5D-18 \[ \frac{13}{14} \]

5D-19 \[ \frac{6}{16} \]

5D-20 \[ \frac{7}{8} \]

5D-21 \[ \frac{5}{4} \]

5D-22 \[ \frac{6}{2} \]

5D-23 \[ \frac{3}{32} \]

5D-24 \[ \frac{22}{16} \]

5D-25 \[ \frac{8}{9} \]
Section 6: Lowest Common Denominator

To be able to add and subtract common fractions, you must determine the lowest common denominator.

6-A Examples

1. Determine the lowest common denominator (LCD) for the fractions $\frac{1}{4}, \frac{3}{16}, \frac{5}{8}$

   solution: Step 1 since the LCD can never be less than the largest denominator, it has to be at least as large as 16.

   Step 2 the other two denominators, 4 and 8, must divide into 16 a whole number of times, or $\frac{16}{4} = 4$, and $\frac{16}{8} = 2$, thus 16 is the LCD for these three fractions.

2. Determine the lowest common denominator (LCD) for the fractions $\frac{3}{11}, \frac{1}{4}, \frac{1}{5}$

   solution: Step 1 since the LCD can never be less than the largest denominator, it has to be at least as large as 11.

   Step 2 the other two denominators 4 and 5, must divide into 11 a whole number of times; let’s check $\frac{11}{4} = 2 \frac{3}{4}$, and $\frac{11}{5} = 2 \frac{1}{5}$. As you can see, each division results in a whole number, plus a remainder. Thus, 11 cannot be the LCD.

   Step 3 we must involve all three denominators: to do this, multiply $11 \times 4 = 44$, and then $44 \times 5 = 220$. Check: $\frac{220}{11} = 20$

   $\frac{220}{4} = 55$

   $\frac{220}{5} = 44$, thus 220 is the LCD for these three fractions.

   Note! Some times, the apparent LCD resulting from multiplying all the
denominators can be reduced further by dividing by 2, or 3, or 5, etc. to meet the requirements of LCD. This will be illustrated in Example 3.

3. Determine the lowest common denominator (LCD) for the fractions \( \frac{1}{15}, \frac{1}{3}, \frac{1}{7} \)

**solution:**

Step 1 \( 15 \times 3 = 45, \text{ and } 45 \times 7 = 315 \)

Step 2 but 315 is not the LCD, since it can be reduced by dividing by 3, or \( \frac{315}{3} = 105 \)

Step 3 \( \frac{105}{15} = 7 \)

\( \frac{105}{3} = 35 \)

\( \frac{105}{7} = 15, \text{ thus } 105 \text{ is the LCD} \)

**NOTE!** A simple method to determine the lowest common denominator (LCD) is as follows:

4. Determine LCD for \( \frac{2}{21}, \frac{3}{14} \)

**solution:**

Step 1 Multiply the denominators, or \( 21 \times 14 = 294 \)

Step 2 Divide the result in Step 1 by the highest common factor for 21 and 14, which is 7; and \( \frac{294}{7} = 42, \text{ the LCD} \).
Now to simplify things:

Steps to follow when determining the Lowest Common Denominator (L.C.D.):

1. Look at your largest number - see if that will work
   e.g. \( \frac{1}{2} \), \( \frac{3}{4} \)

2. If numbers (denominators) are even and odd and the largest number does not work - then go through the multiples of the largest number
   e.g. \( \frac{1}{3}, \frac{3}{8}, \frac{1}{6} \) --- 5, 16, "24"

3. If numbers are all odd, use prime number factoring system - using just prime* numbers
   e.g. \( \frac{1}{3}, \frac{1}{7}, \frac{1}{9} \)
       \[ 3 \times 1, 7 \times 1, 3 \times 3 \]
       \[ 3 \times 3 \times 7 = 63 \]

4. If numbers are all prime, then multiply them together
   e.g. \( \frac{1}{2}, \frac{1}{3}, \frac{1}{7} \)
       \[ 2 \times 3 \times 7 = 42 \]

* A Prime Number is a number larger than 1 that can be evenly divided only by itself and 1. The number 2 is the first prime; it can be evenly divided only by 2 and 1.
You can pick out prime numbers from a list of natural numbers.

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</tbody>
</table>

First, circle 2, the first prime. Then cross out all of the multiples of 2 because they are not primes, such as 4, 6, 8, 10, 12,... Next, circle the next number which is 3 and cross out its multiples such as 6, 9, 12, 15, 18,... This process is continued with succeeding numbers not previously crossed out.

Use of the sieve is not as laborious as it looks. A number less than 100 that is not a prime must have a factor less than 10, because $10 \times 10 = 100$. Thus, sieving out multiples of the primes through 7 will identify all primes from 2-100.

Can you extend the process to find all the primes from 2-200? (You only have to sieve out primes less than 15, because $15 \times 15 = 225$.)
**DRILL SHEET**

**UNIT 2: LOWEST COMMON DENOMINATORS**

Determine the lowest common denominator for each set of fractions.

<p>| 6D-1 | $\frac{1}{4}, \frac{3}{16}, \frac{5}{8} | = | _____ | |
| 6D-2 | $\frac{2}{4}, \frac{1}{3}, \frac{7}{8} | = | _____ | |
| 6D-3 | $\frac{2}{8}, \frac{7}{16}, \frac{3}{32} | = | _____ | |
| 6D-4 | $\frac{1}{5}, \frac{1}{4}, \frac{1}{2} | = | _____ | |
| 6D-5 | $\frac{1}{4}, \frac{3}{8}, \frac{1}{5} | = | _____ | |
| 6D-6 | $\frac{1}{6}, \frac{1}{12}, \frac{3}{24} | = | _____ | |
| 6D-7 | $\frac{2}{16}, \frac{7}{8}, \frac{3}{32} | = | _____ | |
| 6D-8 | $\frac{2}{5}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6} | = | _____ | |
| 6D-9 | $\frac{3}{12}, \frac{1}{8}, \frac{1}{4}, \frac{3}{16} | = | _____ | |
| 6D-10 | $\frac{2}{9}, \frac{1}{27}, \frac{3}{54} | = | _____ | |
| 6D-11 | $\frac{17}{32}, \frac{15}{16}, \frac{19}{64} | = | _____ | |
| 6D-12 | $\frac{1}{9}, \frac{1}{3}, \frac{26}{27} | = | _____ | |
| 6D-13 | $\frac{6}{7}, \frac{11}{14}, \frac{27}{28} | = | _____ | |
| 6D-14 | $\frac{1}{27}, \frac{3}{108}, \frac{7}{9} | = | _____ | |
| 6D-15 | $\frac{29}{64}, \frac{31}{32}, \frac{15}{16} | = | _____ | |</p>
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<tr>
<td>6D-18</td>
<td>$63, 15, 31, 64, 16, 32$</td>
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<tr>
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<td>$23, 7, 11, 16, 32, 64, 4$</td>
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<tr>
<td>6D-22</td>
<td>$2, 7, 49, 3, 17, 51$</td>
<td></td>
</tr>
<tr>
<td>6D-23</td>
<td>$\frac{1}{4}, \frac{1}{8}, \frac{1}{32}, \frac{1}{64}, \frac{1}{16}$</td>
<td></td>
</tr>
<tr>
<td>6D-24</td>
<td>$\frac{3}{16}, \frac{7}{48}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}$</td>
<td></td>
</tr>
<tr>
<td>6D-25</td>
<td>$47, 15, 31, 95, 48, 16, 32, 96$</td>
<td></td>
</tr>
</tbody>
</table>
Section 7: Adding Common Fractions

7-A Examples

1. Add the following fractions:

\[
\frac{1}{8} + \frac{12}{32} + \frac{7}{16} + \frac{3}{4}
\]

solution:

Step 1: arrange fractions in vertical fashion

\[
\begin{array}{cccc}
\frac{1}{8} & & & \\
\frac{12}{32} & & & \\
\frac{7}{16} & & & \\
\frac{3}{4} & & & \\
\end{array}
\]

Step 2: determine lowest common denominator, LCD

\[
\begin{array}{ccc}
\frac{1}{8} = & & \\
\frac{12}{32} = & \frac{32}{32} & \text{LCD} \\
\frac{7}{16} = & & \\
\frac{3}{4} = & & \\
\end{array}
\]

\[
\frac{1}{8} + \frac{12}{32} + \frac{7}{16} + \frac{3}{4}
\]
Step 3  use principle of equivalent fractions to obtain new form of numerators

\[
\frac{1}{8} \times (4) = \frac{4}{32} \\
\frac{19}{32} \times (1) = \frac{19}{32} \\
\frac{7}{16} \times (2) = \frac{14}{32} \\
\frac{3}{4} \times (8) = \frac{24}{32} \\
\]

Step 4  add the numerators and place total over the common denominator

\[
\frac{4}{61} + \frac{19}{61} + \frac{14}{61} = \frac{61}{32}, \text{ Answer}
\]

Step 5  reduce the improper fraction to a mixed number (always to lowest terms)

\[
\frac{61}{32} = 1 \frac{29}{32}
\]

2. \[\frac{1}{3} + \frac{3}{8} + \frac{5}{16} + \frac{1}{4} + \frac{37}{48} = \]

solution:  Step 1

1  \[
\frac{3}{8} \\
\frac{5}{16} \\
\frac{1}{4} \\
\frac{37}{48}
\]
Step 2

\[ \frac{1}{3} = \]

\[ \frac{3}{8} = \]

\[ \frac{5}{16} = \]

\[ \frac{1}{4} = \]

\[ \frac{37}{48} = \frac{48}{48} \]

Step 3

\[ \frac{1}{3} \times (16) = \frac{16}{48} \]

\[ \frac{3}{8} \times (6) = \frac{18}{48} \]

\[ \frac{5}{16} \times (3) = \frac{15}{48} \]

\[ \frac{1}{4} \times (12) = \frac{12}{48} \]

\[ \frac{37}{48} \times (1) = \frac{37}{48} \]

Step 4

16

18

15

12

+ \[ \frac{37}{48} \]

98, \[ \frac{48}{48} \]

Step 5

\[ \frac{98}{48} = 2 \frac{2}{24}, \text{ or } 2 \frac{1}{24}, \text{ Answer} \]
UNIT 2: ADDING FRACTIONS

Add the following fractions and reduce the final answer to lowest terms.

7D-1 \[ \frac{1}{4} + \frac{3}{8} = \]

7D-2 \[ \frac{5}{16} + \frac{9}{32} = \]

7D-3 \[ \frac{1}{2} + \frac{3}{7} + \frac{2}{3} = \]

7D-4 \[ \frac{5}{8} + \frac{3}{16} + \frac{3}{2} + \frac{1}{64} = \]

7D-5 \[ \frac{15}{16} + \frac{7}{8} + \frac{1}{4} + \frac{3}{2} = \]

7D-6 \[ \frac{1}{7} + \frac{3}{8} + \frac{93}{112} = \]

7D-7 \[ \frac{7}{16} + \frac{3}{8} + \frac{63}{64} + \frac{3}{4} + \frac{19}{32} = \]

7D-8 \[ \frac{1}{5} + \frac{3}{10} + \frac{11}{15} + \frac{17}{30} = \]

7D-9 \[ \frac{7}{25} + \frac{43}{50} + \frac{83}{100} = \]

7D-10 \[ \frac{6}{11} + \frac{2}{33} + \frac{17}{99} = \]

7D-11 \[ \frac{14}{17} + \frac{11}{17} + \frac{2}{17} + \frac{15}{17} = \]

7D-12 \[ \frac{3}{10} + \frac{1}{5} + \frac{3}{4} = \]

7D-13 \[ \frac{2}{3} \div \frac{7}{9} + \frac{11}{12} = \]

7D-14 \[ \frac{9}{32} + \frac{1}{4} + \frac{5}{24} = \]

7D-15 \[ \frac{1}{2} + \frac{5}{6} + \frac{11}{12} = \]

7D-16 \[ \frac{14}{15} + \frac{3}{10} + \frac{29}{30} = \]

73
7D-17 \[ \frac{1}{5} + \frac{1}{7} + \frac{1}{4} = \]

7D-18 \[ \frac{6}{7} + \frac{1}{3} + \frac{1}{2} = \]
Section 8: Adding Fractions, Mixed Numbers, and Whole Numbers

Add fractions, mixed numbers, and whole numbers. Reduce answers to lowest terms.

8-A Examples

1. \[ \frac{3}{16} + 9 + \frac{14}{64} = \]

solution: 
Step 1 
\[ \frac{3}{16} \]

Step 2 
\[ 9 + \frac{14}{64} + \frac{27}{64} = \frac{26}{64} \]

Answe

2. \[ 23 \frac{1}{8} + \frac{36}{32} = \]

solution: 
Step 1 
\[ 23 \frac{1}{8} + \frac{36}{32} \]

Step 2 
\[ 23 \frac{1}{8} = 23 \frac{4}{32} \]
\[ + \frac{36}{32} = \frac{59}{32} \]

Answe
3. \( \frac{13}{16} + 73 + \frac{3}{4} + 19 \frac{17}{32} = \) __________

solution: Step 1

\[
\begin{align*}
6 \frac{13}{16} & \\
73 & \\
27 \frac{3}{4} & \\
19 \frac{17}{32} & + \\
\end{align*}
\]

Step 2

\[
\begin{align*}
6 \frac{13}{16} & = 6 \frac{26}{32} \\
73 & = 73 \\
27 \frac{3}{4} & = 27 \frac{24}{32} \\
19 \frac{17}{32} & = 19 \frac{17}{32} \\
\end{align*}
\]

\[
\begin{align*}
125 \frac{67}{32} & \\
\end{align*}
\]

Note! \( \frac{67}{32} \) is an improper fraction, i.e., \( 32 \frac{32}{32} = 1 \)
greater than a whole \( 32 \frac{32}{32} \).

Step 3

\[
\begin{align*}
125 \frac{67}{32} & = 125 \\
2 \frac{3}{32} & + \\
127 \frac{2}{32} & = \text{Answer}
\end{align*}
\]
UNIT 2: ADDING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

Add and reduce answers to lowest terms.

8D-1 \[ 7 + \frac{7}{8} = \]

8D-2 \[ 10 \frac{1}{16} + \frac{7}{16} = \]

8D-3 \[ 4 \frac{3}{16} + 12 \frac{3}{8} = \]

8D-4 \[ 8 + \frac{7}{4} + \frac{8}{3} + \frac{7}{2} = \]

8D-5 \[ 9 \frac{1}{6} + 8 \frac{5}{9} + 7 = \]

8D-6 \[ 14 \frac{3}{4} + 30 \frac{1}{2} + 4 = \]

8D-7 \[ 7 \frac{2}{3} + 9 \frac{3}{4} + 11 \frac{1}{2} = \]

8D-8 \[ 3 \frac{3}{4} + 5 \frac{4}{7} + 2 \frac{9}{14} + 7 \frac{1}{2} = \]

8D-9 \[ 37 \frac{31}{32} + 19 \frac{15}{16} + 11 \frac{3}{4} + 4 \frac{1}{2} = \]

8D-10 \[ 29 + 11 \frac{5}{6} + 37 + 3 \frac{7}{12} = \]

8D-11 \[ 132 \frac{1}{32} + 3 + 11 \frac{23}{32} + 19 \frac{7}{32} = \]

8D-12 \[ 16 + 165 + 3 \frac{1}{64} = \]

8D-13 \[ \frac{23}{32} + 23 \frac{23}{32} + 11 \frac{13}{16} = \]
8D-14  \[ \frac{17}{64} \times 11 + 9 \times \frac{57}{64} + 3 \times \frac{1}{16} \]
8D-15  \[ \frac{23}{11} \times 9 + 17 \times \frac{29}{33} + 4 \]
8D-16  \[ \frac{6}{13} \times 5 + 13 \times \frac{27}{39} + 65 \]
8D-17  \[ \frac{16}{15} \times 11 + 3 \times \frac{23}{30} + 7 \times \frac{1}{5} \]
8D-18  \[ \frac{13}{17} \times 9 + \frac{11}{51} + 63 \]
8D-19  \[ 22 + 23 + 169 \times \frac{11}{64} \]
8D-20  \[ \frac{17}{35} \times 99 + 3 \times \frac{6}{7} + 19 \times \frac{35}{35} \]
8D-21  \[ 11 + \frac{1}{9} + 33 \times \frac{1}{3} \]
8D-22  \[ 37 + 37 + 37 \]
8D-23  \[ 65 + 37 \times \frac{1}{37} + 69 \times \frac{36}{37} \]
8D-24  \[ 125 \times \frac{11}{25} + 6 \times \frac{4}{5} + 71 \]
8D-25  \[ \frac{33}{19} \times 3 + 16 \times \frac{55}{57} \]
Section 9: Subtracting Fractions

Subtract fractions. Reduce answers to lowest terms.

9-A Examples

1. \( \frac{13}{15} - \frac{2}{20} = \)

solution: Step 1  arrange in vertical form

\[
\begin{array}{c}
15 \\
\hline
15 \\
20
\end{array}
\]

Step 2  determine lowest common denominator (LCD)

\[
\frac{13}{15} = \frac{52}{60}
\]

\[
\frac{2}{20} = \frac{27}{60}
\]

Step 3  subtract

\[
\begin{array}{c}
52 \\
\hline
27
\end{array}
\]

\[
\begin{array}{c}
52 \\
\hline
27
\end{array}
\]

\[
\begin{array}{c}
25 = 5 \\
\hline
60
\end{array}
\]

Answer

2. \( \frac{19}{20} - \frac{4}{5} = \)

solution: Step 1  \( \frac{19}{20} \)

\[
\begin{array}{c}
20 \\
\hline
4
\end{array}
\]

Step 2  \( \frac{19}{20} = \frac{19}{20} \)

\[
\begin{array}{c}
4 = 16 \\
\hline
5
\end{array}
\]

79
3. \( 67 - 33 = \frac{4}{13} \)

solution: Step 1

\[
\begin{align*}
67 \\
33 & \quad 4 \\
- & \quad 13
\end{align*}
\]

Step 2

\[
\begin{align*}
67 \\
66 & \quad = \quad 66 \frac{13}{13}
\end{align*}
\]

\[
\begin{align*}
33 & \quad 4 \\
- & \quad 13
\end{align*}
\]

Note above that 1 whole \((13)\) was "borrowed" from 67 in order to provide a number from which to subtract \(\frac{4}{13}\).

solution: Step 3

\[
\begin{align*}
66 \frac{13}{13} \\
33 & \quad 4 \\
- & \quad 13
\end{align*}
\]

\[
\begin{align*}
33 & \quad 9 \\
& \quad 13
\end{align*}
\]

Answer
## DRILL SHEET

### UNIT 2: SUBTRACTING FRACTIONS

Subtract and reduce to lowest terms.

<table>
<thead>
<tr>
<th>9D-1</th>
<th>7 - 3 =</th>
<th>9D-14</th>
<th>13 - 9 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-2</th>
<th>9 - 1 =</th>
<th>9D-15</th>
<th>31 - 11 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
<td>32</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-3</th>
<th>15 - 2 =</th>
<th>9D-16</th>
<th>15 - 1 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>3</td>
<td>39</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-4</th>
<th>19 - 3 =</th>
<th>9D-17</th>
<th>16 - 3 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>51</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-5</th>
<th>5 - 3 =</th>
<th>9D-18</th>
<th>29 - 1 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>16</td>
<td>48</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-6</th>
<th>5 - 1/6 =</th>
<th>9D-19</th>
<th>22 - 2 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>6</td>
<td>33</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-7</th>
<th>12 - 1 =</th>
<th>9D-20</th>
<th>15 - 9 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>3</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-8</th>
<th>57 - 1 =</th>
<th>9D-21</th>
<th>7 - 7 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>2</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-9</th>
<th>31 - 9 =</th>
<th>9D-22</th>
<th>14 - 3 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>16</td>
<td>27</td>
<td>54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-10</th>
<th>23 - 3 =</th>
<th>9D-23</th>
<th>9 - 9 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>10</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-11</th>
<th>1 - 1 =</th>
<th>9D-24</th>
<th>17 - 3 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>18</td>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-12</th>
<th>3 - 17 =</th>
<th>9D-25</th>
<th>1 - 1 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>32</td>
<td>16</td>
<td>64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9D-13</th>
<th>9 - 7 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>
Section 10: Subtracting Fractions, Mixed Numbers, and Whole Numbers

Subtract and reduce to lowest terms.

10-A Examples

1. \( \frac{33}{10} \frac{7}{4} - \frac{17}{3} = \frac{10}{4} \)

solution: Step 1 \( \frac{33}{10} \)

\( \frac{17}{3} \)

\(- \frac{4}{4} \)

Step 2 \( \frac{33}{10} \frac{7}{4} = \frac{33}{10} \frac{14}{20} \)

\( \frac{17}{3} = \frac{17}{4} \frac{15}{20} \)

Note that 15 cannot be subtracted from 14; we need to borrow 1 whole (20) from 33 and add to 14.

(20)

Step 3 \( \frac{32}{33} = \frac{32}{20} \frac{34}{20} \)

\( \frac{17}{15} \)

\( - \frac{15}{20} \)

\( \frac{15}{19} \frac{20}{20} \) Answer

2. \( \frac{16}{33} \frac{11}{11} - \frac{15}{4} = \frac{11}{11} \)

solution: Step 1 - \( \frac{16}{33} \frac{11}{11} \)

\( \frac{15}{4} \)

\(- \frac{11}{11} \)
Step 2

\[
\begin{align*}
16 \frac{11}{33} &= 16 \frac{11}{33} \\
15 \frac{4}{11} &= 15 \frac{12}{33} \\
\hline
15 &- 11 \\
\hline
\end{align*}
\]

Step 3

\[
\begin{align*}
15 \frac{44}{33} &= 16 \frac{12}{33} \\
\hline
15 \frac{12}{33} &= 32 \frac{22}{33} \\
\hline
\end{align*}
\]

Answer

\[
\frac{32}{33}
\]
## DRILL SHEET

### UNIT 2: SUBTRACTING FRACTIONS, MIXED NUMBERS AND WHOLE NUMBERS

Subtract and reduce to lowest terms.

<table>
<thead>
<tr>
<th>10D-1</th>
<th>21 - 9 = [\frac{12}{16}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10D-2</td>
<td>107 - 5 = [\frac{9}{9}]</td>
</tr>
<tr>
<td>10D-3</td>
<td>31 - 29 [\frac{7}{12}]</td>
</tr>
<tr>
<td>10D-4</td>
<td>9 [\frac{3}{8}] - [\frac{1}{8}]</td>
</tr>
<tr>
<td>10D-5</td>
<td>27 [\frac{31}{32}] - [\frac{7}{8}]</td>
</tr>
<tr>
<td>10D-6</td>
<td>53 [\frac{17}{28}] - 12 [\frac{3}{7}]</td>
</tr>
<tr>
<td>10D-7</td>
<td>11 [\frac{1}{4}] - [\frac{13}{16}]</td>
</tr>
<tr>
<td>10D-8</td>
<td>1 [\frac{13}{18}] - [\frac{15}{18}]</td>
</tr>
<tr>
<td>10D-9</td>
<td>19 [\frac{7}{8}] - 18 [\frac{13}{16}]</td>
</tr>
<tr>
<td>10D-10</td>
<td>1 [\frac{27}{28}] - [\frac{1}{56}]</td>
</tr>
<tr>
<td>10D-11</td>
<td>37 [\frac{19}{51}] - 28 [\frac{14}{17}]</td>
</tr>
<tr>
<td>10D-12</td>
<td>62 [\frac{6}{7}] - 53 [\frac{3}{14}]</td>
</tr>
<tr>
<td>10D-13</td>
<td>13 - [\frac{27}{32}]</td>
</tr>
<tr>
<td>10D-14</td>
<td>101 - 100 [\frac{31}{64}]</td>
</tr>
<tr>
<td>10D-15</td>
<td>65 [\frac{9}{11}] - 24 [\frac{3}{33}]</td>
</tr>
<tr>
<td>10D-16</td>
<td>24 [\frac{1}{16}] - [\frac{8}{32}]</td>
</tr>
</tbody>
</table>

---

84
10D-17  \[\frac{54}{64} - \frac{1}{64} = \frac{52}{64}\]

10D-18  \[\frac{1}{64} + \frac{16}{64} = \frac{17}{64}\]

10D-19  \[\frac{37}{13} - \frac{22}{13} = \frac{15}{52}\]

10D-20  \[\frac{1}{8} - \frac{5}{8} = \frac{64}{64}\]

10D-21  \[\frac{18}{23} - \frac{17}{23} = \frac{1}{46}\]

10D-22  \[\frac{1}{64} - \frac{1}{64} = \frac{4}{64}\]

10D-23  \[\frac{3}{4} - \frac{3}{8} = \frac{6}{16}\]

10D-24  \[\frac{16}{6} - \frac{13}{6} = \frac{7}{12}\]

10D-25  \[\frac{22}{10} - \frac{19}{15} = \frac{3}{5}\]
DRILL SHEET

Section 11: Adding and Subtracting Fractions In Practical Applications

Perform the required operation and reduce to lowest terms.

11D-1 The cafeteria baker prepares a ALPO anniversary cake mix which weighs 100 pounds. The cake mix consists of shortening and other ingredients. The weights of the other ingredients are 20 1/2 pounds of flour, 29 3/4 pounds of sugar, 18 1/8 pounds of milk, 16 pounds of whole eggs, and a total of 5 3/4 pounds of flavoring, salt, and baking powder. How many pounds of shortening are used in the mix?

11D-2 Determine dimensions A, B, C, D, and E of the photo-resist mask shown in the figure below (all dimensions are in.):

\[
\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.} \\
\text{d.} \\
\text{e.} \\
\end{array}
\]

11D-3 Before starting a wiring job for the new switchboard in ALPO’s guard shack, the electrician takes an inventory of materials and finds that 4,550 feet of BX cable are in stock. The following lengths of cable are removed from stock for the job: 275 1/4 feet; 48 1/2 feet; 56 feet; 212 3/4 feet; and 148 feet. Upon completion of the job, 87 1/4 feet are left over and returned to stock. How many feet of cable are in stock after completing the job?

11D-4 A truck is loaded at a structural steel supply house for a delivery to ALPO’s construction site.
The order calls for 125 feet of channel iron which weighs 3 3/4 tons, 140 feet of I beam which weighs 4 3/10 tons, and 80 feet of angle iron which weighs 2 1/5 tons. The maximum legal tonnage permitted to be hauled by the truck is 9 1/2 tons. All of the channel iron and I beam are loaded. Only part of the angle iron is loaded so that the maximum legal tonnage is met but not exceeded. By how many tons of angle iron will the delivery be short of the order?

11D-5 In ALPO's warehouse, cartons move along a conveyor belt that is 423 1/2 feet long. Only 117 3/8 feet of the conveyor belt is at ground level; the rest is overhead. How much of the belt is overhead?

11D-6 What is the difference between a piece of clean room floor grating 1 7/16 inches and a piece 2 17/32 inches thick?

11D-7 A piece of round silicon rod is 7 3/4 inch in diameter. How much must be removed to get a diameter of 6 37/64 inch?

11D-8 At the beginning of trading, one share of Bell Atlantic stock was selling at 45 7/8 points. At the end of trading, the stock was selling at 42 1/4 points. How many points did it lose?

11D-9 An experienced machine setter can assemble his equipment in 6 1/3 hours. A learner can do the same job in 9 1/4 hours. How much faster is the experienced machine setter than the learner?

11D-10 ALPO's storage gas tank is 3/4 full of gasoline. If 1/8 is drawn off, what fraction of the tank of gas remains?

11D-11 Last month Bruce weighed 234 1/4 pounds. Now he weighs 227 5/16 pounds. How much weight has he lost?
11D-12 In one 8-hour working day, Al"O's mechanic spent 1 3/4 hours doing a tune-up, 2 1/2 hours doing a breake job, and 3 1/3 hours replacing a clutch. How many hours did he have left to work on a broken axle?

11D-13 Find dimension A in the following sketch?

11D-14 Find dimension Y in the following sketch:
11D-15 Find the dimension X in the following sketch:

11D-16 Carpenters frequently use fractions and mixed numerals. When they build stairs, for example, the rise plus the run should be about 17 1/2 inches for each step. If the rise for a set of steps is 7 3/8 inches, what should the run be?

Solution: 
Rise + Run = 17 1/2

7 3/8 + Run = 17 1/2

Run = 17 1/2 - 7 3/8
Run = 17 4/8 - 7 3/8
Run = 10 1/8 inches

A flight of stairs is to have a total rise of 8 3/4 feet. How many steps will be needed if each rise is to be 7 1/2 inches?
Section 12: Multiplying Fractions

The multiplication of two fractions is defined as the product (from multiplication) of the numerators divided by the product (from multiplication) of the denominators.

12-A Examples

1. \( \frac{3}{7} \times \frac{5}{8} = \frac{3 \times 5}{7 \times 8} = \frac{15}{56} \)

2. \( \frac{2}{3} \times \frac{2}{7} = \frac{2 \times 2 \times 1}{3 \times 7} = \frac{4}{21} \), Answer: \( \frac{4}{21} \)

12-B You can often reduce the amount of multiplication by "cancelling" or dividing one number in the numerator by one number in the denominator, provided the dividing number is the same.

Examples

1. \( \frac{14}{15} \times \frac{3}{49} = \frac{14 \times 3}{15 \times 49} = \frac{4}{35} \), Answer: \( \frac{4}{35} \)

In the above example, the number 14 and 49 were divided by 7, i.e.: \( \frac{14}{7} = 2 \) and \( \frac{49}{7} = 7 \).

Similarly, the 3 and 15 were divided by 3. i.e.: \( \frac{3}{3} = 1 \) and \( \frac{15}{3} = 5 \).

2. \( \frac{7}{18} \times \frac{10}{12} \times \frac{4}{35} = \frac{1}{9} \), Answer: \( \frac{1}{9} \)

Note!

In the above example, first we divided \( \frac{7}{18} \times \frac{10}{12} \times \frac{4}{35} = \frac{1}{9} \), Answer: \( \frac{1}{9} \)

Next, we divided \( \frac{7}{4} = 1 \) and \( \frac{12}{4} = 3 \). Then, we divided \( \frac{10}{2} = 5 \) and \( \frac{18}{2} = 9 \).

And last, we divided \( \frac{5}{5} = 1 \) and \( \frac{5}{5} = 1 \) since we must always reduce our final answer to lowest terms.
UNIT 2: MULTIPLYING FRACTIONS

Multiply the fractions and reduce to lowest terms.

12D-1 \( \frac{7}{4} \times \frac{4}{11} = \)

12D-2 \( \frac{7}{8} \times \frac{3}{5} \times \frac{1}{2} = \)

12D-3 \( \frac{7}{15} \times \frac{25}{9} = \)

12D-4 \( \frac{24}{35} \times \frac{5}{42} = \)

12D-5 \( \frac{36}{7} \times \frac{5}{12} = \)

12D-6 \( \frac{1}{8} \times \frac{1}{8} = \)

12D-7 \( \frac{3}{5} \times \frac{5}{8} = \)

12D-8 \( \frac{4}{5} \times \frac{7}{8} = \)

12D-9 \( \frac{5}{8} \times \frac{5}{16} = \)

12D-10 \( \frac{1}{6} \times \frac{11}{12} = \)

12D-11 \( \frac{5}{6} \times \frac{2}{5} = \)

12D-12 \( \frac{5}{7} \times \frac{14}{25} = \)

12D-13 \( \frac{1}{3} \times \frac{3}{4} \times \frac{8}{9} = \)

12D-14 \( \frac{19}{20} \times \frac{10}{11} \times \frac{2}{9} = \)

12D-15 \( \frac{11}{15} \times \frac{2}{4} \times \frac{9}{22} = \)

12D-16 \( \frac{13}{16} \times \frac{4}{39} \times \frac{3}{5} \times \frac{25}{9} = \)
12D-17 \[ 17 \times 64 \times 81 = \]
\[ 51 \quad 9 \quad 16 \]

12D-18 \[ \frac{125 \times 19 \times 55 \times 6}{11 \quad 6 \quad 57 \quad 25} = \]

12D-19 \[ \frac{22 \times 17 \times 81 \times 128}{9 \quad 64 \quad 57 \quad 11} = \]

12D-20 \[ \frac{108 \times 63 \times 92 \times 1}{23 \quad 9 \quad 7 \quad 3} = \]

12D-21 \[ \frac{17 \times 3 \times 22}{64 \quad 16 \quad 68} = \]

12D-22 \[ \frac{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}}{\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6}} = \]

12D-23 \[ \frac{1 \times 1 \times 18 \times 21}{9 \quad 7 \quad 3 \quad 2} = \]

12D-24 \[ \frac{14 \times 225}{15 \quad 64} = \]

12D-25 \[ \frac{11 \times 9 \times 128 \times 96}{32 \quad 64 \quad 55 \quad 3} = \]
Section 13: Multiplying Fractions, Mixed Numbers, and Whole Numbers

In order to multiply mixed numbers, you must first convert them to improper fractions, then multiply, and reduce to lowest terms.

13-A Examples

1. \[
\frac{1}{3} \times \frac{2}{15} = \frac{5}{3} \times \frac{15}{17} = \frac{25}{17} = \frac{18}{17} = 1 \frac{8}{17}
\]

2. \[
\frac{6}{3} \times \frac{5}{1} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} = 3 \frac{1}{3}
\]

3. \[
\frac{5}{2} \times \frac{1}{13} \times \frac{2}{3} = \frac{11}{2} \times \frac{10}{39} = \frac{110}{39} = 2 \frac{32}{39}
\]

Note! When dividing \(\frac{110}{39}\) on the calculator, you obtain 2 \(\frac{39}{39}\) as a whole number, plus a decimal remainder of 0.8205 etc. To obtain the fractional remainder, multiply the denominator, 39, by the decimal and you obtain \(31.999\) etc., or 32.
Multiply and reduce to lowest terms.

13D-1 \[
\frac{5}{7} \times \frac{3}{4} = \]

13D-2 \[
4 \times \frac{7}{3} = \]

13D-3 \[
6 \frac{2}{3} \times \frac{3}{5} \times \frac{2}{9} = \]

13D-4 \[
\frac{5}{5} \times \frac{2}{13} \times \frac{3}{3} = \]

13D-5 \[
\frac{7}{3} \times \frac{2}{11} \times \frac{1}{4} = \]

13D-6 \[
\frac{3}{8} \times 12 = \]

13D-7 \[
10 \times \frac{7}{15} = \]

13D-8 \[
\frac{5}{9} \times 21 = \]

13D-9 \[
\frac{3}{4} \times \frac{4}{3} = \]

13D-10 \[
3 \frac{7}{9} \times \frac{3}{10} = \]

13D-11 \[
\frac{7}{15} \times \frac{9}{13} = \]

13D-12 \[
3 \frac{1}{5} \times 6 \frac{25}{32} = \]

13D-13 \[
20 \frac{2}{3} \times 5 \frac{1}{8} = \]

13D-14 \[
\frac{3}{8} \times 24 \times 5 \frac{1}{2} = \]

13D-15 \[
19 \times \frac{4}{7} \times 3 \frac{3}{5} = \]

13D-16 \[
3 \frac{1}{2} \times 4 \frac{1}{2} \times \frac{3}{8} = \]
13D-17 \[ \frac{2}{12} \times \frac{4}{3} = \frac{8}{4} \]

13D-18 \[ \frac{4}{6} \times \frac{1}{5} \times \frac{1}{10} = \frac{2}{5} \]

13D-19 \[ \frac{1}{16} \times \frac{2}{3} \times \frac{3}{7} = \frac{2}{1} \]

13D-20 \[ \frac{2}{2} \times \frac{4}{8} \times \frac{1}{1} = \frac{1}{4} \]
Section 14: Dividing Fractions

To divide fractions (or mixed numbers), invert the divisor (the number or fraction that follows the division sign) and then multiply. Because of this, every division problem with fractions becomes a multiplication problem.

14-A Examples

1. $\frac{20}{9} \div \frac{5}{6}$ (which is the same as $20 \div 5$)

   solution: Step 1 invert the divisor, i.e., $\frac{5}{6}$ becomes $\frac{6}{5}$ (Note! $\frac{6}{5}$ is the reciprocal of $\frac{5}{6}$)

   Step 2 multiply $\frac{20}{9} \times \frac{6}{5} = \frac{8}{3}$, or $2 \frac{2}{3}$, Answer $\frac{8}{3}$

2. $\frac{15}{4} \div \frac{3}{8} = \frac{15}{4} \times \frac{8}{3} = \frac{10}{1}$ or $10$, Answer $\frac{10}{1}$

3. $\frac{4}{3} - \frac{6}{3}$, which is the same as $\frac{4}{3} \div \frac{6}{1}$

   since $\frac{6}{1} = 6$. $\frac{4}{3} \times \frac{1}{6} = \frac{2}{9}$, Answer $\frac{2}{9}$
UNIT 2: DIVIDING FRACTIONS

Divide the fractions and reduce to lowest terms.

14D-1 \[ \frac{20}{7} \div \frac{15}{2} = \]

14D-2 \[ \frac{11}{5} \div \frac{22}{5} = \]

14D-3 \[ \frac{3}{8} \div 6 = \]

14D-4 \[ \frac{3}{8} \div \frac{14}{9} = \]

14D-5 \[ \frac{2}{8} \div \frac{17}{18} = \]

14D-6 \[ \frac{3}{4} \div 2 \frac{1}{5} = \]

14D-7 \[ \frac{5}{4} \div 7 = \]

14D-8 \[ \frac{5}{6} \div 4 = \]

14D-9 \[ \frac{1}{12} \div \frac{5}{12} = \]

14D-10 \[ \frac{3}{5} \div 10 = \]

14D-11 \[ \frac{3}{11} \div \frac{10}{15} = \]

14D-12 \[ \frac{7}{9} \div \frac{3}{7} = \]

14D-13 \[ \frac{5}{44} = \]
14D-14  $\frac{14}{15} \div \frac{7}{15} = \frac{2}{25}$

14D-15  $\frac{4}{15} \div \frac{8}{45} = \frac{1}{1}$

14D-16  $\frac{6}{7} \div \frac{18}{23} = \frac{3}{2}$

14D-17  $\frac{3}{6} \div \frac{1}{27} = \frac{1}{2}$

14D-18  $\frac{61}{64} \div \frac{5}{16} = \frac{1}{9}$

14D-19  $\frac{23}{27} \div \frac{1}{9} = \frac{1}{4}$

14D-20  $\frac{1}{4} \div \frac{1}{8} = \frac{3}{5}$

14D-21  $\frac{3}{8} \div \frac{5}{16} = \frac{1}{2}$

14D-22  $\frac{17}{32} \div \frac{51}{64} = \frac{9}{16}$

14D-23  $\frac{17}{32} \div \frac{5}{64} = \frac{9}{8}$

14D-24  $\frac{9}{16} \div \frac{5}{8} = \frac{1}{2}$
Section 15: Dividing Fractions, Mixed Numbers, and Whole Numbers

Divide and reduce to lowest terms.

15-A Examples

1. \( \frac{3}{8} \div \frac{1}{16} = \)

   solution: Step 1 convert the mixed numbers to improper fractions

   \( \frac{31}{8} \div \frac{31}{16} = \)

   Step 2 invert the divisor, 31, and multiply by 16

   \( \frac{1}{2} \times \frac{31}{16} = \frac{2}{31} \) or \( \frac{2}{1} \), Answer

   \( \frac{8}{31} \cdot \frac{1}{1} \)

2. \( \frac{5}{7} \div \frac{3}{14} = \frac{37}{7} \div \frac{45}{14} = \)

   solution: \( \frac{37}{7} \times \frac{14}{45} = \frac{74}{45} \) or \( \frac{29}{45} \), Answer

   \( \frac{1}{45} \)

Note, dividing 74 with the calculator, you obtain 1.644 etc., which means there is 1 whole plus a decimal remainder, or part of a whole "leftover." Multiplying 0.644 x 45 = 28.98, or 29 for the fractional part or numerator.
UNIT 2: DIVIDING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

Divide and reduce to lowest terms.

15D-1 \[ \frac{11}{5} \div \frac{22}{5} = \]

15D-2 \[ \frac{3}{8} \div 6 = \]

15D-3 \[ 2 \frac{1}{8} \div \frac{17}{18} = \]

15D-4 \[ 3 \frac{3}{4} \div 2 \frac{1}{5} = \]

15D-5 \[ 5 \frac{1}{4} \div 7 = \]

15D-6 \[ 14 \div 2 \frac{2}{3} = \]

15D-7 \[ 2 \div 14 \frac{3}{9} = \]

15D-8 \[ 7 \div 14 \frac{3}{9} = \]

15D-9 \[ \frac{32}{35} \div \frac{16}{1} = \]

15D-10 \[ \frac{7}{8} \div 5 \frac{1}{16} = \]

15D-11 \[ 3 \frac{2}{3} \div 11 \frac{11}{24} = \]

15D-12 \[ 2 \frac{19}{32} \div 3 \frac{3}{64} = \]

15D-13 \[ 15 \frac{7}{25} \div 6 = \]

15D-14 \[ 3 \frac{1}{2} \div 7 \frac{5}{6} = \]
15D-15 \quad 12 \div \frac{3}{8} =

15D-16 \quad \frac{14}{3} - 

15D-17 \quad 1 \frac{7}{8} - \frac{3}{4}

15D-18 \quad 2 \frac{1}{2} \div 1 \frac{1}{4} =

15D-19 \quad 33 \div 1 \frac{3}{8} =

15D-20 \quad 98 \div \frac{7}{64} =

15D-21 \quad 3000 \div 7 \frac{1}{2} =

15D-22 \quad 1 \frac{2}{3} \div 1 \frac{2}{3} =

15D-23 \quad 3 \frac{1}{45} \div \frac{7}{45} =

15D-24 \quad 1 \frac{1}{16} \div \frac{1}{64} =

15D-25 \quad 25 \frac{1}{16} \div \frac{1}{25} =
Solve these word problems and reduce answers to lowest terms, and label answers with appropriate unit of measure:

16D-1 As the speed of an automobile increases, the amount of gasoline that is used also increases. The table shown below lists the gasoline mileage (miles per gallon) at various speeds for a particular 6-cylinder automobile. Determine the total number of gallons of gasoline used when the automobile travels the speeds and times indicated:

<table>
<thead>
<tr>
<th>Miles per hour</th>
<th>Miles per gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>35</td>
<td>27 1/2</td>
</tr>
<tr>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>45</td>
<td>23</td>
</tr>
<tr>
<td>50</td>
<td>21</td>
</tr>
<tr>
<td>55</td>
<td>20</td>
</tr>
</tbody>
</table>

a. 30 mi/hr for 1 1/3 hr
b. 40 mi/hr for 3 3/4 hr
c. 50 mi/hr for 7/10 hr
d. 55 mi/hr for 3 1/5 hr

16D-2 The purchasing department of ALPO purchases supplies for the maintenance department as itemized lists. One such list is shown following:

- 6 2/3 boxes of Item A at $3 1/4 per box
- 1/3 yard of Item B at $4 1/2 per yard
- 8 pieces of Item C at $15 3/4 per dozen pieces

Find the total cost of the items listed.

16D-3 Seven machine operators earn $79.84 each per day, 5 days a week. If each operator saves 3/16 of their earnings, how much will all seven operators save in 1 year? (Consider 52 paychecks per operator per year.)
16D-4 A certain machine can cut 6 3/8 in. of silicon rod in 1 minute. How many inches of the rod can this machine cut in 15 seconds. (60 seconds = 1 minute)

16D-5 A new cinder block wall in the test set department is 23 1/2 blocks long and 14 blocks high. With the concrete joint (i.e., the mortar) a block measures 8 1/4 in. by 16 1/4 in. If the maintenance department is to paint this wall, how large an area should they plan to paint? (Area = height x length)

16D-6 GMA's research center has 217 1/2 acres of land. If it is divided into five equal tracts of land, how many acres will each tract contain?

16D-7 How many pieces of candy can you buy with 18 cents if each piece costs 3/4 of a cent?

16D-8 Ten different orders were received for packaging chips from a piece of tubing that was 12-ft. long. How many whole pieces of tubing 3 9/16 in. long can be cut for each order? Each order must get the same amount. (12 in. = 1 foot)

16D-9 A certain piece of cardboard used for packaging devices weighs 2 1/8 oz. If a bundle of these pieces of cardboard weighs 21 1/4 lb., how many pieces are in a bundle? (16 oz. = 1 lb.)

16D-10 A service station manager bought 15 qt. of oil for $8 1. If he sells one quart for $2, how much is his gross profit? (Express your answer in a fraction of a dollar.)

16D-11 What is the weight of 16 pieces of drill rod each 6 1/2 ft. long if drill rod of this kind and size weighs 5/8 lb. per foot?
16D-12 A certain alloy used for making terminal leads is made up of \( \frac{3}{8} \) copper, \( \frac{1}{4} \) lead, \( \frac{1}{16} \) zinc, and \( \frac{5}{16} \) tin by weight. We need 480 lb. of this alloy. How many pounds of each metal must we use?

- Copper _______
- Zinc _______
- Lead _______
- Tin _______

16D-13 The net weight of nails in a keg is 240 lb. The weight of each nail is \( \frac{3}{4} \) oz. How many nails are in the keg?

16D-14 Find the total weight of three castings weighing 76 1/2 lb. each, seven castings weighing 23 1/4 lb. each, and two castings weighing 23 1/8 lb. each.

16D-15 There are two corner latches on the cover in your clean room that are 4 feet apart. If it was decided to add two additional latches between the two corner latches (equally spaced), how far apart should they be?
Section 17: Order of Operations

This is identical to the procedure discussed in Unit 1 for whole numbers:

Do all operations within grouping symbols: parentheses ( ), brackets [ ], and braces { } first. Within the parentheses, etc., do multiplication or division operations (whichever comes first) in order from left to right. Then do addition and subtraction operations in order from left to right.

After completing the work in the grouping symbols, do multiplication or division operations (whichever comes first) in order from left to right.

Last, do addition and subtraction operations in order from left to right.

17-A Examples

1. \( \frac{(5 + 2)}{(9 - 3)} \div \frac{1}{3} = \frac{\_\_\_\_}{\_\_\_\_} \)

   solution: Step 1 \( \frac{5}{9} = \frac{5}{9} \)
               \( \frac{2}{3} = \frac{6}{9} \)
               \( \frac{+ 3}{9} = \frac{11}{9} \)

   Step 2 \( \frac{11}{9} \div \frac{1}{3} = \frac{11}{9} \times \frac{1}{3} = \frac{11}{3} \) or \( \frac{3}{2} \) Answer

2. \( \frac{5}{9} + \frac{2}{3} \div \frac{1}{3} = \frac{\_\_\_\_}{\_\_\_\_} \)

   solution: Step 1 \( \frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{1} \) or \( \frac{2}{1} \)
Step 2

\[
\begin{array}{c}
\frac{5}{9} \\
\hline
\frac{2}{5}
\end{array}
\]

Answer

3. \((1 \frac{3}{4} + 3 \frac{5}{8} \times 6) \div 18 + \frac{1}{2} = \)

solution: Step 1

\[ [1 \frac{3}{4} + (3 \frac{5}{8} \times 6)] = [1 \frac{3}{4} + (27 \times \frac{3}{8})] \]

\[ = [1 \frac{3}{4} + (\frac{81}{4})] = [1 \frac{21}{4} \text{ or } 20 \frac{1}{4}] \]

\[ = \frac{21}{4} \]

\[ + \frac{20}{4} \text{ or } 22 \]

Step 2

\(22 \div 18 = \frac{11}{9} = 1 \frac{2}{9} \)

Step 3

\(1 \frac{2}{9} \times \frac{2}{2} = \frac{4}{18} \)

\[ + \frac{1}{2} \times \frac{9}{8} = \frac{9}{18} \]

\[ = \frac{13}{18}, \text{ Answer} \]
4. \[ \frac{15}{2} + \frac{25}{3} \] (which is the same as \[ \frac{15}{2} - \frac{2}{\frac{5}{4}} \]) =

solution: Step 1 \[ 15 \div \frac{2}{3} = 15 \times \frac{3}{2} = 15 \times \frac{3}{8} = \frac{45}{8} \]

\[ 5 \frac{5}{8} \]

Step 2 \[ 25 \frac{1}{5} \div 4 = \frac{63}{5} \times \frac{1}{4} = \frac{33}{10} \text{ or } 6 \frac{3}{10} \]

Step 3 \[ 5 \frac{5}{8} + 6 \frac{3}{10} = \]

\[ \frac{5}{8} = 5 \frac{25}{40} \]

\[ \frac{6}{10} = 6 \frac{12}{40} \]

\[ + \]

\[ \frac{11}{40} \text{ Answer} \]

107
UNIT 2: ORDER OF OPERATIONS - FRACTIONS, WHOLE NUMBERS, AND MIXED NUMBERS

17D-1 \( \frac{7}{6} + \frac{5}{16} - \frac{3}{4} = \)

17D-2 \( \frac{(7 \times 14)}{8} \div (\frac{3 \times 6}{8}) = \)

17D-3 \( \frac{7}{8} \times (\frac{7}{8} - \frac{4}{16}) = \frac{3}{4} \)

17D-4 \( \frac{13}{16} \div 4 \times 3 - \frac{1}{4} = \)

17D-5 \( 44 \div 11 \times 4 - \frac{7}{8} = \)

17D-6 \( \frac{7}{8} + 1 \frac{7}{16} = \frac{15}{16} \)

17D-7 \( 23 \frac{1}{3} \div \frac{7}{8} + \frac{15}{16} = \)

17D-8 \( (\frac{15}{16} \div \frac{1}{3} + 2) \div (\frac{11}{32} \times 43 - 9) = \)

17D-9 \( \frac{77}{(10 - \frac{7}{16}) + (\frac{13}{32} \div \frac{1}{3})} = \)

17D-10 \( 35 + \frac{1}{8} - 3 \div 13 \times 5 = \)
UNIT 2: REVIEW

-2R1- \[ \frac{3}{4} + \frac{7}{8} + \frac{5}{16} \]

-2R2- \[ \frac{37}{10} - \frac{19}{4} \]

-2R3- \[ \frac{3 \times 12 \times 1}{4} + \frac{5}{8} \]

-2R4- \[ \frac{5 \times 3}{5} \div \frac{15}{16} \]

-2R5- \[ \frac{8 \times (10 + 5)}{4} \div \frac{(12 + 3)}{16} \]

-2R6- \[ (12 \times 8) \div (8 \times 3) \]

-2R7- \[ (7 \times 4 - \frac{3}{4} + 5) \times 12 \div \frac{3}{16} \]

-2R8- Express \( \frac{9}{16} \) as an equivalent fraction with a denominator of 64

-2R9- Express \( \frac{72}{128} \) in lowest terms

-2R10- Express \( \frac{451}{64} \) as a mixed number

-2R11- Find the lowest common denominator for \( \frac{7}{2}, \frac{2}{3}, \frac{1}{2}, \frac{3}{2} \)

-2R12- Five pieces are cut from the length of silicon rod shown. After the pieces are cut, the remaining length is discarded. What fractional part is discarded? \[ \frac{3}{16} \text{ in.} \]
-2R13- What is the thickness of a chemical room countertop made of 7 in. plywood and 1 in. Formica?

-2R14- Three pieces of steel plate for a curing furnace base are welded together. What is the total thickness if the pieces are 1 ft., 7/8 ft., and 29/32 ft.?

-2R15- If 4 3/8 gallons of water are used to dilute 7 1/8 gallons of acid, how many gallons are in the mixture?

-2R16- Find the missing length (?) in the following sketch:

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3 in.</td>
<td>12 in.</td>
<td>15 in.</td>
</tr>
<tr>
<td></td>
<td>21 in.</td>
<td></td>
</tr>
<tr>
<td>15 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

-2R17- An engineering technician needed 10 pieces of wire each 3 5/8 in. long. What length will be needed for the wire pieces?

-2R18- A fuel tank that holds 75 liters of fuel is 1/4 full. How many liters of fuel are in the tank?

-2R19- A developer subdivided 5 1/4 acres into lots each containing 7/10 of an acre. How many lots were made?
-2R20- A stack of 5 in. thick plywood is 21 7/8 in. high. How many sheets of plywood are in the stack?

-2R21- An order of aluminum trays costs $145, including a $10 shipping fee. If the price of 1 lb. of aluminum is $3, how many pounds of aluminum are in the order?

-2R22- If 7 1/2 kilowatts (KW) of power are distributed equally over 5 resistors, what is the average number of kilowatts per resistor?

-2R23- To convert from a Celsius temperature reading to a Fahrenheit temperature reading, we follow the procedure:

Fahrenheit (F) = \( \frac{9}{5} \times \) Celsius (C) + 32

Find F, when C = 35

-2R24- To convert from a Fahrenheit temperature reading to a Celsius temperature reading, we follow the procedure:

Celsius = \( \frac{5}{9} \times ( \text{Fahrenheit} - 32 ) \)

Find C, when F = 98 3/5
UNIT 3: DECIMALS

OBJECTIVES

After completing this unit, you will be able to:

. understand the meaning of a decimal or part of the whole

. understand the meaning of a decimal place, or the order of position to the right of the decimal point

. understand "rounding" decimals

. express common fractions as decimals

. express decimals as common fractions

. add and subtract decimals

. multiply and divide decimals

. follow order of operations involving decimals

. solve word problems involving decimals

Dictionary:

<table>
<thead>
<tr>
<th>Dictionary</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Decimal</td>
<td>fraction whose denominator is 10 or some power of 10</td>
</tr>
<tr>
<td>2. Decimal point</td>
<td>dot or period</td>
</tr>
<tr>
<td>3. Decimal place</td>
<td>position of number to right of decimal point</td>
</tr>
<tr>
<td>4. Rounding, or rounding off</td>
<td>reporting a number to a specific place after applying 5 and over or under 5 rule</td>
</tr>
<tr>
<td>5. Find</td>
<td>calculate; solve</td>
</tr>
</tbody>
</table>
Section 1: Concept of a Decimal

What is a decimal?

We have already learned in Units 1 and 2 that there are whole numbers and parts of whole numbers which we called common fractions. We also learned in Unit 2 that when we divide the numerator of a proper fraction by the denominator on a calculator we obtain a number to the right of a decimal point (actually a dot) -- this is a decimal of the whole. The whole number is always to the left of the decimal point, and the decimal is to the right of the decimal point.

For example, divide $\frac{7}{8}$ and you obtain 0.875 (when the decimal point indicator on the calculator is positioned at 3).

If you divide $\frac{8}{8}$, you obtain 1.000 (with the decimal point indicator at 3). So 0.875 is a decimal of 1.000, and it is another way of expressing the common fraction $\frac{7}{8}$. 
**Decimal Place**

Illustrated below is the decimal point indicator:

In the above illustration, whole numbers are written to the left of the decimal point, and decimals are written to the right of the decimal point.

The decimal place refers to the position of a number (let's say it is 1 as above) to the right of the decimal point; these places are shown as 1, 2, 3, 4, 5, and 6, and are spoken as first, second, third, fourth, fifth, and sixth. (We can go on and on to the right with additional decimal places, but for our applications, the sixth is sufficient.)

(Also shown in the illustration are the corresponding common fractions for the decimals. Note that the number of zeros for any common fraction is the same as the place, i.e., for \( \frac{1}{1000} \) there are 3 zeros and this is in the third place.)

2-A Examples

In the decimal 0.17563, the numeral 3 is in the sixth place. Indicate the "place" for the following:

\[
\begin{align*}
4 & \quad \quad \\
7 & \quad \quad \\
6 & \quad \quad \\
5 & \quad \quad \\
1 & \quad \quad 
\end{align*}
\]
Section 3: Rounding Off Decimal Numbers

You will often be instructed to give an answer or a result which is correct to a certain number of decimal places. If you have a result of 47.26475 and want it "rounded" to three places, your answer is 47.265; rounded to two places, it is 47.26; rounded to one place, 47.3.

3-A Procedure for rounding off an answer to a given number of decimal places:

1. Set the decimal point indicator of the calculator one digit beyond the number of places the final result is to be rounded.

   Example - If a final answer is to be reported to the second place, set the indicator at 3. (Remember, regardless how many numbers that follow the third place, you disregard all of them and concentrate on the third place only for rounding off.)

2. If the number you are concentrating on is equal to 5 or more (6, 7, 8, 9), add 1 to the number before it. (For our discussion in Step 1, it is added to the number in the second place.)

3. If the number you are concentrating on is 4 or less (3, 2, 1), do not change the number before it. (For our discussion in Step 1, the number in the second place remains unchanged.)

3-B Examples

1. Round 0.73862 to 3 decimal places.

   solution: Step 1 Set decimal indicator on calculator at 4.

   Step 2 Concentrate on the number 6 which is in the fourth place.

   Step 3 Since 6 is more than 5, we add 1 to the number in the third place, and our final answer is 0.739 (to 3 places, as requested).

2. Round 3.805 to 2 places.

   solution: Step 1 Set decimal indicator at 3.

   Step 2 Concentrate on the number 5 which is in the third place.
Step 3 Since this number is equal to 5, we add 1 to the number in the second place, and our final answer is 3.81.

3. Round 0.784 to 2 places.

solution: Step 1 Set decimal indicator at 3.

Step 2 Concentrate on the number in the third place.

Step 3 Since 4 is less than 5, we do not change the second place number, and our final answer is 0.78.

Note! Now that we have embarked into decimal numbers there is a certain language and procedure we follow:

First, in speaking (orally) it is important to emphasize the decimal point. For example, the number 17.565 is spoken as "seventeen point five six five."

Second, when we write a decimal (with no whole number to the left of the decimal point), it is important to place a zero to the left of the decimal point. For example, 0.765. This removes any misunderstanding about the number being a decimal.
UNIT 3: Rounding Decimals

Round the numbers to the indicated decimal places (shown in parentheses):

\[
\begin{array}{ccc}
1D-1 & 0.0855 \ (2) & = & 1D-11 & 7.829 \ (1) & = \\
1D-2 & 0.0063 \ (2) & = & 1D-12 & 14.003 \ (1) & = \\
1D-3 & 0.057 \ (1) & = & 1D-13 & 0.05645 \ (3) & = \\
1D-4 & 76.8999 \ (3) & = & 1D-14 & 0.05645 \ (4) & = \\
1D-5 & 139.0062 \ (2) & = & 1D-15 & 9.37335 \ (3) & = \\
1D-6 & 33.01997 \ (4) & = & 1D-16 & 0.37335 \ (4) & = \\
1D-7 & 1.927 \ (3) & = & 1D-17 & 0.37335 \ (2) & = \\
1D-8 & 1.9254 \ (3) & = & 1D-18 & 33.01997 \ (2) & = \\
1D-9 & 0.0015 \ (3) & = & 1D-19 & 33.01997 \ (3) & = \\
1D-10 & 13.33 \ (1) & = & 1D-20 & 13.0039 \ (3) & =
\end{array}
\]
Section 4: Expressing Common Fractions As Decimals

To express a common fraction as a decimal, divide the numerator by the denominator.

4-A Examples

1. \( \frac{7}{8} \) (or \( 7 \div 8 \)) = 0.875 (to 3 places), Answer

2. \( \frac{1}{16} \) (or \( 1 \div 16 \)) = 0.0625 (to 4 places), Answer

3. \( \frac{23}{24} \) (or \( 23 \div 24 \)) = 0.958 (to 3 places), Answer

4. \( \frac{1}{4} \) (or \( 1 \div 4 \)) = 0.25 (to 2 places), Answer

5. \( \frac{1}{10} \) (or \( 1 \div 10 \)) = 0.1 (to 1 place), Answer

6. \( \frac{63}{64} \) (or \( 63 \div 64 \)) = 0.984 (to 3 places), Answer

7. \( \frac{63}{64} \) (or \( 63 \div 64 \)) = 0.9844 (to 4 places), Answer

8. \( \frac{11}{16} \) (or \( 11 \div 16 \)) = 0.688 (to 3 places), Answer

9. \( \frac{37}{64} \) (or \( 37 \div 64 \)) = 0.578 (to 3 places), Answer

10. \( \frac{37}{64} \) (or \( 37 \div 64 \)) = 0.5781 (to 4 places), Answer

11. \( \frac{37}{64} \) (or \( 37 \div 64 \)) = 0.57813 (to 5 places), Answer

12. \( \frac{37}{64} \) (or \( 37 \div 64 \)) = 0.578125 (to 6 places), Answer
UNIT 3: EXPRESSING COMMON FRACTIONS AS DECIMALS

Express the following as decimals to the places indicated in parentheses:

2D-1 \( \frac{2}{9} = \) _______ (3) 2D-11 \( \frac{15}{6} = \) _______ (4)

2D-2 \( \frac{19}{29} = \) _______ (3) 2D-12 \( \frac{31}{32} = \) _______ (4)

2D-3 \( \frac{5}{12} = \) _______ (3) 2D-13 \( \frac{3}{8} = \) _______ (2)

2D-4 \( \frac{17}{21} = \) _______ (3) 2D-14 \( \frac{17}{64} = \) _______ (3)

2D-5 \( \frac{1}{32} = \) _______ (4) 2D-15 \( \frac{9}{32} = \) _______ (3)

2D-6 \( \frac{1}{8} = \) _______ (3) 2D-16 \( \frac{3}{64} = \) _______ (4)

2D-7 \( \frac{1}{4} = \) _______ (2) 2D-17 \( \frac{17}{1000} = \) _______ (2)

2D-8 \( \frac{1}{2} = \) _______ (3) 2D-18 \( \frac{11}{500} = \) _______ (3)

2D-9 \( \frac{9}{16} = \) _______ (4) 2D-19 \( \frac{13}{75} = \) _______ (4)

2D-10 \( \frac{3}{4} = \) _______ (1) 2D-20 \( \frac{13}{16} = \) _______ (4)

* Note: A functional number whose quotient repeats a particular sequence of number is called a repeating or periodic decimal. A periodic decimal is often indicated by a bar over the digits that repeat.

For example: \( \frac{3}{7} = .428571 \)
Section 5: Expressing Decimals As Common Fractions

To express a decimal as a common fraction follow a four-step procedure. For example, to convert 0.065:

solution: Step 1 - Determine the place of the last digit, 5; it is in the third place.

Step 2 - Since the third place is thousandths, the denominator will be 1000.

Step 3 - Disregard all zeros between the decimal point and the first numeral to the right; then write the number 65 in the numerator, as 65/1000.

Step 4 - Reduce the common fraction to lowest terms, so that 65/1000 = 13/200, Answer, since the numerator and denominator can be divided by the common number 5.

5-A Examples

1. 0.8 = 

solution: Step 1 - The 8 is the first place.

Step 2 - The denominator is 10.

Step 3 - The numerator is 8, and the common fraction is 8/10.

Step 4 - Reduce to lowest terms, so that 4/5 = 4/5, Answer, since the numerator and denominator can be divided by 2.
2. \( 0.006 = \) 

solution: Step 1 - The 6 is in the third place.
Step 2 - The denominator is \( \frac{6}{1000} \).
Step 3 - The numerator is 6, and the common fraction is \( \frac{6}{1000} \).
Step 4 - Reduce to lowest terms, so that \( \frac{6}{1000} = \frac{3}{500} \), Answer, since \( \frac{3}{500} \) numerator and denominator can be divided by 2.
**UNIT 3: EXPRESSING DECIMALS AS COMMON FRACTIONS**

Express the following as common fractions (reduce to lowest terms).

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-1</td>
<td>0.375 =</td>
</tr>
<tr>
<td>3D-2</td>
<td>0.062 =</td>
</tr>
<tr>
<td>3D-3</td>
<td>0.335 =</td>
</tr>
<tr>
<td>3D-4</td>
<td>0.4375 =</td>
</tr>
<tr>
<td>3D-5</td>
<td>0.4375 =</td>
</tr>
<tr>
<td>3D-6</td>
<td>0.2 =</td>
</tr>
<tr>
<td>3D-7</td>
<td>0.12 =</td>
</tr>
<tr>
<td>3D-8</td>
<td>0.235 =</td>
</tr>
<tr>
<td>3D-9</td>
<td>0.93 =</td>
</tr>
<tr>
<td>3D-10</td>
<td>0.75 =</td>
</tr>
<tr>
<td>3D-11</td>
<td>0.3125 =</td>
</tr>
<tr>
<td>3D-12</td>
<td>0.625 =</td>
</tr>
<tr>
<td>3D-13</td>
<td>0.8125 =</td>
</tr>
<tr>
<td>3D-14</td>
<td>0.500 =</td>
</tr>
<tr>
<td>3D-15</td>
<td>0.125 =</td>
</tr>
<tr>
<td>3D-16</td>
<td>0.09375 =</td>
</tr>
<tr>
<td>3D-17</td>
<td>0.1875 =</td>
</tr>
<tr>
<td>3D-18</td>
<td>0.25 =</td>
</tr>
<tr>
<td>3D-19</td>
<td>0.750 =</td>
</tr>
<tr>
<td>3D-20</td>
<td>0.313 =</td>
</tr>
</tbody>
</table>
Section 6: Adding and Subtracting Decimals

The most important step in decimal calculations is to set the decimal point indicator for the proper place. Earlier we learned to set the decimal point indicator one digit to the right of the place we are instructed to report the final result when we are asked to round an answer.

In the absence of any instructions, for reporting a result to a specific decimal place, we must report the entire number resulting from the arithmetic operation. This will be illustrated in the following examples:

6-A Examples of addition

1. \(147.935 + 0.13 = \) 
   
   solution: Step 1 - Observe that the third decimal place is the farthest place, or position from the decimal point for a numeral in either number. Because of this, for addition and subtraction problems on our calculator, set the decimal indicator at 3 (we were not given any instructions to round off).
   
   Step 2 - Enter 147, then press the [ ] key, and then enter 935.
   
   Step 3 - Press [+] key.
   
   Step 4 - Press [ ] key, then enter 13. (Note, although it is important to write a zero before the decimal, it is not necessary to enter the zero in the calculator.) Display shows 148.065.
   
   Answer.

2. \(17 + 0.0134 + 91.02 + 139.001 = \) 
   
   solution: Step 1 - Enter 17 - you do not have to enter the decimal point for any whole number.
   
   Step 2 - Press [+] key.
   
   Step 3 - Press [ ] key and enter 0134.
   
   Step 4 - Press [+] key.
   
   Step 5 - Enter 91, then press [ ] key, and then enter 02.
   
   Step 6 - Press [+] key.
Step 7 - Enter 139, then press [ ] key, and then enter 001.

Step 8 - Press [+] key; display shows 247.0344. Answer

3. \[ 0.0732 + \frac{7}{16} + 0.2323 = \underline{} \]

solution: Step 1 - Set decimal indicator at 4.
Step 2 - Press [ ] key, then enter 0732.
Step 3 - Press [+] key.
Step 4 - Since it is impossible to enter a common fraction in the calculator, we must first convert \( \frac{7}{16} \) to a decimal; in dividing 7 by 16, we obtain 0.4375. Press [ ] key, then enter 4375.
Step 5 - Press [+] key.
Step 6 - Press [ ] key, then enter 2323.
Step 7 - Press [+] key; display shows 0.7430.

6-8 Examples of subtraction

1. \[ 44.6 - 27.368 = \underline{} \]

solution: Step 1 - Observe that the third decimal place is the farthest place, or position from the decimal point for a numeral in either number. Because of this, set the decimal indicator at 3 (again, we were not given any instructions to round off).
Step 2 - Enter 44, then press [ ] key, then enter 6. (Note! The calculator automatically displays two zeros following the 6, thus providing a 3-place number for subtraction.)
Step 3 - Press [+] key
Step 4 - Enter 27, then press [ ] key, then enter 368.
Step 5 - Press [-] key; display shows 17.232, Answer
2. \[ \frac{46}{10} - 33.912 = \]

solution: Step 1 - Set decimal indicator at 3.

Step 2 - Convert the common fraction \( \frac{3}{10} \) of the mixed number \( \frac{46}{10} \) to a decimal; thus, \( \frac{3}{10} \) becomes 0.3. Enter 46, then press \( [ ] \) key, then enter 3.

Step 3 - Press \([+\)] key.

Step 4 - Enter 33, then press \([\)] key, then enter 912.

Step 5 - Press \([-\)] key; display shows 12.388,

Answer
UNIT 3: ADDING AND SUBTRACTING DECIMALS

4D-1  0.317 + 0.029 =
4D-2  8.036 + 16 + 0.7 =
4D-3  0.877 - 0.304 =
4D-4  10.002 - 0.1999 =
4D-5  83.712 + 0.056 + 35 =
4D-6  3.062 - 1.956 =
4D-7  12.002 + 0.018 + 0.003 + 0.017 =
4D-8  305.1 + 43.95 + 0.014 + \frac{1}{8} =
4D-9  0.009 - 0.0086 =
4D-10 26.009 - 25.999 =
4D-11 175 - 173.909 =
4D-12 23 \frac{1}{8} - 0.375 =
4D-13 144 - 11.883 =
4D-14 11.883 - 0.144 =
4D-15 12.01 + 11.003 + 0.0175 =
4D-16 0.3134 - 0.179 =
4D-17 0.347 + 1 \frac{7}{8} + 0.5 + 3.1414 =
4D-18 11.09 + 1.109 + 0.1109 =
4D-19 39 \frac{15}{16} - 17.9435 =
4D-20 63 + 63.63 + 0.63 =
4D-21 1179 - 1011.9755 =
4D-22 12 + 5.005 + 0.1 =
4D-23 0.095 - 0.091 =
4D-24 \( 116.0157 - 16.01 = \)
4D-25 \( 33 + 33.93 + 0.93 + 0.935 = \)
4D-26 \( 67.05 - 37.095 = \)
4D-27 \( 22 \frac{1}{16} + 0.0625 + 0.8750 = \)
4D-28 \( 77.9703 - 64.848 = \)
4D-29 \( 18 - 17 \frac{61}{64} = \)
4D-30 \( 25 + 11.88 + 0.0033 = \)
Section 7: Multiplying and Dividing Decimals

7-A Multiplying Decimals

When multiplying decimals, it is important to follow instructions for the decimal place when reporting the final answer. Otherwise, you must set the decimal place indicator so that the complete result is displayed.

Examples

1. \(60.412 \times 0.53 = \) 

   solution: 
   
   Step 1  Since there are no instructions for the decimal place, you must count the total number of decimal places for both numbers—it is 5. Therefore, set the decimal indicator at 5.
   
   Step 2  Enter 60, then press [ ] key, then enter 412.
   
   Step 3  Press [x] key.
   
   Step 4  Press [ ] key, then enter 53.
   
   Step 5  Press [=] key; display shows 32.01836, Answer

2. \(60.412 \times 0.59 = \) (round to 2 places)

   solution: 
   
   Step 1  We are instructed to round to 2 places, so we set the decimal indicator at 3.
   
   Step 2  Enter 60, press [ ] key, then enter 412.
   
   Step 3  Press [x] key.
   
   Step 4  Press [ ] key, then enter 59.
   
   Step 5  Press [=] key; display shows 35.643; since the third place numeral is less than 5, the number 4 remains unchanged and our final answer is 35.64.

3. \(\frac{1}{16} \times 0.33 \times 4.27 = \) (round to 4 places)

   128
Section 7: Multiplying and Dividing Decimals

7-A Multiplying Decimals

When multiplying decimals, it is important to follow instructions for the decimal place when reporting the final answer. Otherwise, you must set the decimal place indicator so that the complete result is displayed.

Examples

1. \( 60.412 \times 0.53 = \) 
   solution: Step 1 Since there are no instructions for the decimal place, you must count the total number of decimal places for both numbers—it is 5. Therefore, set the decimal indicator at 5.

   Step 2 Enter 60, then press [ ] key, then enter 412.

   Step 3 Press [x] key.

   Step 4 Press [ ] key, then enter 53.

   Step 5 Press [=] key; display shows 32.01836, Answer

2. \( 60.412 \times 0.59 = \) 
   (round to 2 places)
   solution: Step 1 We are instructed to round to 2 places, so we set the decimal indicator at 3.

   Step 2 Enter 60, press [ ] key, then enter 412.

   Step 3 Press [x] key.

   Step 4 Press [ ] key, then enter 59.

   Step 5 Press [=] key; display shows 35.643; since the third place numeral is less than 5, the number 4 remains unchanged and our final answer is 35.64.

128
3. \( \frac{9}{16} \times 0.33 \times 4.27 = \) 

(round to 4 places)

solution:

Step 1 Set decimal indicator at 5.

Step 2 Enter 1, then press [ ], then enter \( \frac{5625}{16} \) (decimal of \( \frac{9}{16} = 0.5625 \)).

Step 3 Press [×] key.

Step 4 Press [ ] key, then enter 33.

Step 5 Press [=] key; display shows 0.51563.

Step 6 Press [×] key.

Step 7 Enter 4, then press [ ] key, then enter 27.

Step 8 Press [=] key; display shows 2.20174; when rounded to 4 places, the answer is 2.2017.

7-B Dividing Decimals

When dividing decimals, it is important to follow instructions for decimal place when reporting the final answer. Otherwise, you set the decimal indicator to the right as far as it can go on our calculator, or at 6.

Examples

1. \( 0.338 - 0.52 = \) 

solution:

Step 1 Since there are no instructions for decimal place in the final answer, set the decimal indicator at 6.

Step 2 Press [.] key, and enter 338.

Step 3 Press [ ] key.

Step 4 Press [.] key, then enter 52.

Step 5 Press [=] key; display shows 0.650000.

Since all the zeros following the 5 are not significant numbers, we drop them and show the answer as 0.65. To test this, multiply 40 by 0.650000, and multiply 40 by 0.65. The answer is the same. 26.

129
2. \(1.7594 \div 6.03\)  
(round to 3 places)

solution:  
Step 1  Set decimal indicator at 4.
Step 2  Enter 1, then press [ ] key, then enter 7594.
Step 3  Press \([\div]\) key.
Step 4  Enter 6, then press [ ] key, then enter 03.
Step 5  Press \([=]\) key; display shows 0.2918; since the fourth place numeral is more than 5, increase the 1 to 2; the final answer is 0.292.
DRILL SHEET

UNIT 3: MULTIPLYING AND DIVIDING DECIMALS

Perform the operations indicated and round to the places shown in parentheses:

5D-1 0.9 x 0.5 (4) =

5D-2 0.8 x 29 (3) =

5D-3 0.63 x 0.16 (4) =

5D-4 7.22 x \frac{3}{8} (3) =

5D-5 0.025 x 0.09 (4) =

5D-6 0.42 x 11 x 0.4 (4) =

5D-7 0.009 x 120 x 6.7 (4) =

5D-8 0.8 \div 0.2 (3) =

5D-9 6.3 \div 0.3 (3) =

5D-10 1.44 \div 0.08 (3) =

5D-11 18.750 \div \frac{3}{4} (3) =

5D-12 0.002 \div 0.91 (3) =

5D-13 153.73 \div 14.27 (4) =

5D-14 0.0084 \div 3.094 (4) =

5D-15 3876.5 \div 5.125 (3) =

5D-16 12.1 x 0.761 x 0.0035 (4) =

5D-17 0.0035 \div 17 (3) =

5D-18 1.7901 \div 40 (1) =

5D-19 35.001 \div 7.2 (2) =

5D-20 \frac{3}{16} x 1.704 x 1 \frac{1}{4} (4) =

5D-21 19 \frac{7}{8} \div 0.0115 (3) =
5D-22 \[ 0.0115 \div 19 \frac{7}{8} (3) = \]

5D-23 \[ 270.0 \div 11.953 (4) = \]

5D-24 \[ 60 \times 0.44 \times 11.1 \times 9.7 (4) = \]

5D-25 Thunder and lighting occur at the same time. Light travels almost instantly, and sound travels about .206 miles per second. To find how far away a storm is, follow these steps.

1. When you see the flash, count the number of seconds it takes before you hear the thunder.
2. Multiply the number of seconds the thunder took to reach you by .206 in order to find the distance in miles.

For example, if it takes 5 seconds for thunder to reach you after you see the lightning, how far away is the storm?

\[
\text{Distance} = 0.206 \times \text{Seconds It Takes Thunder to Reach You} \\
= 0.206 \times 5 \\
= 1.030 \text{Miles}
\]

The storm is about a mile away.

How far is the storm if it takes 8 seconds for thunder to reach you?
Section 8: Order of Operations Involving Decimals

The rules to follow with decimals are exactly the same as discussed for whole numbers and common fractions. We repeat them here:

Do all operations within grouping symbols: parentheses ( ), brackets [ ], and braces { } first. Within the parentheses, etc., do multiplication or division operations (whichever comes first) in order from left to right. Then do addition and subtraction operations in order from left to right.

After completing the work in the grouping symbols, do multiplication or division operations (whichever comes first) in order from left to right.

Last, do addition and subtraction operations in order from left to right.

8-A Examples

1. $9.03 + 2.75 \times 0.9 = \underline{\quad}$
   (round to 2 places)

   **solution:**
   Step 1 Set decimal indicator at 3.
   Step 2 Since multiplication appears in the problem, we do this first, and our result is 2.475.
   Step 3 Add 2.475 to 9.03; display shows 11.505; rounded to 2 places, our final answer is 11.51.

2. $12.4 \times (13.88 - 0.07 \times 0.5) = \underline{\quad}$
   (round to 2 places)

   **solution:**
   Step 1 Set decimal indicator at 3.
   Step 2 We do the operations within the parentheses first, and within we do the multiplication first; our result is 0.035.
   Step 3 Subtract 0.035 from 13.88; our result is 13.845.
   Step 4 Multiply 13.845 by 12.4; our result is 171.678; rounded to 2 places, our final answer is 171.68.
Perform the indicated operations and report final answers to the decimal places shown in parentheses:

6D-1  \[ 1.31 \times 6 - 3.2 \div 3.4 \ (2) = \]

6D-2  \[ 67 + 13 - 9 \times 4 \div 3 \ (2) = \]

6D-3  \[ 24 \div 6.7 \times 3.9 \ (2) = \]

6D-4  \[ 42.090 \times 3.045 + 99 \div 3 \times 4 \ (3) = \]

6D-5  \[ 0.009 \div 3 \times 11.11 + 0.047 \ (3) = \]

6D-6  \[ 25 + 11 \times 5 \div 5.1 \ (1) = \]

6D-7  \[ 64.08 - 19.009 \div 4 \ (3) = \]

6D-8  \[ 134.7 \times 0.01 \div 0.5 \ (2) = \]

6D-9  \[ 0.02 + 0.002 \times 0.002 \div 0.002 \ (3) = \]

6D-10 \[ 1.24 + 1.35 + 9.05 \div 0.15 \ (3) = \]

6D-11 \[ 12 + 32.09 - 9.04 \div 2 \times 4.1 \ (4) = \]

6D-12 \[ 14 \frac{15}{16} - 13 + 4.7 \times 3 \div 1 \ (2) = \]

6D-13 \[ 0.07 - .004 \times 13.019 \div 7.9 \ (3) = \]

6D-14 \[ 14.7 \div 2 \times 3 + 8 \div 4.2 \ (2) = \]

6D-15 \[ 0.019 \times 0.1 \div 0.01 \ (2) = \]

6D-16 \[ 1 \frac{7}{32} + 19 \frac{3}{32} + 0.195 - 0.79 \ (3) = \]

6D-17 \[ \frac{11}{32} \times 3.5 \times 4 + 19.04 - \frac{3}{16} \ (3) = \]

6D-18 \[ 10.9 \times 10.9 \div 10.9 + 10.9 - 10.9 \ (3) = \]

6D-19 \[ 0.525 \times 0.505 \times 0.525 \div 0.505 \ (3) = \]

6D-20 \[ 113.013 + 0.013 \times 9.08 \ (3) = \]

6D-21 The cost of electricity is based on the number of kilowatt hours. A kilowatt is 1000 watts, and a kilowatt hour is the amount of electricity used by a 1000-watt appliance in one hour. To
find the number of kilowatt hours used by an appliance, use this formula:
\[ \text{kw} = \frac{\text{Watts}}{1000} \times \text{No. of Hours}. \]

How much does it cost to run a 210-watt television for 60 hours a month if the cost of electricity is $.0525 per kilowatt hour?

\[
\begin{align*}
\text{kw} &= \left(\frac{210}{1000}\right) \times 60 \\
&= .210 \times 60 \\
&= 12.6 \text{ kilowatt hours}
\end{align*}
\]

To find the cost, multiply the number of kilowatt hours by the cost per watt hour.

\[ 12.6 \times .0525 = .66150 = .66 \]

The cost is about $.66.

Find the cost to operate:

1. A 100-watt bulb for 500 hours.
2. A 5750-watt clothes dryer for 20 hours.
UNIT 3: REVIEW

-3R1- Write each of the word statements as a decimal number:

a. Forty-four and four tenths
b. Sixty point six-six
c. One hundred one point zero-one
d. Two hundred twenty-two and two hundredths

-3R2- In the decimal 0.93758, indicate the place for the following:

a. 3
b. 5
c. 7
d. 8
e. 9

-3R3- Round off to one place:

a. 2.36
c. 17.14
b. 15.1905
d. 139.07

-3R4- Round off to three places:

a. 0.1786
c. 1.3050
b. 15.1905
d. 33.0303

-3R5- Express these common fractions as decimals to 4 places:

a. \(\frac{11}{16}\)
c. \(\frac{17}{1000}\)
b. \(\frac{17}{24}\)
d. \(\frac{39}{64}\)

-3R6- Express these decimals as common fractions:

a. 0.3125
d. 0.890625
b. 0.390625
e. 0.28125
c. 0.65625
During the rainy season, a weather station recorded the following rainfall for one week:

- Sunday: 3.57 in.
- Monday: 2.78 in.
- Tuesday: 2.89 in.
- Wednesday: 3.86 in.
- Thursday: 1.98 in.
- Friday: 4.19 in.
- Saturday: 3.88 in.

What is the total rainfall for the week? 

Ed is a repairman. His wages are $416.45 per week before any deductions. His deductions are:

- Federal Tax: $82.98
- F.I.C.A.: 12.58
- Union dues: 4.87
- Credit union: 25.00
- Health plan: 6.50

What is his take home pay? 

A machine setter uses four shims with thicknesses of:

- 0.0625 in.
- 0.0500 in.
- 0.1375 in.
- 0.00204 in.

What is the total thickness of the four shims? 

a. Find the dimension marked D in the following two sketches:

1,035 in. 1,315 in. 1,036 in.

D
b. Find the dimension marked D in the following sketch:

- 3R11 - From the sum of 5.75 and 2.45 subtract the sum of 1.54 and 3.77.

- 3R12 - How much is left when 66.887 is taken from 87.992?

- 3R13 - \(1.007 \times 2.08 \times 3.19\) (to 3 places)

- 3R14 - A shop drawing shows a part to be 1.75 in. long. If the drawing is enlarged 1.5 times, how long will the part be on the drawing?

- 3R15 - Type M 3-in. copper water tubing weighs 2.68 pounds per foot. How much does 25.8 feet weigh?

- 3R16 - A mechanic determines the total cost of a repair job as $560. Labor costs are $350. What decimal of the total cost is the labor cost?

- 3R17 - An inspector checks 175 pieces of a lot containing 615 pieces. What decimal of the lot has not been inspected?
UNIT 4: PERCENT, AVERAGE, MEDIAN and RANGE

OBJECTIVES

After completing this unit, you will be able to:

- express percents as decimal or common fractions
- express decimal and common fractions as percents
- determine percentage, percent (rate), and base
- solve word problems involving percents
- determine the arithmetic average (\( \bar{x} \))
- determine the median
- determine the range (\( R \))

Dictionary:

<table>
<thead>
<tr>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Percent (%)</td>
</tr>
<tr>
<td>2. Base</td>
</tr>
<tr>
<td>3. Percentage</td>
</tr>
<tr>
<td>4. Rate</td>
</tr>
</tbody>
</table>
Section 1: Concept of a Percent

What is a percent?

Percent is one of the best ways to compare one quantity to another quantity. The word "percent" means in the hundred, or of each hundred.

A percent is designated by the symbol %. Thus, \(10\%\) means 10 percent, or 10 out of one hundred.

Section 2: Expressing Percents as Decimal and Common Fractions

Since a percent cannot be used to perform arithmetic operations, we must convert it to a form that will enable us to do this.

"All percents must be divided by 100 to convert them to decimals."

2-A Examples

1. Express \(37 \frac{1}{2}\%\) as a decimal fraction.

   \[\text{solution:}\]
   \[
   \text{Step 1} \quad \text{Percent means of each hundred, or the second place, set the decimal indicator one place beyond, or at 3.}\n   \]
   \[\text{Step 2} \quad 37 \frac{1}{2}\% = 37.5\% \quad \text{or} \quad \frac{37.5}{2}\]
   \[\text{Step 3} \quad \frac{37.5}{2} = 0.375 \quad \text{Answer}\]

   Note that the instant we divide by 100, the % sign "disappears."

2. Express \(33.6\%\) as a decimal.

   \[\text{solution:}\]
   \[
   \text{Step 1} \quad \text{Set decimal indicator at 3.}\n   \]
   \[\text{Step 2} \quad 33.6\% = 0.336 \quad \text{Answer}\]

3. Express \(147.9\%\) as a decimal.

   \[\text{solution:}\]
   \[
   \text{Step 1} \quad 147.9\% = 1.479 \quad \text{Answer}\]

140
4. Express 323.2% as a decimal.

solution: Step 1 \[
\frac{323.2\%}{100} = 3.232
\]

Answer

2-B Examples

1. Express \(\frac{37}{2}\%\) as a common fraction.

solution: Step 1 Convert percent to decimal, or
\[
\frac{37}{2} \div \frac{2}{100} = 0.375
\]

Step 2 Since 0.375 is in the third decimal place, set up a fraction with 1000 in the denominator as \[
\frac{375}{1000}
\]

Step 3 Reduce to lowest terms, divide the numerator and denominator by 125, so
\[
\frac{375}{1000} = \frac{3}{8}
\]

Answer

2. Express 33% as a common fraction.

solution: Step 1 Convert percent to decimal;
\[
\frac{33\%}{100} = 0.330, \text{ or } 0.33
\]

Step 2 Since 0.33 is in the second decimal place, set up a fraction with 100 in the denominator as \[
\frac{33}{100}
\]

Step 3 Since we cannot reduce to lower terms, our answer is
\[
\frac{33}{100}
\]

3. Express 221.5% as a common fraction.

solution: Step 1 Convert percent to decimal;
\[
\frac{221.5\%}{100} = 2.215
\]

Answer
Our result thus far is a mixed decimal number, i.e., a whole number 2 and a decimal fraction 0.215. We must convert the 0.215 to a common fraction.

Step 2 Since 0.215 is in the third place, set up a fraction with 1000 in the denominator and 215 in the numerator as $\frac{215}{1000}$; the result thus far is $\frac{2}{1000}$.

Step 3 Reduce the common fraction to lowest terms;

$$\frac{215}{1000} = \frac{43}{200}$$

*answer is* $2 \frac{43}{200}$
2-C Type 1

Examples

1. What percent of 60 is 9?

   (60 is the base, or whole; 9 is a fractional part or percentage of the base; we are asked to find the x %)

   solution: Step 1: Set decimal indicator at 3.

   Step 2: Set up a fraction with the base, 60, as the denominator and the fractional part (percentage), 9, as the numerator and divide, \( \frac{9}{60} = 0.150 \), or 0.15

   Step 3: Convert decimal to percent:
   \( 0.15 \times 100 = 15.000 \) or 15%, Answer

   You will note from example 1 and the others that follow, in Type 2 percent problems, the base always follows the word "of," and you will always be given two numbers, not percentages to solve the problem. The second number will be the numerator.

2. What percent of 140 is 105?

   solution: Step 1: Set decimal indicator at 3.

   Step 2: Set up fraction with 140, the base, as the denominator and 105, the fractional part (percentage) as the numerator and divide:
   \( \frac{105}{140} = 0.750 \), or 0.75

   Step 3: Convert decimal to percent:
   \( 0.75 \times 100 = 75.000 \), or 75%, Answer

3. What percent of 8 is 25?

   solution: Step 1: Set decimal indicator at 3.

   Step 2: Note that 8 follows the word "of" which makes it the base, or denominator and 25 is the numerator; set up as a fraction and divide:
   \( \frac{25}{8} = 3.125 \)

   You will note from example 1 and the others that follow, in Type 2 percent problems, the base always follows the word "of," and you will always be given two numbers, not percentages to solve the problem. The second number will be the numerator.
Step 3  Convert decimal to percent:
3.125 \times 100 = 312.500, or 312.5%,
Answer.
UNIT 4: PERCENT PROBLEMS (TYPE I)

Solve for each. Round answers to 2 decimal places:

1D-1 What percent of 87 is 95?
1D-2 What percent of 63 is 44.2?
1D-3 What percent of 19.4 is 8?
1D-4 What percent of 298 is 420?
1D-5 What percent of 40 is 13.1?
1D-6 What percent of 5 is 3? \[
\frac{8}{4}
\]
1D-7 What percent of 11 is 3? \[
\frac{16}{8}
\]
1D-8 What percent of 16 is 4? \[
\frac{16}{16}
\]
1D-9 What percent of 20 is 35?
1D-10 What percent of 116 is 118?
1D-11 What percent of 11 is 6? \[
\frac{32}{8}
\]
1D-12 What percent of 125 is 25?
1D-13 What percent of 25 is 125?
1D-14 What percent of 1.7 is 1? \[
\frac{17}{17}
\]
1D-15 What percent of 7 is 3? \[
\frac{8}{16}
\]
1D-16 What percent of 75 is 70?
1D-17 What percent of 3 is 1?
1D-18 What percent of 1 is 3?
1D-19 What percent of 0.015 is 0.25?
1D-20 What percent of \[
\frac{19}{32}
\] is 2?
2-D Type 2

Examples

1. 9 is 15% of what number?
(9 is the fractional part, or percentage; 15% is the rate; the base, or whole we must solve for)

solution:

Step 1 Set decimal indicator at 3.
Step 2 Write a fraction with 9 as the numerator and a question mark as the denominator: \( \frac{9}{?} \)
Step 3 Set up all Type 3 percent problems as follows: \( \frac{9}{?} = 15\% \)
Step 4 Convert the percent, 15%, to a decimal: \( 15\% = 0.150 \text{ or } 0.15 \)
Step 5 Replace the question mark in Step 3 with 0.15 and divide:
\( \frac{9}{0.15} = 60 \), Answer

Note! In Type 2 percent problems, the wording "of what number" will always be the number or quantity we must solve for, and it is replaced as a question mark in the denominator.
DRILL SHEET

UNIT 4: PRACTICAL APPLICATIONS OF PERCENTS

Solve for each. Round answers to 2 decimal places:

2D-1 Jim’s gross pay of $30,100 was 10% less than last year’s gross pay. What was last year’s gross pay?

2D-2 In BIC-4 department, 4,185 usable devices were produced. If 7% of the parts produced were defective, how many parts were produced altogether?

2D-3 If an error of ± 3% is acceptable and the diameter is supposed to be 2.25", how much could the diameter vary from this value?

2D-4 A certain alloy is 78% tin. If a piece of this alloy weighs 15 pounds, what is the weight of the tin in this alloy?

2D-5 The pitch of a gear was 3% larger than it should have been. If the pitch was 1.8025", what should it have been?

2D-6 An ALPO carpenter estimates that a job requires 550 board feet of lumber, which includes 15% allowance for waste. How many board feet are allowed for waste?

2D-7 If 6 1/2 acres of a 15-acre orchard are harvested, what percent of the orchard is harvested?

2D-8 A machine produces 76 pieces when operating at 80% of its capacity. How many pieces can be produced when the machine is operating at full capacity?

2D-9 It is estimated that a company’s earnings next year will be 125% of this year’s earnings. If the company earned $650,000 this year, what are next year’s estimated earnings?
2D-10 A baker prepares a 130-pound batch of bread dough and uses 120 pounds of the dough. What percent of the batch is used?

2D-11 In an electrical circuit, a certain resistor takes 16% of the total voltage. How many volts are taken by the resistor in a 230-volt circuit?

2D-12 A welder orders 150 square meters of steel plate. If 85 square meters are delivered, what percent of the order is received?

2D-13 A laboratory technician prepares 85% of a total required amount of solution. If the total amount of solution required is 2.8 liters, how many liters are prepared?

2D-14 A mason lays 72 feet of sidewalk which represents 40% of the total job. What is the total length, in feet, of the completed sidewalk?

2D-15 Before stretching, a spring measures 5.84 centimeters. The spring is stretched an additional 1.22 centimeters. What percent of the original length is the amount of stretch?

2D-16 By installing new machinery, a firm increases production by 25%. An average of 1,800 units are produced per day with the new machinery. Determine the average daily production before installation of the new machinery.

D-17 A retailer purchased merchandise at wholesale cost for $1,050. The wholesale cost is 35% less than the retail price. What is the retail price?

2D-18 A building contractor has 4,250 feet of 2" x 4" lumber at the start of a job. At the end of the first week of the job, 32% of the lumber is used. At the end of the second week 40% of the stock that remained at the end of the first week is used. How many feet of lumber remain unused at the end of the second week?
2D-19 A resistor is rated at 1,600 ohms with a tolerance of ± 4%. The resistor is checked and found to have an actual resistance of 1,510 ohms. By how many ohms is the resistor below the acceptable resistance low limit?

2D-20 Brazing solder contains 51.2% copper, 48.3% zinc, 0.1% iron, and lead. How many pounds of lead are required to make 2,000 pounds of solder?

2D-21 The manager of a clothing store computes semi-annual monthly profits as follows: July, $6,250; August, $5,700; September, $7,100; October, $6,000; November, $5,200; and December, $11,050. What percent of the entire semi-annual profit is the December profit?

2D-22 A landscaper estimates that 72 cubic meters of topsoil is required for a job. After the job is completed, it is found that the amount of topsoil estimated was 20% more than what was actually used. How many cubic meters of topsoil were used on the job?

2D-23 In a given volume of solution, 50 milliliters of acid makes up 20% of the solution. For the same volume solution, how many milliliters of acid are required to produce a 28% acid solution?

2D-24 On Monday, a manufacturing firm produced a total of 1,100 units with 3% of the units defective. On Tuesday, the firm produced a total of 1,000 units with 5 1/2% of the units defective. How many more acceptable units were produced on Monday than on Tuesday?

2D-25 A length of copper wire measures 20 feet 6 1/2 inches before being heated. When heated, the wire measures 20 feet 7 3/4 inches. What is the percent of increase in length?
This table shows the standing for one year of the Eastern Division of the National Conference of the NFL. Each team in the division is ranked according to games won. The percent is usually written as a three-place decimal.

St. Louis' Pct. = \frac{\text{Games Won}}{\text{Games Played}}

<table>
<thead>
<tr>
<th>Team</th>
<th>Games Won</th>
<th>Games Lost</th>
<th>Pct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Louis</td>
<td>10</td>
<td>3</td>
<td>.769</td>
</tr>
<tr>
<td>Dallas</td>
<td>9</td>
<td>4</td>
<td>.692</td>
</tr>
<tr>
<td>Washington</td>
<td>11</td>
<td>2</td>
<td>?</td>
</tr>
<tr>
<td>New York</td>
<td>7</td>
<td>6</td>
<td>?</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>?</td>
<td>?</td>
<td>.357</td>
</tr>
</tbody>
</table>

Can you find the missing parts of the table? Remember that all teams play the same number of games in a season.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{20} )</td>
<td>.05</td>
<td>5%</td>
</tr>
<tr>
<td>( \frac{1}{8} )</td>
<td>.10</td>
<td>10%</td>
</tr>
<tr>
<td>( \frac{1}{8} )</td>
<td>( \frac{12}{2} )</td>
<td>12 1%</td>
</tr>
<tr>
<td>( \frac{1}{6} )</td>
<td>( \frac{16}{3} )</td>
<td>16 2%</td>
</tr>
<tr>
<td>( \frac{1}{5} )</td>
<td>.20</td>
<td>20%</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>.25</td>
<td>25%</td>
</tr>
<tr>
<td>( \frac{3}{10} )</td>
<td>.30</td>
<td>30%</td>
</tr>
<tr>
<td>Fraction</td>
<td>Decimal</td>
<td>Percentage</td>
</tr>
<tr>
<td>----------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>1/3</td>
<td>0.33</td>
<td>33.33%</td>
</tr>
<tr>
<td>2/3</td>
<td>0.67</td>
<td>66.67%</td>
</tr>
<tr>
<td>3/4</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>3/5</td>
<td>0.60</td>
<td>60%</td>
</tr>
<tr>
<td>4/5</td>
<td>0.80</td>
<td>80%</td>
</tr>
<tr>
<td>5/6</td>
<td>0.83</td>
<td>83.33%</td>
</tr>
<tr>
<td>5/8</td>
<td>0.62</td>
<td>62.5%</td>
</tr>
<tr>
<td>7/8</td>
<td>0.87</td>
<td>87.5%</td>
</tr>
<tr>
<td>9/10</td>
<td>0.90</td>
<td>90%</td>
</tr>
</tbody>
</table>
Section 3: Arithmetic Average ($\bar{X}$)

### Dictionary:

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Average ($\bar{X}$)</td>
<td>arithmetic average, i.e., sum of all numbers divided by the number count</td>
</tr>
<tr>
<td>2. Median</td>
<td>middle number</td>
</tr>
<tr>
<td>3. Range ($R$)</td>
<td>difference between the smallest and largest values</td>
</tr>
<tr>
<td>4. Graph</td>
<td>a drawn (plotted) representation of past data or statistics</td>
</tr>
<tr>
<td>5. Axes ($X$ and $Y$)</td>
<td>the two boundaries that make up a graph</td>
</tr>
<tr>
<td>6. Chart (or charting)</td>
<td>a pre-printed form with boundaries to plot ongoing data</td>
</tr>
<tr>
<td>7. Plot (or plotting)</td>
<td>placing data points in reference to the $X$ and $Y$ axes</td>
</tr>
</tbody>
</table>

Sometimes we need to use approximate numbers for one reason or another. One commonly used approximation is averaging. Averages are often used for comparisons, such as comparing the average mileage different cars get per gallon of gasoline.

Averages used for our purpose at ALPO will mean arithmetic average. (It is also referred to as the mean in other work places.) Average is designated by the symbol $X$.

#### 3-A Examples

1. Determine the average for this set of numbers:

   $1036, 1002, 1019, 1056, \text{ and } 1011 = \underline{1020}$

   **solution:**  
   **Step 1** Since we are dealing with whole numbers only, set the decimal indicator at 1 (one beyond, or the tenth place).

   At this point, your attention is called to a key on the calculator that has not been used in the past; it is the [N] key at the top of the row of three keys on the right. This is a tally key.
Section 4: Median

Another type of approximation is called median which means "in the middle". It differs from the average (X) in that the numbers being studied are never added; instead we arrange the numbers.

4-A Examples

1. Determine the median for this set of numbers:

\[ 23.6, 22.8, 23.0, 24.5, 25.3, 24.3, 23.2, \text{ and } 24.5. \]

Solution:

Step 1 Arrange the numbers in order from smallest to largest in vertical fashion as follows:

\[
\begin{align*}
22.8 \\
23.0 \\
23.2 \\
23.6 \\
24.3 \\
24.5 \\
24.5 \\
25.3
\end{align*}
\]

Step 2 Count the total in the listing; it is 8, an even number.

Step 3 Since there is no exact middle, or median for an even numbered listing, we select the two innermost numbers from top and bottom - 23.6 is fourth from the top and 24.3 is fourth from the bottom - add these two (with decimal indicator at 2); display shows 47.90.

Step 4 Since we need the average for the two numbers added in Step 3, we divide by 2; display shows 23.95.

Step 5 The median rounded to 1 place is 24.0, Answer.
2. Determine the median for this set of numbers:
3.9, 3.5, 4.2, 3.0, 3.8, 4.1, 3.7.

solution: Step 1  Arrange the numbers in order from smallest to largest in vertical fashion as follows:

3.0
3.5
3.7
3.8
3.9
4.1
4.2

Step 2  Count the total in the listing; it is 7, an odd number.

Step 3  The exact middle, or median, for a listing of 7 is 4 from the top or 4 from the bottom; the median is 3.8.

Answer.

The rule for determining the median is as follows:

a) if there is an odd numbered listing, determine the exact middle in counting from the top or bottom;

b) if there is an even numbered listing, determine the innermost two, one from the top and one from the bottom; divide by two and round off if necessary.
UNIT 4: DETERMINING MEDIANs

Determine the medians for each set of numbers, rounding to the place shown in parentheses:

3D-1  0.098, 0.100, 0.102, 0.097, 0.105, 0.102, 0.096 (3)

3D-2  143, 149, 150, 144, 146, 148, 147, 145 (0)

3D-3  5, 7, 9, 3, 13, 11, 15 (4)

3D-4  1.1, 1.9, 2.0, 1.3, 1.5, 1.7, 1.6, 1.4 (1)

3D-5  1198, 1190, 1199, 1195, 1194, 1193, 1196 (0)

3D-6  7, 9, 3, 1, 8, 5, 6, 4 (0)


3D-8  0.0035, 0.0029, 0.0034, 0.0032, 0.0031 (4)

3D-9  17.9, 17.9, 17.7 (1)

3D-10  35.012, 35.019, 35.017, 35.016 (3)
Section 5: Range (R)

Range (R) is the spread or difference between the smallest value and the largest value.

Before we discussed medians, and you learned that in order to determine the median, it was necessary to arrange the numbers in a vertical listing from smallest to largest. This same procedure could be followed for determining the range (R).

5-A Examples

1. Determine the range (R) for the following numbers:

   13, 11, 12, 10, 12, 14, 13, 11

Solution: Step 1 Arrange in vertical fashion;

   10
   11
   11
   12
   12
   13
   13
   14

   Step 2 We see that the largest number is 14 and the smallest is 10; to obtain the difference between these two, subtract 10 from 14, this equals 4 which is the range (R).

   Note! In order to minimize errors, especially when working with decimal fractions, always enter the larger number in the calculator, press [+] key, enter the smaller number and press [-] key; the display will show the difference, which is the range (R).
UNIT 4: REVIEW

Report answers to two decimal places.

- 4R1 - Express 109.6% as a decimal fraction.
- 4R2 - Express 37 1/2% as a common fraction.
- 4R3 - Express 201.5% as a common fraction.
- 4R4 - Express 0.85% as a decimal fraction.
- 4R5 - Write 0.91 as a percent.
- 4R6 - Express 5/16 as a percent.
- 4R7 - What is 7% of 140?
- 4R8 - 60 is 80% of what number?
- 4R9 - What is 109 3/4% of 75?
- 4R10 - What percent of 130 is 105?
- 4R11 - What percent of 17 is 17/16?
- 4R12 - What percent of 302 is 402?
- 4R13 - 800 parts is 140% of what number of parts?
- 4R14 - 11 is 40% of what number?
- 4R15 - What percent of 30 is 40?

Determine the range, average, and median for the following sets of numbers:

a. 6, 5, 9, 4, 8, 11, 10, 7

Range

Median

b. 0.0098, 0.0087, 0.0092, 0.0087, 0.0098, 0.0082, 0.0093
The shipping department’s number of orders processed for 1985 was as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>Orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>505</td>
</tr>
<tr>
<td>FEB</td>
<td>575</td>
</tr>
<tr>
<td>MAR</td>
<td>480</td>
</tr>
<tr>
<td>APR</td>
<td>600</td>
</tr>
<tr>
<td>MAY</td>
<td>495</td>
</tr>
<tr>
<td>JUN</td>
<td>615</td>
</tr>
<tr>
<td>JUL</td>
<td>500</td>
</tr>
<tr>
<td>AUG</td>
<td>545</td>
</tr>
<tr>
<td>SEP</td>
<td>565</td>
</tr>
<tr>
<td>OCT</td>
<td>615</td>
</tr>
<tr>
<td>NOV</td>
<td>625</td>
</tr>
<tr>
<td>DEC</td>
<td>600</td>
</tr>
</tbody>
</table>

a. Plot a line graph which follows this page.

b. Label the X and Y axes.
UNIT 5: CONCE. OF MEASURE & SIGNED NUMBERS

OBJECTIVES:

. Understanding Conversions with English units of linear measure
. Understanding the Metric Table
. Understanding the Metric Relationship and Conversions

Dictionary:

<table>
<thead>
<tr>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linear Measure</td>
</tr>
<tr>
<td>Measure that pertains to lines or length</td>
</tr>
<tr>
<td>2. Metric</td>
</tr>
<tr>
<td>International System of Units of Measurement</td>
</tr>
</tbody>
</table>

159
Section 1: What is measure?

You have been using measure, or more precisely, units of measure your entire life. For example, you purchased a new car for 15,000 dollars, or you had 400 square feet of carpeting installed; or you purchased 3 dozen eggs; or you bought 1 gallon of milk and 3 pints of cottage cheese; or you worked 44 hours last week at ALPO. All of the underscored are units of measure.

If we would not identify an item or quantity with a unit of measure, there would be chaos because all we would have would be numbers.

IMPORTANT: You will always attach a unit of measure to answers you report in this math module. The only exception will be an exercise involving numbers only.

Keep in mind that it is acceptable to use abbreviations for units of measure. However, the abbreviations must be exactly as shown.
Section 2: Linear Measure

2A - Equivalent English units of linear measure

Examples

1. Express 76.5 inches as feet

solution: Step 1 Since inches and feet imply linear (straight line) measurement, select the appropriate conversion table, which is Table A on page P10.

Observe from Table A that 1 foot = 12 inches, or 12 inches = 1 foot.

Step 2 Set up every conversion as follows:

<table>
<thead>
<tr>
<th>unit began with</th>
<th>conversion = desired unit in answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.5 in</td>
<td>1 ft = 6.38 ft, 12 in</td>
</tr>
</tbody>
</table>

Step 3 Earlier we learned when multiplying fractions, we were able to divide the numerator and denominator by some common number. This same principle applies to units of measure; common units in the numerator and denominator can be divided or cancelled.

Thus, in our example we divide 1 in by 1 in, or 1 in = 1 and the inch units are cancelled, so

76.5 in x 1 ft = 6.38 ft, 12 in

or 76.5 x 1 ft = 6.38 ft, 12

and rounded to 1 decimal place our answer is 6.4 ft.

To repeat, first check with the appropriate conversion table; second set up problem with unit of measure you began with on the left, times the conversion, equals the unit of measure you desire in the answer;
third cancel out the unwanted unit of measure so that only the unit of measure desired remains in the numerator; fourth multiply or divide the numbers to the left of the equal sign and place the result with the remaining unit of measure to the right of the equal sign as your answer.

2. Express 6.375 feet as inches.

solution:  Step 1 First, set decimal indicator on calculator at 4 (one beyond the 3 decimal places given in our problem). Second, we will use Table A as our conversion table.

Step 2 Set up conversion as follows:

\[
6.375 \, \text{ft} \times \frac{12 \, \text{in}}{1 \, \text{ft}} = \text{in.}
\]

Step 3 Cancel (divide) the feet (ft) and multiply the numbers:

\[
6.375 \, \text{ft} \times \frac{12 \, \text{in}}{1 \, \text{ft}} = 76.5000 \, \text{in},
\]
or 76.5 in, Answer.

3. Express \( \frac{7}{18} \) mile as feet

solution:  Step 1 Convert common fraction, \( \frac{7}{18} \), to decimal fraction: \( \frac{7}{18} = 0.3889 \), or 0.389 (to 3 places).

Step 2 From Table A, note that 1 mile = 5,280 feet. Set up conversion:

\[
0.389 \, \text{mi} \times \frac{5,280 \, \text{ft}}{1 \, \text{mi}} = \text{ft}
\]

Step 3 \( 0.389 \, \text{mi} \times \frac{5,280 \, \text{ft}}{1 \, \text{mi}} = 2,053.9200 \, \text{ft}, \)
or 2,053.92 ft, Answer.
4. Express 37.5 yards as feet and inches

Solution: Step 1  When asked to report the answer in multiple units of measure, convert to the larger unit first, as follows:

\[ 37.5 \text{ yd} \times 3 \text{ ft} = 112.50 \text{ ft} \]

Our preliminary result states that we now have 112 whole feet and a fraction, or 0.5 ft.

Step 2  Second, convert the fractional part of a foot to inches, as requested:

\[ 0.5 \text{ ft} \times 12 \text{ in} = 6 \text{ in} \]

Step 3  Third, combine the two units of measure as 112 ft 6 in, Answer.
UNIT 5: EQUIVALENT ENGLISH UNITS OF LINEAR MEASURE

Express each length as indicated; round answers to 3 decimal places:

1D-1 \[ 25 \frac{1}{2} \text{ inches as feet} = \]

1D-2 \[ 16.25 \text{ feet as yards} = \]

1D-3 \[ 3960 \text{ feet as miles} = \]

1D-4 \[ 78 \text{ inches as feet and inches} = \]

1D-5 \[ 47 \text{ feet as yards and feet} = \]

1D-6 \[ 7 \frac{1}{4} \text{ feet as inches} = \]

1D-7 \[ 12 \frac{2}{3} \text{ yards as feet} = \]

1D-8 \[ 0.6 \text{ mile as feet} = \]

1D-9 \[ 2 \frac{1}{4} \text{ yards as inches} = \]

1D-10 \[ 5 \frac{1}{2} \text{ yards as feet and inches} = \]

1D-11 \[ \frac{1}{12} \text{ mile as yards and feet} = \]

1D-12 \[ 6.2 \text{ yards as feet and inches} = \]

1D-13 \[ 17.35 \text{ miles as rods and yards} = \]

1D-14 \[ 153.5 \text{ feet as rods} = \]

1D-15 \[ \frac{3}{5} \text{ mile as feet} = \]

164
Section 3: Arithmetic operations with English compound numbers

Examples:

Perform the indicated arithmetic operation; express the answer in the same units of measure as those given in the exercise. Regroup the answer when necessary.

1. 15 ft 8 in + 6 ft 5 in =

solution: Step 1 Set up as an addition problem in vertical fashion and add:

\[
\begin{align*}
15 \text{ ft} & \quad 8 \text{ in} \\
+ \quad 6 \text{ ft} & \quad 5 \text{ in} \\
\hline
21 \text{ ft} & \quad 13 \text{ in}
\end{align*}
\]

Step 2 Since 13 inches is more than 1 foot (12 in), we regroup by subtracting 12 inches from 13 and add 1 foot to 21.

Step 3

\[
\begin{align*}
15 \text{ ft} & \quad 8 \text{ in} \\
+ \quad 6 \text{ ft} & \quad 5 \text{ in} \\
\hline
21 \text{ ft} & \quad 13 \text{ in} \\
+ \quad 1 \text{ ft} & \quad -12 \text{ in} \\
\hline
22 \text{ ft} & \quad 1 \text{ in}, \text{ Answer}
\end{align*}
\]

2. 14 yd 1 ft \(\frac{1}{2}\) ft - 11 yd 2 ft =

solution: Step 1 Set up as a subtraction problem in vertical fashion:

\[
\begin{align*}
14 \text{ yd} & \quad 1 \text{ ft} \quad \frac{1}{2} \text{ ft} \\
- \quad 11 \text{ yd} & \quad 2 \text{ ft}
\end{align*}
\]

Step 2 Borrow 1 yard from 14 and convert to 3 feet, then add to \(\frac{1}{2}\) ft:

\[
\begin{align*}
13 \text{ yd} & \quad 3 \frac{1}{2} \text{ ft} \\
- \quad 11 \text{ yd} & \quad 2 \text{ ft} \\
\hline
2 \text{ yd} & \quad 1 \frac{1}{2} \text{ ft}, \text{ Answer}
\end{align*}
\]
<table>
<thead>
<tr>
<th>PREFIX (x)</th>
<th>LENGTH</th>
<th>METRIC</th>
<th>ENGLISH</th>
<th>METRIC</th>
<th>ENGLISH</th>
<th>WEIGHT</th>
<th>ENGLISH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>kilo x 100</strong></td>
<td>Kilometer (km)</td>
<td>1 km = .6214 mi = 196.84 rd = 1093.6 yd = 3280.8 ft</td>
<td>Kiloliter (kl)</td>
<td>1 kl = 1000 l = 2.205 lb = 35.28 oz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>hecto x 100</strong></td>
<td>Hectometer (hm)</td>
<td>1 hm = .6214 mi = 196.84 rd = 1093.6 yd = 3280.8 ft</td>
<td>Hectoliter (hl)</td>
<td>1 hl = 10 l = 2.205 lb = 35.28 oz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>deka x 10</strong></td>
<td>Dekameter (dam)</td>
<td>1 dam = 1.988 rd = 10.936 yd = 32.808 ft</td>
<td>Dekaliter (dal)</td>
<td>1 dal = 2.642 gal = 10.570 qt = 21.14 pt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1</strong></td>
<td>Meter (m)</td>
<td>1 m = 3.281 ft = 0.9144 m = 1 yd</td>
<td>Liter (l)</td>
<td>1 l = 0.2642 gal = 10.570 qt = 35.31 ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>deci x 0.1</strong></td>
<td>Decimeter (dm)</td>
<td>1 dm = 3.937 in = 25.4 cm = 1 in</td>
<td>Deciliter (dl)</td>
<td>1 dl = 0.2834 gal = 0.946 L = 1 qt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>centi x 0.01</strong></td>
<td>Centimeter (cm)</td>
<td>1 cm = 0.3937 in = 2.54 cm = 1 in</td>
<td>Centiliter (cl)</td>
<td>1 cl = 0.0283 gal = 0.0946 L = 0.226 pt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>milli x 0.001</strong></td>
<td>Millimeter (mm)</td>
<td>1 mm = 0.03937 in = 25.4 mm = 1 in</td>
<td>Milliliter (ml)</td>
<td>1 ml = 0.03382 oz = 29.583 ml = 1 oz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cu. ft</td>
<td>= 1728 cu in.</td>
<td>1.48 gal = 1 ft</td>
<td>1 gal = 231 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 metric ton = 1.102 ton
1000 kg = .984 long ton = 2205 lb
Examples

1. **Express 0.378 meter as millimeters**

   Referring to metric table on the preceding page we note that

   \[
   10 \times 10 = 100, \text{ and} \\
   100 \times 10 = 1000
   \]

   This tells us there are 1000 millimeters in 1 meter, or that 1 meter is equal to 1000 millimeters.

   **Step 2** Set up conversion:

   \[
   0.378 \times 1000 \text{ mm} = 378 \text{ mm}, \text{ Answer} \\
   \]

2. **Express 472.5 m as km**

   **solution**

   **Step 1** From the left column of Table B, note that millimeter is positioned higher than kilometer, thus kilometer is larger.

   Place left-hand marker above meter in left-hand column. Place right-hand marker below kilometer in right-hand column. There are three 10's between the two markers; multiplying these we get 1000 m = 1 km (or 1 km = 1000 m).

   **Step 2** Set up conversion:

   \[
   472.5 \times \frac{1}{1000} \text{ km} = 0.4725 \text{ km}, \text{ Answer} \\
   \]

   \[167\]

   173
UNIT 5: EQUIVALENT METRIC UNITS OF LINEAR MEASURE

Express each value in the unit indicated:

2D-1 30 mm as cm =
2D-2 8 cm as mm =
2D-3 2460 mm as m =
2D-4 23 m as cm =
2D-5 650 m as km =
2D-6 0.8 km as m =
2D-7 0.75 m as mm =
2D-8 12.2 cm as mm =
2D-9 372.5 m as km =
2D-10 1935 mm as dam =
2D-11 795 dm as hm =
2D-12 3975 mm as km =
2D-13 425 cm as dam =
2D-14 234.5 cm as hm =
2D-15 122.5 dam as km =

168
Section 4: Arithmetic operations with metric lengths

Example

1. \(49.8 \text{ cm} + 14.3 \text{ dm} + 77.75 \text{ mm} = \text{ cm}\)

We see that there are three different metric units of linear measure to be added, and the answer is to be reported in one of these. Therefore, two have to be converted.

solution: Step 1 Convert dm to cm;
\[
14.3 \text{ dm} \times 10 \frac{\text{cm}}{1 \text{ dm}} = 143 \text{ cm}
\]

Step 2 Convert mm to cm;
\[
77.75 \text{ mm} \times 1 \frac{\text{cm}}{10 \text{ mm}} = 7.775 \text{ cm}
\]

Step 3 Add all three cm’s;
\[
\begin{align*}
49.8 \text{ cm} \\
143 \text{ cm} \\
+ 7.775 \text{ cm}
\end{align*}
\]
\[= 200.575 \text{ cm}, \text{ Answer}\]

2. \(728.8 \text{ cm} - 4.1 \text{ m} = \text{ cm}\)

solution: Step 1 Convert m to cm;
\[
4.1 \text{ m} \times 100 \frac{\text{cm}}{1 \text{ m}} = 410 \text{ cm}
\]

Step 2 Subtract;
\[
\begin{align*}
728.8 \text{ cm} \\
- 410 \text{ cm}
\end{align*}
\]
\[= 318.8 \text{ cm}, \text{ Answer}\]

3. \(23 \times 3.3 \text{ m} = \text{ m}\)

solution: Step 1 Since the unit is the same from beginning of our problem to the answer, no conversion is needed; proceed as with usual multiplication:
\[
3.3 \text{ m} \\
\times 23
\]
\[= 75.9 \text{ m}, \text{ Answer}\]
4. $8.64 \text{ dm} - 6 = \underline{\hspace{1cm}} \text{ dm}$

solution:  Step 1  
Again, there is no conversion of units required; proceed as with usual division:

$$\frac{8.64 \text{ dm}}{6} = 1.44 \text{ dm}, \text{ Answer}$$
UNIT 5: EXPRESSING ENGLISH UNITS AS METRIC UNITS OF LINEAR MEASURE AND METRIC UNITS AS ENGLISH UNITS OF LINEAR MEASURE

Report answers to decimal places indicated in parentheses:

3D-1  0.4 m as in = _______ (3)
3D-2  8 in as mm = _______ (2)
3D-3  12 mm as in = _______ (3)
3D-4  23 in as cm = _______ (2)
3D-5  4.5 m as ft = _______ (3)
3D-6  5.8 ft as m = _______ (2)
3D-7  0.18 km as ft = _______ (3)
3D-8  3.4 mi as km = _______ (2)
3D-9  0.75 m as ft = _______ (3)
3D-10 2.7 ft as cm = _______ (2)
3D-11 286 km as mi = _______ (3)
3D-12 71 in as m = _______ (2)
3D-13 4.6 m as yd = _______ (3)
3D-14 3872 ft as km = _______ (2)
3D-15  78 cm as ft =  
3D-16  3 in as mm =  
3D-17  9.8 cm as in =  
3D-18  8 ft 2 1/4 in as m =  
3D-19  19.3 yd as cm =  
3D-20  3/16 mi as m =  
3D-21  234 mm as yd =  
3D-22  0.18 km as yd =  
3D-23  13,984 ft as km =  
3D-24  3 1/11 mi as m =  
3D-25  0.055 mi as m =  
3D-26  542 L as kL =  
3D-27  84 oz as gal =  
3D-28  3.17 hL as L =  
3D-29  0.2 gal as pt =  
3D-30  0.06 L as mL =  
3D-31  1.4 qt as cz =  

172
<table>
<thead>
<tr>
<th>Problem</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-32</td>
<td>$19000 \text{ mL as } \text{ L} =$</td>
</tr>
<tr>
<td>3D-33</td>
<td>$1710 \text{ dL as } \text{ hL} =$</td>
</tr>
<tr>
<td>3D-34</td>
<td>$25000 \text{ cl as } \text{ daL} =$</td>
</tr>
<tr>
<td>3D-35</td>
<td>$1 \text{ hL as } \text{ kL} =$</td>
</tr>
<tr>
<td>3D-36</td>
<td>$125 \text{ mL as } \text{ dL} =$</td>
</tr>
</tbody>
</table>
Section 5: Equivalent English-metric capacity measures

Examples

1. Express 2.5 ounces as milliliters. (Round answer to 3 places.)

   solution: Step 1 Use conversion Table
              Step 2 Set up conversion:
                       \[ 2.5 \text{ oz} \times \frac{29.563 \text{ mL}}{1 \text{ oz}} = \text{ mL} \]
              Step 3 Cancel ounces and multiply numbers:
                       \[ 2.5 \text{ oz} \times 29.563 \text{ mL} = 73.908 \text{ mL}, \text{ Answer} \]

2. Express 73.91 milliliters as ounces. (Round answer to 3 places.)

   solution: Step 1 Use conversion Table
              Step 2 Set up conversion:
                       \[ 73.91 \text{ mL} \times \frac{0.0338 \text{ oz}}{1 \text{ mL}} = \text{ oz} \]
              Step 3 Cancel milliliters and multiply numbers:
                       \[ 73.91 \text{ mL} \times 0.0338 \text{ oz} = 2.498 \text{ oz}, \text{ Answer} \]
DRILL SHEET

UNIT 5: EQUIVALENT ENGLISH-METRIC CAPACITY MEASURES

Express answers in units indicated; round answers to 2 decimal places:

4D-1 10.4 L as qt =
4D-2 4.2 oz as mL =
4D-3 8 gal as L =
4D-4 75 mL as oz =
4D-5 1.75 qt as L =
4D-6 513 L as gal =
4D-7 0.67 L as pt =
4D-8 21 oz as L =
4D-9 175 mL as gal =
4D-10 360 oz as L =
4D-11 305 mL as oz =
4D-12 64 L as qt =
4D-13 0.19 gal as mL =
4D-14 725 oz as L =
4D-15 0.75 qt as mL =
Section 6: Weight Measure

Equivalent English and metric units of weight measure

Examples

1. Express 200 ounces as pounds

   solution:
   Step 1  Set up conversion:
   \[
   200 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{200}{16} \text{ lb} = 12.5 \text{ lb}, \quad \text{Answer}
   \]

2. Express 5400 grams as kilograms

   solution:
   Step 1  Use conversion Table
   Note from table that kilogram is farther down than gram; therefore kilogram is larger. Using left hand, right hand rule, there are three 10's in left-hand column, or 1 kg = 1000 grams.

   Step 2  Set up conversion:
   \[
   5400 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = \frac{5400}{1000} \text{ kg} = 5.4 \text{ kg}, \quad \text{Answer}
   \]
### DRILL SHEET

**UNIT 5: EQUIVALENT ENGLISH AND METRIC UNITS OF WEIGHT MEASURE**

Express each weight in the unit indicated; round answers to 2 decimal places:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5D-1</td>
<td>34 oz as lb</td>
<td>2.12</td>
</tr>
<tr>
<td>5D-2</td>
<td>1880 g as kg</td>
<td>1.88</td>
</tr>
<tr>
<td>5D-3</td>
<td>0.6 lb as oz</td>
<td>9.60</td>
</tr>
<tr>
<td>5D-4</td>
<td>730 mg as g</td>
<td>0.73</td>
</tr>
<tr>
<td>5D-5</td>
<td>48,000 lb as long tons</td>
<td>48.00</td>
</tr>
<tr>
<td>5D-6</td>
<td>2.7 metric tons as kg</td>
<td>2700</td>
</tr>
<tr>
<td>5D-7</td>
<td>9700 lb as short tons</td>
<td>9.70</td>
</tr>
<tr>
<td>5D-8</td>
<td>4.75 g as mg</td>
<td>4750</td>
</tr>
<tr>
<td>5D-9</td>
<td>0.66 short tons as lb</td>
<td>1320</td>
</tr>
<tr>
<td>5D-10</td>
<td>0.21 kg as g</td>
<td>2100</td>
</tr>
<tr>
<td>5D-11</td>
<td>1.08 long tons as lb</td>
<td>2268</td>
</tr>
<tr>
<td>5D-12</td>
<td>310,000 kg as metric tons</td>
<td>310.00</td>
</tr>
<tr>
<td>5D-13</td>
<td>1445 dekagrams as metric tons</td>
<td>1445</td>
</tr>
<tr>
<td>5D-14</td>
<td>3.75 kilograms as hectograms</td>
<td>3750</td>
</tr>
<tr>
<td>5D-15</td>
<td>3.85 kilograms as dekagrams</td>
<td>3850</td>
</tr>
</tbody>
</table>
6-A Equivalent English-metric weight measures

Examples

1. Express 35 pounds as kilograms

   solution:  Step 1  Use conversion Table
             Step 2  Set up conversion:
                     $35\text{ lb} \times \frac{0.454\text{ kg}}{1\text{ lb}} = \text{ kg}$
             Step 3  Cancel pounds and multiply numbers:
                     $35\text{ lb} \times \frac{0.454\text{ kg}}{1\text{ lb}} = 15.89$, Answer

2. Express 15.89 kg as pounds. (Round answer to nearest whole pound.)

   solution:  Step 1  Use conversion Table
             Step 2  Set up conversion:
                     $15.89\text{ kg} \times \frac{2.205\text{ lb}}{1\text{ kg}} = \text{ lb}$
             Step 3  Cancel kilograms and multiply:
                     $15.89\text{ kg} \times \frac{2.205\text{ lb}}{1\text{ kg}} = 35\text{ lb}$, Answer
Express each weight in the unit indicated; round answers to 3 decimal places:

6D-1 154 kg as lb =
6D-2 0.9 oz as g =
6D-3 220 g as oz =
6D-4 964.8 lb as kg =
6D-5 3.07 metric tons as lb =
6D-6 40.4 short tons as metric tons =
6D-7 15,000 oz as kg =
6D-8 13,800 g as short tons =
6D-9 0.75 short tons as kg =
6D-10 11.02 t as lb =
SIGNED NUMBERS

OBJECTIVES

After completing this section, you will be able to:

- express word statements as signed numbers
- write signed number values using a number scale
- add, subtract, multiply and divide signed numbers
- solve combined operations of signed number expressions
- solve signed number problems

Dictionary:

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Signed Number</td>
<td>a number, prefixed with a plus (+) or minus (-) sign which indicates direction from zero</td>
</tr>
<tr>
<td>2. Positive Number</td>
<td>a number with no prefixed sign or with a plus (+) sign</td>
</tr>
<tr>
<td>3. Negative Number</td>
<td>a number prefixed with a minus (-) sign</td>
</tr>
<tr>
<td>4. Absolute Value</td>
<td>a number without its sign - it is technically the distance between the number and zero on the number line. The absolute value of 0 is 0. The absolute value of -7 is 7. The absolute value of 7 is 7.</td>
</tr>
<tr>
<td>5. Number Scale</td>
<td>increasing/decreasing positive and negative numbers</td>
</tr>
</tbody>
</table>
Concept of a Signed Number

What is a signed number?

Signed numbers are used to indicate direction and distance from a reference point. Opposites such as up and down, left and right, north and south, and clockwise and counterclockwise, may be expressed using positive and negative signs. For example, 100 feet above sea level may be expressed as +100 feet, and 100 feet below sea level as -100 feet. Sea level in this case is the zero reference point.

In business applications, a profit of $1000 is expressed as +$1000, whereas a loss of $1000 is expressed as -$1000. Closing prices for stocks are indicated as up (+) or down (-) from the previous day's closing prices.

Shown below are two thermometers with the Celsius scale (°C). On the left we read "20 degrees above zero", and we write +20°C. On the right we read "20 degrees below zero", and we write -20°C.

Signed numbers are used in programming operations for probing of wafers. From a reference point, the probe movements are expressed as + and - directions.
1-A Expressing word statements as signed numbers

Examples

1. A speed increase of 12 miles per hour is expressed as +12 mi/hr. Express a speed decrease of 8 miles per hour.

solution: Step 1 The key words are increase and decrease, one is the opposite of the other.

Step 2 Since an increase was expressed as +12 mi/hr, a decrease will be the opposite, or -8 mi/hr, Answer.

2. The reduction of a person’s daily calorie intake by 400 calories is expressed as -400 calories. Express a daily intake increase of 350 calories.

solution: Step 1 Since a calorie decrease was expressed as -400 calories, the opposite of decrease or increase is expressed as +350 calories, Answer.

3. A company’s assets of $73,600 are expressed as +$73,600. Express company liabilities of $48,000.

solution: Step 1 Since we expressed assets as +$73,600, the opposite of asset, a liability, is expressed as -$48,000, Answer.
Rules for Signed Numbers:

Addition

**Like Signs**
- Add the absolute value of the numbers.
- Give the sum the same sign as the numbers.

**Unlike Signs**
- Subtract the number with the smaller absolute value from that of the larger.
- Give the answer the same sign of that of the number with the largest absolute value.

Subtraction
- Change the sign of the number being subtracted (subtrahend)
- Add the numbers using the rules for adding signed numbers
EXPRESSING WORD STATEMENTS AS SIGNED NUMBERS

Express the answer to each word problem as a signed number:

7D-1 Traveling 50 kilometers west is expressed as -50 Km. How is traveling 75 kilometers east expressed?

7D-2 A wage increase of $25 is expressed as +$25. Express a wage decrease of $18.

7D-3 An increase of 30 pounds per square inch of pressure is expressed as +70 lb/sq in. Express a pressure decrease of 28 pounds per square inch.

7D-4 A circuit voltage loss of 7.5 volts is expressed as -7.5 volts. Express a voltage gain of 9 volts.

7D-5 A savings account deposit of $140 is expressed as +$140. Express a withdrawal of $280.

7D-6 A 0.75 percent contraction of a length of wire is expressed as -0.75%. Express a 1.2 percent expansion.

7D-7 A 15 pound weight gain is expressed as +15 lb. Express a weight loss of 10 pounds.
In construction the height of a 25-foot roof is expressed as +25 feet. Express the depth of the 15 foot basement floor.

A passing grade in school is expressed as +70. Express the failing grade of 55.

The weight of participants undergoing dieting/exercise is recorded. How do you show:

(a) a loss of 5 lb?
(b) a gain of 6 lb?
(c) a loss of 3 lb?
(d) no change to 120 lb weight?

Use signed numbers to show the following bank transactions:

(a) a deposit of $56
(b) a withdrawal of $45
(c) a withdrawal of $23
(d) no change to $1500 in the account

Use signed numbers to show:

(a) a depth of 182 feet below sea level
(b) a loss of $185 on the stock market
Section 2: Number Scale

A number scale (illustrated below) shows the relationship between positive and negative numbers. The scale shows both distance and direction between numbers. Considering any number as a starting point and counting to a number to the right represents a positive (+) direction with numbers increasing in value. Counting to the left represents negative (-) direction with numbers decreasing in value:

Examples of usage of number scale:

Example 1. Starting at 0 and counting to the right to +5 represents 5 units in a positive (+) direction; +5 is 5 units greater than 0.

Example 2. Starting at 0 and counting to the left to -5 represents 5 units in a negative (-) direction; -5 is 5 units less than 0.

Example 3. Starting at -4 and counting to the right to +3 represents 7 units in a positive (+) direction; +3 is 7 units greater than -4.

Example 4. Starting at +3 and counting to the left to -4 represents 7 units in a negative (-) direction; -4 is 7 units less than +3.

Example 5. Starting at -2 and counting to the left to -10 represents 8 units in a negative (-) direction; -10 is 8 units less than -2.

Example 6. Starting at -9 and counting to the right to 0 represents 9 units in a positive (+) direction; 0 is 9 units greater than -9.
Decimals and fractions, because they represent parts of whole numbers or combinations of whole numbers and parts of whole numbers, can also be positive and negative. The number scale below illustrates decimal numbers and fractions along with whole numbers:

As with whole numbers, the positive sign (+) is usually not expressed, except for clarity or emphasis. The numbers \( \frac{1}{2} \) and \( +\frac{1}{2} \) are equivalent, just as 0.5 and -0.5 are equivalent. But the negative sign is always expressed as \( -\frac{1}{2} \), (-0.5), \( -\frac{1}{2} \), \( -3.5 \), and so on.

Zero, 0, has no sign.
DRILL SHEET

NUMBER SCALE

Using the number scale below, give the direction (+ or -) and the number of units counted going from the first to the second number:

8D-1  0 to +6 =  
8D-2  0 to (-6) =  
8D-3  (-2) to 0 =  
8D-4  +2 to 0 =  
8D-5  (-3) to +5 =  
8D-6  (-7) to +1 =  
8D-7  +8 to (-3) =  
8D-8  +6 to (-6) =  
8D-9  (-10) to (-4) =  
8D-10 +9 to (-10) =  

8D-11 +10 to (-10) =  
8D-12 (-10) to +10 =  
8D-13 +6 to (-5) =  
8D-14 (-9) to +8 =  
8D-15 (-3) to (-7) =  
8D-16 (-9) to (-4) =  
8D-17 +4 to +10 =  
8D-18 +7 to +2 =  
8D-19 (-4) to +7 =  
8D-20 +6 to (-4) =

188
Place the following whole numbers, decimal numbers, and fractions on the number scale below:

(a) 1  
(b) (-1)  
(c) 3  
(d) (-1 \frac{1}{2})  
(e) (-6)  
(f) \frac{1}{2}  
(g) 2.5  
(h) 2  
(i) 4  
(j) (-2)  
(k) 6  
(l) (-5)  
(m) 4 \frac{1}{2}  
(n) 5.5  
(o) (-2.5)  
(p) (-3)  
(q) 0.5  
(r) (-4)  
(s) 5  
(t) (-4 \frac{1}{2})  
(u) (-0.5)  
(v) 3 \frac{1}{2}  
(w) 1 \frac{1}{2}  
(x) (-5.5)  
(y) (-5 \frac{1}{2})  
(z) (-1 \frac{1}{2})
Select the greater of each of the two signed numbers and indicate the number of units by which it is greater:

(a) \( +3, (-2) = \)

(b) \( (-6), 0 = \)

(c) \( (-5), +1 = \)

(d) \( (-12), (-4) = \)

(e) \( (-28), (-73) = \)

(f) \( +18, +14 = \)

(g) \( (-18), (-14) = \)

(h) \( +10, (-12) = \)

(i) \( (-2.5), +2.5 = \)

(j) \( (-86 \frac{3}{4}), 0 = \)

(k) \( +18.3, (-20.6) = \)

(l) \( (-23 \frac{1}{4}), (-15 \frac{3}{8}) = \)

(m) \( +1 \frac{1}{16}, (-1 \frac{7}{8}) = \)

(n) \( (-50.23), (-41.76) = \)

(o) \( + \frac{3}{16}, (-9 \frac{3}{32}) = \)
Section 3: Absolute Value

In order to solve signed number problems, it is important to understand the meaning of absolute value. The absolute value of a number is the number without any sign; it is indicated by placing the number between a pair of vertical bars (or lines).

The absolute values of +8 and (-8) are written as follows:

+8 = |8| and (-8) = |8|

The absolute values of (-15) and +5 are written as follows:

(-15) = |-15| and +5 = |5|.

The absolute value of (-15) is 10 greater than the absolute value of +5.
### Absolute Value

Express each of the pairs of signed numbers as absolute values and report the difference (subtract the smaller absolute value from the larger absolute value):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9D-1</strong></td>
<td>+15, (-10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-2</strong></td>
<td>(-15), +10</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-3</strong></td>
<td>(-6), +2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-4</strong></td>
<td>(-14), +14</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-5</strong></td>
<td>(-9), 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-6</strong></td>
<td>+9, 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-7</strong></td>
<td>(-23), +22</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-8</strong></td>
<td>+18, (-18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-9</strong></td>
<td>+18, +18</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-10</strong></td>
<td>(-18), (-18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-11</strong></td>
<td>+6 1/4, (-3 3/4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-12</strong></td>
<td>(-3 1/2), (-12 7/8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-13</strong></td>
<td>+12.7, (-9.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-14</strong></td>
<td>+10.54, (-12.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9D-15</strong></td>
<td>(-0.03), (-0.007)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 4: Addition of Signed Numbers

Procedure For Addition of Signed Numbers:

1. For like signs, add the absolute values of the numbers and affix the same sign to the result:

\[(+ ) + (+ ) = + \text{(result)}\]

and

\[(- ) + (- ) = - \text{(result)}\]

2. For unlike signs, find the difference between the absolute values of the numbers and affix the sign of the larger number to the result:

\[(+ ) + (- ) = \text{diff.}\]

or

\[(- ) + (+ ) = \text{diff.}\]

\[\frac{\text{|}}{\text{|}} \rightarrow \frac{\text{|}}{\text{|}} = \text{sign larger number (result)}\]
4-A Addition of numbers with the same, or "like" signs.

Examples

1. Find the sum of two positive numbers

\[ (+5) + (+4) = \] __________

solution: Step 1 Add the absolute values of the two numbers:

\[ |5| + |4| = 9 \]

add

Step 2 Since the signs for the two numbers are the same, or "like", affix this same sign to the answer as

\[ (+5) + (+4) = +9, \text{ Answer} \]

The preceding addition problem can be illustrated on the number scale below:

2. Find the sum of two negative numbers

\[ (-5) + (-4) = \] __________

solution: Step 1 Add the absolute values of the two numbers:

\[ |-5| + |-4| = 9 \]

add

Step 2 Since signs for the two numbers are like (both -), affix this same sign to the answer as

\[ (-5) + (-4) = -9, \text{ Answer} \]

The preceding addition problem can be illustrated on the number scale below:
4-B Addition of numbers with different or "unlike" signs

1. Find the sum of a positive and negative number

\((+5) + (-3) = \) 

solution: Step 1 Find the diff. between the absolute values of the two numbers:

\[
\begin{array}{c|c}
5 & 3 \\
\hline
\end{array}
\]

Note: To find the difference between any two numbers, enter the larger number in the calculator and press [+ ] key. Then enter the smaller number and press [- ] key. For our example, the difference displayed is 2.

Step 2 Since the sign of the larger number in our problem was +, affix this sign to the answer as

\((+5) + (-3) = +2, \text{ Answer}\)

2. Find the sum of a negative and positive number

\((-5) + (+3) = \) 

solution: Step 1 Find the difference between the absolute values of the two numbers

\[
\begin{array}{c|c}
5 & 3 \\
\hline
\end{array}
\]

Step 2 Since the sign of the larger number in our problem was -, affix this sign to the answer as

\((-5) + (+3) = -2, \text{ Answer}\)
3. Find the sum of the following:

\[(+11) + (+7) + (-3) + (-9) + (+7) = \__\__\_

solution:

Step 1 Add all positive numbers:

\[(+11) + (+7) + (+7) = +25\]

Step 2 Add all negative numbers:

\[(-3) + (-9) = -12\]

Step 3 Add the grouped results:

\[(+25) + (-12) = +13, \text{ Answer}\]
DRILL SHEET

ADDITION OF SIGNED NUMBERS

Add the signed numbers as indicated:

10D-1  (+6) + (+9) =

10D-2  (+15) + (+8) =

10D-3  (+4) + (+20) =

10D-4  (+7) + (+18) + (+2) =

10D-5  0 + (+25) =

10D-6  (-12) + (-7) =

10D-7  (-8) + (-15) =

10D-8  0 + (-16) =

10D-9  (-14) + (-4) + (-11) =

10D-10 (-3) + (-6) + (-17) =

10D-11 (+12) + (-5) =

10D-12 (+18) + (-26) =
10D-13 \((-20) + (+17) =\)

10D-14 \((+46) + (-14) =\)

10D-15 \((-23) + (+17) =\)

10D-16 \((+25) + (-3) =\)

10D-17 \((-25) + (+3) =\)

10D-18 \((-18) + (-25) =\)

10D-19 \((-4) + (-31) =\)

10D-20 \((+27) + (-27) =\)

10D-21 \((-15.3) + (-5.5) =\)

10D-22 \((-15.3) + (+3.5) =\)

10D-23 \((-16.4) + (+2.7) =\)

10D-24 \((+37.9) + (-40.4) =\)

10D-25 \((-9 \frac{1}{4}) + (-3 \frac{3}{4}) =\)
10D-26  
\[
\frac{18}{8} + \frac{-21}{4} =
\]

10D-27  
\[-13 + \frac{-3}{16} =
\]

10D-28  
\[-13 + \frac{3}{16} =
\]

10D-29  
\[-4.25 + (-7) + (-3.22) =
\]

10D-30  
\[+18.07 + (-17.64) =
\]

10D-31  
\[+16 + (-4) + (-11) =
\]

10D-32  
\[-21 + (-6) + (+14) + (+12) =
\]

10D-33  
\[+30 + (-7) + (-8) + (+3) =
\]

10D-34  
\[-10.2 + (-9) + (-7.6) + (+14.7) =
\]

10D-35  
\[+8 + (+16.7) + (-4.1) + (+9.5) =
\]

10D-36  
\[\frac{1}{4} + \left(-\frac{1}{2}\right) + \left(-\frac{3}{4}\right) + \left(+\frac{1}{2}\right) =
\]
Section 5: Subtraction of Signed Numbers

Procedure for Subtraction of Signed Numbers:

1. Re-arrange subtraction problem in vertical fashion:
   from \((-7) - (+8) =\)
   to \((-7) - (+8) =\)

2. Change sign of lower number (subtrahend) to opposite sign; and change function sign from subtraction to addition:
   from \((-7) - (+8) =\)
   to \((-7) + (-8) =\)

3. Since every subtraction problem becomes an addition problem with the function sign change, re-arrange in horizontal fashion:
   from \((-7) + (+8) =\)
   to \((-7) + (-8) =\)

4. Follow rules for addition to obtain answer.
5-A Subtraction of numbers with like signs

Examples

1. Subtract (+8) from (+5) = _____

Solution: Step 1. Set up as subtraction problem in vertical fashion:

\[
\begin{array}{c}
(+5) \\
- (+8)
\end{array}
\]

Step 2. Change sign of lower number (subtrahend) to the opposite sign, and change function sign from subtraction (-) to addition (+):

\[
\begin{array}{c}
(+5) \\
+ (-8)
\end{array}
\]

Step 3. As can be seen in Step 2, every subtraction problem with signed numbers becomes an addition problem. Since we are now adding, apply the rule for adding unlike signs, or

\[
(+5) + (-8) =
\]

\[
|5| \quad \text{diff.} \quad |8| = 3
\]

Step 4. Affix sign of larger number to result as

\[
(+5) + (-8) = -3, \text{ Answer}
\]
2. Subtract (-8) from (-5) = ______

solution:

Step 1 Set up as subtraction problem in vertical fashion:

\[ (-5) \]
\[ - (-8) \]

Step 2 Change sign of lower number to opposite sign, and change function sign from subtraction to addition

\[ (-5) \]
\[ + (+8) \]

Step 3 Add

\[ (-5) + (+8) = \]

\[ |5| \quad \text{diff.} \rightarrow |8| = 3 \]

Step 4 Affix sign of larger number

\[ (-5) + (+8) = 3, \text{ Answer} \]
5-B Subtraction of numbers with unlike signs

Examples

1. Subtract (+8) from (-5) = ____

   solution: Step 1  
              (-5)   
              - (+8)

   Step 2  
            (-5) 
            + (+8)

   Step 3  
            (-5) + (-8) = -13, Answer

2. Subtract (-8) from (+5) = ____

   solution: Step 1  
              (+5)   
              - (-8)

   Step 2  
            (+5) 
            + (-8)

   Step 3  
            (+5) + (+8) = +13, Answer
Subtraction of Signed Numbers

Subtract the signed numbers as indicated:

11D-1  \((-10) - (-8) = \)

11D-2  \((+10) - (+8) = \)

11D-3  \((+5) - (-13) = \)

11D-4  \((+5) - (+13) = \)

11D-5  \((-22) - (-14) = \)

11D-6  \((+17) - (+8) = \)

11D-7  \((+3) - (-19) = \)

11D-8  \((+26) - (+31) = \)

11D-9  \((+40) - (+40) = \)

11D-10 \((-40) - (-40) = \)

11D-11 \((-40) - (+40) = \)

11D-12 \((-25) - 0 = \)

11D-13 \(0 - (+7) = \)

11D-14 \(0 - (-7) = \)

11D-15 \((+36) - (+41) = \)
UNIT 5 : REVIEW

-5R1- Express 42 inches as feet

-5R2- Add 5 ft 9 in and 8 ft 3 \( \frac{3}{4} \) in

-5R3- Multiply 2 ft 8 \( \frac{1}{2} \) in by 3

-5R4- 17 yd 2 ft 10 in \( \frac{3}{4} \) 4

-5R5- 11 yd 1 ft - 7 yd 2 ft

-5R6- Express 616 meters as kilometers

-5R7- Find the sum of 4.6 cm and 17.8 mm in millimeters

-5R8- Express 12 mm to the nearer thousandth inch

-5R9- Write 1.6 cu ft as cubic inches

-5R10- Write 25 oz as pints (round to 2 decimal places)

-5R11- Express 2.3 L as milliliters

-5R12- Express 5.2 oz as milliliters (round to 3 decimal places)

-5R13- Write 0.17 kg as grams
List each set of signed numbers in order of increasing value starting with the smallest value:

a. \(-5, -23, +8.0, +1, -18, +7.9 = \)

b. \(-7.5, 0, +7.5, -2.3, +0.5, -0.3 = \)

c. \(+21.3, 0, +20.6, -4.6, -7, -23.4 = \)

d. \(+3 \frac{1}{2}, -3, +6 \frac{3}{4}, -6 \frac{7}{8}, -6 \frac{13}{16} = \)

Add or subtract the signed numbers as indicated:

-5R15- \(+14 + (-6) = \)

-5R16- \(50 + (-23) = \)

-5R17- \(-37 - (-31) = \)

-5R18- \(-30.7 - (+5.5) = \)

-5R19- \(15 + (-8) + (-15) + (-10) = \)

-5R20- \(-2.9 + 1.6 + 3.2 + 7.5 = \)

-5R22- \(2 \frac{1}{2} + (-3) + (-1) + \frac{3}{8} = \)

-5R23- \((5+9) - (-3+6) = \)

-5R24- \((-13-6) - (4+7) = \)

-5R25- \((6.48-5.32) - (4.8.31) = \)
UNIT 6: RATIO/PROPORTION AND INTRODUCTION TO ALGEBRA

OBJECTIVE:

After completing this Unit you will be able to:

- Understand the concept of **Ratio**
- Understand the concept of **proportion**
- Understand the difference between "direct" and "indirect" proportions
- Solve each proportion for "X"

**Dictionary:**

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratio</strong></td>
<td>The quotient of two numbers</td>
</tr>
<tr>
<td></td>
<td><em>e.g.</em> ( \frac{5}{7} ) : 6 : 7</td>
</tr>
<tr>
<td><strong>Proportion</strong></td>
<td>A statement where two ratios are equal</td>
</tr>
<tr>
<td></td>
<td>( \frac{5}{7} ) : ( \frac{25}{35} ) as ( 25 : 35 )</td>
</tr>
<tr>
<td><strong>Direct Proportion</strong></td>
<td>A proportion in which the order of the ratios is the same</td>
</tr>
<tr>
<td><strong>Indirect Proportion</strong></td>
<td>A proportion in which the order of the ratios is indirect</td>
</tr>
</tbody>
</table>
Unit 6: Ratio (Direct and Indirect Proportion)

Section 1: Ratio

A ratio is the quotient of two numbers, or comparison, of the same kind. "Of the same kind", means both numbers are either abstract numbers, such as 30 and 10, or both numbers are expressed in the same units of measure, such as 20 feet and 30 feet.

The quotient of two numbers, a : b or a/b is sometimes referred to as a ratio and read "the ratio of a to b". This is a convenient way to compare two numbers.

A statement that two ratios are equal, for example,

\[
\frac{2}{3} = \frac{4}{6}
\]

\[
\frac{a}{b} = \frac{c}{d}
\]

is called a proportion and read "2 is to 3 as 4 is to 6"

The proportion \( a/b = c/d \) is often written in the form

\[
a: b = c: d
\]

which is read "a is to b as c is to d".

Which of the following proportions are true?

a) \( 3 : 4 = 21 : 28 \)

b) \( 20 : 35 = 65 : 91 \)

c) \( 42 : 35 = 70 : 60 \)

d) \( 5 : 8 = 25 : 45 \)
### DRILL SHEET

Express each of the following as a ratio:

<table>
<thead>
<tr>
<th>ID</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D-1</td>
<td>1 in to 9 in</td>
</tr>
<tr>
<td>1D-2</td>
<td>10 cm to 50 cm</td>
</tr>
<tr>
<td>1D-3</td>
<td>5 to 6</td>
</tr>
<tr>
<td>1D-4</td>
<td>6 mm to 12 mm</td>
</tr>
<tr>
<td>1D-5</td>
<td>4 kg to 20 kg</td>
</tr>
<tr>
<td>1D-6</td>
<td>9 to 10</td>
</tr>
<tr>
<td>1D-7</td>
<td>20 to 6</td>
</tr>
</tbody>
</table>
When you write a ratio in the form of a fraction, always write the fraction in its lowest terms. To say that a gear is "twenty to five" is unacceptable. Reduce the fraction \( \frac{20}{5} \) to 4 then say the ratio is "four to one". When writing answers to problems try to be clear and use correct terms.

Two important types are probability and percent ratios. The study of probability has become increasingly important and will help you understand many statements about everyday affairs, such as weather predictions and odds on the World Series. For instance, probability provides the foundation upon which insurance rates are based.

Consider this:
A drawer contains 24 red socks and 8 blue socks. If you randomly select a sock from the drawer, what is the probability you will select a blue sock?

Probability Ratio = \( \frac{\text{Successful Ways}}{\text{All Possible Ways}} \)

\[
\frac{8}{32} = \frac{1}{4}
\]

The probability you will select a blue sock drawn at random is \( \frac{1}{4} \). What does that mean?

The mechanical efficiency of a motor, steam engine, shop tool, and other mechanical or electrical devices that use and deliver useful energy is the ratio:

\[
\text{Mechanical Efficiency (ME)} = \frac{\text{(Useful output)}}{\text{(Total) output}}
\]

The output is the useful energy delivered by the machine, and the input is the amount of energy delivered to the machine. (The ratio is usually written as a percentage rather than a common fraction).

Example: An auto mechanic wants to find the efficiency of an automobile engine before a tune-up. From the engine specification chart he finds that the engine is rated to deliver 315 horsepower at 4400 r.p.m. When he tests the engine, he finds that it delivers only 270 horsepower at 4400 r.p.m. Find the efficiency.

Solution: Efficiency = \( \frac{270 \text{ hp}}{315 \text{ hp}} = \frac{6}{7} \)

Answer: The engine is producing six sevenths of its rated horsepower.
One of the most common examples of ratio used in everyday mathematics is pi ($\pi$), the 3.1416 ratio of the circumference of a circle to its diameter. It applies no matter whether the circle is measured in feet, inches, rods or whatever. Ratio even finds its way into law. A U.S. flag, for example, is to have a 1.9 ratio of length to width - whatever its size - its length should be 1.9 times its width.

Section 1. Direct proportion is one in which the order of the ratios is the same.

For example, small is to large as small is to large.

Example:

\[
\frac{4}{16} \text{ is to } \frac{x}{24}
\]

\[
\frac{4}{16} = \frac{x}{24}
\]

\[
\frac{4}{16} = \frac{x}{24}
\]

Means

\[
\frac{4}{16} = \frac{x}{24}
\]

Extremes

Make into an equation and calculate:

\[
16 \times x = 4 \times 24
\]

\[
16 \times x = 96
\]

\[
x = 6
\]

* The product of the means equals the product of the extremes.

Solve each proportion for $X$

Sample Problem

a) \[
\frac{4}{5} \times \frac{x}{20}
\]

\[
(4)(20) = 5x
\]

\[
80 = 5x
\]

\[
x = 16 \quad \text{Answer}
\]
### DRILL SHEET

<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D-1</td>
<td>( \frac{3}{X} = \frac{1}{5} )</td>
<td>( X = 15 )</td>
</tr>
<tr>
<td>2D-2</td>
<td>( \frac{2}{7} = \frac{X}{28} )</td>
<td>( X = 8 )</td>
</tr>
<tr>
<td>2D-3</td>
<td>( \frac{X}{21} = \frac{5}{7} )</td>
<td>( X = 15 )</td>
</tr>
<tr>
<td>2D-4</td>
<td>( \frac{6}{11} = \frac{X}{22} )</td>
<td>( X = 12 )</td>
</tr>
<tr>
<td>2D-5</td>
<td>( \frac{3}{4} = \frac{X}{7} )</td>
<td>( X = \frac{21}{4} )</td>
</tr>
<tr>
<td>2D-6</td>
<td>( \frac{X}{21} = \frac{5}{7} )</td>
<td>( X = 15 )</td>
</tr>
<tr>
<td>2D-7</td>
<td>( \frac{12}{5} = \frac{X}{7} )</td>
<td>( X = \frac{84}{5} )</td>
</tr>
<tr>
<td>2D-8</td>
<td>( \frac{9}{2} = \frac{18}{X} )</td>
<td>( X = 4 )</td>
</tr>
<tr>
<td>2D-9</td>
<td>( \frac{20}{30} = \frac{2}{X} )</td>
<td>( X = 15 )</td>
</tr>
<tr>
<td>2D-10</td>
<td>( \frac{45}{50} = \frac{X}{10} )</td>
<td>( X = 9 )</td>
</tr>
</tbody>
</table>
PRACTICAL APPLICATIONS:

1. Two gears are in the ratio of 3:5. If the larger gear has 40 teeth, how many teeth does the smaller gear have?

2. The power-to-weight ratio for a certain automobile engine is 7:8. If the engine weighs 400 lbs. How much power does it produce?

3. The odds of getting a pair of cards (two cards with the same face value) in a single deal in a power game are 3:4. (In four deals you can expect to get a pair of cards three times). How many pairs can you expect to get in 24 deals of the cards?

4. A road has a grade of 7 per cent which means that it rises 7 feet for every 100 feet measured horizontally. How much does the road rise in a mile? Figure the answer to the nearest foot?

5. A telephone pole 17 feet high casts a shadow 10 feet long. How tall is a church steeple which at the same time, casts a shadow 50 feet long?

6. A car travels at the rate of 50 miles per hour. How many feet per second is this?
Section 3: Indirect Proportion

An indirect proportion is one in which the order of the ratios is indirect (inverse).
For example, large is to small, as small is to large.

In a proportion, the product means is equal to the product of the extremes.
* (Note that this rule is the same for both direct and indirect proportion).

1. Example: What are the r.p.m. of the big gear on the right

Use the formula \( T : t = r : F \)

\[
\begin{align*}
\frac{45}{r} &= \frac{36}{90} \\
\frac{r}{?} &= \frac{214}{220}
\end{align*}
\]
**INTRODUCTION TO ALGEBRA**

**OBJECTIVES:**

After completing this unit, you will be able to:

- Understand the meaning of an algebraic equation
- Write a simple algebraic equation
- Solve equations
- Understand simultaneous equations

**Dictionary:**

<table>
<thead>
<tr>
<th>Word</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Algebra</td>
<td>A type of mathematics that uses numbers and letters that stand for numbers to solve problems.</td>
</tr>
<tr>
<td>2. Equation</td>
<td>An equation is a sentence that contains an equal sign (=); a statement that two amounts are equal.</td>
</tr>
<tr>
<td>3. Equivalent</td>
<td>Two equations are said to be equivalent if they have exactly the same solutions.</td>
</tr>
<tr>
<td>4. Variable</td>
<td>A lower case letter such as x, y, z, a, b, is commonly used to represent a member of a set of numbers.</td>
</tr>
</tbody>
</table>
Section 1: Introduction to Algebra

Algebra is a part of mathematics that uses letters to represent numbers in the forms of equations and formulas calculated according to the rules of arithmetic.

A formula is a statement of a rule in the language of algebra. Rule: Area equals length times width.

Formula: \[ A = lw \]

A = 8 \times 6

The set of relations existing among these known and unknown numbers which go to make up or specify a certain condition may then be expressed by means of the algebraic signs for additions, subtraction, powers, roots, etc., and the resulting form of statement of the condition will be an algebraic equation.
3D-1 Write the formula for the rule: The resistance in an electrical circuit is equal to the voltage divided by the current.

3D-2 Write a formula for the rule:
The volume of a cube is equal to the cube of the length of a side.

3D-3 Write a formula for the rule:
The sum of the three interior angles of a triangle is equal to 180°.
Section 2: Solving Equations in Algebra

One of the most important methods for problem solving is the equation.

The mathematical statement $2x + 1 = 6$ is an equation. Solutions are numbers that make equations true statements.

Example: Show that 2 is a solution to the equation.

$3x + 4 = 10$

Substitute 2 for the $x$ in the equation

$3(2) + 4 = 10$

In the equation $b = c - a$, solve for $a$.

1) Subtraction is indicated. Therefore get the unknown $(c)$ alone by addition. Add $a$ to both sides.

\[
\begin{align*}
    b &= c - a \\
    +a &+a \\
    \hline \\
    a + b &= c \\
    a = b &= c \quad \text{answer}
\end{align*}
\]

Check by substituting the value of the unknown $(c)$ and compute the equation.

\[
\begin{align*}
    b &= c - a \\
    b &= (a + b) - a \\
    b &= b \quad \text{Answer is OK because both sides are equal.}
\end{align*}
\]
DRILL SET

Use either the rule of addition or subtraction to find the solutions to these equations.

4D-1 \[ x + 20 = 60 \]

4D-2 \[ x - 10 = 100 \]

4D-3 \[ x - 2.6 = 4.7 \]

4D-4 Now try to use the rule of subtraction to solve for the variable.

a. \[ x + 10 = 20 \]

b. \[ b + 8 = 10 \]

c. \[ s + 1 = 11 \]
Section 3: Rule for Division in Equations

Divide both sides of an equation by the same quantity to keep the equation in balance.

\[ 4 \ x = 16 \]

Divide both sides by 4

\[ \frac{4 \ x}{4} = \frac{16}{4} \]

\[ x = 4 \]

Use the rule of division to solve for the variable.

1. \( 18 \ t = 36 \)

2. \( 4 \ t = 1.6 \)

3. \( 10 \ a = 100 \)

4. \( 0.5 \ x = 10 \)

5. \( \frac{1}{3} \ x = 9 \)
Express each word problem algebraically and solve.

1. The product of a number and 5 is 65.

2. A number divided by 3 is equal to 17.

3. A voltage \( E \) when doubles equals 628 V.

4. The quotient of a current \( I \) and 10 is 146.

5. A conductor is divided into four equal parts. Each part is 1.4 m long. Find the original length \( L \).

6. Write a problem to fit each equation:
   a) \( \frac{5}{2} + n = \frac{10}{4} \)
b) \( \frac{n - 3}{3} = 3 \)

c) \( (4 \times n) + 6 = 26 \)

7. Write an equation for each problem. The ALPO bus has enough room for 34 employees. If the driver started with a full load, stopped at R&D (Research and Development) to let off 18 employees, and picked up 9 employees going to the Training Room, how many employees were left on the bus?

8. Jerry came home from the Annual ALPO baseball game and was hungry. He found a note from his wife telling him to help himself to a piece of pie. Someone else had already eaten part of the pie, because only \( \frac{3}{4} \) of it was left in the pan. After Jerry cut a piece for himself, only \( \frac{1}{2} \) of the remained. What part of the whole pie did Jerry eat?
Section 4: Simultaneous Equations

Considering two equations: \(x - y = 11\), the requirement that both equations are true for the same set of values for \(x\) and \(y\) defines them as SIMULTANEOUS equations. In simpler words -- only one value for \(x\) and one value for \(y\) will make these equations true.

Section 5: Solving Simultaneous Equations By Addition or Subtraction

Addition

Consider again the two equations: \(x - y = 3\) and \(x + y = 11\). If we think of them as balanced scales, we can show the following:

\[
\begin{align*}
\text{and} & \\
\text{Note that the scale is still in balance!!!}
\end{align*}
\]

This can be shown only in equation form

\[
\begin{align*}
x - y &= 3 \\
\text{plus} \quad x + y &= 11
\end{align*}
\]

Combining or adding each side together, we see that \(-y\) and \(+y\) cancel each other, or:
\[2x = 14\]
\[
x = \frac{14}{2} \quad \text{or} \quad 7
\]

Now substituting 7 for \(x\) in either equation and solving for \(y\), we can readily show that \(y = 4\).

**Subtraction**

Remembering the rule that if we subtract an equal amount from both sides of a balanced equation, the equation still balances. We can subtract either of these two equations from the other and solve for \(x\) and \(y\). That is:

\[
\begin{align*}
  x - y &= 3 \\
  -(x + y) &= -11
\end{align*}
\]

\[
\begin{array}{c}
  x - y = 3 \\
  -(x + y) = -11
\end{array}
\]

\[
\begin{array}{c}
  x - y = 3 \\
  -(x + y) = -11
\end{array}
\]

\[
\begin{array}{c}
  \underline{0} \\
  -2y = -8
\end{array}
\]

Therefore \(y = \frac{-8}{-2} = 4\) or 4

And again, substituting 4 for \(y\) in either equation will result in \(x\) equaling 7.

The main purpose of deciding whether to subtract or add is to eliminate one of the unknowns so that a value can be found for the other. For example, in the set of equations following:

\[
\begin{align*}
  2x + y &= 10 \\
  2x - 5y &= 40
\end{align*}
\]

one can see that in order to eliminate an unknown, \(x\) and its numerical coefficient 2 are exactly the same in both equations. Therefore, it will be advantageous to subtract one from the other and then combine.

\[
\begin{align*}
  2x + y &= 10 \\
  -(2x - 5y) &= -(40)
\end{align*}
\]

\[
\begin{array}{c}
  2x + y = 10 \\
  -2x + 5y = -40
\end{array}
\]

\[
\begin{array}{c}
  \underline{0} + 6y = -30
\end{array}
\]

\[
y = -5
\]

Substituting -5 for \(y\) in one of the original equations; it can be seen that \(x = 7.5\).

It follows then that in the set of equations:

\[
\begin{align*}
  5x + 2y &= 20 \\
  3x - 2y &= 12
\end{align*}
\]
one would simply add (combine) since +2y and -2y would drop out and x can easily be evaluated.

**Multiplication/Addition/Subtraction**

When solving simultaneous equations, it becomes necessary to multiply both sides of an equation (or sometimes both equations) by some quantity in order to eliminate one of the unknowns.

The main purpose is to get either one of the unknowns to have the same numerical coefficient but the opposite sign in order to eliminate it when combining.

Consider the equations:

\[
\begin{align*}
2a - 4b &= 21 \\
3a - b &= 11
\end{align*}
\]

One can multiply either the second equation by -4 to eliminate b; or multiply the first equation by 3 and the second by -2 to eliminate a.

Choosing the first method:

\[
\begin{align*}
2a - 4b &= 21 \\
4(3a - b &= 11) \\
-12a - 4b &= -44
\end{align*}
\]

\[
\begin{align*}
\text{combining (add)} & \quad -10a = -23 \\
& \quad a = 2.3 \\
b &= -4.1
\end{align*}
\]

**Steps in solving simultaneous equations by addition or subtraction.**

1. Multiply one or both of the given equations, if necessary, by some factor, or factors, that will make the coefficients of one variable numerically equal.

2. Eliminate this variable by addition or subtraction.

3. Solve the resulting equation for one variable.

4. Find the value of the other variable by substituting the known value of one variable in one of the equations. (The second variable may be found in the same way as the first.)
DRILL SHEET

5D-1 \[ 3x + 2y = 8 \] \[ x - 4y = 5 \]

5D-2 \[ 4x - 7y = -2 \] \[ 3x + 4y = 17 \]

5D-3 \[ 2r - 5s = 13 \] \[ 7r - 3s = 2 \]

5D-4 \[ 3c - 4d = -5 \] \[ 7d + 4c = -10 \]

5D-5 \[ 7x + 3y = 9 \] \[ 7y - 3x = -8 \]

5D-6 \[ 2x - 3y = 19 \] \[ 5x + 4y = 13 \]

5D-7 \[ 5x + y = 1 \] \[ x + 2y = 11 \]

5D-8 \[ 4m - 3n = 11 \] \[ 7n + 3m = -1 \]

5D-9 \[ 4h + 5k = -3 \] \[ 7k = 3h - 8 \]

5D-10 \[ 3y - 7z = 3 \] \[ 5z - 7y = 10 \]

5D-11 \[ 3x + 2y = 8 \] \[ 7x - y = 6 \]

5D-17 \[ 7x - 10y = 20 \] \[ 11x - 15y = 30 \]
5D-12  \(3x - 4y = 7\)  
\(2x - 7y = -3\)

5D-13  \(3I + 2i = -9\)  
\(4I - 7i = 1\)

5D-14  \(4R - 5r = 7\)  
\(3R - 2R = 10\)

5D-15  \(5R - 6r = 7\)  
\(9r + 4r = 10\)

5D-16  \(12x - 11y = 25\)  
\(15y + 16x = 80\)

5D-17  \(21.3x - 9.5w = 6\)  
\(9.5x - 21w = 0\)

5D-18  \(3E - 5e = 4\)  
\(2E - 7e = -8\)

5D-19  \(24a - 15B = 40\)  
\(15a - 24B = 40\)

5D-20  \(3a - 8b = 7\)  
\(5a - 12b = 3\)

5D-21  \(2y = 7x + 6 = 0\)  
\(5x + 3y - 1 = 0\)

5D-22  \(2.3x + 7.2y = 10\)  
\(5.1x - 4.3y = 10\)

5D-27  \(3.2R - 1.5r = 25\)  
\(2.1R - 2.5r = 5\)
5D-24 \[ 50I + 15i = 80 \]
\[ 15I + 25I = 80 \]

5D-28 \[ 1.3R + 1.5r = 6 \]
\[ 1.5R + 1.3r = 8 \]

5D-25 \[ ax + by = e \]
\[ cx + dy = f \]

5D-29 \[ bx - ky = a \]
\[ cx + ky = n \]

5D-26 \[ mx + ny = 5c \]
\[ nx - my = 3b \]

5D-30 \[ 2ma - 3mb = 4 \]
\[ 5ma - 4mb = 15 \]
UNIT 6: REVIEW

Solve each of the equations for the variable shown.

6R-1 \( Q = CV \), solve for \( C \)

6R-2 \( R_2 = Z_2 - X_2 \), solve for \( Z \)

6R-3 \( I = \frac{E}{Z} \), solve for \( Z \)

6R-4 \( X_1 = 2fL \), solve for \( L \)

6R-5 \( R = \frac{P}{I^2} \), solve for \( I \)
6R-6 \[ Xc = \frac{1}{2\pi f c} \], solve for c

6R-7 \[ R2 = Rt - R1 - Ra \], Solve for Rt

6R-8 \[ pf = \frac{R}{X} \], solve for R

6R-9 \[ P = \frac{120f}{N} \], solve for f

6R-10 \[ Ns = \frac{EsNp}{Ep} \], solve for Np

6R-11 \[ u = gmrp \], solve for rp
6R-12 \[ Z_p = \frac{N_p Z_s}{N_s}, \text{ solve for } Z_p \]

6R-13 \[ E_s I_s = E_p I_p, \text{ solve for } E_p \]

6R-14 \[ M = k \sqrt{L_1 L_2}, \text{ solve for } k \]

6R-15 \[ C = \frac{0.088}{4kA(N-1)}, \text{ solve for } A \]
### UNIT 7: COMPUTER LITERACY USING ALGEBRA SOFTWARE

**OBJECTIVES:**

After completing this unit, you will be able to:

- Understand the basic functions of the computer
- Appreciate the computer as a valuable aid in the learning process therefore alleviating any technological fears.
- Understand that different computers have different ways of operating.

**Dictionary:**

<table>
<thead>
<tr>
<th></th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Monitor</td>
<td>a T.V. set-like display without a channel tuner. Input is from a &quot;video&quot; signal.</td>
</tr>
<tr>
<td>2. Disk Drive</td>
<td>a persphiral unit used to run programs of floppy disks.</td>
</tr>
<tr>
<td>3. Keyboard</td>
<td>the set of keys on an input device used for encoding characters by the depression of keys, as in a typewriter.</td>
</tr>
<tr>
<td>4. Hardware</td>
<td>the physical components of a computer system as opposed to the programs (software)</td>
</tr>
</tbody>
</table>
In this unit, students from both companies had one to two classes at the Math Computer Lab at the Community College. Students experienced working on both an Apple IIe and IBM computer.

Software consisted of:

- IBM Algebra I and Algebra II
- Preparing for Geometry and Algebra (P.G.A.)

Lesson 1: Measurement

1) Reading a Ruler
2) Units of Length
3) Units of Volume and Weight

Lesson 2: Introduction to Geometry

1) Terms in Geometry
2) Angles
3) Perimeter
4) Area and Volume

Lesson 3: Graphs

1) Miscellaneous Graphs
2) Bar Graphs or Bar Charts
3) Fractional Parts or Sets

Lesson 4: Integers

1) Absolute Value
2) Addition and Subtraction
3) Multiplication and Division

Lesson 5: Introduction to Algebra

1) Exponents and Square Roots
2) Scientific Notation
3) Operations with Exponents
4) Simplifying Expressions

Lesson 6: Mixed Topics

1) Money
2) Time and Calendar
3) Temperature
4) Roman Numerals
5) Sequences
Unit Examinations
Observe the instructions in solving the following problems:

1. Write the correct number for each word statement.
   a. Seven hundred thousand, seven hundred seven
   
   ______________________
   
   b. Four hundred thousand and ninety-two thousandths.
   
   ______________________

2. 4,793 - 404 =

3. The sun is about 93,000,000 miles from the earth. Light travels at a speed of 186,000 miles per second.
   a. How many seconds does it take the light from the sun to reach the earth?
   
   ______________________
   
   b. How many minutes does it take the sunlight to reach the earth?
   
   ______________________

4. 7,867 x 403 =

5. Divide 13,701 by 1101

235
6. \[58 + \frac{13+32-9}{3} - 17 = \]

7. In estimating the time required to complete a job, a Section Chief determines that a total of 840 hours are needed. If four utility operators each work five days per week for seven hours per day, how many weeks are required to complete the job?

8. If a machine setter earns $23,820 annually, what is the monthly salary?

9. In order to make the fixture shown below, a machinist must determine dimensions A, B, C, and D; solve for dimensions A, B, C, and D:

```
A
B
C
D
```
10. Which total volume is greater, four drums containing 72, 45, 39, and 86 liters, or three drums containing 97, 115, and 74 liters?

By how much is it greater?

11. ALPO contracted to have its trucks repainted. If 17 trucks cost $159 each, and 43 trucks cost $267 each, how much money will ALPO be billed to have all the trucks painted?

12. A technician has a silicon rod that totals 243 inches in length. If he must cut 9 equal pieces, how long will each piece be?

13. \[2478 - 726 + 598 \times 12 =\]

14. \[72 \times 38 + 86,526 - 69 =\]
15. A high-speed stamping machine can produce small flat parts at a rate of 96 per minute. If an order calls for 297,600 parts to be stamped, how many hours will it take to complete the job? (60 minutes = 1 hour)
UNIT 2 EXAMINATION

Date ____________
Shift ____________
Name _______________________
Grade ____________

MATHMATICS FOR INDUSTRY
Observe the instructions in solving the following problems; always reduce to lowest terms:

1. \[ \frac{3}{4} + \frac{7}{8} + \frac{5}{16} = \]

2. \[ \frac{37}{10} - \frac{19}{4} = \]

3. \[ \frac{3}{4} \times 12 \times 1 \frac{5}{8} = \]

4. \[ \frac{5}{5} - \frac{3}{15} = \]

5. \[ \frac{8}{4} \times (\frac{6}{16} + 5 \frac{1}{16}) - (5 + \frac{3}{8}) = \]

6. Express \( \frac{9}{10} \) as an equivalent fraction with a denominator of 60.
7. Express \( \frac{72}{128} \) in lowest terms.

8. Express \( \frac{3}{4} \) as a fraction.

9. Express \( \frac{451}{64} \) as a mixed number.

10. Find the lowest common denominator for \( \frac{7}{12}, \frac{2}{3}, \frac{1}{2} \).

11. Find the total length of 12 pieces of wire each \( \frac{3}{16} \) inches long.

12. What would be the total length of the silicon rod formed by fusing together the five pieces of rod shown below:

\[
\begin{align*}
10 \frac{3}{4} & \\
11 \frac{6}{15} & \\
12 \frac{4}{7} & \\
2 \frac{6}{21} & \\
13 \frac{2}{13} &
\end{align*}
\]
13. An electrical wiring job requires the following lengths of BX cable: seven pieces each 6 \( \frac{1}{2} \) inches long, four pieces each 34 \( \frac{3}{4} \) inches long, and nine pieces each 19 \( \frac{2}{8} \) inches long. What is the total length of cable needed.

14. A fixture is shown below. Find the indicated dimensions:

Dimension A = ________    Dimension D = ________

Dimension B = ________    Dimension E = ________

Dimension C = ________    Dimension F = ________

Dimension G = ________
15. Find the missing dimension A in the following sketch:

16. When aligning a motor, the following pieces of steel are used under the base: one piece 5\(\frac{1}{2}\)" thick; one piece \(\frac{1}{8}\)" thick; one piece \(\frac{1}{16}\)" thick; one piece \(\frac{1}{32}\)" thick; and one piece \(\frac{1}{64}\)" thick. What is the total thickness of all these pieces?
Mathematics for Industry

Unit 3 Examination

Date ____________
Shift ____________
Name __________________
Grade ____________
Observe the instructions in solving the following problems; always reduce to lowest terms or round the numbers to the indicated decimal places (shown in parenthesis).

Write each of the word statements as a decimal number.

1. Five Hundred Four and Twenty-Two Hundredths ________.
2. Two and Five Tenths ____________________________.
3. One Hundred Fifty and Three Thousandths ____________.
4. 16.7091 (2) ________________________________.
5. 42.01997 (4) ________________________________.
6. .8999 (3) ________________________________.
7. 104.01997 (3) ________________________________.

Express these common fractions as decimals to four places.

8) \( \frac{4}{9} \) ________ 9) \( \frac{11}{18} \) ________

10) \( \frac{41}{50} \) ________ 11) \( \frac{24}{27} \) ________

Express these decimals as common fractions.

12) .21 ________ 13) .0303 ________

14) .625 ________

15) From the sum of 22.825 and 35.099 subtract sum of 19.01 and .1258 ____________________________.

16) 22.08 x 19.23 x .092 = ____________________________.
17) An assembler checks 630 parts for the total week. On Wednesday of that particular week, he checked 205. What decimal of the total checked is the amount checked on Wednesday? 

18) Complete the sequences
   a) .14, .16, .18, _____, _____
   b) .43, .49, .55, _____, _____
   c) .56, .50, .44, _____, _____
   d) .80, .40, .20, .10, _____, _____
Observe instructions in solving the following problems:

1. Express 19.6% as a decimal fraction.

2. Write 0.91 as a percent.

3. What is 7% of 140?

4. 60 is 80% of what number?

5. Find 25% of 120.

6. Express 218.7% as a decimal fraction (or mixed decimal).
7. Express 109.8% as a common fraction (or mixed number).

8. Convert 0.8125 to percent. (Round to 2 decimal places).

9. What percent of 140 is 105?

10. Determine the range (R), average (X), and median for the following sets of numbers:
    a. 6, 5, 9, 4, 8, 11, 10, 7
        R ________
        X ________
        Median ________
    b. 0.0098, 0.0087, 0.0092, 0.0087, 0.0084, 0.0098
        R ________
        X ________
        Median ________
MATHEMATICS FOR INDUSTRY

________________________________________________________

UNIT 5 EXAMINATION

Date
Shift
Name
Grade
Observe instructions in solving the following problems:

1. Express 42 inches as feet.

2. Multiply \(2 \text{ ft } 8 \frac{1}{2} \text{ in}\) by 3.

3. Express 616 meters as kilometers.

4. Find the sum of 4.6 cm and 17.8 mm in millimeters.

5. Write 1.6 cu. ft. as cubic inches.

6. Write 25 oz. as pints. (Round to 2 decimal places.)

247

258
7. Express 2.3 L as milliliters.

8. Express 5.2 oz. as milliliters. (Round to 3 decimal places.)

9. Write 0.17 Kg as grams.

10. Hot air passes through a duct at a rate of 500 cubic inches per second. Compute the number of cubic feet of hot air passing through the duct in one minute. (Round answer to 2 decimal places.)

11. Add: 61.9 cm + 17.9 dm + 74.3 mm + 0.08 m.

12. Express 47.8 rods as yards and inches.

14. Select the greater of each of the two signed numbers and indicate the number of units by which it is greater:

a. \((-51.79), (-39.35) = \) 

b. \((-29 \frac{5}{16}), (-16 \frac{5}{8}) = \) 

15. Express each of the pairs of signed numbers as absolute values and report the difference:

a. \(+10.54, (-7.46) = \) 

b. \((-0.08), (-0.009) = \) 

16. List each of the following sets of numbers in order of increasing value, starting with the smallest value:

a. \(-17.6, +14.1, +2.3, -11.4, -7 \) 

b. What is the range of the above set of value?

17. a. \(0, -7 \frac{1}{2}, -9 \frac{5}{32}, -10 \frac{1}{4}, -1 \frac{7}{16} \) 

b. How far is it and what direction from the maximum to the minimum value?
UNIT 6 EXAMINATION

Date ____________
Shift ____________
Name ____________________________
Grade ____________
Express the following as an algebraic expression.

1. Divide 25 by \( b \)

2. Increase \( e \) by 12

3. Reduce \( y \) by 75

4. The square root of \( X \) plus 6.8

5. One half \( X \) minus four times \( y \)

6. The cube root of \( h \) times the square of \( y \)

7. Divide \( x \) by the product of 25 and \( y \)

8. Take the square root of \( r \), add \( s \), and subtract the product of \( z \) and \( t \):

9. In a series circuit the total resistance (\( R_t \)) of a circuit is equal to the sum of the three individual resistances \( R_1, R_2, R_3 \). The circuit has a total resistance of 200 ohms. Express the resistance \( R_1 \).

10. Impedance (\( Z \)) of a circuit is computed by adding the square of resistance (\( R \)) to the square of the reactance (\( X \)), then taking the square root of the sum. Express the circuit impedance.

250

262
11. The electrical power (P) dissipated by an incandescent lamp is directly proportional to the square of the voltage drop (\( V \)) across the lamp and inversely proportional to the electric resistance (R) of the lamp.

12. \( X_1 = 2 \sqrt{fL} \), solve for L

13. \( A = \frac{\pi D^2}{4} \), solve for D

14. \( X_c = \frac{1}{2\pi fC} \), solve for C

15. \( M = k\sqrt{L_1 L_2} \), Solve for \( L_1 \)

Substitute numbers for letters and evaluate the following.

16. If \( x = 6 \) and \( y = 2 \)
   a) \( 2xy - y = \)
   b) \( (x + y)(x - y) = \)
   c) \( \frac{x + y}{x - y} = \)
   d) \( 2x - x^3 = \)
   e) \( 2x + xy - 4y = \)
Suggested Instructional Strategies/General
Suggestions for Using Manual
Unit 1: Suggested Instructional Strategies/General Suggestions For Using Manual

WHOLE NUMBERS

1. Place Value

For this topic (every student, no matter what level can benefit)

Additional drill sheets with examples like the ones cited below should be provided for the students.

Students should be given a copy of the Numerical Periods Chart. (see Appendix)

Examples:

a. One Hundred Twenty Three
b. Five Hundred One
c. Two Hundred Twenty-Five and Twenty-Five Thousandths **

* Usually in oral communication errors will be made "One Hundred and Twenty Five". Many students are not aware of the fact that 'and' is used only in place of the decimal point. (cite for example writing a check)

** Use English Numbering System Chart (which appears in Manual)

Additional drill (like the examples shown below) should also be provided.

Write the words for each number listed below.

A. 225 ________________________________
B. 4035 ________________________________
C. 59 ________________________________
2. "The Whole is Equal to the Sum of its Parts" - Using Graphics

Most effective if overhead projection is used.

Note: Some students will have a great deal of trouble interpreting diagrams. Additional examples should be given.

Examples:

F = 12 "  A" 2"  E = ?

3. Some students (this especially holds true when the class is not using the calculator (see Appendix for "Thoughts on Use of the Calculator") will need the aid of the Multiplication Chart (see Appendix).
**Unit 2: Suggested Instructional Strategies/General Suggestions For Using Manual**

**COMMON FRACTIONS**

1. Concept of a Fraction

   This is a good time to emphasize that anytime in math when there is a line separating either a number, fraction, decimal, etc. from another number, fraction, decimal, etc. it signifies that the numerator is divided by the denominator. e.g. \( \frac{1}{6} \) can represent the fact that "1" cookie is being divided among "6" people; therefore, you will just receive a fraction of the cookie, or \( \frac{1}{6} \) of it. Re-emphasize this throughout the entire unit.

   Allowing for differentiation among a heterogeneous group of students, more work should be done with lower-skilled students on the concept of a fraction. Using fraction strips (see Appendix) construct examples illustrating the concept of fraction. This can be a tie in with equivalent fractions.

   **Example:**

   The maintenance Department at ALPO have decided to establish two teams. Each team will work the same amount of time each week. Team "A" decided to divide the 24 hour work day into 2 shifts, while "B" decided to divide the 24 hours into 4 shifts. (display a circle divided into 4 parts on transparency using an overhead projector). Superimpose a circle divided into halves onto the transparency divided into fourths.
2. Dictionary Terms - (lower-skilled group)

Since the terms numerator and denominator are frequently used, a good way to remember is "D" for down \(\frac{1}{6}\) the "6" is down - denominator; the upper number is therefore the numerator.

3. Lowest Common Denominator

With a lower to middle-skilled group of students, work through the steps illustrated in the curriculum guide. When working with a more advanced group, challenge with concentration on the prime number factoring system.

4. Addition of Common Fractions

Principle of equivalent fractions may or may not be used when obtaining new forms of numerators (may be more confusing than beneficial)

Lower-skilled students may be initially very confused with the different concepts of fractions. This might be a good time to quiz them on what they have already learned. (see Appendix) This method could and should be employed throughout the entire manual when the instructor feels it's necessary.

5. Adding Fractions, Mixed Numbers, and Whole Numbers

Students may experience some difficulty with transfer of improper fraction to whole number. Additional drill may be necessary.
6. Subtracting Fractions

Encourage students (using red pencils) to circle sign so as not to add when they should be subtracting.

Lower skilled students will experience difficulty with "borrowing" from whole number in order to be able to subtract the subtrahend from a sufficient amount.

7. Multiplication and Division of Fractions

Students will find the multiplication and division of fractions much easier than the addition and subtraction. When multiplying fractions, and "cancelling" demonstrate that a number can’t be used more than once but, continue to cancel as long as there is a numerator and denominator that can be cancelled. e.g. \[ \frac{3}{164} \times \frac{4}{15} \times \frac{204}{6} = \frac{1}{6} \]

Remind students that it doesn’t always have to be diagonally worked - you must always be cancelling a numerator with a denominator.
Unit 3: S. Tested Instructional Strategies/General Suggestions for Using Manual

DECIMALS

If using this manual in a heavy manufacturing environment where employees spend a great deal of time measuring with a micrometer, keep in mind that "shop talk" often confuses the reading of a decimal. Very often, .0065 is referred to as 6.5 and not sixty five thousandths, because they work in thousandths and often verbally abbreviate.

When expressing decimals as common fractions, you can have your students check their answer. e.g. 0.065

Disregard all zeros between the decimal point and the first number to the right - place the number "65" in the numerator's place - count how many numbers there were including the zeros - in this case "3" - that's the number of zeros you write and then add a $1 - 3$ zeros plus a $1 = 65$. When rounding, instruct students to wait to the end to round.

1. Division of Decimals

Special emphasis should be placed on the understanding of division of a "tenth" of a number yielding a whole number e.g. $0.8 \div 0.4 = 2$

Students will have a hard time understanding why you have a whole number for an answer. Explanations should be given that there are "2" four tenths in every group of eight tenths. It's how many four tenths there are.
PERCENTS, MEDIAN and RANGE

1. Expressing Percents as Decimals and Fractions

   When instructing students to express percents as decimals and fractions, show them how to move the decimal point in accordance to $\div$ or $\times$. Two places to the right (X), as opposed to two places to the left ($\div$).

   \begin{tabular}{|c|c|}
   \hline
   \% & Decimal \\
   \hline
   Move decimal point two places to the left & (\div 100) \\
   \hline
   Decimal & \% \\
   \hline
   Fraction & Move decimal point two places to the right & (\times 100) \\
   \hline
   \end{tabular}

2. Type of Percent Problems

   Students will definitely have own way of calculating Type I percent problems (perfectly alright) - will start to see the benefit of "formula" when working Type II and Type III problems.

   Formula should be emphasized:

   \[
   \frac{\text{part}}{\text{whole}} = \frac{\text{rate}}{100}
   \]
Particular attention should be paid to the comprehension of the word problems in the Practical Application of Percents section. For example, students should understand that in problem #1 that Jim's gross pay of $30,100 was 10% less than last year's gross pay or 90% of the whole.

When discussing Average, Median and Range be sure to note, especially in workplaces where additional training such as SPC (Statistical Process Control) will follow, that average and "mean" are synonymous.
Unit 5: Suggested Instructional Strategies/General Suggestions for Using Manual

CONCEPT OF MEASURE AND SIGNED NUMBERS

1. Numbers

A great deal of time should be spent on the Metric Chart. Students are often confused with the relationship and correlation from one unit of measure to another. This is a good time to reiterate the movement of the decimal point. e.g. one centimeter is equal to .3937 inches, while one decimeter is equal to 3.937 inches (the decimeter is .1 larger).

2. Signed Numbers

Concept of Signed Numbers should be illustrated vertically (as in the reading of thermometers) and also horizontally (as in the number line). Have students keep "Rules for Signed Numbers" handy when doing operations. This concept can be difficult and reinforcement is possible in the Computer Literacy unit.
Unit 6: Suggested Instructional Strategies/General Suggestions For Using Manual

RATIO AND INTRODUCTION TO ALGEBRA

Students have an automatic aversion just to the word "algebra". (even the best of them) Starting out very slowly, and discussing "unknowns" works best. Refer at this time to the many places that we do use Algebraic formula in order to solve for the unknown. Just in Unit 4, we used a formula to solve for the part, the whole or the rate.

Emphasize that when solving simultaneous equations a balance must always be maintained.

Students are often confused with the unknown not always on either the right hand side or either the left hand side of the equation.
Survey your students in order to find out the level of computer literacy. Students will usually fall in one of the following categories:

1. No computer experience
2. Limited experience
3. Some training and experience
4. Computer at home

Prior to computer class, group students according to the survey above (mixing different levels of literacy as much as possible). When grouping students at the individual computers (preferably two at the computer) have the student with the least amount of experience turn the computer and the monitor on, etc. while giving initial literacy instructions.

After students begin working on the software, content will become more important to them, and their uneasiness with the machine itself will start to dissipate.
Thoughts on Use of the Calculator

The calculator can be used:

- To encourage students to be inquisitive with mathematical concepts.

- To act as a flexible answer key to verify the results of computations.

- As a resourceful tool which promotes student independence in problem solving.

- To solve problems that have been too time consuming or impractical to be done with paper and pencil.
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Table for Basic Multiplication Facts
## Fraction Equivalent Strips

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UNIT 2 QUIZ

NAME____________________________

1. In the fraction \( \frac{2}{5} \), the 2 is the numerator or denominator?

2. In the fraction \( \frac{2}{5} \), the 5 is the numerator or denominator?

Convert each improper fraction to a mixed number:

3. \( \frac{67}{5} \) = __________ 5. \( \frac{42}{4} \) = __________

4. \( \frac{92}{12} \) = __________ 6. \( \frac{13}{2} \) = __________

Express each common fraction as an equivalent fraction:

7. \( \frac{2}{16} \) = _______ 8. \( \frac{2}{15} \) = _______

Express each common fraction in lowest terms:

9. \( \frac{6}{36} \) = _____ 10. \( \frac{15}{25} \) = _____ 11. \( \frac{4}{8} \) = _____

Express these mixed numbers as fractions:

12. \( \frac{2\frac{3}{8}}{1\frac{1}{6}} \) = _____ 13. \( \frac{5\frac{4}{6}}{} \) = _____

14. \( \frac{2\frac{1}{2}}{} \) = _____ 15. \( \frac{3\frac{1}{7}}{} \) = _____

\( \frac{3\frac{1}{8}}{} \) + \( \frac{1\frac{5}{6}}{} \) + \( \frac{1\frac{1}{2}}{} \) + \( \frac{5\frac{2}{3}}{} \) = _____